Flavor Physics in the LHC Era, 8-12 November 2010, Singapore

Charged Lepton Mass Relation and Related Topics

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Today, I would like to talk about a charged lepton mass relation

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}$$

In the charged lepton sector, we know that the observable quantities are only three (m_e, m_μ, m_τ) and since we discuss only the rations among them, we have only two input values.

Therefore, the present topic seems to be quite narrow and shortsighted. Nevertheless, you will find that the topic can provide a promising clue to a beyond standard model.

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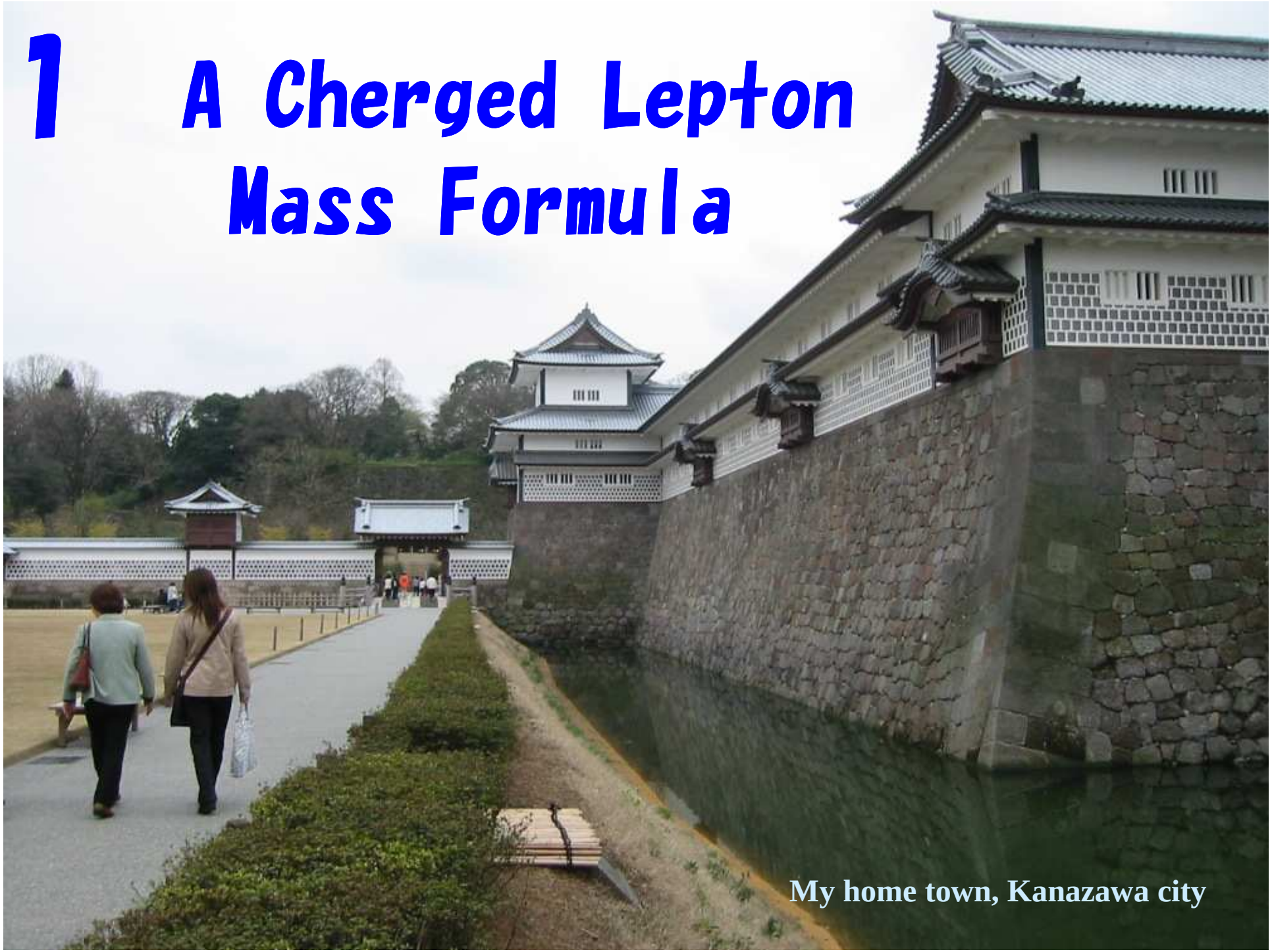


My home town, Kanazawa city

Whether quarks and leptons are really fundamental or composite?

- If quarks and leptons are composite particles, we will find a mass formula which is approximately satisfied as the Gell-Mann-Okubo mass formula for hadrons.
- If quarks and leptons are really fundamental particles, we will find a mass formula which is exactly satisfied and which is beautiful as the Balmer formula in the hydrogen atom.
- If the case is the latter, a beautiful formula will be found in the charged lepton sector, because it will be difficult to observe such a formula in quark sector because of gluon corrections even if it exists.

1.1 Charged lepton mass formula

In 1982, I have proposed a charged lepton mass relation

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 \qquad (1.1)$$

YK, Lett.Nuov.Cim 34, 201 (1982); PLB 120, 161 (1983)

Prediction: $m_\tau = 1776.97 \ \text{MeV}$

Experimental value: $(m_\tau^{exp})_{old} = 1784.2 \pm 3.2 \ \ \text{MeV}$

Note that the prediction was in poor agreement with the observed tau lepton mass value of those days.

Ten years after: $(m_\tau^{exp})_{new} = 1776.99^{+0.29}_{-0.26} \ \ \text{MeV}$

ARGUS, BES, CLEO (1992)

Present: $m_\tau^{exp} = 1776.84 \pm 0.17 \ \ \text{MeV}$

(PDG2010)

Characteristics of the formula

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \qquad (1.2)$$

(i) K is invariant under any exchange among m_{ei}

(permutation symmetry S_3)

(ii) K is invariant under a scale transformation

$$m_{ei} \to m_{ei}(1 + \varepsilon_o + \varepsilon_i) \text{ with } \varepsilon_i = 0 \qquad (1.3)$$

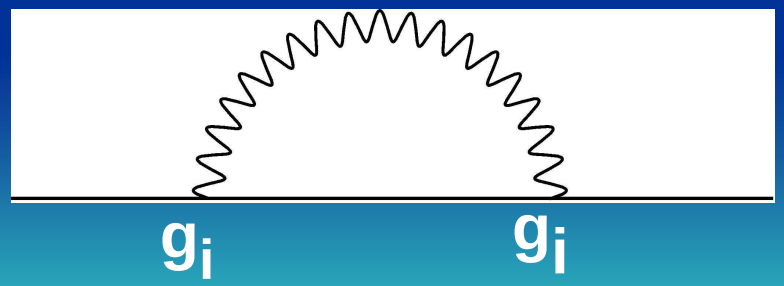
1.2 How to derive the formula: 1982

YK, Lett.Nuov.Cim 34, 201 (1982)

The formula has first been speculated from **a composite model of quarks and leptons**.

In most models, families are understood as a triplet of a symmetry group, while, in this model, **families are understood as (π, η, σ) in SU(3) 8+1**

We suppose a radiative-type mass



$$m_{ei} = m_0 g_i^2 \quad (1.4)$$

where

$$g_i = \tilde{g}_i + g_0 \quad (1.5)$$

$$\begin{array}{ccc} \pi & \eta & \sigma \\ \begin{pmatrix} \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} \\ 0 \end{pmatrix} & \begin{pmatrix} \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{6}} \\ -\frac{2}{\sqrt{6}} \end{pmatrix} & \begin{pmatrix} \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} \end{pmatrix} \end{array}$$

$\tilde{g}_i$: contribution from SU(3) octet

$$\tilde{g}_1 + \tilde{g}_2 + \tilde{g}_3 = 0 \qquad (1.6)$$

g_0 : contribution from SU(3) singlet

$$3g_0^2 = \tilde{g}_1^2 + \tilde{g}_2^2 + \tilde{g}_3^2 \qquad (1.7)$$

Then,

$$\sum m_{ei} = m_0 \sum (\tilde{g}_i + g_0)^2 = m_0 (\sum \tilde{g}_i^2 + 3g_0^2) = 6 m_0 g_0^2$$

$$\sum \sqrt{m_{ei}} = \sqrt{m_0} \sum (\tilde{g}_i + g_0) = \sqrt{m_0} \cdot 3 g_0$$

Therefore, we can obtain the relation as follows

$$K = \frac{\sum m_{ei}}{(\sum \sqrt{m_{ei}})^2} = \frac{6 m_0 g_0^2}{(3\sqrt{m_0} g_0)^2} = \frac{2}{3} \qquad (1.8)$$

1.3 How to derive the formula: 1990

-- Prototype model of a yukawaon model later --

In 1990

YK, MPL A5, 2319 (1990)

I have again tried to derive the same formula on the basis of a potential model, which is a prototype model of the so-called yukawaon model later:

Here, the Yukawa coupling constant Y_e is understood by a vacuum expectation value (VEV) of new scalar Φ_e of U(3) 8+1: $\langle \Phi \rangle = \mathrm{diag}(v_1, v_2, v_3)$

$$Y_e^{eff} \propto \langle \Phi \rangle \langle \Phi \rangle \quad (1.9)$$

$$K = \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{v_1^2 + v_2^2 + v_3^2}{(v_1 + v_2 + v_3)^2} = \frac{\mathrm{Tr}[\langle \Phi_e \rangle \langle \Phi_e \rangle]}{(\mathrm{Tr}[\langle \Phi_e \rangle])^2}$$

(1.10)

How to obtain VEV of Φ

We assume the following scalar potential

$$V = \mu^2 \mathrm{Tr}[\Phi\Phi] + \frac{1}{2}\lambda(\mathrm{Tr}[\Phi\Phi])^2 + \frac{1}{3}\lambda'(\mathrm{Tr}[\Phi])^2 \mathrm{Tr}[\hat{\Phi}\hat{\Phi}]$$

$$\Phi: \ \mathbf{8}+\mathbf{1} \ \text{of} \ \mathrm{U}(3) \qquad (1.11)$$

where Φ is a nonet scalar of U(3) family symmetry:

We assume that the potential is written in terms of Φ.

(we call it as a "nonet hypothesis")

However, the nonet hypothesis is broken only by the third term.

$$\hat{\Phi} = \Phi - \frac{1}{3}\mathrm{Tr}[\Phi] \ : \ \mathbf{8} \ \text{of} \ \mathrm{U}(3)$$

(Note that the third term is still U(3) invariant.)

Since

$$\frac{1}{3}\lambda'(\mathrm{Tr}[\Phi])^2\left(\mathrm{Tr}[\Phi\Phi]-\frac{1}{3}(\mathrm{Tr}[\Phi])^2\right)=\frac{1}{3}\lambda'\left(\mathrm{Tr}[\Phi])^2\mathrm{Tr}[\Phi\Phi]-\frac{1}{3}\mathrm{Tr}[\Phi])^4\right)$$

we can evaluate a minimizing condition of V as

$$\begin{aligned}\frac{\partial V}{\partial\Phi}&=\mu^2 2\Phi+\frac{1}{2}\lambda 2\mathrm{Tr}[\Phi\Phi]2\Phi\\ &+\frac{1}{3}\lambda'\left((\mathrm{Tr}[\Phi])^2 2\Phi+\mathrm{Tr}[\Phi\Phi]2\mathrm{Tr}[\Phi]-\frac{4}{3}(\mathrm{Tr}[\Phi])^3\right)\\ &=\left(\mu^2+\lambda\mathrm{Tr}[\Phi\Phi]+\lambda'(\mathrm{Tr}[\Phi])^2\right)\Phi\\ &+\frac{2}{3}\lambda'\left(\mathrm{Tr}[\Phi\Phi]-\frac{2}{3}(\mathrm{Tr}[\Phi])^2\right)1\end{aligned} \qquad \textbf{(1.12)}$$

If we demand that the eigenvalues (v_1, v_2, v_3) of $\langle\Phi\rangle$ are different from each other and they are non-zero, the coefficients of Φ and 1 must be zero, so that we obtain the mass formula.

1.4 Yukawaon model

Standard Model

The origin of the mass spectra lies in the Yukawa coupling constants Y_f which are fundamental constants in the theory. I cannot believe that the nature needs such so many fundamental constants.

Yukawaon Model (a kind of flavon model)

The mass spectra are understood as dynamical quantities, i.e. VEV matrices of scalars Y_f :

$$Y_f^{eff} = \frac{y_f}{\Lambda}\langle Y_f \rangle \qquad (1.13)$$

We refer to these scalars Y_f as "yukawaons" .

VEV relations are obtained from SUSY vacuum conditions.

Another merit of Yukawaon model

- **Thus, by separating the origin of the flavor mixing from the origin of the quark and lepton masses, we can be freed from the FCNC problem in a multi-Higgs model.**

Origin of the Q & L masses:

conventional Higgs scalars $H_{u/d}$:

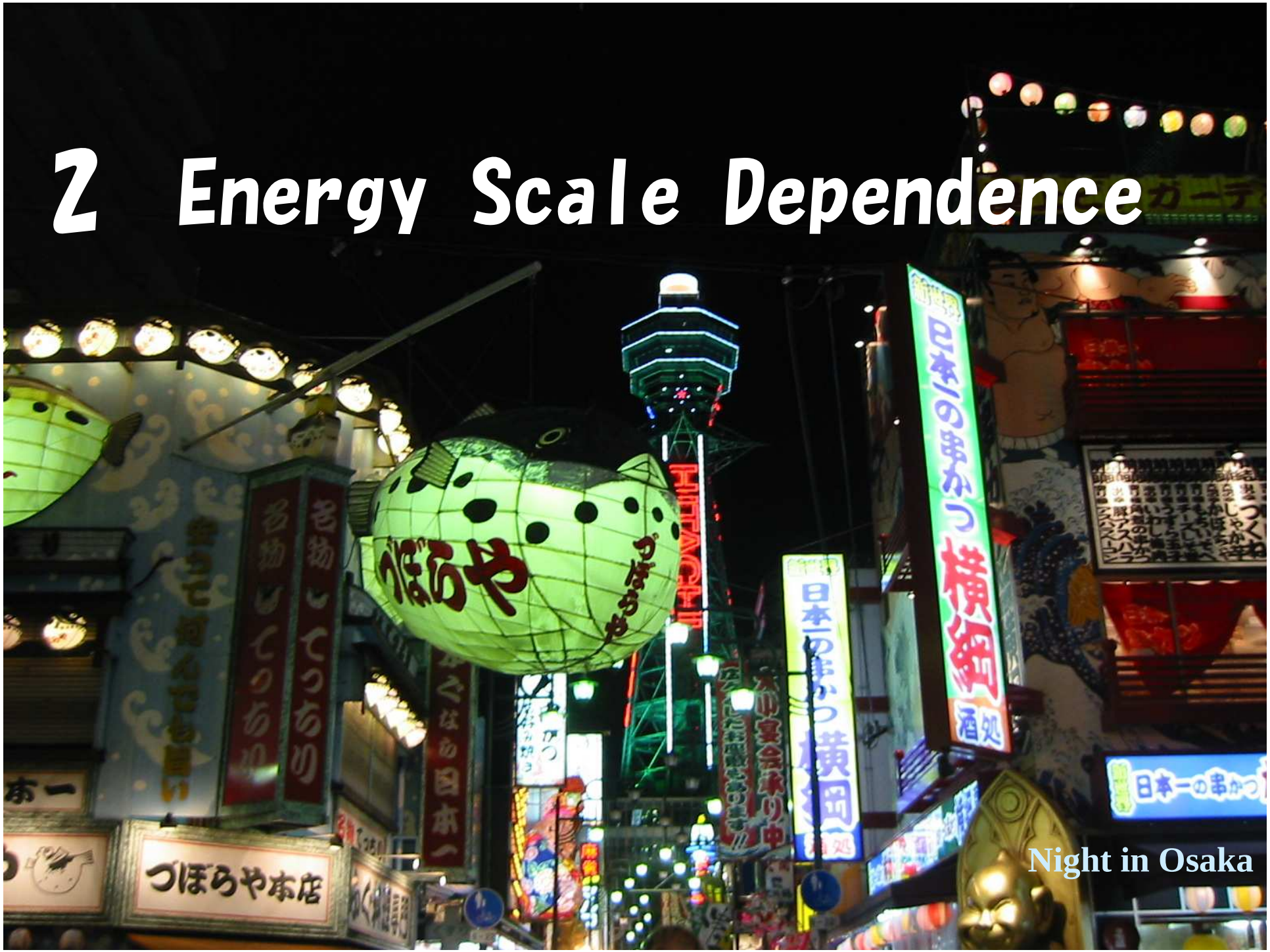
$$m(H_{u/d}) \sim \Lambda_{EW} \sim 10^2 \text{ GeV}$$

Origin of the flavor mixing:

Yukawaons Y_f:

$$m(Y_f) \sim \Lambda \sim 10^{15} \text{ GeV}$$

2 Energy Scale Dependence



Night in Osaka

2.1 Energy scale dependence of K

Anyhow, the relation K=2/3 was derived on the basis of a model at an energy scale $\mu = \Lambda$.

In general, the value of K is dependent on the scale μ.

The relation (1.2) is satisfied with an accuracy of 10^{-5} for the observed charged lepton masses (pole masses)

$$K^{pole} = \frac{2}{3} \times (0.999989 \pm 0.000014) \qquad (2.1)$$

However, if we use the running mass values, the formula is only valid with an accuracy of 10^{-3} for the running masses, for example, at $\mu = m_Z$

$$K^{run}_{\mu=M_Z} = \frac{2}{3} \times (1.00189 \pm 0.00002) \qquad \textbf{(2.2)}$$

- It is useful to define ξ as

$$K(\mu) = K^{pole}(1 + \xi(\mu))$$

- **If we admit K^{pole} = 2/3, we must build a model which can give**

$$K(\Lambda) = \frac{2}{3}(1 + \xi(\Lambda)) \qquad \xi \sim 10^{-3} \tag{2.3}$$

I have tried to build a model which can give (2.3) (not K = 2/3),

e.g. YK, PLB 681, 68 (2009); PLB 687, 219 (2010)

However, in 2009, another approach was proposed by Sumino, as I give a short review later on.

2.2 Another useful parameter

It is also useful to define the following quantity

$$\kappa = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3}$$

$$= \frac{v_1 v_2 v_3}{(v_1 + v_2 + v_3)^3} = \frac{\det\langle \Phi_e \rangle}{(\mathrm{Tr}[\langle \Phi_e \rangle])^3} \qquad (2.4)$$

We can completely determine the charged lepton mass spectrum by giving the two parameters $\xi(\mu)$ and $\kappa(\mu)$.

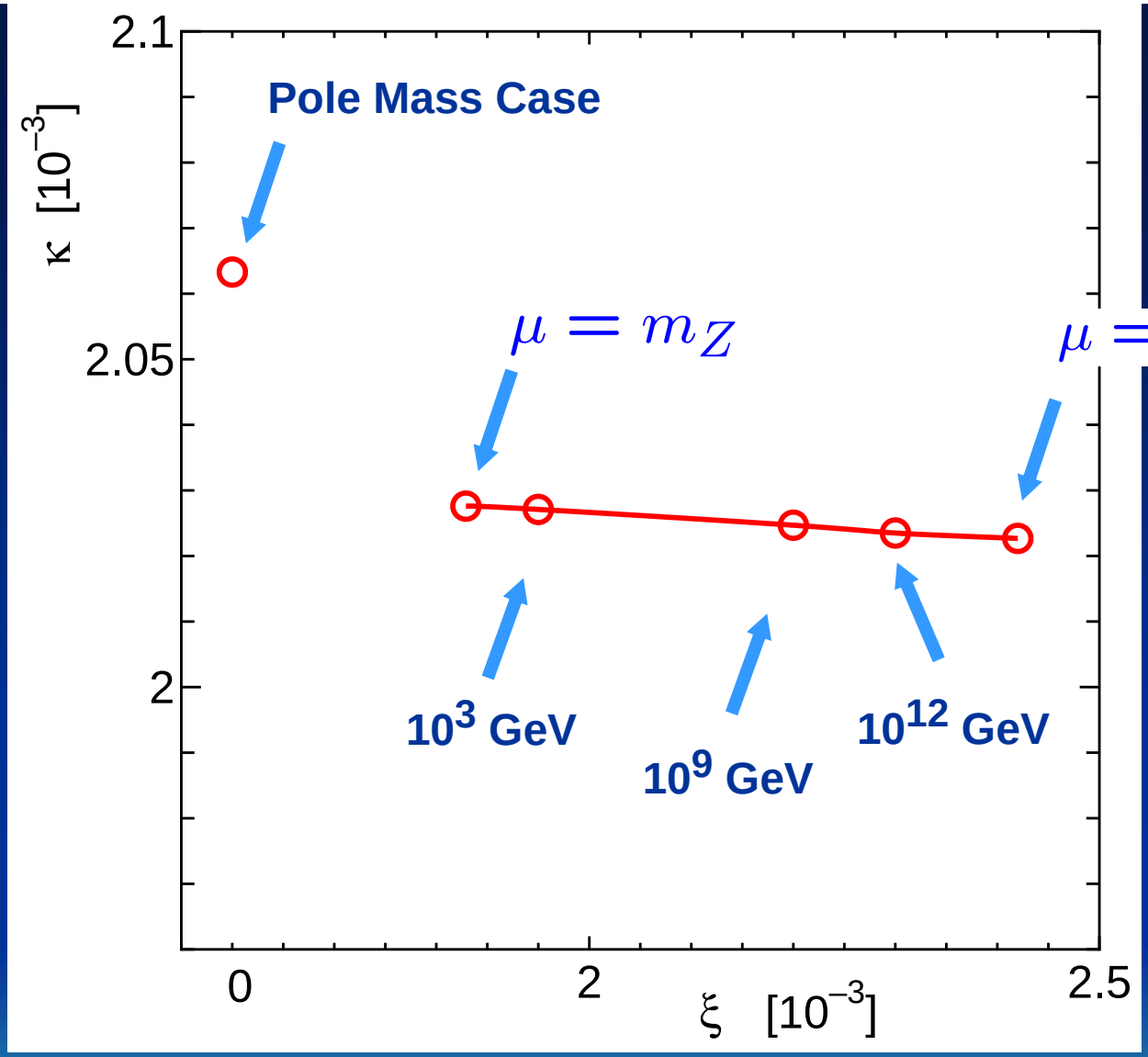
The observed value of κ is of an order of 10^{-3}, i.e. order of ξ

As shown in the next slide,

$\xi(\mu)$ versus $\kappa(\mu)$ have a linear relation.

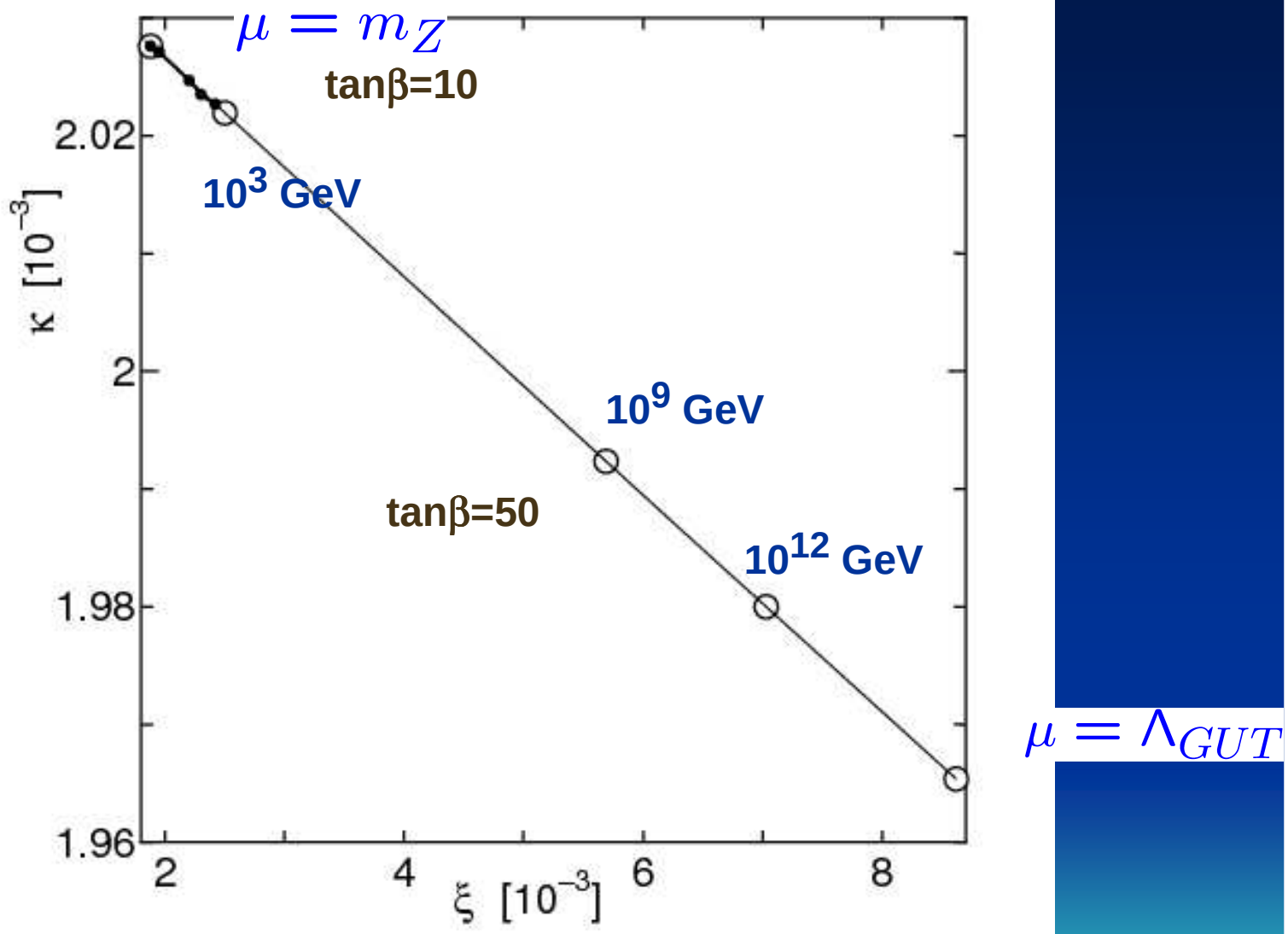
The linear relation is independent of $\tan\beta$

Y.K. PLB687, 219 (2010)



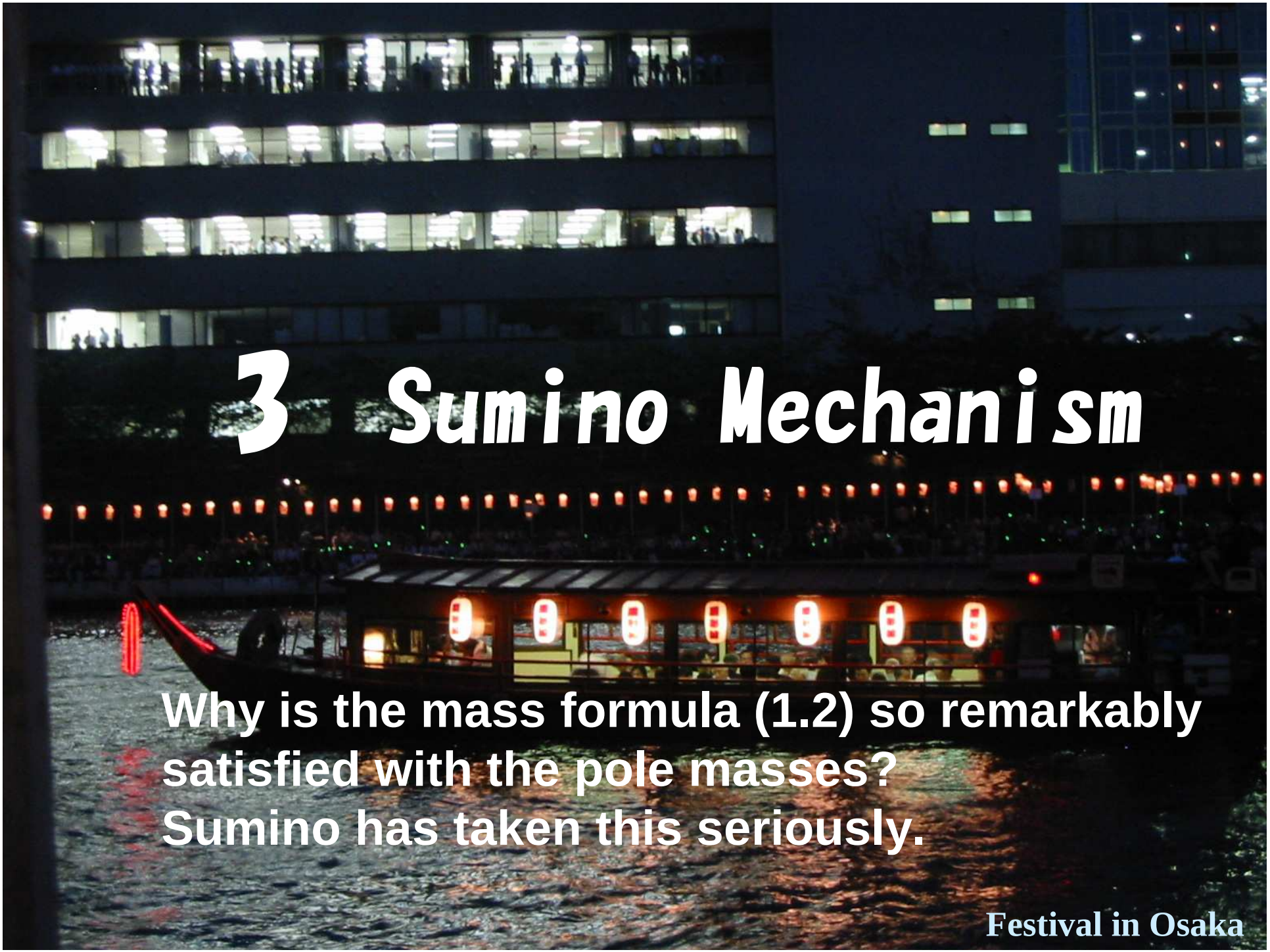
$\xi(\mu)$-$\kappa(\mu)$ **Relation in the MSSM with tanβ=10**

Running mass values have been quoted from Z.-z. Xing, et al, PRD 77, 113016 (2008)



The linear relation is independent of tanβ

Running mass values have been quoted from
Z.-z. Xing, et al, PRD 77, 113016 (2008)



3 Sumino Mechanism

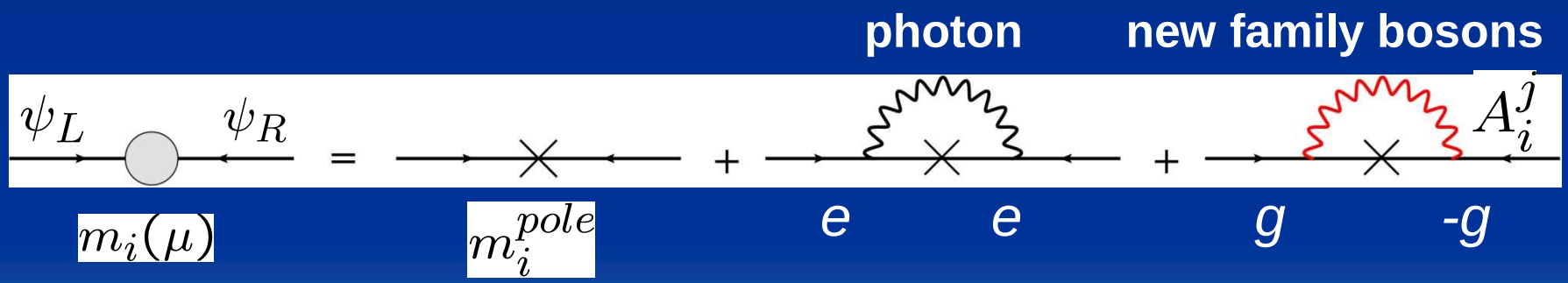
Why is the mass formula (1.2) so remarkably satisfied with the pole masses?
Sumino has taken this seriously.

Festival in Osaka

3.1 Sumino mechanism

[Y.S., PLB 671, 477 (2009); JHEP 0905, 075 (2009)].

The deviation of $K^{run}(\mu)$ from K^{pole} is caused by a term $m_i \log(\mu/m_i)$ in the radiative correction due to photon. He considers that a family symmetry is gauged, and the logarithmic term due to photon is exactly canceled by that due to family gauge bosons.



$(\psi_L, \psi_R) = (\mathbf{3}, \mathbf{3}^*)$

Therefore, we can obtain $K(\mu) = K^{pole}$

In the Sumino model, K=2/3 is exactly given at μ=Λ, and then we can obtain $K^{pole} \simeq K(\Lambda) = \frac{2}{3}$ due to above mechanism

3.2 How different from conventional family gauge symmetries

(i) In order to work the cancellation mechanism correctly , it is essential that the fermion fields are assigned as $(e_L, e_R) = (3, 3^*)$ of U(3)

(ii) Family gauge coupling constant takes a relation

$$\frac{1}{2} g_f = e = g_2 \sin^2 \theta_W$$

(iii) Family gauge boson masses are not free, and they have masses $m_{fij} \equiv m(A_i^j) \propto \sqrt{m_{ei} + m_{ej}}$

(iv) Effective interaction with $|\Delta N_f| = 2$: $(V - A) \cdot (V + A)$ appears, for example, $\mathcal{L}^{eff} = \frac{G_{f12}}{\sqrt{2}} 4 \left(\bar{\mu}_L \gamma_\rho e_L\right) \left(\bar{\mu}_R \gamma^\rho e_R\right) + h.c.,$

because of $(J_\rho)_1^2 = \bar{\mu}_L \gamma_\rho e_L - \bar{e}_R \gamma_\rho \mu_R$

3.3 Experimental test of the Sumino gauge bosons

Sumino gauge bosons can bring fruitful effects in TeV region physics, e.g.

$e^- + e^- \to \mu^- + \mu^-$ at a future ILC

$p + p \to A_1^1 + X \to e^+ e^- + X$

(no peak for $\mu^+\mu^-$) at LHC

and so on.

For phenomenological aspects of the Sumino model, see our recent paper

Y.K, Sumino & Yamanaka, arXive:1007.4739 [hep-ph]

4 New Developments in Sumino model and Yukawaon model

Sumino model can bring fruitful by-products to new physics

in Kyoto

4.1 Another charged lepton mass formula

Sumino has derived a new charged lepton mass relation

$$v_2^3 + v_1 v_2 v_3 - v_1 v_3 (v_1 + v_3) = 0 \quad (v_i \propto \sqrt{m_{ei}})$$

which is invariant under $v_1 \leftrightarrow v_3$ (4.1)

and which predicts

$$\kappa = 2.30869 \times 10^{-3} \quad (4.2)$$

c.f. Experimental value:

$$\kappa^{pole} = (2.0633 \pm 0.0001) \times 10^{-3} \quad (4.3)$$

Sumino, JHEP 05, 075 (2009)

Recall that κ was defined as

$$\kappa = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3}$$

Stimulated by his work, I have proposed a slightly modified relation from the Sumino relation

$$v_2^3 + v_1 v_2 v_3 - v_1 v_3 (v_1 + v_3) + \rho \Delta(v_1, v_2, v_3) = 0 \tag{4.4}$$

$$\Delta(v_1, v_2, v_3) \equiv \frac{(v_2^2 + 2v_1 v_3)(v_2^2 - v_1 v_3) v_2}{v_1^2 + v_2^2 + v_3^2} \tag{4.5}$$

with ρ=3/2.

The revised relation predicts

$$\kappa(\xi = 0) = 2.0653 \times 10^{-3} \tag{4.6}$$

Y.K. PLB 687, 219 (2010)

c.f. Experimental value:

$$\kappa^{pole} = (2.0633 \pm 0.0001) \times 10^{-3}$$

4.2 A model for neutrino mixing

- In the Sumino model, would-be Yukawa coupling for the charged lepton sector is given by

$$H_e = \frac{y_e}{\Lambda^2}\bar{\ell}_L^i(\Phi_e)_{i\alpha}(\Phi_e^T)_{\alpha j}e_R^j H \tag{4.7}$$

The leptons $\ell_L \sim (3,1)$ and $e_R \sim (3^*,1)$ of $U(3)\times O(3)$ do not compose an anomaly-free fermion set.

Of course, his model is an effective theory, so that it does not need to require anomaly-free.

Nevertheless, it is interesting to require that the model is anomaly free. **(For example, we usually regard the standard model for quarks and leptons as an effective theory model, and, at the same time, we know that the model is anomay free.)**

- **If we consider a SUSY version of the Sumino model**

$$W = \frac{y_e}{\Lambda}\ell_i(\Phi_e)^{i\alpha}(\Phi_e^T)^{\alpha j}e_j^c \qquad (4.8)$$

the model can be anomaly free, because we have

ℓ_i	e_i^c	$(\Phi_e)^{i\alpha}$	
two $\mathbf{3}$	one $\mathbf{3}$	three $\mathbf{3}^*$	of U(3)

Then, note that we cannot assign ν^c to 3 of U(3) in order to keep anomaly free. We must assign ν^c to 3 of O(3).

As a result, we can obtain the following superpotential with an anomaly free fermion set

$$W = \frac{y_e}{\Lambda^2}\ell_i Y_e^{ij} e_j^c H_d + \frac{y_\nu}{\Lambda}\ell_i(\Phi_e)^{i\alpha}\nu_\alpha^c H_u + y_R\nu_\alpha^c(Y_R)^{\alpha\beta}\nu_\beta^c$$

$$+\mu_e(Y_e)^{ij}(\Theta_e)_{ji} + \lambda_e(\Phi_e)^{i\alpha}(\Phi_e^T)^{\alpha j}(\Theta_e)_{ji} \qquad \textbf{(4.9)}$$

Therefore, the model suggests a seesaw-type neutrino mass matrix $M_\nu = m_D M_R^{-1} m_D^T$ **with**

$$m_D \propto \langle\Phi_e\rangle \text{ and } M_R \propto \langle Y_R\rangle \qquad \textbf{(4.10)}$$

- The problem is whether we can find or not a suitable form of $\langle Y_R \rangle$ which gives the observed tribimaximal neutrino mixing.

  Answer is: Yes, we can.

- We already know a neutrino mass matrix model $M_\nu = m_D M_R^{-1} m_D^T$ with

$$m_D = M_e \text{ and } M_R \propto M_u^{1/2} M_e + M_e M_u^{1/2} \qquad (4.11)$$

which can lead to the nearly tribimaximal mixing without assuming any discrete symmetry for neutrino sector. Y.K. PLB 665, 227; PLB 680, 76 (2009)

Therefore, we can build a similar model by taking M_R as

$$M_e^{1/2} M_R M_e^{1/2} \propto M_u^{1/2} M_e + M_e M_u^{1/2} \qquad (4.12)$$

Model for the quark sector

Now, let us extend our speculation to quark sector.

We assume

$$W_q = \frac{y_u}{\Lambda} u_i^c (Y_u)^{i\beta} q_\beta H_u + \frac{y_d}{\Lambda} d_\alpha^c (Y_d^T)^{\alpha\beta} q_\beta H_d \quad (4.13)$$

Note that the U(3) family gauge bosons cannot couple to down-quark sector, and only do to up-quarks u^c.

Therefore, the U(3) gauge bosons are free from constraints from the observed kaon physics.

We can build a model with anomaly free for quark sector as well as in the lepton sector.

We can also give reasonable up-quark mass ratios and neutrino mixing, i.e. “nearly tribimaximal mixing” .

For further details, see Y.K. arXiv:1011.1064 [hep-ph].

Appendix

Our neutrino mass matrix is given by

$$M_\nu \propto M_e^{1/2}\left\{M_e^{-1/2}\left[M_u^{1/2}M_e + M_e M_u^{1/2} + \xi_\nu \mathrm{Tr}[M_u^{1/2}]M_e\right]M_e^{-1/2}\right\}^{-1} M_e^{1/2}$$

where

$$M_u^{1/2} = M_e^{1/2}(\mathbf{1} + a_u X)M_e^{1/2}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$$

a_u= - 1.78 predicts reasonable up-quark mass ratios, and

ξ_ν = 0.01 can give the observed “nearly tribimaximal mixing”

ξ_ν	$\tan^2\theta_{solar}$	$\sin^2 2\theta_{atm}$	$\lvert U_{13}\rvert^2$
0	0.6995	0.9872	1.72×10^{-4}
0.009	0.4610	0.9902	2.28×10^{-4}
0.010	0.4408	0.9905	2.35×10^{-4}

5 Concluding Remarks

Thus, the present topics apparently seems to be narrow and shortsighted, but the topics can provide promising hints and clues to new physics. Especially, it should be worthwhile to notice future developments of the Sumino model.

Thank you