University of Shizuoka

US-07-01

A_4 Symmetry and Lepton Masses and Mixing

Yoshio Koide

Department of Physics, University of Shizuoka, 52-1 Yada, Shizuoka 422-8526, Japan
E-mail address: koide@u-shizuoka-ken.ac.jp

Abstract

Stimulated by Ma's idea which explains the tribimaximal neutrino mixing by assuming an A_4 flavor symmetry, a lepton mass matrix model is investigated. A Frogatt-Nielsen type model is assumed, and the flavor structures of the masses and mixing are caused by the VEVs of $SU(2)_L$-singlet scalars ϕ_i^u and ϕ_i^d $(i = 1, 2, 3)$, which are assigned to $\mathbf{3}$ and $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ of A_4, respectively.

1 Introduction

It is generally considered that masses and mixings of the quarks and leptons will obey a simple law of nature. Then, it is also likely that the masses and mixings of those fundamental particles will be governed by a symmetry. However, even if there is such a simple relation in the quark sector, it is hard to see such a relation in the quark sector, because the original symmetry will be spoiled by the gluon cloud. Therefore, in the present paper, we will confine ourselves to the investigation of the lepton masses and mixings.

It is well known that the observed neutrino mixing is nearly described by the so called tribimaximal mixing [1]

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \tag{1.1}$$

In order to understand the tribimaximal mixing, Ma [2] has, recently, proposed a neutrino mass matrix model based on a non-Abelian discrete symmetry A_4. The symmetry A_4 seems to be very promising for a model of the leptons.

On the other hand, it is also well known that the observed charged lepton masses satisfy the relation [3, 4]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \tag{1.2}$$

with remarkable precision. The mass formula (1.2) is invariant under any exchange $\sqrt{m_i} \leftrightarrow \sqrt{m_j}$ $(i, j = e, \mu, \tau)$. This suggests that the lepton mass matrix model will be described by a permutation symmetry S_3 [5].

In order to understand the formula (1.2), a seesaw-type mass matrix model [4, 6, 7] has been proposed:

$$M_e = m_L^e M_E^{-1} m_R^e. \tag{1.3}$$

Here, M_E is a mass matrix of hypothetical heavy leptons E_i $(i = 1, 2, 3)$, and we have assumed $M_E \propto \mathbf{1} \equiv \mathrm{diag}(1, 1, 1)$. The matrices m_L^e and m_R^e are mass matrices defined by $\overline{e}_L m_L^e E_R$ and $\overline{E}_L m_R^e e_R$, respectively, and we assume $m_L^e = m_R^e/k = y_e \mathrm{diag}(v_1, v_2, v_3)$ (k is a constant), where v_i are vacuum expectation values (VEVs) of 3 Higgs scalars $\phi_{Li} = (\phi_{Li}^\dagger, \phi_{Li}^0)$, and they satisfy the relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2. \tag{1.4}$$

The relation (1.4) can be derived from the following Higgs potential [8, 9]

$$V = \mu^2(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3) + \frac{1}{2}\lambda_1(\phi_1^\dagger \phi_1 + \phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3)^2$$

$$+\lambda_2(\phi_\sigma^\dagger \phi_\sigma)(\phi_\pi^\dagger \phi_\pi + \phi_\eta^\dagger \phi_\eta) + V_{SB}, \tag{1.5}$$

where ϕ_i $(i = 1, 2, 3)$ are 3 objects of S_3 (fundamental basis), (ϕ_π, ϕ_η) and ϕ_σ are doublet and singlet of S_3, respectively, which are defined by

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{1.6}$$

and V_{SB} is a soft symmetry breaking term [9] which does not affect the derivation of the relation (1.4). The minimizing condition of the potential (1.5) leads to the VEV relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2. \tag{1.7}$$

The relation (1.7) gives the relation (1.4) because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2. \tag{1.8}$$

(Note that although the Higgs potential (1.5) is invariant under the S_3 symmetry, but it is not a general one of the S_3 invariant form. As pointed out in Ref. [9], a Higgs potential with the general form cannot lead to the relation (1.7). We need an additional requirement.) For a recent S_3 model of the lepton masses and mixings, see Ref. [10].

Considering the scenario for the charged lepton mass spectrum, the S_3 symmetry is also attractive, but, in the present paper, we will investigate an A_4 model by noticing Ma's model [2] for the tribimaximal neutrino mixing. In the next section, we will show that the S_3 scenario for the charged lepton masses can be translated into the language of A_4. In Sec.3, we will give a Frogatt-Nielsen [11] type model of the leptons based on an A_4 flavor symmetry. The mass matrix structures in the charged lepton and neutrino sectors are discussed in Secs.4 and 5, respectively. In Sec.6, a speculation about the neutrino masses will be given. In Sec.7, a SUSY version of the Higgs potential (1.5) [(2.16)] will be proposed. Finally, Sec.8 is devoted to the summary. In order to obtain the tribimaximal mixing (1.1) and the charged lepton mass relation (1.2), we will need further phenomenological assumptions, (i) $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry and (ii) the universality of the coupling constants, in addition to the A_4 symmetry.

2 From $\mathbf{S}_3$ into $\mathbf{A}_4$

When we define $\overline{\psi} = (\overline{\psi}_1, \overline{\psi}_2, \overline{\psi}_3)$ and $\psi = (\psi_1, \psi_2, \psi_3)$ as $\mathbf{3}$ of A_4, we can compose $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$ of A_4 as follows:

$$(\overline{\psi}\psi)_{\mathbf{1}} = \frac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2 + \overline{\psi}_3\psi_3), \tag{2.1}$$

$$(\overline{\psi}\psi)_{\mathbf{1}'} = \frac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2\omega + \overline{\psi}_3\psi_3\omega^2), \tag{2.2}$$

$$(\overline{\psi}\psi)_{\mathbf{1}''} = \frac{1}{\sqrt{3}}(\overline{\psi}_1\psi_1 + \overline{\psi}_2\psi_2\omega^2 + \overline{\psi}_3\psi_3\omega), \tag{2.3}$$

where

$$\omega = e^{i\frac{2}{3}\pi} = \frac{-1 + i\sqrt{3}}{2}. \tag{2.4}$$

The expressions (2.1)–(2.3) can be rewritten as

$$(\overline{\psi}\psi)_{\mathbf{1}} = (\overline{\psi}\psi)_{\sigma}, \tag{2.5}$$

$$(\overline{\psi}\psi)_{\mathbf{1}'} = \frac{1}{\sqrt{2}}[(\overline{\psi}\psi)_{\eta} - i(\overline{\psi}\psi)_{\pi}], \tag{2.6}$$

$$(\overline{\psi}\psi)_{\mathbf{1}''} = \frac{1}{\sqrt{2}}[(\overline{\psi}\psi)_{\eta} + i(\overline{\psi}\psi)_{\pi}], \tag{2.7}$$

where

$$\begin{pmatrix} (\overline{\psi}\psi)_{\sigma} \\ (\overline{\psi}\psi)_{\eta} \\ (\overline{\psi}\psi)_{\pi} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \begin{pmatrix} \overline{\psi}_1\psi_1 \\ \overline{\psi}_2\psi_2 \\ \overline{\psi}_3\psi_3 \end{pmatrix}. \tag{2.8}$$

It is useful to define the following $(\phi_\sigma, \phi_\eta, \phi_\pi)$ basis correspondingly to (2.5)–(2.7),

$$\phi_{\mathbf{1}} = \phi_{\sigma}, \tag{2.9}$$

$$\phi_{\mathbf{1}'} = \frac{1}{\sqrt{2}}(\phi_\eta - i\phi_\pi), \tag{2.10}$$

$$\phi_{\mathbf{1}''} = \frac{1}{\sqrt{2}}(\phi_\eta + i\phi_\pi), \tag{2.11}$$

where the scalars $\phi_{\mathbf{1}}$, $\phi_{\mathbf{1}'}$ and $\phi_{\mathbf{1}''}$ are $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$ of A$_4$. Then, A$_4$-invariant Yukawa interactions which are composed of $\overline{\psi}, \psi$ and ϕ are expressed as follows:

$$(\overline{\psi}\psi)_{\mathbf{1}}\phi_{\mathbf{1}} = (\overline{\psi}\psi)_\sigma\phi_\sigma, \tag{2.12}$$

$$(\overline{\psi}\psi)_{\mathbf{1}'}\phi_{\mathbf{1}''} = \frac{1}{2}[(\overline{\psi}\psi)_\eta\phi_\eta + (\overline{\psi}\psi)_\pi\phi_\pi + i(\overline{\psi}\psi)_\eta\phi_\pi - i(\overline{\psi}\psi)_\pi\phi_\eta], \tag{2.13}$$

$$(\overline{\psi}\psi)_{\mathbf{1}''}\phi_{\mathbf{1}'} = \frac{1}{2}[(\overline{\psi}\psi)_\eta\phi_\eta + (\overline{\psi}\psi)_\pi\phi_\pi - i(\overline{\psi}\psi)_\eta\phi_\pi + i(\overline{\psi}\psi)_\pi\phi_\eta]. \tag{2.14}$$

Hereafter, we will always assume a $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry, so that we obtain

$$(\overline{\psi}\psi)_{\mathbf{1}'}\phi_{\mathbf{1}''} + (\overline{\psi}\psi)_{\mathbf{1}''}\phi_{\mathbf{1}'} = (\overline{\psi}\psi)_\eta\phi_\eta + (\overline{\psi}\psi)_\pi\phi_\pi. \tag{2.15}$$

In the S$_3$-invariant Higgs potential (1.5), the existence of the λ_2-term was essential for the derivation of the VEV relation (1.7). In the present A$_4$ model, if we adopt the basis $\phi = (\phi_\sigma, \phi_\eta, \phi_\pi)$ which are defined by Eqs.(2.9)–(2.11), we can regard the Higgs potential (1.5) as an A$_4$-invariant one:

$$\begin{aligned} V &= \mu^2(\phi^\dagger_{\mathbf{1}}\phi_{\mathbf{1}} + \phi^\dagger_{\mathbf{1}'}\phi_{\mathbf{1}''} + \phi^\dagger_{\mathbf{1}''}\phi_{\mathbf{1}'}) + \frac{1}{2}\lambda_1(\phi^\dagger_{\mathbf{1}}\phi_{\mathbf{1}} + \phi^\dagger_{\mathbf{1}'}\phi_{\mathbf{1}''} + \phi^\dagger_{\mathbf{1}''}\phi_{\mathbf{1}'})^2 \\ &\quad +\lambda_2(\phi^\dagger_{\mathbf{1}}\phi_{\mathbf{1}})(\phi^\dagger_{\mathbf{1}'}\phi_{\mathbf{1}''} + \phi^\dagger_{\mathbf{1}''}\phi_{\mathbf{1}'}) + V_{SB} \\ &= \mu^2(\phi^\dagger_\sigma\phi_\sigma + \phi^\dagger_\eta\phi_\eta + \phi^\dagger_\pi\phi_\pi) + \frac{1}{2}\lambda_1(\phi^\dagger_\sigma\phi_\sigma + \phi^\dagger_\eta\phi_\eta + \phi^\dagger_\pi\phi_\pi)^2 \\ &\quad +\lambda_2(\phi^\dagger_\sigma\phi_\sigma)(\phi^\dagger_\pi\phi_\pi + \phi^\dagger_\eta\phi_\eta) + V_{SB}. \end{aligned} \tag{2.16}$$

When we define (ϕ_1, ϕ_2, ϕ_3) by Eq.(1.6), we can obtain the VEV relation (1.4) for the VEVs $v_i = \langle\phi_i\rangle$, so that we obtain the charged lepton mass relation (1.2) from the A$_4$-invariant Yukawa interaction

$$\begin{aligned} (\overline{e}E)_{\mathbf{1}}\phi_{\mathbf{1}} + (\overline{e}E)_{\mathbf{1}'}\phi_{\mathbf{1}''} + (\overline{e}E)_{\mathbf{1}''}\phi_{\mathbf{1}'} &= (\overline{e}E)_\sigma\phi_\sigma + (\overline{e}E)_\eta\phi_\eta + (\overline{e}E)_\pi\phi_\pi \\ &= \overline{e}_1E_1\phi_1 + \overline{e}_2E_2\phi_2 + \overline{e}_3E_3\phi_3, \end{aligned} \tag{2.17}$$

where e_L and E_R have been assigned to $\mathbf{3}$ of A$_4$. (However, in the next section, we will not adopt the seesaw model (1.3), but do a Frogatt-Nielsen type model without the heavy leptons E_i.)

Note that, in the S$_3$ model, (ϕ_1, ϕ_2, ϕ_3) were 3 objects of S$_3$ and $(\phi_\sigma, \phi_\eta, \phi_\pi)$ were (singlet, doublet) of S$_3$, while, in the present A$_4$ model, $(\phi_\sigma, \phi_\eta, \phi_\pi)$ and (ϕ_1, ϕ_2, ϕ_3) are merely linear combinations of $(\phi_{\mathbf{1}}, \phi_{\mathbf{1}'}, \phi_{\mathbf{1}''})$, and they are not irreducible representations of A$_4$.

Thus, we have a possibility that we can build a model which leads not only to the tribimaximal mixing for the neutrinos, but also to the mass relation (1.2) for the charged leptons by developing Ma's idea.

3 Model

So far, we have considered that 3 scalars ϕ_i are SU(2) doublets. However, such a model with multi-Higgs doublets causes a flavor changing neutral current (FCNC) problem. Therefore, in the present paper, we assume a Frogatt-Nielsen [11] type model

$$H_{eff} = y_e \bar{l}_L H_L^d \frac{\phi^d}{\Lambda_d}\frac{\phi^d}{\Lambda_d} e_R + y_\nu \bar{l}_L H_L^u \frac{\phi^u}{\Lambda_u}\nu_R + y_R \overline{\nu}_R \Phi \nu_R^*, \tag{3.1}$$

where ℓ_{iL} are $\mathrm{SU(2)}_L$-doublet leptons $\ell_{iL} = (\nu_{iL}, e_{iL})$, H_L^d and H_L^u are conventional $\mathrm{SU(2)}_L$ doublet Higgs scalars, ϕ^d and ϕ^u are $\mathrm{SU(2)}_L$ singlet scalars, and Λ_d and Λ_u are scales of the effective theory. We consider that $\langle\phi^f\rangle/\Lambda_f$ $(f = u, d)$ are of the order of 1. Here, we have not adopted the seesaw type model (1.3) for the charged lepton sector, because the existence of $M_E \propto \mathbf{1}$ in Eq.(1.3) did not play any essential role in the flavor structure of the charged lepton mass matrix M_e. The scalar Φ has been introduced in order to generate the Majorana mass M_R of the right-handed neutrinos ν_R. The model is essentially unchanged compared with the seesaw model as far as the flavor structures are concerned. However, the scenario for the energy scale of the symmetry breaking is considerably changed, i.e. we consider that the VEVs of ϕ_i^f are of the order of the Planck mass scale although we have considered $\langle\phi_i\rangle \sim 10^2$ GeV in the seesaw model [12]. In other words, in the Frogatt-Nielsen-type model, the A_4-broken structure of the effective Yukawa coupling constants is formed at the Planck mass scale. However, this is not serious problem, because the formula (1.1) is not so sensitive to the renormalization group equation (RGE) effects as far as the lepton sector is concerned [13]. Although the relation (1.2) is in remarkable agreement with the observed charged lepton masses (the pole masses), the standpoint in the present paper is that the remarkable coincidence is accidental and the relation (1.2) will be satisfied only approximately at a low energy scale.

Ma has assigned the scalars ϕ^d to $\mathbf{3}$ of A_4 in Ref.[2]. However, as we have shown in Sec.2, since the scalars which can give the VEV relation (1.7) [or (1.4)] are not $\mathbf{3}$ of A_4, but $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ of A_4, we regard ϕ^d as $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ of A_4 in the present paper. Also, we will change the assignment of e_R and ν_R from those in the Ma model. Of course, the essential idea to obtain the tribimaximal mixing is indebted to the Ma model. The A_4 assignments in the present model are listed in Table 1. In order to forbid unwelcome combinations $\bar{l}_L H_L^d (\phi^d)^n (\phi^u)^m e_R$ except for $(n, m) = (2, 0)$ and $\bar{l}_L H_L^u (\phi^d)^n (\phi^u)^m \nu_R$ except for $(n, m) = (0, 1)$, for example, we assume the following U(1) charge assignments: $Q(l_L) = Q(\nu_R) = Q(e_R) = Q(\Phi) = 0$, $Q(\phi^d) = -\frac{1}{2}Q(H_l^d) = q_d > 0$, and $Q(\phi^u) = -Q(H_L^u) = q_u > 0$, where $q_d/q_u \neq n/2$ and $q_u/q_d \neq n$ $(n = 0, 1, 2, 3, \cdots)$.

4 Charged lepton sector

In the charged lepton sector, the possible A_4-invariant interactions $(\bar{e}_L e_R)\phi^d\phi^d$, i.e., $(\mathbf{3} \times \mathbf{3}) \times (\mathbf{1}, \mathbf{1}', \mathbf{1}'') \times (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$, are given by

$$H_e = (\bar{e}e)_{\mathbf{1}}[y_0 \phi_{\mathbf{1}}\phi_{\mathbf{1}} + y_1(\phi'_{\mathbf{1}}\phi''_{\mathbf{1}} + \phi''_{\mathbf{1}}\phi'_{\mathbf{1}})] + y_2[(\bar{e}e)'_{\mathbf{1}}\phi'_{\mathbf{1}}\phi'_{\mathbf{1}} + (\bar{e}e)''_{\mathbf{1}}\phi''_{\mathbf{1}}\phi''_{\mathbf{1}}]$$

$$+ y_3[(\bar{e}e)'_{\mathbf{1}}(\phi''_{\mathbf{1}}\phi_{\mathbf{1}} + \phi_{\mathbf{1}}\phi''_{\mathbf{1}}) + (\bar{e}e)''_{\mathbf{1}}(\phi'_{\mathbf{1}}\phi_{\mathbf{1}} + \phi_{\mathbf{1}}\phi'_{\mathbf{1}})]$$

$$= (\bar{e}e)_\sigma[y_0\phi_\sigma^2 + y_1(\phi_\pi^2 + \phi_\eta^2)] + \frac{1}{\sqrt{2}}y_2[(\bar{e}e)_\eta(\phi_\eta^2 - \phi_\pi^2) - 2(\bar{e}e)_\pi\phi_\eta\phi_\pi]$$

$$+2y_3[(\overline{e}e)_\eta\phi_\eta\phi_\sigma + (\overline{e}e)_\pi\phi_\pi\phi_\sigma], \tag{4.1}$$

where, for convenience, we have dropped the index d, and we have assumed the $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry. Furthermore, if we assume the universality of the coupling constants,

$$y_0 = y_1 = y_2 = y_3, \tag{4.2}$$

we obtain

$$\begin{aligned} H_e &= y_0\left[(\overline{e}e)_\sigma(\phi_\sigma^2+\phi_\pi^2+\phi_\eta^2) + \frac{1}{\sqrt{2}}(\overline{e}e)_\eta(\phi_\eta^2-\phi_\pi^2+2\sqrt{2}\phi_\eta\phi_\sigma) + (\overline{e}e)_\pi(-2\phi_\eta\phi_\pi+2\sqrt{2}\phi_\pi\phi_\sigma)\right] \\ &= y_0\left[(\overline{e}e)_\sigma(\phi_1^2+\phi_2^2+\phi_3^2) + \frac{1}{\sqrt{2}}(\overline{e}e)_\eta(2\phi_1^2-\phi_2^2-\phi_3^2) + \sqrt{\frac{3}{2}}(\overline{e}e)_\pi(\phi_3^2-\phi_2^2)\right] \\ &= \sqrt{3}y_0(\overline{e}_1e_1\phi_1^2+\overline{e}_2e_2\phi_2^2+\overline{e}_3e_3\phi_3^2), \end{aligned} \tag{4.3}$$

where ϕ_i $(i=1,2,3)$ are defined by Eq.(1.6). As we discussed in Sec.2, since we can write the Higgs potential (1.5) for $\phi^d = (\phi_\pi^d, \phi_\eta^d, \phi_\sigma^d)$, we can obtain the VEV relation (1.7) [i.e. (1.4) for $v_i = \langle\phi_i^d\rangle$]. Therefore, from Eqs.(4.3) and (1.4), we can obtain the charged lepton mass relation (1.2).

5 Neutrino sector

Since $\overline{\nu}_L\phi^u\nu_R \sim \mathbf{3}\times\mathbf{3}\times(\mathbf{1},\mathbf{1}',\mathbf{1}'')$, the A$_4$-invariant Yukawa interactions are as follows:

$$H_\nu = y_0^\nu(\overline{\nu}_L\phi^u)_\sigma\nu_{R\sigma} + y_1^\nu\left[(\overline{\nu}_L\phi^u)_\eta\nu_{R\eta} + (\overline{\nu}_L\phi^u)_\pi\nu_{R\pi}\right], \tag{5.1}$$

so that we obtain the mass matrix m_L^ν which is defined by

$$(\overline{\nu}_1 \ \ \overline{\nu}_2 \ \ \overline{\nu}_3)_L m_L^\nu \begin{pmatrix} \nu_\sigma \\ \nu_\eta \\ \nu_\pi \end{pmatrix}_R , \tag{5.2}$$

as follows:

$$m_L^\nu = \begin{pmatrix} \frac{1}{\sqrt{3}}y_0^\nu v_i^u & y_1^\nu\frac{2}{\sqrt{6}}v_1^u & 0 \\ \frac{1}{\sqrt{3}}y_0^\nu v_2^u & -y_1^\nu\frac{1}{\sqrt{6}}v_2^u & -y_1^\nu\frac{1}{\sqrt{2}}v_2^u \\ \frac{1}{\sqrt{3}}y_0^\nu v_3^u & -y_1^\nu\frac{1}{\sqrt{6}}v_3^u & y_1^\nu\frac{1}{\sqrt{2}}v_3^u \end{pmatrix}. \tag{5.3}$$

When we again assume the universality of the coupling constants, $y_0^\nu = y_1^\nu$, we obtain

$$m_L^\nu = DU_{TB}, \tag{5.4}$$

$$D = y_0^\nu \mathrm{diag}(v_1^u, v_2^u, v_3^u), \tag{5.5}$$

and U_{TB} is the tribimaximal mixing matrix (1.1), where we have changed the basis of ν_R from $(\nu_\sigma, \nu_\eta, \nu_\pi)_R$ to $(\nu_\eta, \nu_\sigma, \nu_\pi)_R$.

Although the VEVs of the scalars $\phi^d = (\phi^d_\pi, \phi^d_\eta, \phi^d_\sigma)$ satisfy the VEV relation (1.7), the VEVs $v^u_i = \langle \phi^u_i \rangle$ do not have such a relation, because we cannot write an A$_4$-invariant term which corresponds to the λ_2-term, i.e. $\phi^2_\sigma(\phi^2_\pi + \phi^2_\eta)$. The potential for the scalars $\phi^u = (\phi^u_1, \phi^u_2, \phi^u_3)$ is symmetric for any exchange $\phi^u_i \leftrightarrow \phi^u_j$. Therefore, we consider

$$\langle \phi^u_1 \rangle = \langle \phi^u_2 \rangle = \langle \phi^u_3 \rangle \equiv v_u. \tag{5.6}$$

Then, the mass matrix m^ν_L is diagonalized as

$$U^T_{TB} m^\nu_L = y^\nu_0 v_u \mathbf{1}. \tag{5.7}$$

If the Majorana mass matrix M_R is diagonal on the basis $(\nu_\eta, \nu_\sigma, \nu_\pi)_R$, i.e.

$$M_R = \mathrm{diag}(M_\eta, M_\sigma, M_\pi), \tag{5.8}$$

we obtain the mixing matrix U_{MNS} and the eigenvalues $m_{\nu i}$ of the neutrino mass matrix $M_\nu = m^\nu_L M^{-1}_R (m^\nu_L)^T$,

$$U_{MNS} = U_{TB}, \tag{5.9}$$

$$m_{\nu 1} = (y^\nu_0 v_u)^2 \frac{1}{M_\eta}, \quad m_{\nu 2} = (y^\nu_0 v_u)^2 \frac{1}{M_\sigma}, \quad m_{\nu 3} = (y^\nu_0 v_u)^2 \frac{1}{M_\pi}. \tag{5.10}$$

The explicit structure of $M_R = \mathrm{diag}(M_\eta, M_\sigma, M_\pi)$ will be discussed in the next section.

6 Speculation on the neutrino mass spectrum

In order to speculate on the neutrino mass spectrum, let us assume that the Majorana masses are generated by the following interaction with the scalars $\Phi = (\Phi_{\mathbf{1}}, \Phi_{\mathbf{1}'}, \Phi_{\mathbf{1}''})$,

$$H_R = \left[y^R_0 \overline{\nu}_{\mathbf{1}} \nu^*_{\mathbf{1}} + y^R_1 (\overline{\nu}_{\mathbf{1}'} \nu^*_{\mathbf{1}''} + \overline{\nu}_{\mathbf{1}''} \nu^*_{\mathbf{1}'}) \right] \Phi_{\mathbf{1}} + y^R_2 \left(\overline{\nu}_{\mathbf{1}'} \nu^*_{\mathbf{1}'} \Phi_{\mathbf{1}'} + \overline{\nu}_{\mathbf{1}''} \nu^*_{\mathbf{1}''} \Phi_{\mathbf{1}''} \right)$$

$$= \left[y^R_0 \overline{\nu}_\sigma \nu^*_\sigma + y^R_1 (\overline{\nu}_\pi \nu^*_\pi + \overline{\nu}_\eta \nu^*_\eta) \right] \Phi_\sigma + \frac{1}{\sqrt{2}} y^R_2 \left[(\overline{\nu}_\eta \nu^*_\eta - \overline{\nu}_\pi \nu^*_\pi) \Phi_\eta - (\overline{\nu}_\pi \nu^*_\eta + \overline{\nu}_\eta \nu^*_\pi) \Phi_\pi \right]. \tag{6.1}$$

We assume that the VEVs of $\Phi \sim (\mathbf{1}, \mathbf{1}', \mathbf{1}'')$ satisfy the same relation as (1.7) for ϕ^d

$$\langle \Phi_\pi \rangle^2 + \langle \Phi_\eta \rangle^2 = \langle \Phi_\sigma \rangle^2, \tag{6.2}$$

because ϕ^d and Φ are assigned to the same multiplets $(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$. However, if we consider $\langle \Phi_\pi \rangle \neq 0$, the matrix M_R cannot become diagonal. Therefore, we assume $\langle \Phi_\pi \rangle = 0$, so that we will take

$$\langle \Phi_\pi \rangle = 0, \quad \langle \Phi_\eta \rangle = \langle \Phi_\sigma \rangle. \tag{6.3}$$

This assumption corresponds to that, although we have already assumed the $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry, we have assumed this symmetry for the VEV values of Φ, not for the fields, i.e. $\langle \Phi_{\mathbf{1}'} \rangle = \langle \Phi_{\mathbf{1}''} \rangle = \langle \Phi_\eta \rangle / \sqrt{2}$. Then, we obtain the eigenvalues of M_R as follows:

$$M_\eta = \left(y^R_1 + \frac{1}{\sqrt{2}} y^R_2 \right) \langle \Phi_\sigma \rangle, \quad M_\sigma = y^R_0 \langle \Phi_\sigma \rangle, \quad M_\pi = \left(y^R_1 - \frac{1}{\sqrt{2}} y^R_2 \right) \langle \Phi_\sigma \rangle. \tag{6.4}$$

In order to speculate the neutrino masses $m_{\nu i}$, we must more reduce the number of the parameters. Therefore, let us assume that the fermion terms which couple to the scalars Φ_σ, Φ_η and Φ_π are normalized as

$$H_R = y_R \left[\left(\sin\alpha\, \overline{\nu}_\sigma \nu_\sigma^* + \cos\alpha\, \frac{\overline{\nu}_\pi \nu_\pi^* + \overline{\nu}_\eta \nu_\eta}{\sqrt{2}} \right) \Phi_\sigma + \frac{\overline{\nu}_\eta \nu_\eta^* - \overline{\nu}_\pi \nu_\pi^*}{\sqrt{2}} \Phi_\eta - \frac{\overline{\nu}_\pi \nu_\eta^* + \overline{\nu}_\eta \nu_\pi^*}{\sqrt{2}} \Phi_\pi \right]. \quad (6.5)$$

Then, we can write the eigenvalues (6.4) as follows:

$$M_\eta = \frac{y_R}{\sqrt{2}} \langle \Phi_\sigma \rangle (1 + \cos\alpha), \quad M_\sigma = y_R \langle \Phi_\sigma \rangle, \quad |M_\pi| = \frac{y_R}{\sqrt{2}} \langle \Phi_\sigma \rangle (1 - \cos\alpha), \quad (6.6)$$

so that we obtain the neutrino mass spectrum

$$m_{\nu 1} = \frac{1}{1+\cos\alpha} m_{\nu 0}, \quad m_{\nu 2} = \frac{1}{\sqrt{2}\sin\alpha} m_{\nu 0}, \quad m_{\nu 3} = \frac{1}{1-\cos\alpha} m_{\nu 0}, \quad (6.7)$$

where $m_{\nu 0} = (y_0^\nu v_u)^2 / (y_R \langle \Phi_\sigma \rangle / \sqrt{2})^2$. For the observed ratio [14, 15]

$$R_{obs} \equiv \frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} = \frac{(7.9^{+0.6}_{-0.5}) \times 10^{-5} \mathrm{eV}^2}{(2.74^{+0.44}_{-0.26}) \times 10^{-3} \mathrm{eV}^2} = (2.9 \pm 0.5) \times 10^{-2}, \quad (6.8)$$

we obtain the predicted ratio

$$R = \frac{\Delta m^2_{21}}{\Delta m^3_{32}} = \frac{m^2_{\nu 2} - m^2_{\nu 1}}{m^2_{\nu 3} - m^2_{\nu 2}} = \frac{(3\cos\alpha - 1)(1 - \cos\alpha)}{(3\cos\alpha + 1)(1+\cos\alpha)}. \quad (6.9)$$

For example, for $\alpha = \pi/6$, we obtain

$$R = \frac{(3\sqrt{2} - 2)(2 - \sqrt{3})}{(3\sqrt{2}+2)(2+\sqrt{3})} = 0.0319. \quad (6.10)$$

The value is in good agreement with the observed value (6.8). By putting $m_{\nu 3} = \sqrt{\Delta m^2_{atm}}$, we obtain $m_{\nu 1} = (0.38 \pm 0.02) \times 10^{-2}$ eV, $m_{\nu 2} = (0.99^{+0.08}_{-0.05}) \times 10^{-2}$ eV, and $m_{\nu 3} = (5.23^{+0.40}_{-0.25}) \times 10^{-2}$ eV.

However, the theoretical reason for $\alpha = \pi/6$ is unclear. Since we have assumed the universality of the coupling constants in the interactions (4.1) and (5.2), rather, the case with the universal coupling $y_0^R = y_1^R$,

$$H_R = y_R \left(\frac{\overline{\nu}_\sigma \nu_\sigma^* + \overline{\nu}_\pi \nu_\pi^* + \overline{\nu}_\eta \nu_\eta^*}{\sqrt{3}} \Phi_\sigma + \frac{\overline{\nu}_\eta \nu_\eta^* - \overline{\nu}_\pi \nu_\pi^*}{\sqrt{2}} \Phi_\eta - \frac{\overline{\nu}_\pi \nu_\eta^* + \overline{\nu}_\eta \nu_\pi^*}{\sqrt{2}} \Phi_\pi \right), \quad (6.11)$$

is likely. The case corresponds to $\cos\alpha = \sqrt{2/3}$, and it predicts

$$R = \frac{4\sqrt{6}-9}{4\sqrt{6}+9} = 0.0424. \tag{6.12}$$

The value (6.12) is somewhat large compared with the observed value (6.8), but, at present, the case cannot be ruled out within three sigma. Again, by putting $m_{\nu 3} = \sqrt{\Delta m^2_{atm}}$, we obtain

$$m_{\nu 1} = (0.53^{+0.04}_{-0.03}) \times 10^{-2}\mathrm{eV}, \quad m_{\nu 2} = (1.17^{+0.08}_{-0.05}) \times 10^{-2}\mathrm{eV}, \quad m_{\nu 3} = (5.23^{+0.40}_{-0.25}) \times 10^{-2}\mathrm{eV}. \tag{6.13}$$

Anyhow, the predicted value of $m_{\nu 1}$ in the present A_4 model is relatively large compared with that in the S_3 model [10]. We would like to expect the detection from future double beta experiments.

7 Superpotential for 3 flavor scalars

So far, we have not considered the supersymmetric version of the present model. Recently, Ma has proposed a SUSY version [16] of the Higgs potential (1.5) which can lead to the VEV relation $v_\pi^2 + v_\eta^2 = v_\sigma^2$, (1.7). In a similar way, we can write the superpotential W for the superfields $\phi^d = (\phi^d_{\mathbf{1}}, \phi^d_{\mathbf{1}'}, \phi^d_{\mathbf{1}''})$ (hereafter, for convenience, we will drop the index d) by assuming as follows:

(i) The field ϕ_a ($a = \mathbf{1}, \mathbf{1}', \mathbf{1}''$) to the power nth, $(\phi_a)^n$ ($n = 1, 2, 3$), appears always accompanied with the factor $1/n!$ in the superpotential W.

(ii) In order to forbid unwelcome A_4 invariant terms, we require that W is invariant under the transformation

$$\phi_{\mathbf{1}'} \to -\phi_{\mathbf{1}'}, \quad \phi_{\mathbf{1}''} \to -\phi_{\mathbf{1}''}. \tag{7.1}$$

Under this requirement, the terms $(\phi_{\mathbf{1}'})^3$ and $(\phi_{\mathbf{1}''})^3$ are forbidden, but $\phi_{\mathbf{1}'}\phi_{\mathbf{1}''}$, $(\phi_{\mathbf{1}'})^2$ and $(\phi_{\mathbf{1}''})^2$ are not forbidden.

(iii) The A_4 symmetry is softly broken by a term W_{SB}.

As a result, we obtain the superpotential

$$W = m\left(\phi_{\mathbf{1}'}\phi_{\mathbf{1}''} + \frac{1}{2!}\phi_{\mathbf{1}}^2\right) + \lambda\left(\phi_{\mathbf{1}}\phi_{\mathbf{1}'}\phi_{\mathbf{1}''} + \frac{1}{3!}\phi_{\mathbf{1}}^3\right) + W_{SB}, \tag{7.2}$$

$$W_{SB} = \varepsilon m\left(-\phi_{\mathbf{1}'}\phi_{\mathbf{1}''} + \frac{1}{2!}e^{i\theta}\phi_{\mathbf{1}'}^2 + \frac{1}{2!}e^{-i\theta}\phi_{\mathbf{1}''}^2\right). \tag{7.3}$$

Here, although the first and second terms in W_{SB} do not break the A_4 symmetry, we have added those to W_{SB} in order to keep the result (1.7) independent of W_{SB}. We will show below that the superpotential (7.2) can lead to the VEV relation (1.7) independently of W_{SB} and the parameter θ in W_{SB} determines the ratio v_π/v_η, so that the charged lepton mass spectrum is completely determined only by the parameter θ.

The superpotential (7.2) can be rewritten in terms of the superfields $(\phi_\pi, \phi_\eta, \phi_\sigma)$ defined by Eqs.(2.9)–(2.11) as follows:

$$W = m(1-\varepsilon)(\phi_\eta^2 + \phi_\pi^2) + \frac{1}{2}m\phi_\sigma^2 + \varepsilon m e^{i\theta}(\phi_\eta^2 - \phi_\pi^2 - 2i\phi_\eta\phi_\pi)$$

$$+\varepsilon m e^{-i\theta}(\phi_\eta^2 - \phi_\pi^2 + 2i\phi_\eta\phi_\pi) + \frac{1}{2}\lambda\phi_\sigma\left(\phi_\eta^2 + \phi_\pi^2 + \frac{1}{3}\phi_\sigma^2\right). \tag{7.4}$$

Since

$$\frac{\partial W}{\partial \phi_\pi} = \left[m(1-\varepsilon) + \lambda\phi_\sigma - 2\varepsilon m\cos\theta\right]\phi_\pi + 2\varepsilon m \sin\theta\phi_\eta, \tag{7.5}$$

$$\frac{\partial W}{\partial \phi_\eta} = \left[m(1-\varepsilon) + \lambda\phi_\sigma + 2\varepsilon m\cos\theta\right]\phi_\eta + 2\varepsilon m \sin\theta\phi_\pi, \tag{7.6}$$

$$\frac{\partial W}{\partial \phi_\sigma} = m\phi_\sigma + \frac{1}{2}\lambda(\phi_\eta^2 + \phi_\pi^2 + \phi_\sigma^2), \tag{7.7}$$

the minimization conditions of the potential leads to the relations

$$\tan\theta = \frac{2v_\eta v_\pi}{v_\eta^2 - v_\pi^2}, \tag{7.8}$$

$$v_\pi^2 + v_\eta^2 = v_\sigma^2, \tag{7.9}$$

$$m + \lambda v_\sigma = 0. \tag{7.10}$$

Note that the derivation of the relation (7.8) is independent of the explicit values of m, λ and ε, and the derivation of the relation (7.9) is independent of the explicit values of m, λ, ε and θ. [Also note that the conditions (7.5)–(7.7) can lead to an alternative solution with $(1-2\varepsilon)(v_\pi^2 + v_\eta^2) = (1+2\varepsilon)v_\sigma^2$ and $(1-2\varepsilon)m + \lambda\phi_\sigma = 0$ instead of Eqs.(7.9) and (7.10), respectively. However, we have taken only the solution which is independent of the parameter ε.]

From the observed values [17] of the charged lepton masses, we obtain the numerical values $z_1 = 0.016473$, $z_2 = 0.236869$ and $z_3 = 0.971402$, where the parameters z_i are defined by $\sqrt{m_{ei}} = z_i v_d$ with $z_1^2 + z_2^2 + z_3^2 = 1$, so that, for the VEVs of ϕ_a defined by Eq.(1.6), we obtain $z_\pi = 0.519393$, $z_\eta = -0.479824$ and $z_\sigma = 0.707106$. Therefore, we can obtain the value of θ as follows:

$$\tan\frac{\theta}{2} = \frac{v_\pi}{v_\eta} = \sqrt{3}\frac{z_3 - z_2}{2z_1 - z_2 - z_3} = -1.082466, \tag{7.11}$$

i.e. $\theta = -94.5354^\circ$. When we express the parameter z_i as

$$\begin{aligned} z_1 &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}}\sin\xi_e, \\ z_2 &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}}\sin(\xi_e + \tfrac{2}{3}\pi), \\ z_3 &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}}\sin(\xi_e + \tfrac{4}{3}\pi), \end{aligned} \tag{7.12}$$

the angle θ is related to the parameter ξ_e by

$$\frac{\theta}{2} = \xi_e - \frac{\pi}{2}. \tag{7.13}$$

Note that the model gives $m_e \to 0$ in the limit of $\theta \to -\pi/2$.

8 Summary

In conclusion, on the basis of the A_4 symmetry, we have investigated a Frogatt-Nielsen type model (3.1). The Higgs potential (2.16) for the scalars ϕ_i^d which are assigned to $(\phi_{\mathbf{1}}, \phi_{\mathbf{1}'}, \phi_{\mathbf{1}''})$ of A_4 can lead to the VEV relation (1.7), $v_\pi^2 + v_\eta^2 = v_\sigma^2$, i.e. to the relation (1.4) for the VEVs $\langle \phi_i^d \rangle$ defined in (1.6). Since the charged lepton interactions $\overline{\ell}_L H_L^d \phi^d \phi^d e_R$ give $m_{ei} \propto \langle \phi_i^d \rangle^2$, we have obtained the charged lepton mass relation (1.2). For the neutrino sector, we have obtained the tribimaximal mixing (1.1) by assuming $\mathbf{3}$ of A_4 for the scalars ϕ^u.

However, it should be noted that, in order to obtain the above results, we have needed to assume the following requirements in addition to the A_4 symmetry: (i) the $\mathbf{1}' \leftrightarrow \mathbf{1}''$ symmetry and (ii) the universality of the coupling constants. Those assumptions are phenomenological ones at present. On the other hand, recently, Ma [16] has also proposed a model which can lead not only to the tribimaximal mixing (1.1), but also to the charged lepton mass relation (1.2) by assuming a symmetry $\Sigma(81)$. In Ma's $\Sigma(81)$ model, such an additional assumption except for the symmetry $\Sigma(81)$ has not been required. However, in his model, we need somewhat unfamiliar and complicated symmetry $\Sigma(81)$. In contrast with the Ma model, in the present model, we have adopted a familiar symmetry A_4, and, instead, we have put some intuitive assumptions (i) and (ii). Such an approach with phenomenological assumptions, at present, seems to be still useful for future extension of the model rather than a rigid theoretical model.

In Secs.5 and 6, a possible neutrino mass spectrum has been discussed by assuming $\langle \phi_1^u \rangle = \langle \phi_2^u \rangle = \langle \phi_3^u \rangle$, where ϕ_i^u belong to $\mathbf{3}$ of A_4. By assuming the structure (6.3) of the right-handed neutrino Yukawa interaction, we can speculate the neutrino mass spectrum (6.7). The case $\alpha = \pi/6$ is interesting from the phenomenological point of view, because the case predicts the ratio $\Delta m^2_{solar}/\Delta m^2_{atm} = 0.0319$. However, the numerical predictions in Sec.6 are not conclusive because the model has needed some speculative assumptions.

In Sec.7, a SUSY version for the Higgs potential of ϕ^d has been proposed. The essential idea is indebted to the Ma model based on a $\Sigma(81)$ symmetry [16].

The present model will give a suggestive hint on seeking for a more plausible model which leads to the tribimaximal mixing (1.1) and the charged lepton mass relation (1.2).

Acknowledgements

The author would like to thank T. Singai for helpful conversations on the A_4 symmetry. He also thank E. Ma for informing his recent paper [16] prior to putting it on the hep-ph arXive. The author is much indebted to his recent paper [16], especially, for Sec.7. This work is supported by the Grant-in-Aid for Scientific Research, Ministry of Education, Science and Culture, Japan (No.18540284).

References

[1] S. Pakvasa and H. Sugawara, Phys. Lett. **B82** (1979) 105; Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. **D25** (1982) 1895; P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B458** (1999) 79; Phys. Lett. **B530** (2002) 167; Z. Z. Xing, Phys. Lett. **B533** (2002) 85; P. F. Harrison and W. G. Scott, Phys. Lett. **B535** (2003) 163; Phys. Lett. **B557** (2003) 76; E. Ma, Phys. Rev. Lett. **90** (2003) 221802; C. I. Low and R. R. Volkas, Phys. Rev. **D68** (2003) 033007; X.-G. He and A. Zee, Phys. Lett. **B560** (2003) 87.

[2] E. Ma, Phys. Rev. **D73** (2006) 057304.

[3] Y. Koide, Lett. Nuovo Cimento **34** (1982) 201; Phys. Lett. **B120** (19836) 161; Phys. Rev. **D28** (1983) 252.

[4] Y. Koide, Mod. Phys. Lett. **A5** (1990) 2319.

[5] S. Pakvasa and H. Sugawara, Phys. Lett. **B73** (1978) 61; H. Harari, H. Haut and J. Weyers, Phys. Lett. **B78** (1978) 459; E. Derman, Phys. Rev. **D19** (1979) 317; D. Wyler, Phys. Rev. **D19** (1979) 330.

[6] Y. Koide and H. Fusaoka, Z. Phys. **C71** (1996) 459.

[7] Y. Koide and M. Tanimoto, Z. Phys. **C72** (1996) 333.

[8] Y. Koide, Phys. Rev. **D60** (1999) 077301.

[9] Y. Koide, Phys. Rev. **D73** (2006) 057901. Also, see Y. Koide, Phys. Rev. **D60** (1999) 077301.

[10] Y. Koide, arXive: hep-ph/0612058.

[11] C. Frogatt and H. B. Nielsen, Nucl. Phys. **B147** (1979) 277.

[12] Y. Koide, Eur. Phys. J. **C48** (2006) 223.

[13] N. Li and B. Q. Ma, Phys. Rev. **D73**, 013009 (2006); Z. Z. Xing, H. Zhang, Phys. Lett. **B635**, 107 (2006).

[14] B. Aharmim *et al.* SNO Collaboration, Phys. Rev. **C72** (2005) 055502; T. Araki *et al.* KamLAND Collaboration, Phys. Rev. Lett. **94** (2005) 081801.

[15] D. G. Michael *et al.* MINOS Collaboration, Phys. Rev. Lett. **97** (2006) 191801. Also see, J. Hosaka *et al.* the Super-Kamiokande Collaboration, arXiv: hep-ex/0604011.

[16] E. Ma, arXive: hep-ph/0612022.

[17] Particle Data Group, J. Phys. G: Nucl. Phys. **33** (2006) 1.

Table 1 A_4 assignments of the fields

Fields	A_4	U(1)
ℓ_L	$\mathbf{3}$	0
ν_R	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	0
e_R	$\mathbf{3}$	0
ϕ^u	$\mathbf{3}$	q_u
ϕ^d	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	q_d
Φ	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	0
H_L^u	$\mathbf{1}$	$-q_u$
H_L^d	$\mathbf{1}$	$-2q_d$