GUT2012, March 15, 2012 @YITP, Kyoto U

Family Gauge Bosons with an Inverted Mass Hierarchy

Based on ArXiv:1203. 2028

Yoshio Koide (Osaka University)

in collaboration with

Toshifumi Yamashita (Maskawa Insititute, KSU)

This workshop is on the Grand Unification Theory.
Sorry, this talk is not a topic based on GUT.
However, I think that a topic of family symmetries may give a suggestive hint on the flavor problem in GUT

Contents

1. **Motive to consider the inverted mass hierarchy***

   Brief review of the Sumino's cancellation mechanism

2. **Outline of the model** **(The details will be skipped.)**
3. **How different from the Sumino model?**
4. **How to observe the gauge boson effects**

   (Note that the lowest gauge boson is A_3^3 .)

5. **Summary**

*** A family gauge bosons with the inverted mass hierarchy has been proposed by Grinstein *et al.* (2010) based on the triple SU(3) model. However, the phenomenology is quite different from ours because of the current structures.**

1. Motive to consider the inverted mass hierarchy

1.1 Sumino's motive to consider a family gauge symmetry

2009: Sumino has seriously taken why the mass formula

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

is so remarkably satisfied with the pole masses:

$$K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$$

while if we take the running masses, the ratio becomes

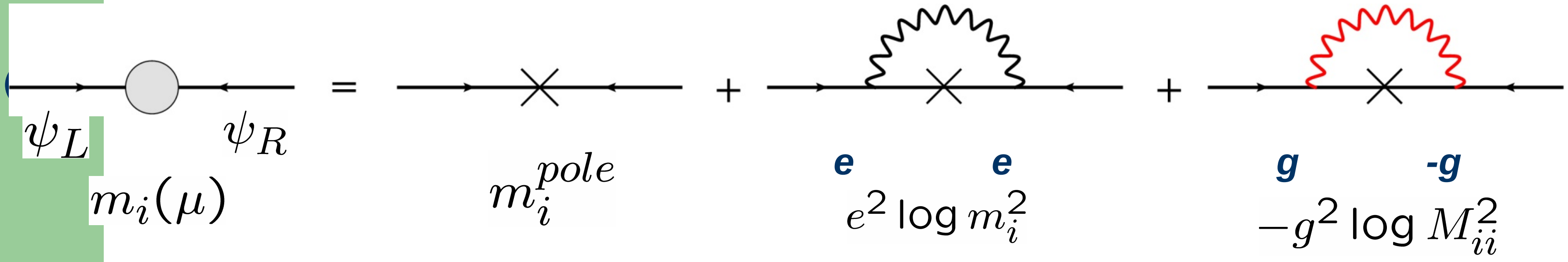
$$K(\mu) = (2/3) \times (1.00189 \pm 0.000002) \quad \text{at } \mu = m_Z$$

Under $m_i \to m_i(1 + \varepsilon_o + \varepsilon_i)$, **K is invariant if** $\varepsilon_i = 0$.

The deviation comes from the QED radiative correction

$$\delta m_i = -\frac{\alpha(\mu)}{\pi} m_i \left(1 + \frac{3}{4} \log \frac{\mu^2}{m_i^2}\right)$$

Therefore, Sumino has proposed an idea that the $e^2 \log m_i^2$ term is cancelled by a contribution from family gauge bosons (2009)



In order to work the Sumino mechanism correctly,

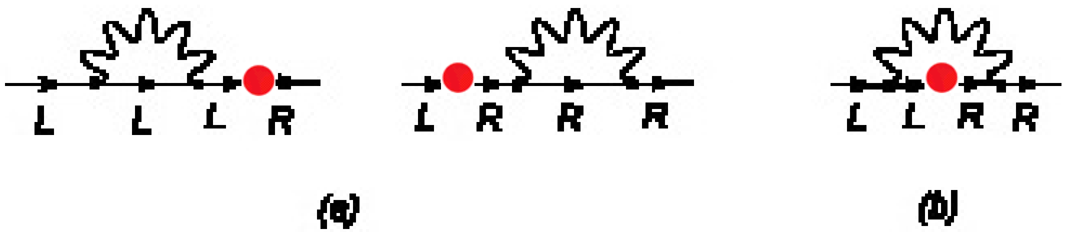
the following conditions are essential:

(i) $(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$ of the U(3) family symmetry

(ii) Masses of the gauge bosons A_i^j :

$$M_{ij} \equiv m(A_i^j) \propto \sqrt{m_{ei} + m_{ej}}$$

Note that **the contribution of (b) is zero in a SUSY model:**



Therefore, **the Sumino mechanism cannot apply to a SUSY model.**

1.2 Our motive

We would like to consider a Sumino-like mechanism which can work in a SUSY model.

We propose a Sumino-like mechanism for the diagram (a).

(Wave function renormalization diagram)

photon family gauge bosons

$$\varepsilon_i = \rho \left(\log \frac{m_{ei}^2}{\mu^2} + \zeta \sum_j \log \frac{M_{ij}^2}{\mu^2} \right)$$

$$\rho = \frac{3}{4}\frac{\alpha_{em}}{\pi}, \quad \zeta = \frac{2}{3}\frac{\alpha_F}{\alpha_{em}},$$

In order that the cancellation works correctly,

since $\zeta > 0$, we consider $M_{ij}^2 \equiv m^2(A_j^i) \propto \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right)$

differently from the Sumino's gauge boson mass relation

$$M_{ij}^2 \equiv m^2(A_j^i) \propto m_{ei} + m_{ej}$$

2. Outline of the model

By introducing two types of scalars $\Phi\ (\bar{\Phi})$ and $\Psi\ (\bar{\Psi})$ we build our model as follows:

(i) The charged lepton mass matrix M_e is given by

$$M_e \propto \langle\bar{\Phi}\rangle\langle\Phi\rangle$$

(ii) Family gauge boson masses M_{ij} are dominantly given by the VEV $\langle\Psi\rangle\ (\langle\bar{\Psi}\rangle)$

Therefore, we must show $|\langle\Psi\rangle| \gg |\langle\Phi\rangle|$

(iii) Gauge bosons take an inverted mass hierarchy:

We must show $\langle\Phi\rangle\langle\Psi\rangle \propto 1$

For more details, see YK and TY, arXiv:1203.2028

No time! Skip this page!

(i) Charged lepton mass matrix $M_e \propto \langle \bar{\Phi} \rangle \langle \Phi \rangle$

We assume the superpotential

$$W_Y = y_\ell \ell_i \bar{\Phi}^i_\alpha \bar{L}^\alpha + y_H L_\alpha H_d \bar{E}^\alpha$$

$$+ y_e E_\alpha \Phi^\alpha_j e^{cj} + M_E E_\alpha \bar{E}^\alpha + M_L L_\alpha \bar{L}^\alpha$$

and we obtain

$$W_Y^{eff} = \frac{y_H y_\ell y_e}{\lambda_E M_E M_L} \ell_i \bar{\Phi}^i_\alpha \Phi^\alpha_j e^{cj} H_d$$

where $\Phi^\alpha_i \sim (3, 3^*)$ of $\mathrm{U}(3) \times \mathrm{U}(3)'$

No time! Skip this page!

(ii) Relation $\langle \Phi \rangle \langle \Psi \rangle \propto 1$

- We assume the superpotential

$$W_{\Phi\Psi} = \left(\lambda_A \bar{\Psi}^i_\alpha \Phi^\alpha_j + \bar{\lambda}_A \bar{\Phi}^i_\alpha \Psi^\alpha_j\right)(\Theta_A)^j_i + \left(\lambda'_A \bar{\Psi}^i_\alpha \Phi^\alpha_i + \bar{\lambda}'_A \bar{\Phi}^i_\alpha \Psi^\alpha_i - \mu_A S\right)(\Theta_A)^j_j$$

$$+ \left(\lambda_B \Phi^\alpha_i \bar{\Psi}^i_\beta + \bar{\lambda}_B \Psi^\alpha_i \bar{\Phi}^i_\beta\right)(\Theta_B)^\beta_\alpha + \left(\lambda'_B \Phi^\alpha_i \bar{\Psi}^i_\alpha + \bar{\lambda}'_B \Psi^\alpha_i \bar{\Phi}^i_\alpha - \mu_B S\right)(\Theta_B)^\beta_\beta$$

- We assume that $W_{\Phi\Psi}$ is invariant under the exchange

U(3) $\leftrightarrow$ U(3)$'$. Then, we obtain

$$\langle \Phi \rangle = \langle \bar{\Phi} \rangle = v_\Phi Z, \qquad \langle \Psi \rangle = \langle \bar{\Psi} \rangle = v_\Psi Z^{-1}$$

We take a diagonal form of Z as follows

$$Z = \begin{pmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \end{pmatrix} \propto \begin{pmatrix} \sqrt{m_e} & 0 & 0 \\ 0 & \sqrt{m_\mu} & 0 \\ 0 & 0 & \sqrt{m_\tau} \end{pmatrix}$$ with $z_1^2 + z_2^2 + z_3^2 = 1$

$z_1 = 0.016473$, $z_2 = 0.23688$, $z_3 = 0.97140$

No time! Skip this page!

(iii) Relation $v_\Phi / v_\Psi \sim O(\varepsilon)$

- We assume the superpotential by introducing a field S

$$W_\Phi = \lambda_1 \Phi_i^\alpha \bar{\Phi}_\alpha^i \theta_\Phi - \lambda_2 S^2 \theta_\Phi$$

- Here, S is a family singlet field, and U(1) is softly broken by

$$W_{br} = \mu_S S \theta_S - \varepsilon \mu_S^2 \theta_S \longrightarrow \langle S \rangle = \varepsilon \mu_S$$

We can conclude that $\langle \Phi \rangle \sim \varepsilon \mu_S$

- Thus, by neglecting the contribution of <Φ>, we obtain

$$M_{ij}^2 \equiv m^2(A_j^i) \simeq g_F^2 v_\Psi^2 \left(\frac{1}{z_i^2} + \frac{1}{z_j^2} \right) \propto \left(\frac{1}{m_i} + \frac{1}{m_j} \right)$$

3. How different from the Sumino model?

	Sumino	K-Y
	Non-SUSY	**SUSY**
U(3) assignment of (e_L, e_R)	**~ (3,3*)**	**~ (3,3)**
Anomaly	**a model with anomaly**	**an anomaly-less model**
Gauge boson masses	**Normal** $M_{ij} \propto \sqrt{m_i + m_j}$	**Inverted** $M_{ij} \propto \sqrt{\frac{1}{m_i} + \frac{1}{m_j}}$
Family currents	$(J_\mu)_i^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} - \bar{\psi}_{Ri} \gamma_\mu \psi_R^j$	$(J_\mu)_i^j = \bar{\psi}^j \gamma_\mu \psi_i$
Effective ΔN_F=2 int.	**appear even if U_q=1**	**not appear in the limit of U_q=1**

The gauge boson model with the inverted mass hierarchy will bring a new view in the low energy (below TeV scale) phenomenology.

4. How to observe the gauge boson effects

- Note that **the family number is defined on the diagonal basis of the charged lepton mass matrix M_e.**

Hadronic modes are in general dependent on the quark mixing matrices U_u and U_d.

We know the observed values of $U_{CKM} = U_u^\dagger U_d$ but we do not know values of U_u and U_d individually.

- In the present model, we do not give an explicit model for quark mixings on the diagonal basis of M_e.

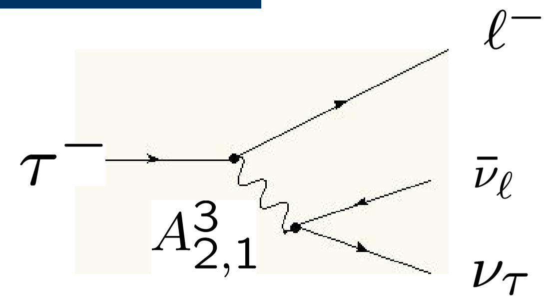
A constraint from K^0-$\bar{K}^0$ mixing is highly dependent on a model of U_d. We will not discuss it in this talk.

The pure-leptonic decays are independent of a model of the quark mixing

4.1 Deviation from the e-μ universality in the tau decays

We have family current-current interactions

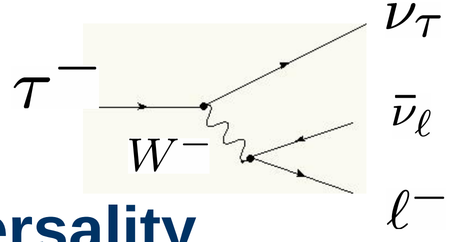
$$\frac{G_{ij}}{\sqrt{2}}(\bar{\nu}_i \gamma_\mu \nu_j)(\bar{e}_j \gamma^\mu e_i) \qquad \frac{G_{ij}}{\sqrt{2}} = \frac{g_F^2}{2M_{ij}^2} \simeq \frac{z_j^2}{2v_\Psi^2}$$



$(z_1, z_2, z_3) = (0.016473, 0.23688, 0.97140)$

in addition to the conventional weak interactions

$$\frac{G_F}{\sqrt{2}}(\bar{e}_j \gamma_\mu (1-\gamma_5)\nu_j)(\bar{\nu}_i \gamma^\mu (1-\gamma_5) e_i) \qquad \frac{G_F}{\sqrt{2}} = \frac{g_W^2}{8M_W^2} = \frac{1}{2v_W^2}$$



We predict a deviation from the e-μ universality

$$R_\tau \equiv \frac{1+\epsilon_\mu}{1+\epsilon_e} = \left[\frac{B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{f(m_e/m_\tau)}{f(m_\mu/m_\tau)}\right]^{1/2}$$

$$f(m_e/m_\tau)/f(m_\mu/m_\tau) = 1.028215 \qquad \epsilon_j \simeq \frac{1}{4} z_j^2 (v_W/v_\Psi)^2$$

$$R_\tau \equiv \frac{1+\epsilon_\mu}{1+\epsilon_e} = \left[\frac{B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{f(m_e/m_\tau)}{f(m_\mu/m_\tau)}\right]^{1/2}$$

$r = v_W/v_\Psi$

$v_W = 246$ GeV

$$\epsilon_\mu \simeq \frac{1}{4} z_2^2 r^2 = 1.4 \times 10^{-2} r^2 \qquad \epsilon_e \simeq \frac{1}{4} z_1^2 r^2 = 6.8 \times 10^{-5} r^2$$

Present experimental values

$$B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = (17.39 \pm 0.04)\%$$
$$B(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = (17.82 \pm 0.04)\%$$

lead to $R_\tau^{exp} = 1.0017 \pm 0.0016$

This result is in favor of the inverted gauge boson mass hierarchy. **(For a normal mass hierarchy, R_τ will show $R_\tau < 1$.)**

$\Delta R_\tau \simeq \epsilon_\mu \sim 0.0017$ **suggests** $r \sim 0.35$ **i.e.** $v_\Psi \sim 0.7$ TeV

This value $v_\Psi \sim 0.7$ TeV **seems to be somewhat low.**

We speculate $r \sim 10^{-1}$ **(** $v_\Psi \sim$ **a few TeV)** $\Delta R_\tau \equiv R_\tau - 1 \sim 10^{-4}$

The value will be confirmed by a tau factory in the near future.

4.2 Family number conserved semileptonic decays of ps-mesons

Branching ratios of family number conserved semileptonic decays are not sensitive to explicit values of the quark mixings U_u and U_d.

We predict those

in the limit of U_u =1 and U_d=1

$$B(K^+ \to \pi^+ \mu^+ e^-) \simeq 4.88 \times 10^{-8} r^4$$

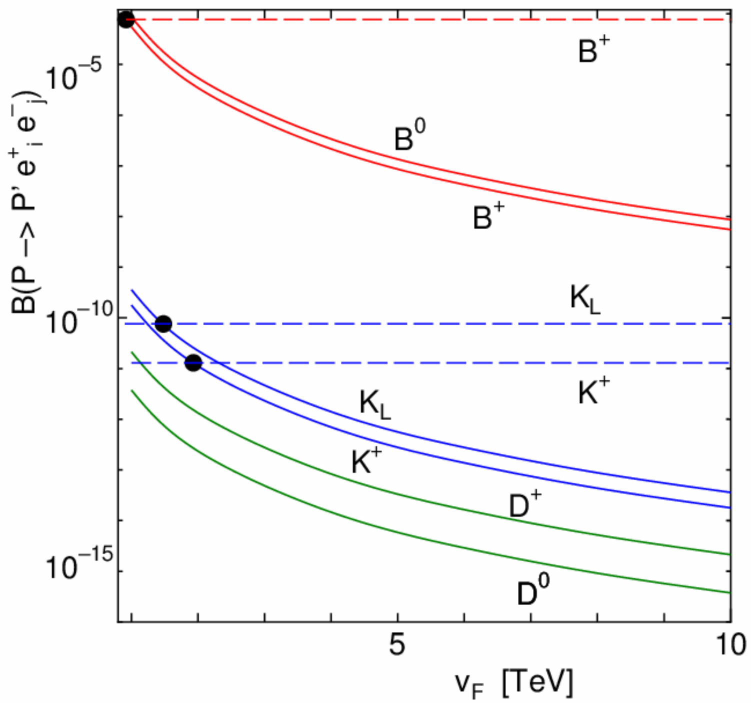
$$B(K_L \to \pi^0 \mu^\pm e^\mp) \simeq 9.82 \times 10^{-8} r^4$$

$$B(D^+ \to \pi^+ \mu^- e^+) \simeq 5.83 \times 10^{-9} r^4$$

$$B(D^0 \to \pi^0 \mu^- e^+) \simeq 1.03 \times 10^{-9} r^4$$

$$B(B^+ \to K^+ \mu^- \tau^+) \simeq 1.51 \times 10^{-2} r^4$$

$$B(B^0 \to K^0 \mu^- \tau^+) \simeq 2.37 \times 10^{-2} r^4$$



$r = v_W/v_F \ (v_W = 246 \text{ GeV})$

Observations of the K- and B-decays will be within our reach.

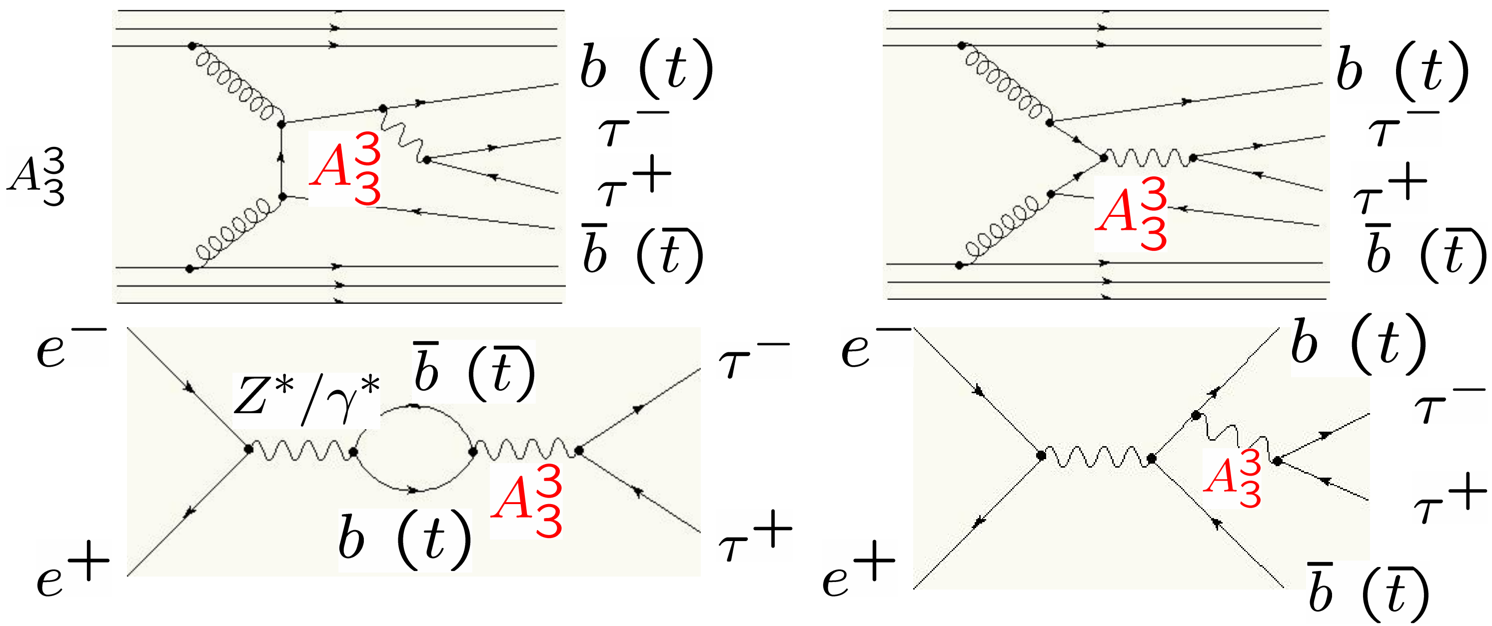
we speculate

4.3 Direct searches at LHC and ILC

for the lightest gauge boson with a mass of a few TeV

- The searches are similar to Z' searches,
- but we see only peak in $\tau^+\tau^-$ channel

(no peaks in e^+e^- and $\mu^+\mu^-$ channels)



5. Summary

- **The Sumino mechanism does not work for a SUSY model.**

In order to work a Sumino-like mechanism for a SUSY model, we must consider a family gauge boson model with an inverted mass hierarchy.

- Whether the gauge boson masses are inverted or normal is confirmed by observing the deviation form the e-μ universality in the pure leptonic tau decays, i.e. $R_\tau > 1$ or $R_\tau < 1$.
- **The present observed values show** $R_\tau^{exp} = 1.0017 \pm 0.0016$ **which is in favor of the inverted mass hierarchy.**

Since we speculate that the lightest gauge boson mass is a few TeV, we expect the deviation $\Delta R_\tau \equiv R_\tau - 1 \sim 10^{-4}$

A tau factory in the near future will confirm this deviation.

- We also expect a direct observation of $\tau^+\tau^-$ in the LHC and the ILC.


Thank you for your kind attention