What Does μ-τ Symmetry Imply about Leptonic CP Violation?

Teppei Baba
Tokai University

In collaboration with Masaki Yasue'

Based on :hep-ph/0612034 (2006)

Content

1. Introduction
2. What's μ - τ symmetry ?
3. μ - τ symmetry-breakings and CP phase
4. An example
5. Summary

1.Introduction

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix}$$

? ? ? → Dirac CP phase

(*) Charged lepton masses are diagonalized

We don't know which masses give Dirac CP phase

However, there is an ambiguity, where phases of M_{ij} (ij=e,μ,τ) are not uniquely determined because of the redefinition of phases of the neutrinos.

Observed quantities such as the mixing angles and the Dirac phase are independent of this ambiguity.

We can give the Dirac phase in terms of phases M_{ij} (ij=e,μ,τ).

We study general property of leptonic CP violation without referring to specific relations among M_{ij}.

The mixing angles and δ are to be given as functions of M_{ij}.

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix} \Rightarrow \begin{cases} \theta_{13}\left(M_{ij}\right), \theta_{23}\left(M_{ij}\right), \theta_{13}\left(M_{ij}\right), \delta\left(M_{ij}\right) \\ \text{functions of } M_{ij} \ \ (i, j = e, \mu, \tau) \end{cases}$$

<u>Experiment data give useful constraints on M_{ij}.</u>

$$\sin^2\theta_{13} = \left(0.9\,{}^{+2.3}_{-0.9}\right)\times 10^{-2} \qquad \sin^2\theta_{23} = 0.44\left(1\,{}^{+0.41}_{-0.22}\right) \qquad \sin^2\theta_{12} = 0.314\left(1\,{}^{+0.18}_{-0.15}\right)$$

$$\theta_{23}\left(M_{ij}\right) \xrightarrow{\text{exp}} \sim \frac{\pi}{4} \qquad \theta_{12}\left(M_{ij}\right) \xrightarrow{\text{exp}} \approx \frac{\pi}{4} \qquad \theta_{13}\left(M_{ij}\right) \xrightarrow{\text{exp}} \sim 0 \qquad \delta\left(M_{ij}\right) \xrightarrow{\text{exp}} ???$$

Constraints on $M_{ij} \Rightarrow$ Constraints on δ

2.What's μ - τ symmetry ?

μ - τ symmetry is the constraint that Lagrangian is invariant under transformation of $\nu_\mu \to -\sigma \nu_\tau$, $\nu_\tau \to -\sigma \nu_\mu$ ($\sigma = \pm 1$)

(*) –sign is just our convention.

Problem

μ - τ symmetry gives consistent results with experimental data. But, It can not give Dirac CP Violation. Why?

Why does μ - τ symmetry give no Dirac CP violation?

μ - τ symmetry

$\sin\theta_{13}e^{-i\delta} = 0$

$\cos\theta_{23} = \sigma\sin\theta_{23} = \frac{1}{\sqrt{2}} \quad (\sigma = \pm 1)$

We need μ - τ Symmery Breaking Part

CP Violation (δ) can not be obtained

experiment:

$$\sin\theta_{13} \approx 0$$

$$\cos\theta_{23} \approx \sigma\sin\theta_{23} \approx \frac{1}{\sqrt{2}}$$

extended to

μ - τ Symmetric Part

+

μ - τ Symmery Breaking Part

We clarify which flavor neutrino mass determines δ as general as possible.

Definition of mass matrix

We can fomally devide $M\nu$ into:

$$M_\nu = M_{sym} + M_b = \begin{pmatrix} M_{ee} & M_{e\mu}^{(+)} & -\sigma M_{e\mu}^{(+)} \\ M_{e\mu}^{(+)} & M_{\mu\mu}^{(+)} & M_{\mu\tau} \\ -\sigma M_{e\mu}^{(+)} & M_{\mu\tau} & M_{\tau\tau}^{(+)} \end{pmatrix} + \begin{pmatrix} 0 & M_{e\mu}^{(-)} & \sigma M_{e\mu}^{(-)} \\ M_{e\mu}^{(-)} & M_{\mu\mu}^{(-)} & 0 \\ \sigma M_{e\mu}^{(-)} & 0 & M_{\tau\tau}^{(-)} \end{pmatrix}$$

$$M_{e\mu}^{(+)} = \frac{M_{e\mu} + \sigma M_{e\tau}}{2} \quad M_{e\mu}^{(-)} = \frac{M_{e\mu} - \sigma M_{e\tau}}{2} \quad M_{\mu\mu}^{(+)} = \frac{M_{\mu\mu} + M_{\tau\tau}}{2} \quad M_{\mu\mu}^{(-)} = \frac{M_{\mu\mu} - M_{\tau\tau}}{2}$$

μ - τ symmetric part

μ - τ symmetry breaking part

With $\left(\mathbf{M} \equiv M_\nu^\dagger M_\nu = \begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix} \right)$

$$\mathbf{M} = \mathbf{M}_{\rm sym} + \mathbf{M}_{\rm b} = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & E_+ \\ -\sigma B_+^* & E_+ & D_+ \end{pmatrix} + \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-^* & D_- & iE_- \\ \sigma B_-^* & -iE_- & -D_- \end{pmatrix}$$

μ - τ symmetric part

This gives $\theta_{23}=\dfrac{\pi}{4}$, $\theta_{13}=0$ and no Dirac CP violation. We calculate θ_{12} :

The phase η: $B_+ \equiv e^{i\eta}\left|B_+\right|$

$$\mathbf{M}_{sym} = \begin{pmatrix} A & e^{i\eta}\left|B_+\right| & -\sigma e^{i\eta}\left|B_+\right| \\ e^{i\eta}\left|B_+\right| & D_+ & E_+ \\ -\sigma e^{i\eta}\left|B_+\right| & E_+ & D_+ \end{pmatrix} \quad (A, D_+, E_+ : \text{real})$$

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13}e^{i\rho} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}e^{-i\rho} - c_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i(\delta+\rho)} & s_{23}c_{13} \\ s_{23}s_{12}e^{-i\rho} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i(\delta+\rho)} & c_{23}c_{13} \end{pmatrix} \quad (c_{13} = \cos\theta_{13}, \text{ etc}) \Rightarrow J_{CP} \propto \sin\theta_{13}\sin(\delta+\rho)$$

diagonalized by U_{sym} gives U

$$X_\pm = \frac{A - D_+ + \sigma E_+ \pm \sqrt{\left(A - D_+ + \sigma E_+\right)^2 + 8\left|B_+\right|^2}}{2\left|B_+\right|}$$

$$U_{sym} = \frac{1}{\sqrt{2}}\left(\begin{pmatrix} \sqrt{2}\cos\chi \\ -\sin\chi e^{-i\eta} \\ \sigma\sin\chi e^{-i\eta} \end{pmatrix}, \begin{pmatrix} \sqrt{2}\sin\chi e^{i\eta} \\ \cos\chi \\ -\sigma\cos\chi \end{pmatrix}, \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix}\right)$$

$$\cos\chi \equiv \frac{X_-}{\sqrt{2+X_-^2}} = \sqrt{\frac{2}{2+X_+^2}}$$

$$\sin\chi \equiv \sqrt{\frac{2}{2+X_-^2}} = \frac{X_+}{\sqrt{2+X_+^2}}$$

$$\boxed{\theta_{12} = \chi},\ \rho = \eta;\ s_{13} = 0,\ c_{23} = \sigma s_{23} = \frac{1}{\sqrt{2}} \Rightarrow J_{CP} \propto \sin\theta_{13}\sin(\delta+\rho) = 0$$

3. μ - τ symmetry-breaking and CP phase

We estimate Dirac CP violation induced by μ–τ symmetry breakings

1. First, we use perturbation with **M**$_b$ treated as a perturbative part to estimate δ.
2. Next, we formulate exact estimation of δ that gives the perturbative results.

3-1. Perturbation with $\mathbf{M}_b = \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-^* & D_- & iE_- \\ \sigma B_-^* & -iE_- & -D_- \end{pmatrix}$

$$B_+ \equiv e^{i\eta}\left|B_+\right|$$

$$|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}c_{12} \\ -s_{12}e^{-i\eta} \\ \sigma s_{12}e^{-i\eta} \end{pmatrix} + a_{13}^{(1)}\begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \qquad s_{13}e^{-i\delta} \approx \sigma \frac{\sqrt{2}\left(2 - R\cos 2\theta_{12}\right)B_- + R\sin 2\theta_{12}\left(D_- + \sigma iE_-\right)e^{i\eta}}{2\Delta m_{atm}^2}$$

Δ and γ can be calculated

$$|2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}s_{12}e^{i\eta} \\ c_{12} \\ -\sigma c_{12} \end{pmatrix} + a_{23}^{(1)}\begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix}$$

and

$$\Delta \approx \frac{R\sqrt{2}\sin 2\theta_{12}\,\mathrm{Re}\left(B_- e^{-i\eta}\right) + \left(R\cos 2\theta_{12} + 2\right)D_-}{2\sqrt{2}\Delta m_{atm}^2}$$

$$\gamma \approx \frac{R\sqrt{2}\sin 2\theta_{12}\,\mathrm{Im}\left(B_- e^{-i\eta}\right) + \left(R\cos 2\theta_{12} + 2\right)\sigma E_-}{2\sqrt{2}\Delta m_{atm}^2}$$

$$a_{13}^{(1)} = \sigma \frac{\sqrt{2}c_{12}B_-^* - s_{12}\left(D_- - i\sigma E_-\right)e^{-i\rho}}{m_1^2 - m_3^2}, \quad R \equiv \frac{\Delta m_\odot^2}{\Delta m_{atm}^2} (<<1)$$

These δ、Δ and γ consistently describe |1> and |2>

$$a_{23}^{(1)} = \sigma \frac{\sqrt{2}s_{12}B_-^* e^{i\rho} + c_{12}\left(D_- - i\sigma E_-\right)}{m_2^2 - m_3^2}$$

$$|3\rangle = \begin{pmatrix} s_{13}e^{-i\delta} \\ s_{23}e^{i\gamma} \\ c_{23}e^{-i\gamma} \end{pmatrix} \approx \frac{1}{\sqrt{2}}\begin{pmatrix} \sqrt{2}s_{13}e^{-i\delta} \\ 1-\Delta+i\gamma \\ 1+\Delta-i\gamma \end{pmatrix}$$

$$s_{23} \equiv \frac{1-\Delta}{\sqrt{2\left(1+\Delta^2\right)}} \qquad c_{23} \equiv \frac{1+\Delta}{\sqrt{2\left(1+\Delta^2\right)}}$$

$$\Delta << 1, \ \gamma << 1$$

Suggested U_{PMNS}

$$U = \left(|1\rangle, |2\rangle, |3\rangle\right)$$

$$B_+ \equiv e^{i\eta} |B_+|$$

$$= \frac{1}{\sqrt{2}} \left(\begin{pmatrix} \sqrt{2} c_{12} \\ -s_{12}(1+\Delta+i\gamma)e^{-i\eta} - \sigma c_{12} s_{13} e^{i\delta} \\ \sigma s_{12}(1-\Delta-i\gamma)e^{-i\eta} - c_{12} s_{13} e^{i\delta} \end{pmatrix}, \begin{pmatrix} \sqrt{2} s_{12} e^{i\eta} \\ c_{12}(1+\Delta+i\gamma) - s_{12} s_{13} e^{i(\eta+\delta)} \\ -\sigma c_{12}(1-\Delta-i\gamma) - \sigma s_{12} s_{13} e^{i(\eta+\delta)} \end{pmatrix}, \begin{pmatrix} \sqrt{2} s_{13} e^{-i\delta} \\ 1-\Delta+i\gamma \\ 1+\Delta-i\gamma \end{pmatrix} \right)$$

If $\gamma \ll 1$, $\Delta \ll 1$

We guess the appropriate form of the PMNS matrix

This expression gives perturbation result

$$U_{PMNS}(\delta,\rho,\gamma) \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma} & 0 \\ 0 & 0 & e^{-i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12}e^{i\rho} & 0 \\ -s_{12}e^{-i\rho} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} K(\beta_1, {}_2, {}_3)$$

3-2. Exact result

Use redefined masses to control phase-ambiguities:

$$B' = e^{i(\gamma-\rho)}B,\ C' = e^{-i(\gamma+\rho)}C\ ,\ E' = e^{-2i\gamma}E \text{ with } \delta' = \delta + \rho$$

$$U^\dagger{}_{PMNS}(\delta,\rho,\gamma)\mathbf{M}U_{PMNS}(\delta,\rho,\gamma) = \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix}$$

This equation gives the following formula:

$$\tan 2\theta_{12} e^{i\rho} = \frac{2}{\lambda_2 - \lambda_1}\frac{c_{23}B' - s_{23}C'}{c_{13}} \Rightarrow \boxed{\theta_{12},\rho}$$

$$\tan 2\theta_{13} e^{-i\delta} = \frac{2}{\lambda_3 - A}\left(s_{23}B' + c_{23}C'\right) \Rightarrow \boxed{\theta_{13},\delta}$$

$$\mathrm{Re}(E')\cos 2\theta_{23} + D_{-}\sin 2\theta_{23} + i\,\mathrm{Im}(E')$$

$$= -s_{13}e^{i(\rho+\delta)}\frac{c_{23}B' - s_{23}C'}{c_{13}} \Rightarrow \boxed{\theta_{23},\gamma}$$

$$\lambda_1 \equiv \frac{c_{13}{}^2 A - s_{13}{}^2\lambda_3}{c_{13}{}^2 - s_{13}{}^2}$$

$$\lambda_2 \equiv c_{23}{}^2 D + s_{23}{}^2 F - 2s_{23}c_{23}\,\mathrm{Re}(E')$$

$$\lambda_3 \equiv s_{23}{}^2 D + c_{23}{}^2 F + 2s_{23}c_{23}\,\mathrm{Re}(E')$$

Exact result for δ'

$\delta' \equiv \delta + \rho$ with

$$\delta = -\arg\left(s_{23}e^{i\gamma}B + c_{23}e^{-i\gamma}C\right)$$

$$\rho = \arg\left(c_{23}e^{i\gamma}B - s_{23}e^{-i\gamma}C\right)$$

$$\begin{cases} B = B_+ + B_- \\ -\sigma C = B_+ - B_- \end{cases}$$

$\gamma << 1$

$\Delta << 1$

$$(*)\, c_{23} \approx \frac{1+\Delta}{\sqrt{2}},\ s_{23} \approx \frac{1-\Delta}{\sqrt{2}}$$

$$\delta \approx -\arg\left(B_-\right),\ \ \rho \approx \arg\left(B_+\right)$$

$\delta' = \delta + \rho$ receive main contribution from B_+ & B_-

Exact result for θ_{23}

$E' = e^{-2i\gamma} E$ with $\delta' = \delta + \rho$

$$\mathrm{Re}(E')\cos 2\theta_{23} + D_- \sin 2\theta_{23} + i\,\mathrm{Im}(E') = -s_{13} e^{i\delta'} X'$$

Re part : $\mathrm{Re}(E')\cos 2\theta_{23} + D_- \sin 2\theta_{23} = -s_{13}\cos\delta' |X'| (\equiv -x)$

$$\cos\theta = \sqrt{\frac{\mathrm{Re}^2\left(e^{-2i\gamma}E\right) + D_-^{\,2} - x^2}{\mathrm{Re}^2\left(e^{-2i\gamma}E\right) + D_-^{\,2}}} \qquad \sin\theta = \frac{\sigma x}{\sqrt{\mathrm{Re}^2\left(e^{-2i\gamma}E\right) + D_-^{\,2}}}$$

$$\theta_{23} = \sigma\frac{\pi}{4} + \frac{\theta + \phi}{2}$$

$$\cos\phi = \frac{\mathrm{Re}\left(e^{-2i\gamma}E\right)}{\sqrt{\mathrm{Re}^2\left(e^{-2i\gamma}E\right) + D_-^{\,2}}} \qquad \sin\phi = \frac{\kappa D_-}{\sqrt{\mathrm{Re}^2\left(e^{-2i\gamma}E\right) + D_-^{\,2}}}$$

Maximal atmospheric mixing ⇒ x=0($s_{13}\cos\delta'$=0) & D_=0(**M**$_{\mu\mu}$=**M**$_{\tau\tau}$)

⇒ **Maximal CP violation** if **M**$_{\mu\mu}$=**M**$_{\tau\tau}$

Im part : $\mathrm{Im}(E') = s_{13}\sin(\rho + \delta) X' (\equiv x')$

$$\cos\theta' = \frac{\sqrt{|E|^2 - x'^2}}{|E|} \qquad \sin\theta' = \frac{x'}{|E|}; \quad \cos\phi' = \frac{\mathrm{Re}(E)}{|E|} \qquad \sin\phi' = \frac{\kappa'|\mathrm{Im}(E)|}{|E|}$$

$$\gamma = \frac{\phi' - \theta'}{2}$$

Which masses give which phases

If the textures are approximately μ–τ symmetric

$$s_{13}e^{-i\delta} \approx \sigma \frac{\sqrt{2}B_-}{\Delta m^2_{atm}}$$

$$\rho \approx \arg(B_+)$$

$$\Delta \approx -\frac{D_-}{\sqrt{2}\Delta m^2_{atm}}$$

$$\gamma \approx \frac{\sigma E_-}{\sqrt{2}\Delta m^2_{atm}}$$

δ **depends on B_**

ρ **depends on B+**

Δ **depends on D_**

γ **depends on E_**

$$\mathbf{M} = M_\nu^\dagger M_\nu = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & E_+ \\ -\sigma B_+^* & E_+ & D_+ \end{pmatrix} + \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-^* & D_- & iE_- \\ \sigma B_-^* & -iE_- & -D_- \end{pmatrix}$$

(∗) $\delta + \rho$ is **Dirac CP Violating phase**

What are the redefined masses?

The redefined masses are given by

$$\mathbf{M}'_{e\mu} = e^{i(\gamma-\rho)}\mathbf{M}_{e\mu}$$

$$\mathbf{M}'_{e\tau} = e^{-i(\gamma+\rho)}\mathbf{M}_{e\tau}$$

$$\mathbf{M}'_{\mu\tau} = e^{-2i\gamma}\mathbf{M}_{\mu\tau}$$

The Jarlskog invariant in terms of the redefined masses:

$$J'_{CP} = \frac{\mathrm{Im}\left(\mathbf{M}'_{e\mu}\mathbf{M}'_{\mu\tau}\mathbf{M}'^{*}_{e\tau}\right)}{\Delta m_{12}{}^2 \Delta m_{23}{}^2 \Delta m_{31}{}^2} = \frac{\mathrm{Im}\left(e^{i(\gamma-\rho)}\mathbf{M}_{e\mu} e^{-2i\gamma}\mathbf{M}_{\mu\tau}\left(e^{-i(\gamma+\rho)}\mathbf{M}_{e\tau}\right)^*\right)}{\Delta m_{12}{}^2 \Delta m_{23}{}^2 \Delta m_{31}{}^2}$$

$$= \frac{\mathrm{Im}\left(\mathbf{M}_{e\mu}\mathbf{M}_{\mu\tau}\mathbf{M}_{e\tau}{}^*\right)}{\Delta m_{12}{}^2 \Delta m_{23}{}^2 \Delta m_{31}{}^2}$$

The Jarlskog invariant is reassured to be weak-base invariant quantity by the use of the redefined masses.

Three versions of **M** and U_{PMNS}

For the redefined masses, we have the PDG version of U_{PMNS}:

$$\mathbf{M}=\begin{pmatrix} A & Be^{-i(\rho-\gamma)} & Ce^{-i(\rho+\gamma)} \\ B^*e^{i(\rho-\gamma)} & D & Ee^{-2i\gamma} \\ C^*e^{i(\rho+\gamma)} & E^*e^{2i\gamma} & F \end{pmatrix} \Rightarrow U_{PMNS}^{PDG}=\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i(\delta+\rho)} \\ -s_{12}c_{23}-s_{23}c_{12}s_{13}e^{i(\delta+\rho)} & c_{23}c_{12}-s_{23}s_{12}s_{13}e^{i(\delta+\rho)} & s_{23}c_{13} \\ s_{23}s_{12}-c_{23}c_{12}s_{13}e^{i(\delta+\rho)} & -s_{23}c_{12}-c_{23}s_{12}s_{13}e^{i(\delta+\rho)} & c_{23}c_{13} \end{pmatrix}$$

There are other two versions

1) The original one:

$$\mathbf{M}=\begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix} \Rightarrow U_{PMNS}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma} & 0 \\ 0 & 0 & e^{-i\gamma} \end{pmatrix}\begin{pmatrix} c_{12}c_{13} & e^{i\rho}s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}e^{-i\rho}-s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12}-s_{23}s_{12}s_{13}e^{i(\delta+\rho)} & s_{23}c_{13} \\ s_{23}s_{12}e^{-i\rho}-c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{12}s_{13}e^{i(\delta+\rho)} & c_{23}c_{13} \end{pmatrix}$$

2) The intermediate one:

$$\mathbf{M}=\begin{pmatrix} A & Be^{i\gamma} & Ce^{-i\gamma} \\ B^*e^{-i\gamma} & D & Ee^{-2i\gamma} \\ C^*e^{i\gamma} & E^*e^{2i\gamma} & F \end{pmatrix} \Rightarrow U_{PMNS}=\begin{pmatrix} c_{12}c_{13} & e^{i\rho}s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23}e^{-i\rho}-s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12}-s_{23}s_{12}s_{13}e^{i(\delta+\rho)} & s_{23}c_{13} \\ s_{23}s_{12}e^{-i\rho}-c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12}-c_{23}s_{12}s_{13}e^{i(\delta+\rho)} & c_{23}c_{13} \end{pmatrix}$$

5. An Example

R.N.Mohapatra,S.Nasri and Hai-Bo Yu
Phys.Rev.D72 (2005) 033007

We consider the simplest mass matrix, which is approximately μ-τ symmetric:

$$M_\nu = \begin{pmatrix} a\varepsilon^2 & b\varepsilon & -\sigma b\varepsilon e^{i\alpha} \\ b\varepsilon & 1+\varepsilon & \sigma \\ -\sigma b\varepsilon e^{i\alpha} & \sigma & 1+\varepsilon \end{pmatrix} \quad (\varepsilon << 1) \;\longrightarrow\; \begin{cases} B = B_+ + B_- \approx \left(2ie^{-i\frac{\alpha}{2}}\sin\frac{\alpha}{2} + \varepsilon\right) b\varepsilon \\ C = -\sigma\left(B_+ - B_-\right) \approx \sigma\left(2ie^{-i\frac{\alpha}{2}}(1+\varepsilon)\sin\frac{\alpha}{2} - \varepsilon\right) b\varepsilon \\ E \approx -\sigma\left(b^2\varepsilon^2 e^{i\alpha} - 2d\right) \end{cases}$$

which reported $\delta = \frac{\alpha}{2} - \frac{\pi}{2}$. This is not correct and should be $\delta + \rho$

Two versions of Jarlskog invariant

$$J_{CP}\left(\equiv \frac{\mathrm{Im}\left(\mathbf{M}_{e\mu}\mathbf{M}_{\mu\tau}\mathbf{M}^*_{e\tau}\right)}{\Delta m^2_{12}\Delta m^2_{23}\Delta m^2_{31}}\right) = \frac{\mathrm{Im}\left(BC^*E\right)}{\Delta m^2_{12}\Delta m^2_{23}\Delta m^2_{31}} \approx -\frac{\sqrt{2}b\varepsilon}{8}\sin 2\theta_{12}\sin\frac{\alpha}{2}$$

$$J_{CP}\left(\equiv J_{CP}\left(U_{PMNS}\right)\right) = -s_{12}c_{12}s_{13}c_{13}^2 s_{23}c_{23}\sin(\delta+\rho) \approx \frac{\sqrt{2}b\varepsilon}{8}\sin 2\theta_{12}\sin\frac{\alpha}{2}\sin\delta'$$

same result

This shows that

$$\delta \approx -\arg\left(B_-\right), \quad \rho \approx \arg\left(B_+\right)$$

$$\rho = -\frac{\alpha}{2}, \quad \delta = \frac{\alpha}{2} - \frac{\pi}{2}, \quad \delta' = \delta + \rho = -\frac{\pi}{2}$$

$$\rho = -\frac{\alpha}{2}, \quad \delta = \frac{\alpha}{2} - \frac{\pi}{2}, \quad \delta' = \delta + \rho = -\frac{\pi}{2}$$

$$\sin\left(\delta + \rho\right) = -1 \quad \left(\delta + \rho = -\frac{\pi}{2}\right)$$

5.Summary

- **We can determine θ_{23}, and the phase of ρ and δ**

$$\theta_{23} = \sigma\frac{\pi}{4} + \frac{\theta+\phi}{2}, \text{with} \quad \sin\theta \propto \sin\theta_{13}\cos(\delta+\rho)\Delta m_e^2, \quad \sin\phi \propto M_{\mu\mu} - M_{\tau\tau}$$

$$\delta = -\arg\left(s_{23}e^{i\gamma}B + c_{23}e^{-i\gamma}C\right) \ , \quad \rho = \arg\left(c_{23}e^{i\gamma}B - s_{23}e^{-i\gamma}C\right)$$

Maximal atmospheric mixing conditions are given by

$$\delta'(=\delta+\rho) = \frac{\pi}{2} \quad and \quad \mathbf{M}_{\mu\mu} = \mathbf{M}_{\tau\tau}$$

- **We are able to determine which masses provide which phases.**
  - **δ depends on the B_**
  - **ρ depends on the B+**
  - **γ depends on the E_**

$$\mathbf{M} = M_\nu^\dagger M_\nu = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & E_+ \\ -\sigma B_+^* & E_+ & D_+ \end{pmatrix} + \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-^* & D_- & iE_- \\ \sigma B_-^* & -iE_- & -D_- \end{pmatrix}$$

- **Redefined flavor masses given by**

$$\mathbf{M}'_{e\mu} = e^{i(\gamma-\rho)}\mathbf{M}_{e\mu}, \ \mathbf{M}'_{e\tau} = e^{-i(\gamma+\rho)}\mathbf{M}_{e\tau}, \ \mathbf{M}'_{\mu\tau} = e^{-2i\gamma}\mathbf{M}_{\mu\tau}$$

reassure the weak-base independence of the Jarlskog invariant:

$$J'_{CP} = \frac{\mathrm{Im}\left(\mathbf{M}'_{e\mu}\mathbf{M}'_{\mu\tau}\mathbf{M}'^{*}_{e\tau}\right)}{\Delta m_{12}{}^{2}\Delta m_{23}{}^{2}\Delta m_{31}{}^{2}} = \frac{\mathrm{Im}\left(\mathbf{M}_{e\mu}\mathbf{M}_{\mu\tau}\mathbf{M}_{e\tau}{}^{*}\right)}{\Delta m_{12}{}^{2}\Delta m_{23}{}^{2}\Delta m_{31}{}^{2}}$$

END

3. Redefinition of the neutrino

$$\nu_{flavor}{}' = e^{-i\rho}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i(\rho+\gamma)} & 0\\ 0 & 0 & e^{i(\rho-\gamma)}\end{pmatrix}\nu_{flavor} \equiv \Omega(\rho,\gamma)\nu_{flavor}$$

⇩

$$U'_{PMNS}(\delta+\rho,0,0) = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i(\delta+\rho)} \\ \left(-s_{12}c_{23} - s_{13}c_{12}s_{23}e^{i(\delta+\rho)}\right) & \left(c_{23}c_{12} - s_{13}s_{12}s_{23}e^{i(\delta+\rho)}\right) & s_{23}c_{13} \\ \left(s_{12}s_{23} - s_{13}c_{12}c_{23}e^{i(\delta+\rho)}\right) & \left(-s_{23}c_{12} - s_{13}s_{12}c_{23}e^{i(\delta+\rho)}\right) & c_{23}c_{13} \end{pmatrix}\begin{pmatrix} e^{i(\beta_1-\rho)} & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & e^{i\beta_3}\end{pmatrix}$$

$U_{PMNS}(\delta,\rho,\gamma)$ is able to turn $U_{PMNS}^{PDG}(\delta,0,0)$

公式と摂動との比較

$$\mathrm{Re}\left(e^{-2i\gamma}E\right)\cos 2\theta_{23} + D_{\_}\sin 2\theta_{23} + i\,\mathrm{Im}\left(e^{-2i\gamma}E\right) = -s_{13}e^{i(\rho+\delta)}X"$$

$$\cos 2\theta_{23} \approx 2\Delta \approx -\frac{\left(R\cos 2\theta_{12}+2\right)D_{\_} + \sigma s_{13}\cos(\delta+\rho)\sin 2\theta_{12}\Delta m_e{}^2}{\Delta m_{atm}{}^2}$$

$$\gamma \approx \frac{\left(R\cos 2\theta_{12}+2\right)\sigma E_{\_} - s_{13}\sin(\delta+\rho)\sin 2\theta_{12}\Delta m_e{}^2}{2\Delta m_{atm}{}^2}$$

and

$$2s_{13}e^{-i\delta} \approx \sigma\frac{\sqrt{2}\left(2-R\cos 2\theta_{12}\right)B_{\_} + (i\gamma-\Delta)\sin 2\theta_{12}e^{i\rho}\Delta m_e{}^2}{\Delta m_{atm}{}^2}$$

Already ,

Perturbation gave Δ and γ

$$s_{13}e^{-i\delta} \approx \sigma\frac{\sqrt{2}\left(2-R\cos 2\theta_{12}\right)B_{\_} + R\sin 2\theta_{12}\left(D_{\_}+\sigma iE_{\_}\right)e^{i\rho}}{2\Delta m_{atm}{}^2}$$

$$\Delta \approx -\frac{R\sqrt{2}\sin 2\theta_{12}\,\mathrm{Re}\left(B_{\_}e^{-i\rho}\right) + \left(R\cos 2\theta_{12}+2\right)D_{\_}}{2\Delta m_{atm}{}^2}$$

$$\gamma \approx \frac{R\sqrt{2}\sin 2\theta_{12}\,\mathrm{Im}\left(B_{\_}e^{-i\rho}\right) + \left(R\cos 2\theta_{12}+2\right)\sigma E_{\_}}{2\Delta m_{atm}{}^2}$$

γ is put on PMNS Unitary matrix like that

$$P_\gamma = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma} & 0 \\ 0 & 0 & e^{-i\gamma} \end{pmatrix}$$

$$\Omega^\dagger \Omega = 1$$

An Example

$$B \approx \left(2ie^{-i\frac{\alpha}{2}} \sin\frac{\alpha}{2} + \varepsilon \right) b\varepsilon$$

$$C \approx \sigma \left(2ie^{-i\frac{\alpha}{2}} (1+\varepsilon) \sin\frac{\alpha}{2} - \varepsilon \right) b\varepsilon$$

$$E = -\sigma \left(b^2 \varepsilon^2 e^{i\alpha} - 2d\sigma e \right)$$

$$M_\nu = \begin{pmatrix} a\varepsilon^2 & b\varepsilon & -\sigma b\varepsilon e^{i\alpha} \\ b\varepsilon & 1+\varepsilon & \sigma \\ -\sigma b\varepsilon e^{i\alpha} & \sigma & 1+\varepsilon \end{pmatrix}$$

$$-\arg(Y) = \delta = \frac{\alpha}{2} - \frac{\pi}{2}$$

$$\delta' = \delta + \rho = -\frac{\pi}{2}$$

$$c_{13} X \approx \sqrt{2} b\varepsilon^2 e^{-i\frac{\alpha}{2}} \frac{m_0^2}{4} \cos\frac{\alpha}{2}$$

$$Y \approx 2\sqrt{2} i\sigma b\varepsilon \frac{m_0}{4} e^{-i\frac{\alpha}{2}} \sin\frac{\alpha}{2}$$

$$\arg(c_{13} X) = \rho = -\frac{\alpha}{2}$$

$$J_P = -\frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{13} \sin 2\theta_{23} c_{13} \sin(\delta + \rho) \approx \frac{\sqrt{2} b\varepsilon}{8} \sin 2\theta_{12} \sin\frac{\alpha}{2} \sin\delta$$

Redefinition of the neutrino

$$\nu_{flavor}{}' = \begin{pmatrix} e^{i\alpha_e} & 0 & 0 \\ 0 & e^{i\alpha_\mu} & 0 \\ 0 & 0 & e^{i\alpha_\tau} \end{pmatrix} \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = e^{i\alpha_e} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\alpha_+ + \alpha_-)} & 0 \\ 0 & 0 & e^{i(\alpha_+ - \alpha_-)} \end{pmatrix} \nu_{flavor}$$

$$\Downarrow$$

$$M_\nu{}' = e^{-2i\alpha_e} \begin{pmatrix} a & e^{-i(\alpha_+ + \alpha_-)} b & e^{-i(\alpha_+ - \alpha_-)} c \\ e^{-i(\alpha_+ + \alpha_-)} b & e^{-2i(\alpha_+ + \alpha_-)} d & e^{-2i\alpha_+} e \\ e^{-i(\alpha_+ - \alpha_-)} c & e^{-2i\alpha_+} e & e^{-2i(\alpha_+ - \alpha_-)} f \end{pmatrix}$$

$$U_{PMNS}\left(\delta,\rho,0\right)=U_{\nu}\left(\delta,\rho,\gamma\right)K\left(\beta_1,\beta_2,\beta_3\right)$$

$$=\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\gamma} & 0\\ 0 & 0 & e^{-i\gamma}\end{pmatrix}\begin{pmatrix}c_{13}c_{12} & c_{13}s_{12}e^{i\rho} & s_{13}e^{-i\delta}\\ -s_{12}c_{23}e^{-i\rho}-s_{13}c_{12}s_{23}e^{i\delta} & c_{23}c_{12}-s_{13}s_{12}s_{23}e^{i(\delta+\rho)} & s_{23}c_{13}\\ s_{12}s_{23}e^{-i\rho}-s_{13}c_{12}c_{23}e^{i\delta} & -s_{23}c_{12}-s_{13}s_{12}c_{23}e^{i(\delta+\rho)} & c_{23}c_{13}\end{pmatrix}\begin{pmatrix}e^{i\beta_1} & 0 & 0\\ 0 & e^{i\beta_2} & 0\\ 0 & 0 & e^{i\beta_3}\end{pmatrix}$$

$$e^{i\alpha_e}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i(\rho-\gamma)} & 0\\ 0 & 0 & e^{i(\rho+\gamma)}\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\ 0 & e^{i\gamma} & 0\\ 0 & 0 & e^{-i\gamma}\end{pmatrix}\begin{pmatrix}c_{13}c_{12} & c_{13}s_{12}e^{i\rho} & s_{13}e^{-i\delta}\\ -s_{12}c_{23}e^{-i\rho}-s_{13}c_{12}s_{23}e^{i\delta} & c_{23}c_{12}-s_{13}s_{12}s_{23}e^{i(\delta+\rho)} & s_{23}c_{13}\\ s_{12}s_{23}e^{-i\rho}-s_{13}c_{12}c_{23}e^{i\delta} & -s_{23}c_{12}-s_{13}s_{12}c_{23}e^{i(\delta+\rho)} & c_{23}c_{13}\end{pmatrix}\begin{pmatrix}e^{i\beta_1} & 0 & 0\\ 0 & e^{i\beta_2} & 0\\ 0 & 0 & e^{i\beta_3}\end{pmatrix}$$

$$=e^{i\alpha_e}\begin{pmatrix}c_{13}c_{12} & c_{13}s_{12}e^{i\rho} & s_{13}e^{-i\delta}\\ \left(-s_{12}c_{23}-s_{13}c_{12}s_{23}e^{i(\delta+\rho)}\right) & e^{i\rho}\left(c_{23}c_{12}-s_{13}s_{12}s_{23}e^{i(\delta+\rho)}\right) & e^{i\rho}s_{23}c_{13}\\ \left(s_{12}s_{23}-s_{13}c_{12}c_{23}e^{i(\delta+\rho)}\right) & e^{i\rho}\left(-s_{23}c_{12}-s_{13}s_{12}c_{23}e^{i(\delta+\rho)}\right) & e^{i\rho}c_{23}c_{13}\end{pmatrix}\begin{pmatrix}e^{i\beta_1} & 0 & 0\\ 0 & e^{i\beta_2} & 0\\ 0 & 0 & e^{i\beta_3}\end{pmatrix}$$

$$=e^{i(\alpha_e-\rho)}\begin{pmatrix}e^{-i\rho}c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i(\delta+\rho)}\\ e^{-i\rho}\left(-s_{12}c_{23}-s_{13}c_{12}s_{23}e^{i\rho}e^{i\delta}\right) & \left(c_{23}c_{12}-s_{13}s_{12}s_{23}e^{i(\delta+\rho)}\right) & s_{23}c_{13}\\ e^{-i\rho}\left(s_{12}s_{23}-s_{13}c_{12}c_{23}e^{i\rho}e^{i\delta}\right) & \left(-s_{23}c_{12}-s_{13}s_{12}c_{23}e^{i(\delta+\rho)}\right) & c_{23}c_{13}\end{pmatrix}\begin{pmatrix}e^{i\beta_1} & 0 & 0\\ 0 & e^{i\beta_2} & 0\\ 0 & 0 & e^{i\beta_3}\end{pmatrix}$$

$$c_{13}X\approx\sqrt{2}b\varepsilon^2 e^{-i\frac{\alpha}{2}}\frac{m_0{}^2}{4}\cos\frac{\alpha}{2}\longrightarrow\arg\left(c_{13}X\right)=\rho=-\frac{\alpha}{2}$$

$$Y\approx 2\sqrt{2}i\sigma b\varepsilon\frac{m_0{}^2}{4}e^{-i\frac{\alpha}{2}}\sin\frac{\alpha}{2}\longrightarrow-\arg\left(Y\right)=\delta=\frac{\alpha}{2}-\frac{\pi}{2}$$

$$\delta'=\delta+\rho=-\frac{\pi}{2}$$

$$\tan 2\theta_{12} = \frac{2X'}{\lambda_2 - \lambda_1}\ ,\ \tan 2\theta_{13} e^{-i\delta'} = \frac{2Y'}{\lambda_3 - A}$$

$$\lambda_2 = c_{23}{}^2 D + s_{23}{}^2 F - 2 s_{23} c_{23} \operatorname{Re}\left(E''\right)$$
$$\lambda_3 = s_{23}{}^2 D + c_{23}{}^2 F + 2 s_{23} c_{23} \operatorname{Re}\left(E''\right)$$

$$\lambda_1 = \frac{c_{13}{}^2 A - s_{13}{}^2 \lambda_3}{c_{13}{}^2 - s_{13}{}^2}$$

$$\cos\theta = \sqrt{\frac{\operatorname{Re}^2\left(e^{-2i\gamma} E\right) + D_-{}^2 - x^2}{\operatorname{Re}^2\left(e^{-2i\gamma} E\right) + D_-{}^2}} \quad \sin\theta = \frac{\sigma x}{\sqrt{\operatorname{Re}^2\left(e^{-2i\gamma} E\right) + D_-{}^2}}$$

$$\cos\phi = \frac{\operatorname{Re}\left(e^{-2i\gamma} E\right)}{\sqrt{\operatorname{Re}^2\left(e^{-2i\gamma} E\right) + D_-{}^2}} \quad \sin\phi = \frac{\kappa D_-}{\sqrt{\operatorname{Re}^2\left(e^{-2i\gamma} E\right) + D_-{}^2}}$$

$$\operatorname{Re}\left(e^{-2i\gamma} E\right) \cos 2\theta_{23} + D_- \sin 2\theta_{23} = -s_{13} \cos\left(\rho + \delta\right) \left|X\right| \left(\equiv -x\right)$$

$$\Omega\left(\rho, \gamma\right) \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i(\rho+\gamma)} & 0 \\ 0 & 0 & e^{i(\rho-\gamma)} \end{pmatrix}$$

$$\cos\theta' = \frac{\sqrt{|E|^2 - x'^2}}{|E|} \qquad \sin\theta' = \frac{x'}{|E|}$$

$$\cos\phi' = \frac{\mathrm{Re}(E)}{|E|} \qquad \sin\phi' = \frac{\kappa'|\mathrm{Im}(E)|}{|E|}$$

$$\gamma = \frac{\phi' - \theta'}{2}$$

$$\left.\begin{aligned} \tan 2\theta_{12} e^{i\rho} &= \frac{2X'}{\lambda_2 - \lambda_1} \Rightarrow \boxed{\theta_{12}} \\ \tan 2\theta_{13} e^{-i\delta} &= \frac{2Y'}{\lambda_3 - A} \Rightarrow \boxed{\theta_{13}} \end{aligned}\right\} \Rightarrow \not{CP} = \boxed{\delta + \rho}$$

END

Formula ③

$$\tan 2\theta_{13} e^{-i\delta} \approx 2 s_{13} e^{-i\delta} \approx \sigma \frac{\sqrt{2}(2 - R\cos 2\theta_{12}) B_- + (i\gamma - \Delta)\sin 2\theta_{12} e^{i\rho} \Delta m_e{}^2}{\Delta m_{atm}{}^2}$$

From previous formula

$$\cos 2\theta_{23} \approx 2\Delta \approx -\frac{(R\cos 2\theta_{12} + 2) D_- + \sigma s_{13}\cos(\delta + \rho)\sin 2\theta_{12} \Delta m_e{}^2}{\Delta m_{atm}{}^2}$$

$$\gamma \approx \frac{(R\cos 2\theta_{12} + 2)\sigma E_- - s_{13}\sin(\delta + \rho)\sin 2\theta_{12} \Delta m_e{}^2}{2\Delta m_{atm}{}^2}$$

Corresponding perturbation !

Conditions

$$s_{13} << 1, \gamma << 1, \Delta << 1, D_- << 1, E_- << 1$$

$$X = \frac{e^{i\rho}\sin 2\theta_{12} \Delta m_e{}^2}{2}, \quad \sigma \,\mathrm{Re}\left(e^{-2i\gamma} E\right) = \frac{1}{2}\Delta m_{atm}{}^2 \left(1 - \frac{R\cos 2\theta_{12}}{2}\right)$$

The Dirac CP phase δ and mixing angles θ_{ij} depend on phase of M_{ij}

But , The redefinitions of M_{ij} phase don't change physics because of

$$M_{e\mu}'M_{\mu\tau}'M_{e\tau}'^{*} = M_{e\mu}M_{\mu\tau}M_{e\tau}^{*}$$

$$J_{CP} = \frac{\mathrm{Im}\left(M_{e\mu}M_{\mu\tau}M_{e\tau}^{*}\right)}{\Delta m_{12}^{2}\Delta m_{23}^{2}\Delta_{31}m^{2}}$$

Which one is the phase source

If the textures are approximately μ–τ symmetric

$$s_{13}e^{-i\delta} \approx \sigma \frac{\sqrt{2}B_-}{\Delta m^2_{atm}}$$

$$\rho \approx \arg\left(B_+\right)$$

$$\Delta \approx -\frac{D_-}{\sqrt{2}\Delta m^2_{atm}}$$

$$\gamma \approx \frac{\sigma E_-}{\sqrt{2}\Delta m^2_{atm}}$$

- **δ depend on the B_**
- **ρ is the B_+ phase**
- **Δ depend on the D_**
- **γ depend on the E_**

$$\mathbf{M} = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+{}^* & D_+ & E_+ \\ -\sigma B_+{}^* & E_+ & D_+ \end{pmatrix} + \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-{}^* & D_- & iE_- \\ \sigma B_-{}^* & -iE_- & -D_- \end{pmatrix}$$

CP violation depend on $\delta + \rho$