Neutrino 2012, June 3 - 9, 2012, at Kyoto, Japan

Neutrino Mass Matrix Composed of M_e and M_u Only

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The pre-abstract given in Neutrino 2012 was based on a work published in EPJC 72, 1933 (2012).
In this presentation, we will report more recent works.

Contents

1. Overview

In a seesaw type neutrino mass matrix

$$M_\nu = M_D M_R^{-1} M_D^T,$$

we take the following form as M_R

$$M_R = k_R(M_u^{1/2} M_e + M_e M_u^{1/2}).$$

Then, by using the observed charged lepton mass values as inputs, we can obtain reasonable PMNS mixing parameters and up-quark mass ratios.

- Characteristic of this model:

The model has quite few parameters because the observed charged lepton mass values are used as input values, so that we do not count those values as adjustable parameters.

Yukawaon model

- Yukawaon model = a kind of flavon model

Yukawa coupling constants Y_f^{eff} is given by

$$Y_f^{eff} = \frac{y_f}{\Lambda}\langle Y_f \rangle$$

Y_f: "yukawaon"

Relations among VEVs of yukawaons are derived from SUSY vacuum conditions under a family symmetry.

At present, the relations are phenomenological ones, because the superpotential form depends on the R charge assignments for fields.

2. Earlier models

- A prototype of the model was given by

  YK, PLB 665, 227 (2008); JPG 35, 125004 (2008).

- Model A: an unified model of quark and lepton mass matrices: **YK, PLB 680, 76 (2009).**

$$M_e = \Phi_e \Phi_e$$

$$\Phi_e = \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$$

$$M_D = M_e$$

$$M_u = M_u^{1/2} M_u^{1/2}$$

$$M_u^{1/2} = \Phi_e (1 + a_u X) \Phi_e$$

$$M_d = \Phi_e (1 + a_d X) \Phi_e$$

$$M_R = M_u^{1/2} P_u M_e + M_e P_u M_u^{1/2} + \xi_\nu\text{-term}$$

Here and hereafter, we omit common coefficients, because we discuss only the relative ratios.

where

$$1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

P_u=diag(+1,-1,+1) at the diagonal basis of M_u

ξ-term $= \xi_\nu(P_u M_e \Phi_u + \Phi_u Y_e P_u)$

By adjusting only two parameters as a_u=-1.78 and ξ_ν=0.005, we obtain excellent predictions

$$r_{12}^u \equiv \sqrt{\frac{m_u}{m_c}} = -0.0425 \qquad r_{23}^u \equiv \sqrt{\frac{m_c}{m_t}} = -0.0570$$

$$(r_{12}^u)_{obsv} = -0.045^{+0.013}_{-0.010} \qquad (r_{23}^u)_{obsv} = 0.060 \pm 0.005$$

$$\sin^2 2\theta_{atm} = 0.9819 \qquad \tan^2 \theta_{solar} = 0.4486$$

except for $\sin^2 2\theta_{13} = 6.0 \times 10^{-4}$

Problems: The model can derive a nearly tribimaximal mixing, but we cannot give a sizable value of θ_{13} .

Besides, P_u is unnatural and ξ-term is not beautiful.

3. S_3 model

- Where does the form 1+a X come from?

We consider a superpotential

$$W_f = \left[\mu(Y_f)^{ij} + \frac{\lambda_f}{\Lambda}\Phi_0^{i\alpha} S^f_{\alpha\beta}\Phi_0^{T\beta j}\right]\Theta^f_{ji}$$

(6*,1) (3*,3*) (1,6) (3*,3*) (6,1)

Here, we have assumed family symmetries

$\mathrm{U(3)\times U(3)'}$ **which break into** $\mathrm{U(3)\times S_3}$ **at** $\mu = \Lambda'$ **.**

Then, $\Phi_0^{i\alpha} S^f_{\alpha\beta}\Phi_0^{T\beta j} \to \Phi_0^{i\alpha}\langle S^f_{\alpha\beta}\rangle\Phi_0^{T\beta j}$

Note that Φ_0 **still takes no VEV at** $\mu = \Lambda'$ **.**

Φ_0 **becomes** $\langle\Phi_0\rangle$ **at** $\mu = \Lambda < \Lambda'$ **.**

In order that W_f^{eff} is invariant under S_3,

$\langle S^f \rangle$ must be a form $\langle S^f \rangle = v_f(1 + a_f X)$

We consider that the forms of $M_u^{1/2}$ and M_d come from this mechanism.

For M_e in the previous model, we may consider that it is a specific case $a_e=0$.

For a model with $a_e \neq 0$, see our recent paper

YK and HN, PLB 712, 396 (2012), arXiv:1202.5815 [hep-ph]

Model B : S_3 model with non-diagonal M_e

YK and HN, PLB 712, 396 (2012).

However, there is no reason that we take a_e=0 for M_e.

We investigate another model with $a_e \neq 0$

$$
\begin{aligned}
&PM_eP \propto \langle Y_e \rangle \propto \langle \Phi_0 \rangle (1 + a_e X) \langle \Phi_0 \rangle, \\
&PM_dP \propto \langle Y_d \rangle \propto \langle \Phi_0 \rangle (1 + a_d X) \langle \Phi_0 \rangle + m_{0d} \mathbf{1}, \\
&\quad P = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, +1), \\
&M_u^{1/2} \propto \langle Y_u^{1/2} \rangle \propto \langle \Phi_0 \rangle (1 + a_u X) \langle \Phi_0 \rangle.
\end{aligned}
$$

Inputs: 8 parameters $a_e, a_u, \alpha_u, a_d, m_d^0, \xi_\nu, (\phi_1, \phi_2)$

Predictions: 15 observables; we can obtain reasonable quark mass ratios, neutrino mass ratios, PMNS mixing, and CKM mixing except for $\sin^2 2\theta_{13} = 0.027$

e.g. $\sin^2 2\theta_{23} = 0.978, \ \tan^2 \theta_{solar} = 0.522$

$|V_{us}| = 0.2240, |V_{cb}| = 0.0404, |V_{ub}| = 0.00409$

$|V_{td}| = 0.00823$, and so on.

Model C : S_3 model with $M_D \neq M_e$ (preliminary)

- Inputs: 3 parameters

$$a_e = 8.0, a_D = 7.7, a_u = -1.285$$

- Predictions:

$$r^u_{12} = 0.0415,\ r^u_{23} = 0.0586$$

$$\sin^2 2\theta_{23} = 0.931,\ \tan^2\theta_{12} = 0.450$$

$$\sin^2 2\theta_{13} = 0.0329$$

$$R_\nu \equiv (m_2^2 - m_1^2)/(m_3^2 - m_2^2) = 0.0339$$

- Comments

It is likely that all M_f take the same form.

We do not need ξ_ν term and P_u, but we need

$P_e = \mathrm{diag}(1, 1, -1)$ **in** $U_{PMNS} = U_e^\dagger P_e U_\nu$

θ_{13} is still small.

4. Model D : An attempt to obtain large θ_{13} (preliminary)

Regrettably, so far, the yukawaon model cannot obtain the observed value $\sin^2 2\theta_{13} \simeq 0.09$

We propose a new model:

$$M_e^{ij} = \Phi_0^{i\alpha}(1 + a_e X_3)_{\alpha\beta}\Phi_0^{T\beta j}$$

$$M_D^{ij} = \Phi_0^{i\alpha} E_\alpha^a (1 + a_D X_2)_{ab} E_\beta^b \Phi_0^{T\beta j}$$

$$(M_u^{1/2})^{ij} = \Phi_0^{i\alpha}(1 + a_u X_3)_{\alpha\beta}\Phi_0^{T\beta j}$$

$$(P_d)_k^i M_d^{kl} (P_d^T)_l^j = \Phi_0^{i\alpha} E_\alpha^a (1 + a_d X_2)_{ab} E_\beta^b \Phi_0^{T\beta j}$$

where (3*, 3*,1) (1, 3, 3*) (1, 1, 6) of $[U(3)]^3$

$$X_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \quad X_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Here, we have assumed S_2 symmetry in addition to S_3.

Inputs: Only 3 parameters (preliminary)

$$a_e = 8.5,\ a_D = 9.68,\ a_u = -1.29$$

Predictions:

$$r^u_{12} = 0.0440,\ r^u_{23} = 0.0586$$

$$\sin^2 2\theta_{23} = 0.959,\ \tan^2\theta_{12} = 0.448$$

$$\sin^2 2\theta_{13} = 0.0943$$

Comments:

We do not need ξ_ν term, P_u, and P_e

In order to fit R_ν we need a constant term m^0_ν 1 **in M_ν.**

The predicted value θ_{13} is insensitive to the parameters.

The reason that M_D takes X_2 (not X_3) is unknown.

5. Concluding remarks

- According to historical order, we have introduced yukawaon models in which neutrino mass matrix is given by the form

  $M_\nu = M_D M_R^{-1} M_D^T$ and $M_R = k_R(M_u^{1/2} M_e + M_e M_u^{1/2} + \ldots)$.
- For convenience, we have denoted relations among mass matrices in terms of M_f, not in terms of VEVs of yukawaons, $\langle Y_f \rangle$, and we have not given the superpotential which derives the VEV relations. For those, please see our papers.
- As you have seen, many versions can be considered starting from the original model, Model A. The characteristic is that the model can describe quark and lepton mass spectra and mixings simultaneously. At present, the preliminary models (Models C and D) are not discussed for down-quark mass ratios and CKM mixing. We will complete those models soon.

Summary Table

Model	A PLB(2009)	B PLB(2012)	C Preliminary	D Preliminary
Parameters *	2 a_u, ξ_ν	4 (6) $a_e, a_u, \alpha_u, \xi_\nu$	3 a_u, a_D, a_u	3 a_e, a_D, a_u
q-mass ratios	OK	OK	OK	OK
$\sin^2 2\theta_2 3$	0.982	0.978	0.931	0.959
$\tan^2 \theta_{12}$	0.449	0.522	0.450	0.448
$\sin^2 2\theta_{13}$	6×10^{-4}	0.027	0.0329	0.0943
R_ν	**	0.044	0.0339	**
others	CKM	CKM, CP phase		

* Onlu parameters in M_e, M_D and M_u are counted.

** We need an additional one parameter without affecting PMNS mixing.