Low energy Effective Lagrangian for SuperSymmetric Seesaw

A.Kato, S. Kang*, T.Morozumi and N. Yokozaki
Hiroshima University, Sogang University*

Internatinal Workshop on Neutrino Masses and Mixings

Shizuoka Dec.18th/2006

Seesaw model is an attractive model as leptogenesis model which may also explain neutrino mass. The supersymmetric seesaw model may be more attractive as a solution of the naturalness problem. Regards to the latter point, recently, the higgs mass correction due to neutrino and sneutrino loops is computed, CaoYang. It has been shown that in the supersymmetric version of seesaw model sneutrino and neutrino loops may give the negative corrections to the mass squared of the lightest neutral higgs mass. The result would be important from the view point of higgs search in Tevatron, LHC and NLC. Here we derive the low energy ($\mu \sim M_W$) effective Lagrangian for supersymmetric seesaw model. Our final goal is derive the loop corrected Higgs potential including CP violating effects of neutrino-sneutrino sector.

,

- General Effective Potentilal for Two higgs doublet model

$$
\begin{aligned}
V &= m_{11}^2 \Phi_1 + m_{22}^2 \Phi_2 - (m_{12}^2 \Phi_{12} + \text{h.c.}) \\
&+ \frac{\lambda_1}{2}\Phi_1^2 + \frac{\lambda_2}{2}\Phi_2^2 + \lambda_3 \Phi_1 \Phi_2 + \lambda_4 \Phi_{12}\Phi_{21} \\
&\quad + \left(\frac{\lambda_5}{2}\Phi_{12}^2 + (\lambda_6 \Phi_1 + \lambda_7 \Phi_2)\Phi_{12} + \text{h.c.}\right)
\end{aligned}
$$

two Higgs doublets $H_1(Y=-1), H_2(Y=1)(\tilde{H}_2 = i\tau_2 H_2^*)$

$$
\Phi_1 = H_1^\dagger \cdot H_1 \quad \Phi_2 = \tilde{H_2}^\dagger \cdot \tilde{H}_2 \quad \Phi_{12} = H_1^\dagger \cdot \tilde{H}_2
$$

In Minimal Super Symmetric Standard Model

$$m_{11}^2 = m_1^2 + |\mu|^2 \quad m_{22}^2 = m_2^2 + |\mu|^2 \quad \lambda_1 = \lambda_2 = \frac{g^2 + g'^2}{4}$$

$$\lambda_3 = \frac{g^2 - g'^2}{4} \quad \lambda_4 = -\frac{g^2}{2} \quad m_{12}^2 = -(B^* \mu^*)$$

$\lambda_5 \sim \lambda_7 = 0$ CP violating complex coupling

- Sneutrino mass matrix and CP violation
  SuperSymmetric Seesaw Model (Super Potential + Soft breaking)
  (Cao Yang, Hisano, Moroi, Tobe Yamaguchi)

$$W = Y_\nu N^c (L \cdot H_2) + \frac{M_R}{2} N^c N^c + \mu (H_2 \cdot H_1) + Y_t t^c (Q \cdot H_2)$$

$$
\begin{aligned}
\mathcal{L}_W &= -|Y_\nu \tilde{L} \cdot H_2 + M_R \tilde{N}^*|^2 - (Y_\nu N^* \tilde{L} - \mu H_1)^\dagger \cdot (Y_\nu N^* \tilde{L} - \mu H_1) \\
&- |\mu|^2 H_2^\dagger \cdot H_2 - |Y|^2 |N|^2 H_2^\dagger \cdot H_2.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_D &= -\frac{g'^2}{8}(H_1^\dagger \cdot H_1 + L^\dagger \cdot L - H_2^\dagger \cdot H_2)^2 \\
&- \frac{g^2}{8}(H_1^\dagger \tau^a H_1 + H_2^\dagger \tau^a H_2 + \tilde{L}^\dagger \tau^a \tilde{L})^2.
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{L}_{\mathrm{soft}} &= -m_{\tilde{L}}^2 |\tilde{L}|^2 - m_{\tilde{N}}^2 |\tilde{N}|^2. - \frac{1}{2} B_N^* M_R^* \tilde{N}^2 - \mathrm{h.c.} \\
&- \mathrm{Re}(B \mu H_1 \cdot H_2) - m_1^2 H_1^\dagger \cdot H_1 - m_2^2 H_2^\dagger \cdot H_2 \\
&+ A Y (H_2 \cdot \tilde{L}) \tilde{N}^* + \mathrm{h.c.}.
\end{aligned}
$$

slepton vs lepton doublets $\tilde{L} \leftrightarrow l = (\nu_L, e_L)$

Singlet sneutrino vs neutrino $\tilde{N} \leftrightarrow N_R$

- neutrino sneutrino loop corrections to Higgs sector
  The loop corrected effective Higgs potential
  For the purpose, we integrate out heavy fields $\tilde{N} N_R$, $\tilde{L}$ and hard modes ($k > \mu_{\mathrm{W}}$)of higgs fields and lepton doublets l We

first split Higgs fields into two parts as,

$$\mathcal{L} = \mathcal{L}(H_1^0, H_2^0) - \frac{1}{2}(L^\dagger, L^T, N^*, N)(\Box + M_B^2)\begin{pmatrix} L \\ L^* \\ N \\ N^* \end{pmatrix}$$

$$M_B^2(H_1^0, \tilde{H}_2{}^0) =$$

$$\begin{pmatrix} m_{\tilde{L}}^2 + |Y|^2 \tilde{H}_2^0 \tilde{H}_2^{0\dagger} & \mathbf{0} & Y^*(A^* \tilde{H}_2^0 - \mu H_1^0) & Y^* M_R \tilde{H}_2^0 \\ \mathbf{0} & m_{\tilde{L}}^2 + |Y|^2 \tilde{H}_2^{0*} \tilde{H}_2^{0T} & Y M_R^* \tilde{H}_2^{0*} & Y(A_\nu \tilde{H}_2^{0*} - \mu^* H_1^{0*}) \\ * & * & |M_R|^2 + m_{\tilde{N}}^2 + |Y|^2 \Phi_2 & B_N M_R \\ * & * & B_N^* M_R^* & |M_R|^2 + m_{\tilde{N}}^2 + |Y|^2 \Phi_2 \end{pmatrix}$$

$$+ \quad \frac{1}{2}\left(\overline{l_L}, \overline{(l_L)^c}, \overline{N_R}, \overline{(N_R)^c}\right)\begin{pmatrix} i\partial\!\!\!/ & 0 & -Y\tilde{H}_2^0 & 0 \\ 0 & i\partial\!\!\!/ & 0 & -Y^*\tilde{H}_2^{0*} \\ -Y^*\tilde{H}_2^{0\dagger} & 0 & i\partial\!\!\!/ & -M_R \\ 0 & -Y\tilde{H}_2^{0T} & -M_R^* & i\partial\!\!\!/ \end{pmatrix}\begin{pmatrix} l_L \\ (l_L)^c \\ N_R \\ (N_R)^c \end{pmatrix}$$

$$\begin{aligned} \Gamma_{\mathrm{eff}}(H_1, H_2) &= \int d^4x \mathcal{L}(H_1, H_2) \\ &+ \frac{i}{2} \mathrm{TrLn} D_B^{-1}(H_1, H_2) - \frac{i}{2} \mathrm{TrLn} S_F^{-1}(H_1, H_2) \end{aligned}$$

- Singlet sneutrino (N, N^*) mass matrix and eigenstates

  $\mathcal{L}_{\text{soft}} = -m_{\tilde{N}}^2 |N|^2 - Re(B_N M_R N^{*2})$

$$\frac{1}{2}(N, N^*) \begin{pmatrix} |M_R|^2 + m_{\tilde{N}}^2 & M_R B_N \\ M_R^* B_N^* & |M_R|^2 + m_{\tilde{N}}^2 \end{pmatrix} \begin{pmatrix} N^* \\ N \end{pmatrix}$$

  Mass eigenstates

$$\begin{aligned} m_5^2 &= |M_R|^2 + m_{\tilde{N}}^2 + |B_N M_R| \\ m_6^2 &= |M_R|^2 + m_{\tilde{N}}^2 - |B_N M_R| \end{aligned}$$

  Without lepton number violating ($N \to N \exp(i\alpha)$) soft terms $B_N M_R$, N is just one complex field. With the soft terms, two real scalars masses N_5 N_6 are non- degenerate.

- Low energy effective Lagrangian The corrections to mass squared terms:

$$-\mathcal{L} = m_{11}^2 H_1^\dagger \cdot H_1 + m_{22}^2 \tilde{H}_2^\dagger \cdot \tilde{H}_2 - 2\mathrm{Re.}(m_{12}^2 H_1^\dagger \cdot \tilde{H}_2)$$

$$\begin{aligned}\delta m_{22}^2 &= \frac{|Y|^2}{32\pi^2}\Bigg(-2(m_{\tilde{L}}^2 + m_{\tilde{N}}^2 + |A|^2)\log\frac{\Lambda^2}{|M_R|^2} + 2m_{\tilde{N}}^2 \\ &+ 4\mathrm{Re}(AB_N^*)\Bigg)\end{aligned}$$

No quadratic dipendence on Λ(cut off) nor M_R(singlet Majorana neutrino). All proportional to the soft breaking terms of susy.

$$\delta m_{22}(\text{non-susy}) \sim -\frac{|Y|^2}{8\pi^2}(\Lambda^2 - |M_R|^2\log\frac{\Lambda^2}{|M_R|^2})$$

$$
\begin{aligned}
m_{22}^2 &= \frac{|Y|^2}{2}\int\frac{d^4k}{(2\pi)^4 i}\left(\frac{2}{m_{\tilde{L}}^2-k^2}+\frac{1}{m_5^2-k^2}+\frac{1}{m_6^2-k^2}-4\frac{1}{|M_R|^2-k^2}\right)\\
&- \frac{|Y|^2}{2}\int\frac{d^4k}{(2\pi)^4 i}\left(|M_R|^2+|A|^2\right)\times\\
&\left(\frac{1}{(m_{\tilde{L}}^2-k^2)(m_5^2-k^2)}+\frac{1}{(m_{\tilde{L}}^2-k^2)(m_6^2-k^2)}\right)\\
&- |Y|^2\int\frac{d^4k}{(2\pi)^4 i}\mathrm{Re}(AM_R e^{-2i\theta})\\
&\left(\frac{1}{(m_{\tilde{L}}^2-k^2)(m_5^2-k^2)}-\frac{1}{(m_{\tilde{L}}^2-k^2)(m_6^2-k^2)}\right)
\end{aligned}
\tag{1}
$$

$$\delta m^2_{12} = \frac{1}{16\pi^2}|Y|^2(B^*_N - A^* \log \frac{\Lambda^2}{|M_R|^2})\mu^*$$
$$\delta m^2_{11} = -\frac{1}{16\pi^2}|Y|^2|\mu|^2 \log \frac{\Lambda^2}{|M_R|^2}$$

The corrections to quartic terms $-\mathcal{L} = \frac{\lambda_2}{2}(H_2^\dagger \cdot H_2)^2 + ...$

$$
\begin{aligned}
\Phi_1^2 \delta\lambda_1 &= -\frac{|Y|^4}{8\pi^2}\frac{|\mu|^4}{|M_R|^4} \log \frac{|M_R|^2}{m_{\tilde{L}}^2} \\
\Phi_2^2 \delta\lambda_2 &= -\frac{|Y|^4}{16\pi^2} \log \frac{|M_R|^2}{m_{\tilde{L}}^2} \\
\Phi_1 \Phi_2 \delta\lambda_3 &= \frac{|Y|^4}{16\pi^2}\frac{|\mu|^2}{|M_R|^2} \\
|\Phi_{12}|^2 \lambda_4 &= \frac{3|Y|^4}{16\pi^2}\frac{|A\mu|^2}{|M_R|^4} \log \frac{|M_R|^2}{m_{\tilde{L}}^2} \\
\Phi_{12}^2 \lambda_5 &= \frac{|Y|^4}{32\pi^2}\frac{M_R^2 \mu^{*2}}{|M_R|^4} \log \frac{|M_R|^2}{m_{\tilde{L}}^2} \\
\Phi_{12} \Phi_1 \lambda_6 &= -\frac{|Y|^4}{8\pi^2}\frac{|\mu|^2 \mu^* A^*}{|M_R|^4} \log \frac{|M_R|^2}{m_{\tilde{L}}^2} \\
\Phi_{12} \Phi_2 \lambda_7 &= -\frac{|Y|^4}{16\pi^2}\frac{A^* \mu^*}{|M_R|^2}
\end{aligned}
$$

- Summary

- We derived effective lagrangian (Higgs sector) for supersymmetric seesaw model.

- neutrino and sneutrino loops corrections to quadratic terms $m^2_{11}, m^2_{22}, m^2_{12}$ of higgs and quartic couplings ($\lambda_{1\sim 7}$) are computed.

- As for the corrections to quadratic terms, the corrections are proportional to either soft breaking terms (as $\delta m_{22}, \delta m_{12}$) or it is proportional to suppersymmetric mass terms $|\mu|^2$ (as $\delta m^2_{11} \sim |\mu|^2 \log$) There is no quadratic dependence on $|M_R|$.

- As for quartic terms, except λ_2 (quartic coplings for H_2, the corrections are suppressed as $\left(\frac{m^2_{\mathrm{soft}}}{|M_R|^2}\right)^n$ $(n = 1, 2)$. The corrections to λ_2 is negative as

$$\lambda_2 = \frac{g^2 + g'^2}{4} - \frac{|Y|^4}{8\pi^2} \log \frac{|M_R|^2}{m^2_{\tilde{L}}} \tag{2}$$

  It may lower the higgs mass. (c.f. Cao and Yang)

- CP violating effect of higgs sector $H \leftrightarrow A$ mixing can be studied.