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SO(10)×SO(10) Universal Seesaw Model and its Intermediate Mass Scales

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Abstract

On the basis of the universal seesaw mass matrix model, which is a promising model of the unified description of the quark and lepton mass matrices, the behaviors of the gauge coupling constants and intermediate energy scales in the $SO(10)_L \times SO(10)_R$ model are investigated related to the neutrino mass generation scenarios. The non-SUSY model cannot give favorable values of the intermediate energy scales to explain the smallness of the neutrino masses, while the SUSY model can give the plausible values if the number n_ϕ of the weak doublet Higgs scalars is $n_\phi \geq 3$.

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1 Introduction

Recently, a universal seesaw mass matrix model has considerably attracted us as a unified mass matrix model of the quarks and leptons. The model [?] was proposed in order to understand the question why the masses of quarks (except for top quark) and charged leptons are so small compared with the electroweak scale Λ_L ($\sim 10^2$ GeV). The model has hypothetical fermions F_i in addition to the conventional quarks and leptons f_i (flavors $f = u, d, \nu, e$; family indices $i = 1, 2, 3$), and those are assigned to f_L = (2,1), f_R = (1,2), F_L = (1,1) and F_R = (1,1) of SU(2)$_L\times$ SU(2)$_R$. The 6×6 mass matrix which is sandwiched between the fields $(\overline{f}_L, \overline{F}_L)$ and (f_R, F_R) is given by

$$M^{6\times 6} = \left(\begin{array}{cc} 0 & m_L \\ m_R & M_F \end{array} \right) , \tag{1.1}$$

where m_L and m_R are universal for all fermion sectors ($f = u, d, \nu, e$) and only M_F have structures dependent on the flavors f. For $\Lambda_L < \Lambda_R \ll \Lambda_F$, where $\Lambda_L = O(m_L), \Lambda_R = O(m_R)$ and $\Lambda_F = O(M_F)$, the 3×3 mass matrix M_f for the fermions f is given by the well-known seesaw expression $M_f \simeq -m_L M_F^{-1} m_R$, so that the quarks and lepton masses $m_{q,l}$ are given with a suppression factor Λ_R/Λ_F. In order to understand the observed heavy top quark mass value $m_t \sim \Lambda_L$, we put an additional condition det$M_F = 0$ on the up-quark sector ($F = U$) [?, ?]. Then, since one of the fermion masses $m(U_i)$ ($i = 1, 2, 3$) is zero [say, $m(U_3) = 0$], so that the seesaw mechanism does not work for the third family, and the fermions (u_{3L}, U_{3R}) and (U_{3L}, u_{3R}) acquire masses of $O(m_L)$ and $O(m_R)$, respectively, without the suppression factor Λ_R/Λ_F. We identify (u_{3L}, U_{3_R}) as the top quark (t_L, t_R). An explicit model for the matrix forms m_L, m_R and M_F has been proposed by Fusaoka and the author [?], and they have successfully obtained the numerical results on the quark masses and mixings, where those quantities are described in terms of the charged lepton masses by assuming simple structures of m_L, m_R and M_F.

For the neutrino mass matrix, we start the following 12×12 mass matrix:

$$\left(\overline{\nu}_L\ \overline{\nu}^c_R\ \overline{N}_L\ \overline{N}^c_R \right) \left(\begin{array}{cccc} 0 & 0 & 0 & m_L \\ 0 & 0 & m_R^T & 0 \\ 0 & m_R & M_R & M_D \\ m_L^T & 0 & M_D^T & M_L \end{array} \right) \left(\begin{array}{c} \nu^c_L \\ \nu_R \\ N^c_L \\ N_R \end{array} \right) . \tag{1.2}$$

The mass matrix (1.2) leads to different scenarios of the neutrino phenomenology correspondingly to the different structure of the intermediate mass scales

$\Lambda_{NL} = O(M_L)$, $\Lambda_{NR} = O(M_R)$, and $\Lambda_D = O(M_D)$ together with to Λ_L, Λ_R, and Λ_F ($F \neq N$). For example, for the case $\Lambda_{NL}, \Lambda_{NL} \geq \Lambda_D$, the neutrino mass matrix is approximately given by $M_\nu \simeq -m_L M_L^{-1} m_L^T$, so that the neutrino masses are suppressed by the factors Λ_L/Λ_R and Λ_F/Λ_{NL} compared with the quark and charged lepton masses $m_{q,l}$.

In spite of such phenomenological successes, there is a reluctance to recognize the model, because the model needs extra fermions F. In most unification models, there are no rooms for the fermions F. Whether we can built a unification model in which the fermions F are reasonably embedded will be a touchstone for the great future of the universal seesaw mass matrix model.

For this problem, there is an attractive idea [?]. We can consider that the fermions F_R^c ($\equiv C\overline{F}_R^T$) together with the fermions f_L belong to **16** of SO(10), and also F_L^c together with f_R belong to **16** of another SO(10), i.e.,

$$(f_L + F_R^c) \sim (16, 1) \ , \quad (f_R + F_L^c) \sim (1, 16) \ , \tag{1.3}$$

of SO(10)$_L\times$SO(10)$_R$. In order to examine the idea (1.3), in the present paper, we investigate the evolution of the gauge coupling constants on the basis of the SO(10)$_L\times$SO(10)$_R$ model and estimate the intermediate energy scales Λ_R, Λ_F and Λ_N together with the unification energy scale Λ_X. The evolutions of the gauge coupling constants under SO(10)$_L\times$SO(10)$_R$ symmetries have already been done by Davidson, Wali and Cho [?]. The case of the symmetry breaking SO(10)$_L$ $\times$ SO(10)$_R$ $\to$ [SU(5)$\times$U(1)]$_L\times$[SU(5)$\times$U(1)]$_R$ is easily ruled out phenomenologically. On the contrary, it is not clear that the case SO(10)$_L$ $\times$ SO(10)$_R$ $\to$ [SU(2) $\times$ SU(2)$'$ $\times$ SU(4)]$_L$ $\times$ [SU(2) $\times$ SU(2)$'$ $\times$ SU(4)]$_R$ is ruled out or not, because there are many symmetry breaking patterns which were not discussed in the Ref. [?]. In the present paper, we will systematically investigate the intermediate mass scales for all possible cases (including non-SUSY and SUSY cases), but under a numerical constraint $\Lambda_R/\Lambda_F \simeq 0.02$ [?] which were derived from the observed ratio m_c/m_t under the new scenario of the universal seesaw model.

In Sec. **??**, we introduce the Higgs scalars in the present model and classify the possible cases of the symmetry breaking patterns. In Sec. **??**, we shortly review possible forms of the active neutrinos ν_L under the various cases of the intermediate energy scales. In Sec. **??**, we discuss the evolution of the gauge coupling constants for the case SO(10)$_L$ $\times$ SO(10)$_R$ $\to$ [SU(2) $\times$ SU(2)$'$ $\times$ SU(4)]$_L$ $\times$ [SU(2) $\times$ SU(2)$'$ $\times$ SU(4)]$_R$. The numerical results are presented in Sec. **??**. Although non-SUSY cases give the value $\Lambda_L/\Lambda_R \sim 10^{-5}$, the value is not sufficient to explain the smallness of the neutrino masses. On the other hand, the SUSY case can give a reasonable order of $\Lambda_L/\Lambda_R \sim 10^{-10}$ for $n_\phi = 6$, where n_ϕ is the number of the SU(2) doublet Higgs scalars. However, the model will encounter a new problem, i.e., the flavor-changing neutral currents (FCNC) problem. Sec. ?? will be devoted to the conclusions and remarks.

2 Higgs bosons and possible symmetry breaking patterns

In the present model, we consider the following Higgs scalars:
(i) $\Phi_{XL} = (54, 1)$ and $\Phi_{XR} = (1, 54)$, whose vacuum expectation values (VEV) break the symmetries $\mathrm{SO}(10)_L$ and $\mathrm{SO}(10)_R$ into $[\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_L$ and $[\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_R$, respectively;
(ii) $\Phi_{NL} = (126, 1)$ and $\Phi_{NR} = (1, 126)$, whose VEV break the symmetries $[\mathrm{SU}(2)' \times \mathrm{SU}(4)]_L$ and $[\mathrm{SU}(2)' \times \mathrm{SU}(4)]_R$ into $[\mathrm{U}(1) \times \mathrm{SU}(3)]_L$ and $[\mathrm{U}(1) \times \mathrm{SU}(3)]_R$, respectively, and generate the Majorana mass terms $\overline{N}_R^c M_L N_R$ and $\overline{N}_L M_R N_L^c$, respectively;
(iii) $\Phi_F = (\overline{16}, 16)$, whose VEV break $\mathrm{SU}(3)_L \times \mathrm{SU}(3)_R$ into $\mathrm{SU}(3)_{LR}$ and $\mathrm{U}(1)_L \times \mathrm{U}(1)_R$ into $\mathrm{U}(1)_{LR}$, and generate the Dirac mass terms $\overline{F}_L M_F F_R$;
(iv) $\phi_L = (10, 1)$ and $\phi_R = (1, 10)$, whose VEV break $\mathrm{SU}(2)_L$ and $\mathrm{SU}(2)_R$ and generate the mass terms $\overline{f}_L m_L F_R$ and $\overline{F}_L m_R f_R$, respectively.

For example, we consider the following case of the symmetry breaking pattern:
Case RLRL: $\Lambda_{XR} > \Lambda_{XL} > \Lambda_{NR} > \Lambda_{NL} = \Lambda_D > \Lambda_F$

$$\mathrm{SO}(10)_L \times \mathrm{SO}(10)_R$$

$$\downarrow \quad \text{at } \mu = \Lambda_{XR}$$

$$\mathrm{SO}(10)_\mathrm{L} \times [\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_\mathrm{R}$$

$$\downarrow \quad \text{at } \mu = \Lambda_{XL}$$

$$[\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_L \times [\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_R$$

$$\downarrow \quad \text{at } \mu = \Lambda_{NR}$$

$$[\mathrm{SU}(2) \times \mathrm{SU}(2)' \times \mathrm{SU}(4)]_L \times [\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{SU}(3)]_R$$

$$\downarrow \quad \text{at } \mu = \Lambda_{NL} = \Lambda_D$$

$$[\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{SU}(3)]_L \times [\mathrm{SU}(2) \times \mathrm{U}(1) \times \mathrm{SU}(3)]_R$$

$$\downarrow \quad \text{at } \mu = \Lambda_F$$

$$\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{LR} \times \mathrm{SU}(3)_{LR}$$

$$\downarrow \quad \text{at } \mu = \Lambda_R$$

$$\mathrm{SU(2)}_L \times \mathrm{U(1)}_Y \times \mathrm{SU(3)}_{LR}$$

$$\downarrow \quad \text{at } \mu = \Lambda_L$$

$$\mathrm{U(1)}_{em} \times \mathrm{SU(3)}_{LR} \ , \tag{2.1}$$

where $\mathrm{SU(3)}_{LR}$ means the color gauge symmetry $\mathrm{SU(3)}_c$.

For convenience, we define the following ranges of the energy scale μ: Range 1 ($\Lambda_L < \mu \leq \Lambda_R$); Range 2 ($\Lambda_R < \mu \leq \Lambda_F$); Range 3 ($\Lambda_F < \mu \leq \Lambda_{NL}$); Range 4 ($\Lambda_{NL} < \mu \leq \Lambda_{NR}$); Range 5 ($\Lambda_{NR} < \mu \leq \Lambda_{XL}$); Range 6 ($\Lambda_{XL} < \mu \leq \Lambda_{XR}$). Here, we regard $\Lambda_D = \Lambda_{NL}$. Also, for convenience of the next section, let us define the following parameters

$$x_1 = \log(\Lambda_R/\Lambda_L) \ , \quad x_2 = \log(\Lambda_F/\Lambda_R) \ , \quad x_3 = \log(\Lambda_{NL}/\Lambda_F) \ ,$$

$$x_4 = \log(\Lambda_{NR}/\Lambda_{NL}) \ , \quad x_5 = \log(\Lambda_{XL}/\Lambda_{NR}) \ , \quad x_6 = \log(\Lambda_{XR}/\Lambda_{XL}) \ . \tag{2.2}$$

The values of x_i must be positive or zero. Especially, the value of x_1 must roughly be $x_1 \geq 1$ from the experimental lower limit [?] of the right-handed weak boson mass.

We also investigate the following cases:

Case RLLR: $\Lambda_{XR} > \Lambda_{XL} > \Lambda_{NL} > \Lambda_{NR} = \Lambda_D > \Lambda_F$;

Case LRLR: $\Lambda_{XL} > \Lambda_{XR} > \Lambda_{NL} > \Lambda_{NR} = \Lambda_D > \Lambda_F$;

Case LRRL: $\Lambda_{XL} > \Lambda_{XR} > \Lambda_{NR} > \Lambda_{NL} = \Lambda_D > \Lambda_F$;

Case RRLL: $\Lambda_{XR} > \Lambda_{NR} > \Lambda_{XL} > \Lambda_{NL} = \Lambda_D > \Lambda_F$;

Case LLRR: $\Lambda_{XL} > \Lambda_{NL} > \Lambda_{XR} > \Lambda_{NR} = \Lambda_D > \Lambda_F$.

We consider a model without Φ_{NL}:

Case RLLD: $\Lambda_{XR} > \Lambda_{XL} > \Lambda_{NL} > \Lambda_D > \Lambda_F$;

Case LRLD: $\Lambda_{XL} > \Lambda_{XR} > \Lambda_{NL} > \Lambda_D > \Lambda_F$;

a model without Φ_{NL}:

Case RLRD: $\Lambda_{XR} > \Lambda_{XL} > \Lambda_{NR} > \Lambda_D > \Lambda_F$;

Case LRRD: $\Lambda_{XL} > \Lambda_{XR} > \Lambda_{NR} > \Lambda_D > \Lambda_F$;

and a model without Φ_{NL} and Φ_{NR}

Case RLD: $\Lambda_{XR} > \Lambda_{XL} > \Lambda_D > \Lambda_F$;

Case LRD: $\Lambda_{XL} > \Lambda_{XR} > \Lambda_D > \Lambda_F$.

In addition to these cases, we investigation the following cases, which lead to a model with pseudo-Dirac neutrino states which was pointed out by Bowes and Volkas [?]:

Case RLDN: $\Lambda_{XR} > \Lambda_{XL} > \Lambda_D > \Lambda_F > \Lambda_{NR/NL}$;

Case LRDN: $\Lambda_{XL} > \Lambda_{XR} > \Lambda_D > \Lambda_F > \Lambda_{NR/NL}$.

For each case, in a similar way to the case RLRL, we define the energy scale regions and parameters x_i. The definitions for some typical cases are listed in Table **??**. For the model without the scalars Φ_{NL} and Φ_{NR} (the cases RLD and LED) and the Bowes-Volkas model (the cases RLDN and LRDN), we define the ranges without the range 4 (the parameter x_4). The definitions for the other cases which are not given in Table **??** can readily be read by the exchange $L \leftrightarrow R$.

Each case is investigated for the cases of non-SUSY and SUSY. Here, the "SUSY" case means a minimal SUSY model, and for simplicity, we take the SUSY breaking energy scale Λ_{SUSY} as $\Lambda_{SUSY} = \Lambda_L$ in the numerical estimates.

3 Mass matrix for the active neutrinos

Our interest is in the effective mass matrix M_ν for the active neutrinos ν_L. For $\Lambda_D \gg \Lambda_R \gg \Lambda_L$, the mass matrix (1.2) approximately leads to the mass matrix for the neutrinos (ν_L^c, ν_R):

$$M^{6\times 6} \simeq - \left(\begin{array}{cc} m_L M_{22}^{-1} m_L^T & m_L M_{21}^{-1} m_R \\ m_L^T M_{12}^{-1} m_R^T & m_R^T M_{11}^{-1} m_R \end{array} \right) , \tag{3.1}$$

where

$$\left(\begin{array}{cc} M_{11}^{-1} & M_{12}^{-1} \\ M_{21}^{-1} & M_{22}^{-1} \end{array} \right) = \left(\begin{array}{cc} M_R & M_D \\ M_D^T & M_L \end{array} \right)^{-1} , \tag{3.2}$$

$$\begin{array}{l} M_{11} = M_R - M_D M_L^{-1} M_D^T \ , \\ M_{22} = M_L - M_D^T M_R^{-1} M_D \ , \\ M_{12} = M_{21}^T = M_D^T - M_L M_D^{-1} M_R \ . \end{array} \tag{3.3}$$

The scenarios for neutrino masses and mixings are highly dependent on the structure of the intermediate mass scales Λ_{NL}, Λ_{NR}, and Λ_D relative to Λ_F $(F \neq N)$, Λ_R, and Λ_L. In this section, let us review possible matrix forms of the effective mass matrix of the active neutrinos ν_L.

Case A : $M_L, M_R \geq M_D$

For the case $M_L, M_R \geq M_D$, the 6×6 matrix (3.1) becomes

$$M^{6\times 6} \simeq \begin{pmatrix} -m_L M_L^{-1} m_L^T & m_L M_L^{-1} M_D^T M_R^{-1} m_R \\ m_R^T M_R^{-1} M_D M_L^{-1} m_L^T & -m_R^T M_R^{-1} m_R \end{pmatrix} , \tag{3.4}$$

so that the approximate mass matrix $M(\nu_L)$ for the active neutrinos ν_L

$$M(\nu_L) \simeq -m_L M_L^{-1} m_L^T , \tag{3.5}$$

together with the mass matrix for the neutrinos ν_R

$$M(\nu_R) \simeq -m_R^T M_R^{-1} m_R . \tag{3.6}$$

The smallness of the neutrino masses is given by

$$m_\nu \sim \frac{\Lambda_L}{\Lambda_R} \cdot \frac{\Lambda_F}{\Lambda_{NL}} \cdot m_{e,q} . \tag{3.7}$$

For the case $M_R \gg M_L \sim M_D$, especially for the case $\Lambda_L^2/\Lambda_{NL} \sim \Lambda_R^2/\Lambda_{NR}$, we can build an interesting scenario, where the solar neutrino data [?], atmospheric neutrino data [?] and LSND data [?] are explained from a small mixing $\nu_{eL} \leftrightarrow \nu_{eR}^c$, a large mixing $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$, and a small mixing $\nu_{\mu L} \leftrightarrow \nu_{eL}$, respectively. In order to realize the smallness of the neutrino masses, $m_\nu/m_{e,q} \sim 10^{-9}$, the constraint

$$\Lambda_{NL}\Lambda_{NR}/\Lambda_F^2 \sim 10^{18} , \tag{3.8}$$

is required.

Case B : $M_L, M_R \ll M_D$

For the case $M_R, M_L \ll M_D$, the matrix (3.1) leads to

$$M^{6\times 6} \simeq \begin{pmatrix} -m_L M_D^{-1} M_R M_D^{T-1} m_L^T & -m_L M_D^{-1} m_R \\ m_R^T M_D^T m_L^T & -m_R^T M_D^{T-1} M_L M_D^{-1} m_R \end{pmatrix} . \tag{3.9}$$

Since $(M^{6\times 6})_{12} \gg (M^{6\times 6})_{11(22)}$ because of $m_R \gg m_L$, the case leads to a model with the pseudo-Dirac neutrino states $\nu_{i\pm} \simeq (\nu_{Li} \pm \nu_{Ri}^c)/\sqrt{2}$, whose masses are given by the order

$$m(\nu_\pm) \sim \Lambda_L \Lambda_R/\Lambda_D \sim (\lambda_F/\Lambda_D) m_{e,q} . \tag{3.10}$$

This case has been discussed by Bowes and Volkas [?].

Case C: $M_R = 0$

For a model without Φ_{NR}, the matrix (3.1) leads to

$$M^{6\times 6} \simeq \begin{pmatrix} 0 & -m_L M_D^{-1} m_R \\ -m_R^T M_D^{T-1} m_L^T & m_R^T M_D^{T-1} M_L M_D^{-1} m_R \end{pmatrix} . \tag{3.11}$$

When $(\Lambda_R/\Lambda_L)(\Lambda_{NL}/\Lambda_D) \gg 1$, we again obtain the effective mass matrix (3.5) for the active neutrinos ν_L.

Case D: $M_L = 0$

For a model without Φ_{NL}, the matrix (3.1) leads to

$$M^{6\times 6} \simeq \begin{pmatrix} m_L M_D^{-1} M_R M_D^{T-1} m_L^T & -m_L M_D^{-1} m_R \\ -m_R^T M_D^{T-1} m_L^T & 0 \end{pmatrix} . \tag{3.12}$$

When $\Lambda_{NR}/\Lambda_D \ll \Lambda_R/\Lambda_L$, the case gives a model with pseudo-Dirac neutrinos whose masses are given by (3.10). When $\Lambda_{NR}/\Lambda_D \gg \Lambda_R/\Lambda_L$, the case gives

$$M(\nu_L) \simeq m_L M_D^{-1} M_R M_D^{T-1} m_L^T \ . \tag{3.13}$$

Thus, in order to obtain small values of the neutrino masses $(m_\nu/m_{e,q} \sim 10^{-9})$, we need to seek for a model with $(\Lambda_R/\Lambda_L)(\Lambda_R/\Lambda_L) \sim 10^9$, i.e.,

$$x_1 + x_3 \sim 9 \ . \tag{3.14}$$

4 Evolution of the gauge coupling constants

For convenience, let us discuss on the case RLRL. The electric charge operator Q is given by

$$Q = I_3^L + \frac{1}{2} Y \ , \quad \Lambda_L < \mu \leq \Lambda_R \ , \tag{4.1}$$

$$\frac{1}{2} Y = I_3^R + \frac{1}{2} Y_{LR} \ , \quad \Lambda_R < \mu \leq \Lambda_F \ , \tag{4.2}$$

$$\frac{1}{2} Y_{LR} = \frac{1}{2} Y_L + \frac{1}{2} Y_R \ , \quad \Lambda_F < \mu \leq \Lambda_{NL} \ , \tag{4.3}$$

$$\frac{1}{2}Y_L = I_3'^L + \sqrt{\frac{2}{3}}F_{15}^L \ , \qquad \Lambda_{NL} < \mu \le \Lambda_{XL} \ , \tag{4.4}$$

$$\frac{1}{2}Y_R = I_3'^R + \sqrt{\frac{2}{3}}F_{15}^R \ , \qquad \Lambda_{NR} < \mu \le \Lambda_{XR} \ , \tag{4.5}$$

where $I_3'^L$, $I_3'^R$, F_{15}^L and F_{15}^R are generators of $\mathrm{SU}(2)'_L$, $\mathrm{SU}(2)'_R$, $\mathrm{SU}(4)_L$ and $\mathrm{SU}(4)_R$, respectively. We denote the gauge coupling constants corresponding to the operators Q, Y, Y_{LR}, Y_L, Y_R, I^L, I^R, I'^L, I'^R, F^L and F^R as $g_{em} \equiv e$, g_1, g_{1LR}, g_{1L}, g_{1R}, g_{2L}, g_{2R}, g'_{2L}, g'_{2R}, g_{4L} and g_{4R}, respectively. The boundary conditions for these gauge coupling constants at $\mu = \Lambda_L$, $\mu = \Lambda_R$, $\mu = \Lambda_F$, $\mu = \Lambda_{NL}$, and $\mu = \Lambda_{NR}$ are as follows:

$$\alpha_{em}^{-1}(\Lambda_L) = \alpha_{2L}^{-1}(\Lambda_L) + \frac{5}{3}\alpha_1^{-1}(\Lambda_L) \ , \tag{4.6}$$

$$\frac{5}{3}\alpha_1^{-1}(\Lambda_R) = \alpha_{2R}^{-1}(\Lambda_R) + \frac{2}{3}\alpha_{1LR}^{-1}(\Lambda_R) \ , \tag{4.7}$$

$$\frac{2}{3}\alpha_{1LR}^{-1}(\Lambda_F) = \frac{5}{3}\alpha_{1L}^{-1}(\Lambda_F) + \frac{5}{3}\alpha_{1R}^{-1}(\Lambda_F) \ , \tag{4.8}$$

$$\frac{5}{3}\alpha_{1L}^{-1}(\Lambda_{NL}) = \alpha_{2L}'^{-1}(\Lambda_{NL}) + \frac{2}{3}\alpha_{4L}^{-1}(\Lambda_{NL}) \ , \tag{4.9}$$

and

$$\frac{5}{3}\alpha_{1R}^{-1}(\Lambda_{NR}) = \alpha_{2R}'^{-1}(\Lambda_{NR}) + \frac{2}{3}\alpha_{4R}^{-1}(\Lambda_{NR}) \ , \tag{4.10}$$

respectively, correspondingly to Eqs. (4.1) - (4.5), where $\alpha_i \equiv g_i^2/4\pi$. We also have the following boundary conditions at $\mu = \Lambda_F$, $\mu = \Lambda_{NL}$, $\mu = \Lambda_{NR}$, $\mu = \Lambda_{XL}$ and $\mu = \Lambda_{XR}$:

$$\alpha_3^{-1}(\Lambda_F) = \alpha_{3L}^{-1}(\Lambda_F) + \alpha_{3R}^{-1}(\Lambda_F) \ , \tag{4.11}$$

$$\alpha_{3L}^{-1}(\Lambda_{NL}) = \alpha_{4L}^{-1}(\Lambda_{NL}) \ , \tag{4.12}$$

$$\alpha_{3R}^{-1}(\Lambda_{NR}) = \alpha_{4R}^{-1}(\Lambda_{NR}) \ , \tag{4.13}$$

$$\alpha_{2L}^{-1}(\Lambda_{XL}) = \alpha_{2L}'^{-1}(\Lambda_{XL}) = \alpha_{4L}^{-1}(\Lambda_{XL}) \ , \tag{4.14}$$

$$\alpha_{2R}^{-1}(\Lambda_{XR}) = \alpha_{2R}'^{-1}(\Lambda_{XR}) = \alpha_{4R}^{-1}(\Lambda_{XR}) \ . \tag{4.15}$$

The evolutions of the gauge coupling constants g_i at one-loop are given by the equations

$$\frac{d}{dt}\alpha_i(\mu) = -\frac{1}{2\pi}b_i\alpha_i^2(\mu) \ , \tag{4.16}$$

where $t = \ln \mu$.

For example, for the case RLRL, the coefficients b_i are calculated as follows. The quantum numbers of the fermions f and F are assigned as those in Table ??. Note that in the model with $\det M_U = 0$, the heavy fermions F_L and F_R except for U_{3L} and U_{3R} are decoupled for $\mu \leq \Lambda_F$ and the fermions u_{3R} and U_{3L} are decoupled for $\mu \leq \Lambda_R$. Components of the Higgs scalars Φ_{NR}, Φ_F and ϕ_R which contribute to the coefficients b_{iR}, for example, in the energy-scale range 6, are $[1;(1,3,\overline{10})+(3,1,10)]$, $[\overline{16};(1,2,\overline{4})]$, and $[1;(2,2,1)]$ of $\mathrm{SO}(10)_L \times [\mathrm{SU}(2)\times\mathrm{SU}(2)'\times \mathrm{SU}(4)]_R$, respectively. In the range 5, those become $[(1,1,1);(1,3,\overline{10})+(3,1,10)]$, $[\overline{(1,2,\overline{4})};(1,2,\overline{4})]$, and $[(1,1,1);(2,2,1)]$ of $[\mathrm{SU}(2)\times\mathrm{SU}(2)'\times\mathrm{SU}(4)]_L \times [\mathrm{SU}(2)\times \mathrm{SU}(2)'\times\mathrm{SU}(4)]_R$, respectively. The results are listed in Table ??. The coefficients b_i for the other cases can be calculated in a similar way.

For the numerical study, we use the following input values [?]: $\alpha_1 = 0.01683$, $\alpha_2 = 0.03349$, and $\alpha_3 = 0.1189$ at $\mu = m_Z$ instead of those at $\mu = \Lambda_L$, and

$$\frac{\Lambda_R}{\Lambda_F} = 0.02 \ , \quad \text{i.e.,}\ x_2 = \log\frac{\Lambda_F}{\Lambda_R} = 1.70 \ , \qquad (4.17)$$

which was derived from the observed ratio m_c/m_t and the modified universal seesaw model [?] with the constraint $\det M_U = 0$. Since we have four constraint equations (4.14) and (4.15), four of the eight parameters x_1, x_3, x_4, x_5, x_6 and

$$a_1 = \alpha_{1R}^{-1}(\Lambda_F) \ , \quad a_2 = \alpha_{2R}^{-1}(\Lambda_R) \ , \quad a_3 = \alpha_{3R}^{-1}(\Lambda_F) \ , \qquad (4.18)$$

are independent. (For the cases with $x_4 = 0$, the number of the independent parameters are three.) What is of great interest to us is whether we can give a reasonable order of the neutrino masses or not. Therefore, we evaluate the maximal value of x_3 under the constraints $x_1 \geq 1$, $x_4 \geq 0$, $x_5 \geq 0$, $x_6 \geq 0$, $a_1 \geq 1$, $a_2 \geq 1$ and $a_3 \geq 1$. If there is no solution with $(x_3)_{max} \geq 0$, the case will be ruled out. Even if we have a solution with $(x_3)_{max} \geq 0$, which means that there are the unification points Λ_{XL} and Λ_{XR} of the gauge coupling constants, it will be difficult to explain the smallness of the neutrino masses if the numerical results of $x_1 + x_3$ show $x_1 + x_3 < 8$.

5 Numerical results

The numerical study has been done for non-SUSY and SUSY cases with $n_F = 1,2,3,4$ and $n_\phi = 1,2,\cdots,6$, where n_F and n_ϕ are numbers of the Higgs scalars Φ_F and ϕ_L (ϕ_R).

In all the non-SUSY cases with $\Lambda_{XR} > \Lambda_{XL}$, there is no solution with $(x_3)_{max} \geq 0$. Therefore, the cases are ruled out. For the non-SUSY case with $\Lambda_{XL} \geq \Lambda_{XR}$, except for the case LRD which gives $(x_3)_{max} < 0$, we can get positive values $(x_3)_{max} \geq 0$. However, the solutions with $(x_3)_{max} \geq 0$ are allowed only when $n_F = 1$ and $n_\phi \geq 5$. In Table **??**, the values of $(x_3)_{max}$ and $(x_1)_{max}$ for the cases with $n_F = 1$ and $n_\phi = 6$ are demonstrated. As seen in Table **??**, these cases cannot give large values of $(x_1)_{max}$ and/or $(x_3)_{max}$. Therefore, all the non-SUSY cases cannot explain the smallness of neutrino masses m_ν, so that they are ruled out. (A similar study for a non-SUSY case, but for a case with different Higgs scalars, has been done by the author [**?**]. In Ref. [**?**], in spite of his numerical result $x_1 \leq 6.1$ for a case with $\Lambda_L > \Lambda_R$, he concluded that the case cannot be ruled out, because the numerical results should not be taken rigidly. However, the discrepancy between 10^6 and 10^9 is too large to reconcile.)

In the SUSY cases, there are solutions with $(x_3)_{max} > 0$ for the cases $n_\phi \geq 3$, but they are allowed only when $x_4 = x_5 = x_6 = 0$ for the cases RLRL, RLLR, LRLR, LRRL, RRLL, LLRR, RLLD, LRLD, RLRD, LRRD, and only when $x_5 = x_6 = 0$ for the cases RLD, LRD, RLDN, LRDN. This means that $\Lambda_{XL} = \Lambda_{XR} = \Lambda_{NL} = \Lambda_{NR} = \Lambda_D$, and the symmetries $SO(10)_L \times SO(10)_R$ are directly broken into the symmetries $[SU(2) \times U(1) \times SU(3)]_L \times [SU(2) \times U(1) \times SU(3)]_R$ neither via SU(4) nor SU(5). As seen in Table **??**, the results are independent of the number of Φ_F, n_F, for the cases with $x_4 = x_5 = x_6 = 0$. All the cases RLRL, RLLR, $\cdots$, LRDN give the same numerical results for the cases with the same value of n_ϕ. The results are listed in Table **??**. The value of $x_1 + x_3$ for the case $n_\phi = 3$ is somewhat large compared with a desirable value $x_1 + x_3 \sim 9$. If we take the numerical results rigidly, the case $n_\phi = 6$ is favorable to explain $m_\nu / m_{e,q} \sim 10^{-9}$.

Numerical results for a typical SUSY case with $n_\phi = 4$ is as follows: $x_1 = 11.66$, $x_3 = x_4 = x_5 = x_6 = 0$, i.e.,

$$\Lambda_L = 0.912 \times 10^2 \text{ GeV} , \quad \Lambda_R = 4.17 \times 10^{13} \text{ GeV} , \quad \Lambda_F = \Lambda_X = 2.08 \times 10^{15} \text{ GeV} , \tag{5.1}$$

$$\alpha_{1R}^{-1}(\Lambda_F) = \alpha_{2R}^{-1}(\Lambda_F) = 2.41 , \quad \alpha_{3R}^{-1}(\Lambda_R) = 3.66 . \tag{5.2}$$

Since the numerical results are same for all the cases RLRL, RLLR, $\cdots$, the simplest choice will be to consider a model without Majorana mass terms.

6 Conclusion

In conclusion, by using one-loop evolution equation for the gauge coupling constants, we have investigated possible intermediate mass scales Λ_R, Λ_F, Λ_D, Λ_{NL}, Λ_{NR}, and the unification scales Λ_{XL} and Λ_{XR} in the universal seesaw model with the gauge unification $SO(10)_L \times SO(10)_R$. The evolution has systematically been investigated for all the cases in which the symmetries $SO(10)_L \times SO(10)_R$ are broken into $[SU(2) \times SU(2)' \times SU(4)]_L \times [SU(2) \times SU(2)' \times SU(4)]_R$. We have evaluated the maximum values of $x_3 = \log(\Lambda_D/\Lambda_F)$ and $x_1 = \log(\Lambda_R/\Lambda_L)$, because in most cases, the neutrino masses are suppressed compared with the charged lepton and quark masses $m_{e,q}$ by the factor $10^{-(x_1+x_3)}$ as stated in Sec. ??.

We have found that all the cases cannot give a model with $\Lambda_D \gg \Lambda_F$, i.e., we obtain, at most, $(x_3)_{max} = 1.27$ for the SUSY model with $n_\phi = 6$. Therefore, models based on the cases B and D discussed in Sec. ?? are ruled out. The cases A and C and a model without Majorana mass terms are our possible choices.

The non-SUSY model with $\Lambda_{XR} > \Lambda_{XL}$ is also ruled out, because they cannot have the value $(x_3)_{max} \geq 0$. Although the non-SUSY model with $\Lambda_{XL} > \Lambda_{XR}$ can have the value $(x_3)_{max} \geq 0$, the values of x_1 and x_3 are not sufficiently large to explain the smallness of the neutrino masses.

In the SUSY cases, there are solutions with $(x_3)_{max} > 0$, but they are allowed only when $x_4 = x_5 = x_6 = 0$ and $n_\phi \geq 3$. The constraint $x_4 = x_5 = x_6 = 0$ means that the symmetries $SO(10)_L \times SO(10)_R$ are directly broken into the symmetries $[SU(2) \times U(1) \times SU(3)]_L \times [SU(2) \times U(1) \times SU(3)]_R$ neither via SU(4) nor SU(5).

The constraint $n_\phi \geq 3$ is somewhat unwilling, because the cases induce the flavor-changing neutral currents (FCNC). If we take the numerical results in Table ?? rigidly, the case with $n_\phi = 6$ (or $n_\phi = 7$) is favorable. However, the case with $n_\phi = 6$ will fatally bring the FCNC problem to us. If we postpone the FCNC problem to the future, the case with $n_\phi = 6$ ($n_\phi^{up} = 3$ and $n_\phi^{down} = 3$) is favorable, where n_ϕ^{up} and n_ϕ^{down} are the numbers of the SU(2) doublet Higgs scalars which couple with up- and down-quark sectors, respectively. Such a multi-Higgs model may play a role of the hierarchical mass structure within the family, i.e., (m_e, m_μ, m_τ) (for example, see Ref. [?]).

On the other hand, if we take the FCNC problem seriously, we must take the case with $n_\phi = 3$. For example, if we consider a case with $n_\phi^{up} = 2$ and $n_\phi^{down} = 1$, the FCNC appear only in the up-quark sector, so that the damage form the FCNC will be reduced a little. However, the case gives an over-suppression of the neutrino masses because of $x_1 + x_3 \sim 12$. If we want to take the case $n_\phi = 3$, the excuse for the over-suppression is as follows: the present numerical results should not be taken too rigidly. (i) The results were obtained by using the one-loop evolution equation (4.16). The consideration of the higher order corrections may slightly change the numerical results. (ii) The constraint (3.14), i.e., $m_\nu \sim 10^{x_1+x_3} m_{e,q}$, is obtained for the case that $y_L \sim 1$, $y_R \sim 1$ and $y_F \sim 1$, where the Yukawa coupling constants y_L, y_R and y_F are defined by $m_L = y_L \Lambda_L$, $m_R = y_R \Lambda_R$ and $M_F = y_F \Lambda_F$, respectively. For example, if we suppose the case that $y_L \sim 1$, $y_R \sim 1$ and $y_F \sim 10^{-1}$, then the suppression can be reduced by a factor 10^1. (iii) We have used the numerical constraint $\Lambda_R/\Lambda_F = 0.02$. The constraint came from the phenomenological study [**?**] of the quark masses based on the universal seesaw model. The value is model-dependent. [However, it is likely that the value of Λ_R/Λ_F is of the order of m_c/m_t (m_b/m_t) in the framework of the new universal seesaw model. Therefore, the numerical conclusion is still reliable as the order.] Thus, we cannot completely exclude the case with $n_\phi = 3$.

We conclude that when we intend to give a gauge unification of the universal seesaw model, the SUSY $\mathrm{SO(10)}_L \times \mathrm{SO(10)}_R$ mode is the most attractive one, where the symmetries are directly broken into $[\mathrm{SU(2)} \times \mathrm{U(1)} \times \mathrm{SU(3)}]_L \times [\mathrm{SU(2)} \times \mathrm{U(1)} \times \mathrm{SU(3)}]_R$ and the number of the weak doublet Higgs scalars is $n_\phi \geq 3$, although the case still remains problems. It is our next task to investigate whether the Yukawa coupling constants show reasonable behaviors under these symmetries or not.

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Table 1: Surviving symmetries in each energy-scale range and definition of the parameters x_i for typical cases. G_{224} and G_{123} denote $\mathrm{G}_{224} = \mathrm{SU}(2)\times\mathrm{SU}(2)'\times\mathrm{SU}(4)$ and $\mathrm{G}_{123} = \mathrm{U}(1) \times \mathrm{SU}(2) \times \mathrm{SU}(3)$, respectively. For all cases, the symmetries in the ranges 1 and 2 are given by $\mathrm{SU}(2)_L \times \times \mathrm{U}(1)_Y \times \mathrm{SU}(3)_{LR}$ and $\mathrm{SU}(2)_L \times \mathrm{SU}(2)_R \times \mathrm{U}(1)_{LR} \times \mathrm{SU}(3)_{LR}$, respectively, and the parameters x_1 and x_2 are defined by $x_1 = \log(\Lambda_R/\Lambda_L)$ and $x_2 = \log(\Lambda_F/\Lambda_R)$. For all cases, we can read x_3 as $x_3 = \log(\Lambda_D/\Lambda_F)$

Case	Range 3	Range 4	Range 5	Range 6
LRLR	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{123})_R$ $x_3 = \log(\Lambda_{NR}/\Lambda_F)$	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{224})_R$ $x_4 = \log(\Lambda_{NL}/\Lambda_{NR})$	$(\mathrm{G}_{224})_L \times (\mathrm{G}_{224})_R$ $x_5 = \log(\Lambda_{XR}/\Lambda_{NL})$	$(\mathrm{G}_{224})_L \times SO(10)$ $x_6 = \log(\Lambda_{XL}/\Lambda_{X}$
LLRR	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{123})_R$ $x_3 = \log(\Lambda_{NR}/\Lambda_F)$	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{224})_R$ $x_4 = \log(\Lambda_{XR}/\Lambda_{NR})$	$(\mathrm{G}_{123})_L \times SO(10)_R$ $x_5 = \log(\Lambda_{NL}/\Lambda_{XR})$	$(\mathrm{G}_{224})_L \times SO(10)$ $x_6 = \log(\Lambda_{XL}/\Lambda_{N}$
LRLD	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{123})_R$ $x_3 = \log(\Lambda_D/\Lambda_F)$	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{224})_R$ $x_4 = \log(\Lambda_{NL}/\Lambda_D)$	$(\mathrm{G}_{224})_L \times (\mathrm{G}_{224})_R$ $x_5 = \log(\Lambda_{XR}/\Lambda_{NL})$	$(\mathrm{G}_{224})_L \times SO(10)$ $x_6 = \log(\Lambda_{XL}/\Lambda_{X}$
LRRD	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{123})_R$ $x_3 = \log(\Lambda_D/\Lambda_F)$	$(\mathrm{G}_{224})_L \times (\mathrm{G}_{123})_R$ $x_4 = \log(\Lambda_{NR}/\Lambda_D)$	$(\mathrm{G}_{224})_L \times (\mathrm{G}_{224})_R$ $x_5 = \log(\Lambda_{XR}/\Lambda_{NR})$	$(\mathrm{G}_{224})_L \times SO(10)$ $x_6 = \log(\Lambda_{XL}/\Lambda_{X}$
LRD	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{123})_R$ $x_3 = \log(\Lambda_D/\Lambda_F)$		$(\mathrm{G}_{224})_L \times (\mathrm{G}_{224})_R$ $x_5 = \log(\Lambda_{XR}/\Lambda_D)$	$(\mathrm{G}_{224})_L \times SO(10)$ $x_6 = \log(\Lambda_{XL}/\Lambda_{X}$
LRDN	$(\mathrm{G}_{123})_L \times (\mathrm{G}_{123})_R$ $x_3 = \log(\Lambda_D/\Lambda_F)$		$(\mathrm{G}_{224})_L \times (\mathrm{G}_{224})_R$ $x_5 = \log(\Lambda_{XR}/\Lambda_D)$	$(\mathrm{G}_{224})_L \times SO(10)$ $x_6 = \log(\Lambda_{XL}/\Lambda_{X}$

Table 2: Quantum numbers of the fermions f and F for $[\mathrm{SU}(2)\times\mathrm{SU}(2)'\times\mathrm{SU}(4)]_L$ $\times$ $[\mathrm{SU}(2)\times\mathrm{SU}(2)'\times\mathrm{SU}(4)]_R$.

	I_3^L	$I_3'^L$	$\sqrt{\frac{2}{3}}F_{15}^L$		I_3^R	$I_3'^R$	$\sqrt{\frac{2}{3}}F_{15}^R$
u_L	$+\frac{1}{2}$	0	$+\frac{1}{6}$	u_R	$+\frac{1}{2}$	0	$+\frac{1}{6}$
d_L	$-\frac{1}{2}$	0	$+\frac{1}{6}$	d_R	$-\frac{1}{2}$	0	$+\frac{1}{6}$
ν_L	$+\frac{1}{2}$	0	$-\frac{1}{2}$	ν_R	$+\frac{1}{2}$	0	$-\frac{1}{2}$
e_L	$-\frac{1}{2}$	0	$-\frac{1}{2}$	e_R	$-\frac{1}{2}$	0	$-\frac{1}{2}$
D_R^c	0	$+\frac{1}{2}$	$-\frac{1}{6}$	D_L^c	0	$+\frac{1}{2}$	$-\frac{1}{6}$
U_R^c	0	$-\frac{1}{2}$	$-\frac{1}{6}$	U_L^c	0	$-\frac{1}{2}$	$-\frac{1}{6}$
E_R^c	0	$+\frac{1}{2}$	$+\frac{1}{2}$	E_L^c	0	$+\frac{1}{2}$	$+\frac{1}{2}$
N_R^c	0	$-\frac{1}{2}$	$+\frac{1}{2}$	N_L^c	0	$-\frac{1}{2}$	$+\frac{1}{2}$

Table 3: Coefficients b_{iL} and b_{iR} of the gauge-coupling-constant renormalization group equations in the case RLRL. The coefficients b_i without the indices L or R, except for b_1 and b_3 in the regions 1 and 2, denote $b_i \equiv b_{iL} = b_{iR}$.

Range	Non-SUSY	SUSY	
Range 1 $\Lambda_L < \mu \leq \Lambda_R$	$b_3 = 7$ $b_2 = \frac{10}{3} - \frac{1}{6}h_2$ $b_1 = -\left(4 + \frac{1}{5}h_1\right)$	$b_3 = -3$ $b_2 = -\frac{1}{2}h_2$ $b_1 = -\left(6 + \frac{3}{5}h_1\right)$	$h_2 = n_\phi$ $h_1 = \frac{1}{2}n_\phi$
Range 2 $\Lambda_R < \mu \leq \Lambda_F$	$b_3 = \frac{19}{3}$ $b_2 = \frac{10}{3} - \frac{1}{6}h_2$ $b_1 = -\left(\frac{20}{3} + \frac{1}{2}h_1\right)$	$b_3 = 2$ $b_2 = -\frac{1}{2}h_2$ $b_1 = -\left(10 + \frac{3}{2}h_1\right)$	$h_2 = n_\phi$ $h_1 = n_\phi$
Range 3 $\Lambda_F < \mu \leq \Lambda_{NL}$	$b_3 = 7 - \frac{1}{6}h_3$ $b_2 = \frac{10}{3} - \frac{1}{6}h_2$ $b_1 = -\left(4 + \frac{1}{5}h_1\right)$	$b_3 = 3 - \frac{1}{2}h_3$ $b_2 = -\frac{1}{2}h_2$ $b_1 = -\left(6 + \frac{3}{5}h_1\right)$	$h_3 = 6$ $h_2 = n_\phi$ $h_1 = \frac{21}{4} + \frac{1}{2}n_\phi$
Range 4 $\Lambda_{NL} < \mu \leq \Lambda_{NR}$	$b_{4L} = \frac{32}{3} - \frac{1}{6}h_{4L}$ $b_{2L} = \frac{10}{3} - \frac{1}{6}h_{2L}$ $b'_{2L} = \frac{10}{3} - \frac{1}{6}h'_{2L}$ $b_{3R} = 7 - \frac{1}{6}h_{3R}$ $b_{2R} = \frac{10}{3} - \frac{1}{6}h_{2R}$ $b_{1R} = -\left(4 + \frac{1}{5}h_{1R}\right)$	$b_{4L} = 6 - \frac{1}{2}h_{4L}$ $b_{2L} = -\frac{1}{2}h_{2L}$ $b'_{2L} = -\frac{1}{2}h'_{2L}$ $b_{3R} = 3 - \frac{1}{2}h_{3R}$ $b_{2R} = -\frac{1}{2}h_{2R}$ $b_{1R} = -\left(6 + \frac{3}{5}h_{1R}\right)$	$h_{4L} = 36 + 16$ $h_{2L} = 40 + 2n_\phi$ $h'_{2L} = 40 + 32 + 2n_\phi$ $h_{3R} = 16$ $h_{2R} = n_\phi$ $h_{1R} = \frac{43}{3} + \frac{1}{2}n_\phi$
Range 5 $\Lambda_{NR} < \mu \leq \Lambda_{XL}$	$b_4 = \frac{32}{3} - \frac{1}{6}h_4$ $b_2 = \frac{10}{3} - \frac{1}{6}h_2$ $b'_2 = \frac{10}{3} - \frac{1}{6}h'_2$	$b_4 = 6 - \frac{1}{2}h_4$ $b_2 = -\frac{1}{2}h_2$ $b'_2 = -\frac{1}{2}h'_2$	$h_4 = 36 + 16n_F$ $h_2 = 40 + 2n_\phi$ $h'_2 = 40 + 32n_F + 2n_\phi$
Range 6 $\Lambda_{XL} < \mu \leq \Lambda_{XR}$	$b_{4R} = \frac{32}{3} - \frac{1}{6}h_{4R}$ $b_{2R} = \frac{10}{3} - \frac{1}{6}h_{2R}$ $b'_{2R} = \frac{10}{3} - \frac{1}{6}h'_{2R}$	$b_{4R} = 6 - \frac{1}{2}h_{4R}$ $b_{2R} = -\frac{1}{2}h_{2R}$ $b'_{2R} = -\frac{1}{2}h'_{2R}$	$h_{4R} = 36 + 32n_F$ $h_{2R} = 40 + 2n_\phi$ $h'_{2R} = 40 + 64n_F + 2n_\phi$

Table 4: Maximal values of $x_3 = \log(\Lambda_D/\Lambda_F)$ and $x_1 = \log(\Lambda_R/\Lambda_L)$ in the non-SUSY cases with $n_F = 1$ and $n_\phi = 6$.

Case	$(x_3)_{max}$	at x_1	$(x_1)_{max}$	at x_3
LRLR	+0.642	1.00	+5.20	0.00
LRRL	+0.642	1.00	+5.20	0.00
LLRR	+0.301	1.00	+2.11	0.00
LRLD	+0.647	1.00	+5.28	0.00
LRRD	+0.628	1.00	+4.99	0.00
LRD	−0.049	1.00	+0.44	0.00
LRDN	+0.642	1.00	+5.20	0.00

Table 5: Maximal values of $x_3 = \log(\Lambda_D/\Lambda_F)$ and $x_1 = \log(\Lambda_R/\Lambda_L)$ in the SUSY cases with $n_\phi = 3, 4, 6$.

n_ϕ	$(x_3)_{max}$	at x_1	$(x_1)_{max}$	at x_3	$x_1 + x_3$
3	+0.063	12.27	+12.38	0.00	$12.38 \geq x_1 + x_3 \geq 12.33$
4	+0.570	10.66	+11.66	0.00	$11.66 \geq x_1 + x_3 \geq 11.23$
6	+1.269	8.16	+10.41	0.00	$10.41 \geq x_1 + x_3 \geq 9.43$