S_4 Flavor Symmetry Embedded into SU(3) and Lepton Masses and Mixing

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Abstract

Under an assumption that an S_4 flavor symmetry is embedded into SU(3), a lepton mass matrix model is investigated. A Froggatt-Nielsen type model is assumed, and the flavor structures of the masses and mixing are caused by VEVs of $SU(2)_L$-singlet scalars ϕ_u and ϕ_d which are nonets (**8**+**1**) of the SU(3) flavor symmetry, and which are broken into **2**+**3**+**3′** and **1** of S_4. If we require the invariance under the transformation $(\phi^{(8)}, \phi^{(1)}) \to (-\phi^{(8)}, +\phi^{(1)})$ for the superpotential of the nonet field $\phi^{(8+1)}$, the model leads to a beautiful relation for the charged lepton masses. The observed tribimaximal neutrino mixing is understood by assuming two SU(3) singlet right-handed neutrinos $\nu_R^{(\pm)}$ and an SU(3) triplet scalar χ.

1 Introduction

The observed mass spectra and mixings of the fundamental particles will provide promising clues to unified understanding of the quarks and leptons. Especially, in the lepton sector, the following characteristic features have been observed [1]:
(i) The observed charged lepton masses (m_e, m_μ, m_τ) satisfy the relation [2, 3]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \tag{1.1}$$

with remarkable precision;
(ii) The observed neutrino mixing U_ν is nearly given by the so-called tribimaximal mixing [4]

$$U_{TB} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1.2}$$

Such characteristic features have not been seen in the quark sector. For example, the mixing form (1.2) suggests that the mixing can be described by Clebsh-Gordan-like coefficients, while, for the Cabibbo-Kobayashi-Maskawa mixing in the quark sector, such a characteristic feature has not been seen, although we have known some relations among the mixing angles and quark mass ratios. Therefore, for a start, in the present paper, we investigate the lepton masses and mixings.

In order to understand the relation (1.1), for example, we assume that there are three scalars ϕ_i $(i = 1, 2, 3)$, and the values of the charged lepton masses m_{ei} are proportional to the square of the vacuum expectation values (VEVs) $v_i = \langle\phi_i\rangle$ of the scalars ϕ_i, $m_{ei} = kv_i^2$ (in the

Ref.[3, 5, 6], for instance, a seesaw type model $(M_e)_{ij} = \delta_{ij} v_i (M_E)^{-1} v_j$ has been assumed). We define singlet ϕ_σ and doublet (ϕ_π, ϕ_η) of a permutation symmetry S$_3$ [7] by

$$\begin{pmatrix} \phi_\pi \\ \phi_\eta \\ \phi_\sigma \end{pmatrix} = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix}, \tag{1.3}$$

from the three objects (ϕ_1, ϕ_2, ϕ_3), and we consider the following S$_3$ invariant scalar potential $V(\phi)$ [3, 8, 9]:

$$V(\phi) = m^2(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2) + \lambda_1(\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2)^2 + \lambda_2 \phi_\sigma^2(\phi_\pi^2 + \phi_\eta^2). \tag{1.4}$$

The minimizing condition of the potential (1.4) leads to the relation

$$v_\pi^2 + v_\eta^2 = v_\sigma^2. \tag{1.5}$$

The relation (1.5) means

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2, \tag{1.6}$$

because

$$v_1^2 + v_2^2 + v_3^2 = v_\pi^2 + v_\eta^2 + v_\sigma^2 = 2v_\sigma^2 = 2\left(\frac{v_1 + v_2 + v_3}{\sqrt{3}}\right)^2. \tag{1.7}$$

Therefore, we can obtain the mass relation (1.1). Here, note that although the scalar potential (1.4) is invariant under the S$_3$ symmetry, but it is not a general one of the S$_3$ invariant form. As pointed out in Ref. [9], the scalar potential with a general form cannot lead to the relation (1.5). For the derivation of the VEV relation (1.5), it is essential to choose the specific form (1.4) of the S$_3$ invariant terms. Similar formulation is also possible for other discrete symmetries A$_4$ [10] and S$_4$ (see below). However, in such a symmetry, we still need an additional specific selection rule. What is the meaning of such a specific selection? In the present paper, we investigate this problem by assuming that the S$_4$ flavor symmetry is embedded into SU(3).

Recently, a superpotential which leads to the relation (1.5) has proposed by Ma [11] on the basis of a symmetry $\Sigma(81)$. Stimulated by the Ma's idea, the author [10] has also investigated a similar superpotential on the basis of a symmetry A$_4$. Here, based on an S$_4$ flavor symmetry instead of the A$_4$ symmetry, let us review the superpotential W which gives the relation (1.5). We denote singlet and doublet of S$_4$ as ϕ_σ and $\phi_D = (\phi_\pi, \phi_\eta)^T$, respectively, as well as those in S$_3$. In order to write the superpotential for the scalar fields ϕ_σ and doublet ϕ_D of S$_4$, we put the following phenomenological rule [10]: the field ϕ_a $(a = \sigma, D)$ to the power nth, $(\phi_a)^n$ $(n = 1, 2, 3)$, appears always accompanied with the factor $1/n!$ in the superpotential W. Under this phenomenological rule, we can uniquely write the superpotential of ϕ_σ and ϕ_D as

$$W(\phi) = \frac{1}{2!} m \left(\phi_\sigma^2 + \phi_D^T \phi_D\right) + \lambda \left(\frac{1}{2!}\phi_\sigma \phi_D^T \phi_D + \frac{1}{3!}\phi_\sigma^3\right)$$

$$= \frac{1}{2}m\left(\phi_\sigma^2+\phi_\pi^2+\phi_\eta^2\right)+\frac{1}{2}\lambda\left[(\phi_\pi^2+\phi_\eta^2)\phi_\sigma+\frac{1}{3}\phi_\sigma^3\right]. \tag{1.8}$$

The potential (1.8) can also lead the relation (1.5). What is the meaning of this phenomenological rule?

On the other hand, if we assume that the charged lepton fields e_{Li} and e_{Ri} are triplets of S_4, and we assume a Froggatt-Nielsen type model [12], $H_e^{eff} = (\bar{\ell}_L H_L^d(\phi_\sigma,\phi_D)^2 e_R)$, where ℓ_{Li} are the $SU(2)_L$ doublet left-handed lepton fields $\ell_{Li} = (\nu_i, e_i)_L$ and H_d is the conventional $SU(2)_L$ doublet Higgs scalar $H_L^d = (H_L^-, H_L^0)$, then we obtain

$$H_e^{eff} = (\bar{e}_L e_R)_\sigma[\phi_\sigma\phi_\sigma+\sqrt{2}(\phi_D\phi_D)_\sigma]$$

$$+(\bar{e}_L e_R)_\pi[2\phi_\sigma\phi_\pi-(\phi_D\phi_D)_\pi]+(\bar{e}_L e_R)_\eta[2\phi_\sigma\phi_\eta-(\phi_D\phi_D)_\eta]$$

$$=\sqrt{3}\left[\bar{e}_{L1}(\phi_1)^2 e_{R1}+\bar{e}_{L2}(\phi_2)^2 e_{R3}+\bar{e}_{L3}(\phi_3)^2 e_{R3}\right], \tag{1.9}$$

where we have dropped the $SU(2)_L$ structure. Here, $(\bar{e}_L e_R)_\sigma$ and $((\bar{e}_L e_R)_\pi, (\bar{e}_L e_R)_\eta)$ are singlet and doublet composed of the triplets (e_{L1}, e_{L2}, e_{L3}) and (e_{R1}, e_{R2}, e_{R3}), respectively, and those are given by

$$\begin{aligned}(\bar{e}_L e_R)_\sigma &= \tfrac{1}{\sqrt{3}}\left(\bar{e}_{L1}e_{R1}+\bar{e}_{L2}e_{R2}+\bar{e}_{L3}e_{R3}\right),\\ (\bar{e}_L e_R)_\pi &= \tfrac{1}{\sqrt{2}}\left(-\bar{e}_{L2}e_{R2}+\bar{e}_{L3}e_{R3}\right),\\ (\bar{e}_L e_R)_\eta &= \tfrac{1}{\sqrt{6}}\left(2\bar{e}_{L1}e_{R1}-\bar{e}_{L2}e_{R2}-\bar{e}_{L3}e_{R3}\right).\end{aligned} \tag{1.10}$$

Also, $(\phi_D\phi_D)_\sigma$ and $((\phi_D\phi_D)_\pi, (\phi_D\phi_D)_\eta)$ are singlet and doublet composed of the doublet $\phi_D = (\phi_\pi, \phi_\eta)$, respectively, and those are defined by

$$\begin{aligned}(\phi_D\phi_D)_\sigma &= \tfrac{1}{\sqrt{2}}\left(\phi_\pi\phi_\pi+\phi_\eta\phi_\eta\right),\\ (\phi_D\phi_D)_\pi &= \tfrac{1}{\sqrt{2}}\left(\phi_\pi\phi_\eta+\phi_\eta\phi_\pi\right),\\ (\phi_D\phi_D)_\eta &= \tfrac{1}{\sqrt{2}}\left(\phi_\pi\phi_\pi-\phi_\eta\phi_\eta\right),\\ &.\end{aligned} \tag{1.11}$$

Thus, we can obtain the relation (1.1) for the charged lepton masses from Eqs.(1.6) and (1.9). However, in Eq.(1.9), we have again assumed some specific combinations of the S_4 invariant terms. What is the meaning of the selection rule?

Now, let us return the topic of the tribimaximal mixing. From the definition (1.2), we can denote the fields (ψ_1, ψ_2, ψ_3) as

$$\begin{pmatrix}\psi_1\\ \psi_2\\ \psi_3\end{pmatrix} = U_{TB}\begin{pmatrix}\psi_\eta\\ \psi_\sigma\\ \psi_\pi\end{pmatrix}. \tag{1.12}$$

The observed neutrino mixing (1.2) means that when the mass eigenstates of the charged leptons are given by the (ψ_1, ψ_2, ψ_3) basis, the mass eigenstates of the neutrinos are given by the

$(\psi_\eta, \psi_\sigma, \psi_\pi)$ basis. Therefore, the problem is to find a model where the charged lepton mass eigenstates are (e_1, e_2, e_3), while the neutrino mass eigenstates are given by $(\nu_\eta, \nu_\sigma, \nu_\pi)$ with the masse hierarchy $m_\eta^2 < m_\sigma^2 \ll m_\pi^2$ (or $m_\pi^2 \ll m_\eta^2 < m_\sigma^2$). In the present paper, we will investigate such a model based on an S_4 model.

Note that the fermions (ψ_1, ψ_2, ψ_3) is a triplet of S_4, but the basis $(\psi_\eta, \psi_\sigma, \psi_\pi)$ is not in any irreducible representations of S_4, while the scalar (ϕ_1, ϕ_2, ϕ_3) is not irreducible representation of S_4, but (ϕ_π, ϕ_η) and ϕ_σ are doublet and singlet of S_4.

Thus, the characteristic features (1.1) and (1.2) in the lepton sector may be understood from the language of S_4 (also S_3 or A_4). However, as seen from the above review, the characteristic features (1.1) and (1.2) cannot be understood from the S_4 symmetry only. We need some additional assumptions. In this paper, we will investigate these problems under an assumption that the present S_4 symmetry is embedded into an SU(3) symmetry.

2 VEVs of SU(3) nonet scalars

The goal in the present section is to obtain the VEV relation (1.6) [i.e. (1.5)]. As seen in the previous section, in order to obtain the desirable results (1.5), we need assume an equal weight between the doublet and singlet terms of S_4. In the present paper, we assume that the S_4 symmetry is embedded into an SU(3) symmetry, and we consider that the doublet (ϕ_π, ϕ_η) and singlet ϕ_σ of S_4 come from SU(3) octet and singlet, respectively [13]. The essential assumption in the present paper is that the fields ϕ_u and ϕ_d always appear in the theory with the form of the nonet of U(3):

$$\phi = \begin{pmatrix} \phi_1^1 & * & * \\ * & \phi_2^2 & * \\ * & * & \phi_3^3 \end{pmatrix}, \tag{2.1}$$

where

$$\begin{aligned} \phi_1^1 &= \tfrac{1}{\sqrt{3}}\phi_\sigma + \tfrac{2}{\sqrt{6}}\phi_\eta, \\ \phi_2^2 &= \tfrac{1}{\sqrt{3}}\phi_\sigma - \tfrac{1}{\sqrt{6}}\phi_\eta - \tfrac{1}{\sqrt{2}}\phi_\pi, \\ \phi_3^3 &= \tfrac{1}{\sqrt{3}}\phi_\sigma - \tfrac{1}{\sqrt{6}}\phi_\eta + \tfrac{1}{\sqrt{2}}\phi_\pi, \end{aligned} \tag{2.2}$$

and the index f $(f = u, d)$ has been dropped.

The outline to obtain the superpotential form (1.8) in the present scenairo is as follows: The SU(3) invariant superpotential for the nonet fields ϕ_f $(f = u, d)$ are given by

$$W(\phi) = \frac{1}{2}m\mathrm{Tr}(\phi\phi) + \frac{1}{2\sqrt{3}}\lambda\mathrm{Tr}(\phi\phi\phi), \tag{2.3}$$

where, for convenience, we have dropped the index f. Although the term $\mathrm{Tr}(\phi\phi)$ gives the desirable term $\phi_\pi^2 + \phi_\eta^2 + \phi_\sigma^2 + \cdots$ of S_4, the cubic term $\mathrm{Tr}(\phi\phi\phi)$ gives

$$\mathrm{Tr}(\phi\phi\phi) = \sqrt{3}\left[\frac{1}{\sqrt{2}}\left(-\phi_\pi^2 + \frac{1}{3}\phi_\eta^2\right)\phi_\eta + (\phi_\pi^2 + \phi_\eta^2)\phi_\sigma + \frac{1}{3}\phi_\sigma^3\right] + \cdots, \tag{2.4}$$

where the terms "$\cdots$" denote terms which include **3** and **3**$'$ of the subgroup S_4, so that the potential (2.3) cannot give the relation (1.5). Therefore, we must drop the first term in cubic terms (2.4). For this purpose, we introduce Z_2 symmetry, and we assign the Z_2 parity -1 and $+1$ for the octet part $\phi^{(8)}$ and singlet part $\phi^{(1)}$ of the nonet field ϕ, respectively. The symmetry Z_2 breaks U(3) into SU(3). Under the requirement of the Z_2 invariance, i.e. the invariance under the transformation

$$(\phi^{(8)}, \phi^{(1)}) \to (-\phi^{(8)}, +\phi^{(1)}), \tag{2.5}$$

the terms $\mathrm{Tr}(\phi^{(8)}\phi^{(8)}\phi^{(8)})$, i.e. $(-\phi_\pi^2+(1/3)\phi_\eta^2)\phi_\eta+\cdots$, are forbidden. The revised superpotential is not U(3) invariant anymore, but it is still SU(3) invariant. In other words, in the present model, the flavor symmetry U(3) is broken from the begining by the Z_2 symmetry.

The superpotential (2.3) with the constraint (2.5) gives the form (1.8) which leads to the VEV relation (1.5). However, we want to assign Z_3 charges to the fields ϕ_f in order to select some special combinations among the fields in the present model as we see in the next section. If we assign $Q(Z_2) = +1$ to the field ϕ, the term $\lambda\phi\phi\phi$ is allowed, but the term $m\phi\phi$ is forbidden by the Z_3 symmetry. We need the m-term, but also need the Z_3 assignment. Therefore, in the present paper, keeping the idea of (2.3) with (2.5), we propose the following super potential

$$W(\phi) = m\mathrm{Tr}(\phi_u\phi_d) + \lambda_u\mathrm{Tr}(\phi_u\phi_u\phi_u) + \lambda_u\mathrm{Tr}(\phi_d\phi_d\phi_d), \tag{2.6}$$

where the fields ϕ_u and ϕ_d have the charges of the Z_3 transformation, $+1$ and -1, respectively.

Under the constraint (2.5), the superpotential (2.6) becomes

$$W(\phi) = m\left[\mathrm{Tr}(\phi_u^{(8)}\phi_d^{(8)}) + \phi_{u\sigma}\phi_{d\sigma}\right] + \sqrt{3}\lambda_u\phi_{u\sigma}\left[\mathrm{Tr}(\phi_u^{(8)}\phi_u^{(8)}) + \frac{1}{3}\phi_{u\sigma}^2\right]$$

$$+\sqrt{3}\lambda_d\phi_{d\sigma}\left[\mathrm{Tr}(\phi_d^{(8)}\phi_d^{(8)}) + \frac{1}{3}\phi_{d\sigma}^2\right]. \tag{2.7}$$

From the conditions

$$\frac{\partial W}{\partial\phi_{u\sigma}} = m\phi_{d\sigma} + \sqrt{3}\lambda_u\left[\mathrm{Tr}(\phi_u^{(8)}\phi_u^{(8)}) + \phi_{u\sigma}^2\right], \tag{2.8}$$

$$\frac{\partial W}{\partial\phi_{d\sigma}} = m\phi_{u\sigma} + \sqrt{3}\lambda_d\left[\mathrm{Tr}(\phi_d^{(8)}\phi_d^{(8)}) + \phi_{d\sigma}^2\right], \tag{2.9}$$

$$\frac{\partial W}{\partial(\phi_u^{(8)})_i^j} = m(\phi_d^{(8)})_j^i + 2\sqrt{3}\lambda_u\phi_{u\sigma}(\phi_u^{(8)})_j^i, \tag{2.10}$$

$$\frac{\partial W}{\partial(\phi_d^{(8)})_i^j} = m(\phi_u^{(8)})_j^i + 2\sqrt{3}\lambda_d\phi_{d\sigma}(\phi_d^{(8)})_j^i, \tag{2.11}$$

by eliminating ϕ_u and m, we obtain

$$4\phi_{d\sigma}^4 = \left[\mathrm{Tr}(\phi_d^{(8)}\phi_d^{(8)}) + \phi_{d\sigma}^2\right]^2, \tag{2.12}$$

so that we obtain the relation

$$\phi_{d\sigma}^2 = \mathrm{Tr}(\phi_d^{(8)}\phi_d^{(8)}) = \phi_{d\pi}^2 + \phi_{d\eta}^2 + \cdots , \tag{2.13}$$

when, we take the positive sign in $2\phi_{d\sigma}^2 = \pm[\cdots]$, where "$\cdots$" denotes the contributions of **3** and **3′** of S_4.

The result (2.13) is not our goal, because the relation contains the VEVs of the **3** and **3′** of S_4. So far, we have not discussed the splitting among the S_4 multiplets. Now, we bring a symmetry breaking of SU(3) into S_4 with an infinitesimal parameter ε

$$\mathrm{Tr}(\phi_u^{(8)}\phi_d^{(8)}) \Rightarrow \phi_{u\pi}\phi_{d\pi} + \phi_{u\eta}\phi_{d\eta} + (1+\varepsilon)\sum_{i\neq j}(\phi_u^{(8)})_i^j(\phi_d^{(8)})_j^i, \tag{2.14}$$

by hand. (At present, we do not consider the origin of the symmetry breaking. We always consider only the limit of $\varepsilon \to 0$.) Then, we can obtain the solution

$$\langle(\phi_d^{(8)})_j^i\rangle = 0 \quad \text{for all } i \neq j. \tag{2.15}$$

Therefore, we can obtain the desiarable relation (1.5). (However, it is possible that we can also obtain another solution with $\phi_\pi = \phi_\eta = 0$ and $(\phi_d^{(8)})_j^i \neq 0$. The VEV solutions are not unique. The result (1.5) is merely one of the possible solutions.)

3 Model

If we regard the scalars ϕ_i as $SU(2)_L$ doublets, such a model with multi-Higgs doublets causes a flavor changing neutral current (FCNC) problem. Therefore, in the present paper, we assume a Froggatt-Nielsen [12] type model

$$H^{eff} = y_e \bar{\ell}_L H_L^d \frac{\phi_d}{\Lambda}\frac{\phi_d}{\Lambda}\frac{\xi}{\Lambda} e_R + y_\nu \bar{\ell}_L H_L^u \frac{\phi_u}{\Lambda}\frac{\chi}{\Lambda}\nu_R + y_R \overline{\nu}_R \Phi_R \nu_R^*, \tag{2.1}$$

where ℓ_{iL} are $SU(2)_L$ doublet leptons $\ell_{iL} = (\nu_{iL}, e_{iL})$, H_L^d and H_L^u are conventional $SU(2)_L$ doublet Higgs scalars, ϕ_f $(f = u, d)$, ξ and χ are $SU(2)_L$ singlet scalars, and Λ is a scale of the effective theory. We consider that $\langle\phi_f\rangle/\Lambda$, $\langle\xi\rangle/\Lambda$ and $\langle\chi\rangle/\Lambda$ are of the order of 1. The scalar Φ_R has been introduced in order to generate the Majorana mass M_R of the right-handed neutrino ν_R. The role of ξ and χ will be explained later. In order to understand the appearance of the combinations $H_L^d\phi_d\phi_d\xi$ and $H_L^u\phi_u\chi$, we assume that the scalars H_L^d and ϕ_d have charge +1 of a Z_3 symmetry (we denote it by Z_3^d in Table 1), and the scalars H_L^u, ϕ_u and χ have charge +1 of another Z_3 (we denote it by Z_3^u in Table 1). In order to forbid the combinations $H_L^d\phi_d\phi_d\xi^n$ $(n \neq 1)$, we also assume another Z_3 symmetry ($Z_3^{\prime d}$).

On the other hand, in the charged lepton sector, we consider a Froggatt-Nielsen type interaction $H_e^{eff} = \bar{\ell}_L H_L^d \phi_d\phi_d e_R$. Recall that we have already assumed the invariance of the superpotential under the transformation (3.3) in order to drop the cubic part of the octet $\phi^{(8)}$. Therefore, the term $\phi\phi$ means $\phi^{(8)}\phi^{(8)} + \phi^{(1)}\phi^{(1)}$ under the Z_2 invariance. However, in order to

Table 1 SU(3) and S_4 assignments of the fields

Fields	SU(3)	S_4	Z_3^d	$Z_3'^d$	Z_3^u	Z_2
ℓ_L	$\mathbf{3}$	$\mathbf{3}'$	0	0	0	0
e_R	$\mathbf{3}$	$\mathbf{3}'$	0	0	0	0
$\nu_R^{(\pm)}$	$\mathbf{1}$	$\mathbf{1}$	0	0	0	0/+1
ϕ_d	$\mathbf{1}$+$\mathbf{8}$	$\mathbf{1}+(\mathbf{2}+\mathbf{3}+\mathbf{3}')$	+1	+1	0	0
$\xi^{(\pm)}$	$\mathbf{1}$	$\mathbf{1}$	0	+1	0	0/+1
ϕ_u	$\mathbf{1}$+$\mathbf{8}$	$\mathbf{1}+(\mathbf{2}+\mathbf{3}+\mathbf{3}')$	0	0	+1	0
χ	$\mathbf{3}$	$\mathbf{3}'$	0	0	+1	0
Φ_R	$\mathbf{1}$	$\mathbf{1}$	0	0	0	0
H_L^d	$\mathbf{1}$	$\mathbf{1}$	+1	0	0	0
H_L^u	$\mathbf{1}$	$\mathbf{1}$	0	0	+1	0

give $m_{ei} \propto \langle \phi_i^i \rangle^2$, what we want is not $\phi^{(8)}\phi^{(8)}+\phi^{(1)}\phi^{(1)}$, but $\phi^{(8)}\phi^{(8)}+\phi^{(1)}\phi^{(1)}+\phi^{(8)}\phi^{(1)}+\phi^{(1)}\phi^{(8)}$. In order to evade this problem, we have introduced additional fields $\xi^{(+)}$ and $\xi^{(-)}$ whose Z_2 parity are $+1$ and -1, respectively. The revised interactions are given by

$$H_e^{eff} = y_e \bar{e}_L^i (\phi_d)_i^j (\phi_d)_j^k (\xi^{(+)} + \xi^{(-)}) e_{Rk}, \tag{3.4}$$

where we have dropped the Higgs scalar H_L^d since we discuss flavor structure only. The expression (3.4) becomes

$$H_e^{eff} = y_e \bar{e}_L [(\phi_d^{(8)}\phi_d^{(8)} + \phi_d^{(1)}\phi_d^{(1)})\xi^{(+)} + (\phi_d^{(8)}\phi_d^{(1)} + \phi_d^{(1)}\phi_d^{(8)})\xi^{(-)}] e_R. \tag{3.5}$$

Since we have assumed that $\xi^{(+)}$ and $\xi^{(-)}$ appear symmetrically in the theory, we also assume

$$\langle \xi^{(+)} \rangle = \langle \xi^{(-)} \rangle \equiv v_\xi. \tag{3.6}$$

Then, we obtain the effective Hamiltonian for the charged leptons

$$H_e^{eff} = \frac{y_e v_d v_\xi}{\Lambda^3} \sum_i \bar{e}_L^i \langle (\phi_d^{(8+1)})_i^i \rangle^2 e_{Ri}, \tag{3.7}$$

where $v_d = \langle H_L^{d0} \rangle$. Since the fields $(\phi_d)_i^i$ are given by Eq.(2.3), we obtain the charged lepton mass relation (1.1) from the VEV relation (1.6).

However, the present mechanism to obtain $m_{ei} \propto \langle \phi_i^i \rangle^2$ is somewhat artificial. The present mechanism will be improved in the future model. [Of course, there is a possibility that the superpotential (3.1) must exactly be invariance under the Z_2 symmetry, but the effective Hamiltonian (3.3) does not need to be invariance under the Z_2 symmetry. Then, we can consider a model without $\xi^{(\pm)}$.]

By the way, from the superpotential (1.8), we can obtain the VEV relation (1.5) only. In order to fix the three charged lepton masses completely, we must fix the ratio v_π/v_η. The value v_π/v_η will be fixed by introducing an SU(3) (or S_4) symmetry breaking in the mass term of W. In Sec.5, we will demonstrate that we can indeed choose such a soft symmetry breaking term which does not spoil the VEV relation (1.5).

4 Neutrino mixing

In the present model, the right-handed neutrinos $\nu^{(\pm)}$ are singlets of SU(3). Therefore, the structure of the neutrino mass matrix $M_\nu = m_L^\nu M_R^{-1}(m_L^\nu)^T$ is essentially determined by the structure of the neutrino Dirac mass matrix m_L^ν. In order to compensate for the absence of the conventional triplet neutrinos ν_R, a new scalar χ which is a triplet of SU(3) has been introduced. The neutrino Dirac mass terms are given by the following effective Hamiltonian

$$H_{Dirac}^{eff} = y_\nu \frac{v_u}{\Lambda^2} \bar{\nu}_L^i \langle (\phi_u)_i^j \rangle \langle \chi_j \rangle (\nu_R^{(+)} + \nu_R^{(-)}), \tag{4.1}$$

where $v_u = \langle H_L^{u0} \rangle$. It is likely that the scalar potential $V(\chi)$ for the SU(3) triplet χ has a specific VEV solution

$$\langle \chi_1 \rangle = \langle \chi_2 \rangle = \langle \chi_3 \rangle \equiv v_\chi. \tag{4.2}$$

When we assume the VEVs (4.2), we obtain

$$H_{Dirac}^{eff} = y_\nu \frac{v_u v_\chi}{\Lambda^2} (\bar{\nu}_\eta \ \bar{\nu}_\sigma \ \bar{\nu}_\pi)_L \left[\begin{pmatrix} v_\eta \\ 0 \\ v_\pi \end{pmatrix} \nu_R^{(-)} + \begin{pmatrix} 0 \\ v_\sigma \\ 0 \end{pmatrix} \nu_R^{(+)} \right], \tag{4.3}$$

where $v_a = \langle \phi_{ua} \rangle$ $(a = \pi, \eta, \sigma)$ (for convenience, we have dropped the index u). Therefore, we obtain the effective neutrino mass matrix on the (η, σ, π) basis,

$$M_\nu^{(\eta\sigma\pi)} = \frac{1}{M_R^{(-)}} \begin{pmatrix} v_\eta^2 & 0 & v_\pi v_\eta \\ 0 & 0 & 0 \\ v_\pi v_\eta & 0 & v_\pi^2 \end{pmatrix} + \frac{1}{M_R^{(+)}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & v_\sigma^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \tag{4.4}$$

where $M_R^{(\pm)} = y_R^{(\pm)} \langle \Phi_R \rangle$, and we have dropped the common factors $(y_\nu v_u v_\chi/\Lambda^2)^2$. As we state in the next section, the ratio v_π/v_η cannot be determined from the potential (1.8), and the ratio is determined by a soft S_4 symmetry breaking term W_{SB}. Of course, we can choose a solution $v_\pi = 0$ in the superpotential $W(\phi_u)$ by choosing a suitable parameter β in W_{SB}, which is defined in the next section, differently from the case of $W(\phi_d)$. Then, the neutrino mass matrix (4.4) becomes a diagonal form $D_\nu = (1/M_R^{(-)})\mathrm{diag}(v_\eta^2, 0, 0) + (1/M_R^{(+)})\mathrm{diag}(0, v_\sigma^2, 0)$. Since the mass matrix on the $(\nu_1, \nu_2, \nu_3) = (\nu_e, \nu_\mu, \nu_\tau)$ basis is given by

$$M_\nu = U_{TB} M_\nu^{(\eta\sigma\pi)} U_{TB}^T = U_{TB} D_\nu U_{TB}^T, \tag{4.5}$$

we can obtain the tribimaximal mixing

$$U_\nu = U_{TB}, \tag{4.6}$$

and the neutrino masses

$$m_{\nu 1} = k v_\eta^2, \quad m_{\nu 2} = k v_\sigma^2, \quad m_{\nu 3} = 0, \tag{4.7}$$

for the case of $M_R^{(+)} = M_R^{(-)} \equiv M_R$, where $k = (y_\nu v_u v_\chi)^2/M_R\Lambda^4$ and $(\nu_\eta, \nu_\sigma, \nu_\pi)$ has been renamed (ν_1, ν_2, ν_3) according to the conventional naming.

However, since we have taken $v_\pi = 0$, the value of v_η satisfies $v_\eta^2 = v_\sigma^2$ from the relation (1.5), so that the result (4.8) gives $m_{\nu 1} = m_{\nu 2}$. The observed value Δm^2_{solar} is small, but it is not zero. Therefore, we must consider a small deviation between the first and second terms in (4.5) (i.e. $M_R^{(+)} \neq M_R^{(-)}$). Since the value $M_R^{(-)}/M_R^{(+)}$ is free in the present model, we cannot predict an explicit value of the ratio $\Delta m^2_{solar}/\Delta m^2_{atm}$.

Since the present model gives an inverse hierarchy of the neutrino masses, the predicted effective electron neutrino mass

$$\langle m_{\nu_e} \rangle = \left| \sum_i U_{ei}^2 m_{\nu i} \right| \simeq |m_{\nu 1}| \simeq |m_{\nu 2}| \simeq \sqrt{\Delta m^2_{atm}} = 5.23^{+0.25}_{-0.40} \times 10^{-2}\,\mathrm{eV}, \qquad (4.8)$$

where we have used the value [14] $\Delta m^2_{atm} = 2.74^{+0.44}_{-0.26} \times 10^{-3}\,\mathrm{eV}^2$. This value (4.8) is sufficiently sensitive to the next generation experiments of the neutrinoless double beta decay.

5 Summary

In conclusion, on the basis of the S_4 symmetry which is embedded into SU(3), we have investigated a Froggatt-Nielsen type model (2.1). In the derivation of the VEV relation (1.5), the essential assumptions for the superpotential W are the following two: (i) the scalar fields ϕ always appear in terms of the nonet form of U(3), (2.3); (ii) the superpotential W is invariant under the Z_2 transformation (3.3). Then, we have obtained the VEV relation (1.5). Since the charged lepton interactions $\bar{\ell}_L H_L^d \phi_d \phi_d e_R$ give $m_{ei} \propto \langle(\phi_d)_i^i\rangle^2$, we have obtained the charged lepton mass relation (1.1).

For the neutrino sector, we have obtained the tribimaximal mixing (1.2) by introducing an SU(3) triplet scalar χ and the two SU(3) singlet right-handed neutrinos $\nu_R^{(\pm)}$ in addition to the nonet scalar ϕ_u. In the present model, since the right-handed neutrinos are singlets of SU(3), the Majorana neutrino mass matrices $M_R^{(\pm)}$ have no flavor structure. However, the requirement that $\nu_R^{(\pm)}$ are singlets of SU(3) is essential for the structure of the neutrino mass matrix $M_\nu = m_L^\nu M_R^{-1} (m_L^\nu)^T$ which gives the tribimaximal mixing U_{TB}. For the neutrino mass spectrum, since the model gives $m_{\nu 1} = m_{\nu 2}$ in the limit of $M_R^{(+)} = M_R^{(-)}$, we must consider a small deviation $M_R^{(+)} \neq M_R^{(-)}$. Since the value of $M_R^{(-)}/M_R^{(+)}$ is a free parameter in the present model, we cannot predict the value $\Delta m^2_{solar}/\Delta m^2_{atm}$ at present, although the smallness of the ratio $\Delta m^2_{solar}/\Delta m^2_{atm}$ can be understood.

The present model seems to provide suggestive hints on seeking for a model which leads to the tribimaximal mixing (1.2) and the charged lepton mass relation (1.1), although the model has still many points which should be improved. At the same time, the model will provide a clue to the quark mass matrix model from a point of unified view of the quarks and leptons. The extension of the present model to the quark mass matrix model will be given elsewhere.

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References

[1] Particle Data Group, J. Phys. G: Nucl. Phys. **33** (2006) 1.

[2] Y. Koide, Lett. Nuovo Cimento **34** (1982) 201; Phys. Lett. **B120**, 161 (1983); Phys. Rev. **D28**, 252 (1983).

[3] Y. Koide, Mod. Phys. Lett. **A5**, 2319 (1990).

[4] S. Pakvasa and H. Sugawara, Phys. Lett. **B82**, 105 (1979); Y. Yamanaka, H. Sugawara and S. Pakvasa, Phys. Rev. **D25**, 1895 (1982); P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B458**, 79 (1999); Phys. Lett. **B530**, 167 (2002); Z. Z. Xing, Phys. Lett. **B533**, 85 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. **B535**, 163 (2003); Phys. Lett. **B557**, 76 (2003); E. Ma, Phys. Rev. Lett. **90**, 221802 (2003); C. I. Low and R. R. Volkas, Phys. Rev. **D68**, 033007 (2003); X.-G. He and A. Zee, Phys. Lett. **B560**, 87 (2003).

[5] Y. Koide and H. Fusaoka, Z. Phys. **C71**, 459 (1996).

[6] Y. Koide and M. Tanimoto, Z. Phys. **C72**, 333 (1996).

[7] S. Pakvasa and H. Sugawara, Phys. Lett. **B73**, 61 (1978); H. Harari, H. Haut and J. Weyers, Phys. Lett. **B78**, (1978); E. Derman, Phys. Rev. **D19** (1979) 317; 459 D. Wyler, Phys. Rev. **D19**, 330 (1979).

[8] Y. Koide, Phys. Rev. **D60**, 077301 (1999).

[9] Y. Koide, Phys. Rev. **D73**, 057901 (2006). Also, see Y. Koide, Phys. Rev. **D60**, 077301 (1999).

[10] Y. Koide, arXiv: hep-ph/0701018.

[11] E. Ma, arXive: hep-ph/0612022.

[12] C. Froggatt and H. B. Nielsen, Nucl. Phys. **B147**, 277 (1979).

[13] C. Hagedorn, M. Linder and R. N. Mohapatra, JHEP, **0606**, 042 (2006) (hep-ph/0602244).

[14] D. G. Michael *et al.* MINOS Collaboration, Phys. Rev. Lett. **97**, 191801 (2006). Also see, J. Hosaka *et al.* the Super-Kamiokande Collaboration, arXiv: hep-ex/0604011.