S_3 Symmetry and Neutrino Masses and Mixings

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Abstract

Based on a universal seesaw mass matrix model with three scalars ϕ_i, and by assuming an S_3 flavor symmetry for the Yukawa interactions, the neutrino masses and mixings are investigated. Suggested from that the observed neutrino mixing is nearly the tribimaximal mixing, it is assumed that when the charged leptons (e, μ, τ) are regarded as the states (e_1, e_2, e_3) of S_3, the neutrino mass-eigenstates are nearly in the states $(\nu_\eta, \nu_\sigma, \nu_\pi)$, where (ν_π, ν_η) and ν_σ are a doublet and a singlet of S_3, respectively. Possible structures of the Yukawa interactions are investigated systematically.

1 Introduction

It is generally considered that masses and mixings of the quarks and leptons will obey a simple law of nature, so that we expect that we will find a beautiful relation among those values. However, even if there is such a simple relation in the quark sector, it is hard to see such a relation in the quark sector, because the relation will be spoiled by the gluon cloud. We may expect that such a beautiful relation will be found just in the lepton sector. Therefore, in the present paper, we will confine ourselves to the investigation of the lepton masses and mixings. Here, we would like to emphasize that we should search a model which gives a reasonable description of not only the masses, but also the mixings. Especially, we should direct our attention to the mixing pattern rather than to the mass spectrum in the neutrino sector.

It is also considered that the mass matrices of the fundamental particles will be governed by a kind of symmetry. In the present paper, we take notice of a permutation symmetry S_3 [1]. Let us begin with giving a short review how useful a description based on the S_3 symmetry is in the lepton masses and mixings.

The observed neutrino data have strongly suggested that the neutrino mixing is approximately described by the so-called tribimaximal mixing [2]

$$U = \begin{pmatrix} -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \tag{1.1}$$

According to the conventional notations, we define the doublet (ψ_π, ψ_η) and singlet ψ_σ of the permutation symmetry S_3 as

$$\begin{pmatrix} \psi_\pi^A \\ \psi_\eta^A \\ \psi_\sigma^A \end{pmatrix} = A \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} , \tag{1.2}$$

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{1.3}$$

Hereafter, we will call the basis $(\psi_\pi^A, \psi_\eta^A, \psi_\sigma^A)$ defined by Eq.(1.2) the basis A. We can also take another S_3 basis $(\psi_\pi^B, \psi_\eta^B, \psi_\sigma^B)$ (we call it the basis B) which is defined by

$$\begin{pmatrix} \psi_\pi^B \\ \psi_\eta^B \\ \psi_\sigma^B \end{pmatrix} = B \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix}, \tag{1.4}$$

$$B = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{1.5}$$

Of course, an S_3-invariant interaction is invariant under the transformation $U_{AB} \equiv AB^T$, which transforms the basis A into the basis B. Since $U_{AB}^2 = \mathbf{1}$, we can say that the bases A and B are dual each other.

When we take a flavor basis $(e_1, e_2, e_3) = (\tau, \mu, e)$ [and also $(\nu_1, \nu_2, \nu_3) = (\nu_\tau, \nu_\mu, \nu_e)$], if the states $(\nu_\pi^B, \nu_\eta^B, \nu_\sigma^B)$ are mass eigenstates, and if their masses satisfy the relation

$$m_{\nu\eta}{}^2 < m_{\nu\sigma}{}^2 < m_{\nu\pi}{}^2, \tag{1.6}$$

the neutrino mixing matrix U of the basis $(\nu_\eta^B, \nu_\sigma^B, \nu_\pi^B)$ to the basis $(e_1, e_2, e_3) = (e, \mu, \tau)$ is given by the form (1.1). Here, we would like to emphasize that the condition (1.6) is essential to obtaining the tribimaximal mixing (1.1). (Hereafter, since we always discuss the mass matrix form on the basis B, we will drop the index B.)

On the other hand, it is well-known that the observed charged lepton mass spectrum [3] satisfies the relation [4, 5]

$$m_e + m_\mu + m_\tau = \frac{2}{3} \left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2, \tag{1.7}$$

with remarkable precision. The mass formula (1.7) is invariant under any exchange $\sqrt{m_i} \leftrightarrow \sqrt{m_j}$ $(i, j = e, \mu, \tau)$. This, too, suggests that a description by S_3 may be useful for a mass matrix model.

As an explanation of the mass formula (1.7), the author has proposed a model [5, 6, 7] with 3 flavor scalars ϕ_i in the framework of the universal seesaw model [8]: A fermion mass matrix M_f is given by

$$M_f = m_L^f M_F^{-1} m_R^f, \tag{1.8}$$

where M_F is a mass matrix of hypothetical heavy fermions F_i $(i = 1, 2, 3)$. For example, for the charged lepton sector, we assume

$$m_L^e = \frac{1}{\kappa} m_R^e = y_e \mathrm{diag}(v_1, v_2, v_3), \tag{1.9}$$

(κ is a constant with $\kappa \gg 1$) and $M_E \propto \mathbf{1}$, where m_L^e and m_R^e are defined by $\bar{\ell}_L m_L^e E_R$ and $\bar{\ell}_R (m_R^e)^\dagger E_L^c$ and $v_i \equiv \langle \phi_{Li}^0 \rangle = \langle \phi_{Ri}^0 \rangle / \kappa$, $\ell_{L/R} = (\nu_{L/R}, e_{L/R})$, and $\phi_{L/R} = (\phi_{L/R}^+, \phi_{L/R}^0)$. If we assume that the vacuum expectation values (VEV) v_i satisfy the relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3} (v_1 + v_2 + v_3)^2 \, , \tag{1.10}$$

we can obtain the relation (1.7). Of course, here, we have assumed that the Yukawa interaction in the charged lepton sector is given by an S_3 invariant form

$$H_e = y_e \left(\bar{\ell}_{L1} \phi_{L1} E_{R1} + \bar{\ell}_{L2} \phi_{L2} E_{R2} + \bar{\ell}_{L3} \phi_{L3} E_{R3} \right) , \tag{1.11}$$

(and also a similar interaction for $\bar{\ell}_R \phi_R E_L$).

The relation among the VEVs v_i, (1.10), can read

$$v_\pi^2 + v_\eta^2 = v_\sigma^2, \tag{1.12}$$

in terms of S_3, where $(\phi_\pi, \phi_\eta, \phi_\sigma)$ have been defined by Eq.(1.4). For a Higgs potential model based on an S_3 symmetry which leads to the relation (1.12), for example, see Ref. [9]. The S_3 symmetry is again related to the lepton masses and mixings.

Thus, it is likely that the S_3 symmetry (or a higher symmetry which include S_3) plays an essential role on a unified description of the lepton mass matrices. In the present paper, we will assume that, in the universal seesaw model with three flavor scalars, the Yukawa interactions are exactly invariant under the S_3 symmetry, and the S_3 symmetry is broken only by the VEVs v_i of the three scalars ϕ_i. Thereby, we will investigate the masses and mixings of the neutrinos systematically.

Another motivation of the present paper is in a speculation, which has recently been pointed out by Brannen [10], that the neutrino masses also satisfy a relation similar to the relation (1.7) for the charged lepton masses,

$$m_{\nu 1} + m_{\nu 2} + m_{\nu 3} = \frac{2}{3} (-\sqrt{m_{\nu 1}} + \sqrt{m_{\nu 2}} + \sqrt{m_{\nu 3}})^2 \, . \tag{1.13}$$

Of course, we cannot extract the values of the neutrino mass ratios $m_{\nu 1}/m_{\nu 2}$ and $m_{\nu 2}/m_{\nu 3}$ from the neutrino oscillation data Δm_{solar}^2 and Δm_{atm}^2 unless we have more information on the neutrino masses, so that we cannot judge whether the observed neutrino masses satisfy the relation (1.13) or not. However, we interest in investigating what constraints are imposed on the mass matrix parameters when we regard the Brannen relation (1.13) as true. We will conclude that the present S_3 model cannot generally give the relation (1.13), except for a case with a specific relation among the Yukawa coupling constants.

By the way, the seesaw-type model (1.8) with 3 scalars ϕ_{Li} (and ϕ_{Ri}) causes some trouble, for example, the flavor changing neutral currents (FCNC) problem, the spoiling of the asymptotic freedom of the SU(3) color, and so on. Therefore, instead of the model (1.8), we may consider a Frogatt-Nielsen [11] type model with six dimensional operators $\overline{f}_{Li} \phi_{fi} H_L \phi_{fi} f_{Ri}$, where H_L is the conventional $\mathrm{SU}(2)_L$-doublet Higgs scalar, and ϕ_{fi} are 3-family $\mathrm{SU}(2)_L$-singlet scalars.

However, in the present paper, it is essential that the lepton masses m_{fi} are given by a bilinear form

$$m_{fi} = (z_{fi})^2 m_{f0}, \tag{1.14}$$

where the sector-dependent parameters z_{fi} are normalized as $(z_{f1})^2 + (z_{f2})^2 + (z_{f3})^2 = 1$. At present, we will not discuss whether the effective mass matrix form originates in a seesaw model or in a Frogatt-Nielsen model. Since our interest is only in the structure of z_{fi}, for convenience, in the present paper, we will investigate a possible Yukawa interaction form in the framework of the seesaw mass matrix model (1.8). The results in the present paper are also applicable to a Frogatt-Nielsen type model.

2 Mass eigenvalues

In general, an S_3 invariant Yukawa interaction with 3 scalars ϕ_a $(a = \pi, \eta, \sigma)$ is given by

$$H = \left(y_0 \frac{\bar{\psi}_\pi \psi_\pi + \bar{\psi}_\eta \psi_\eta + \bar{\psi}_\sigma \psi_\sigma}{\sqrt{3}} + y_1 \frac{\bar{\psi}_\pi \psi_\pi + \bar{\psi}_\eta \psi_\eta - 2\bar{\psi}_\sigma \psi_\sigma}{\sqrt{6}} \right) \phi_\sigma$$

$$+ y_2 \left(\frac{\bar{\psi}_\pi \psi_\eta + \bar{\psi}_\eta \psi_\pi}{\sqrt{2}} \phi_\pi + \frac{\bar{\psi}_\pi \psi_\pi - \bar{\psi}_\eta \psi_\eta}{\sqrt{2}} \phi_\eta \right)$$

$$+ y_3 \frac{\bar{\psi}_\pi \phi_\pi + \bar{\psi}_\eta \phi_\eta}{\sqrt{2}} \psi_\sigma + y_4 \bar{\psi}_\sigma \frac{\phi_\pi \psi_\pi + \phi_\eta \psi_\eta}{\sqrt{2}}, \tag{2.1}$$

where we read $\bar{\psi} = \bar{\ell}_L \equiv (\bar{\nu_L}, \ \bar{e}_L)$, $\psi = E_R$ and $\phi_a = \phi_a^d = (\phi_a^{d+}, \phi_a^{d0})$ for the charged lepton sector, and $\bar{\psi} = \bar{\ell}_L$, $\psi = N_R$ (or ν_R) and $\phi_a = \phi_a^u = (\phi_a^{u0}, \phi_a^{u-})$ for the neutrino sector. For example, the interaction (1.11) in the charged lepton sector corresponds to the case

$$y_0 = y_e, \quad y_1 = 0, \quad y_2 = \frac{1}{\sqrt{3}} y_e, \quad y_3 = y_4 = \sqrt{\frac{2}{3}} y_e. \tag{2.2}$$

The Yukawa interaction (2.1) gives the mass matrix m_L^f for the basis $(\psi_\pi, \psi_\eta, \psi_\sigma)$,

$$m_L^f = \begin{pmatrix} \left(\frac{y_0}{\sqrt{3}} + \frac{y_1}{\sqrt{6}}\right) v_\sigma + \frac{y_2}{\sqrt{2}} v_\eta & \frac{y_2}{\sqrt{2}} v_\pi & \frac{y_3}{\sqrt{2}} v_\pi \\ \frac{y_2}{\sqrt{2}} v_\pi & \left(\frac{y_0}{\sqrt{3}} + \frac{y_2}{\sqrt{6}}\right) v_\sigma - \frac{y_2}{\sqrt{2}} v_\eta & \frac{y_3}{\sqrt{2}} v_\eta \\ \frac{y_4}{\sqrt{2}} v_\pi & \frac{y_4}{\sqrt{2}} v_\eta & \left(\frac{y_0}{\sqrt{3}} - 2\frac{y_1}{\sqrt{6}}\right) v_\sigma \end{pmatrix}. \tag{2.3}$$

Hereafter, for simplicity, we confine ourselves to investigating a case with a symmetric mass matrix form $(m_L^f)^T = m_L^f$, i.e. with $y_3 = y_4$. Then, we have still 5 parameters, $y_0 v_\sigma, y_1 v_\sigma, y_2 v_\pi, y_3 v_\pi$ and v_π / v_η, in the model, so that the model has no predictability. In the present paper, we do not impose a further symmetry on the model. Alternatively, we will investigate what constraints on the mass matrix parameters (or specific relations among those) are required from the phenomenological studies.

Now let us return to the subject on the neutrino Dirac mass matrix m_L^ν for the basis $(\nu_\pi, \nu_\eta, \nu_\sigma)$ is, in general, given by the form (2.3). As we discussed in the previous section, the present neutrino oscillation date favor to the tribimaximal mixing, so that the neutrino states are approximately in the mass eigenstates $(\nu_\eta, \nu_\sigma, \nu_\pi)$ with $m_\eta^2 < m_\sigma^2 < m_\pi^2$. Therefore, for convenience, we investigate a case in the limit of $y_3 = y_4 = 0$. The mass matrix with $y_3 = y_4 = 0$ is diagonalized by a rotation

$$R(\theta_{\pi\eta}) = \begin{pmatrix} c_{\pi\eta} & s_{\pi\eta} & 0 \\ -s_{\pi\eta} & c_{\pi\eta} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.4}$$

where $c_{\pi\eta} = \cos\theta_{\pi\eta}$ and $s_{\pi\eta} = \sin\theta_{\pi\eta}$, and

$$\tan 2\theta_{\pi\eta} = -\frac{v_\pi}{v_\eta}, \tag{2.5}$$

as

$$R^T(\theta_{\pi\eta}) m_L^\nu R(\theta_{\pi\eta}) = \mathrm{diag}(m_\pi, m_\eta, m_\sigma). \tag{2.6}$$

The mass eigenvalues m_π, m_η and m_σ are given by

$$\begin{aligned} m_\pi &= \left(\tfrac{y_0}{\sqrt{3}} + \tfrac{y_1}{\sqrt{6}}\right) v_\sigma \pm \tfrac{|y_2|}{\sqrt{2}} \sqrt{v_\pi^2 + v_\eta^2}, \\ m_\eta &= \left(\tfrac{y_0}{\sqrt{3}} + \tfrac{y_1}{\sqrt{6}}\right) v_\sigma \mp \tfrac{|y_2|}{\sqrt{2}} \sqrt{v_\pi^2 + v_\eta^2}, \\ m_\sigma &= \left(\tfrac{y_0}{\sqrt{3}} - 2\tfrac{y_0}{\sqrt{6}}\right) v_\sigma, \end{aligned} \tag{2.7}$$

where we have defined

$$\sqrt{2} y_0 + y_1 > 0, \tag{2.8}$$

and the upper and lower signs in $\pm|y_2|$ (and also $\mp|y_2|$) correspond to the cases $y_2 v_\eta > 0$ and $y_2 v_\eta < 0$, respectively.

In the previous section, we have assumed that the VEVs v_i^d of the scalars ϕ_i^d, which couple to the charged leptons, satisfy the relation (1.12). Therefore, we also assume that the VEVs v_i^u of the scalar ϕ_i^u, which couple to the neutrino sector, satisfy the relation

$$(v_\pi^u)^2 + (v_\eta^u)^2 = (v_\sigma^u)^2 \equiv \frac{1}{2} v_u^2, \tag{2.9}$$

where we do not always assume $\langle\phi_i^u\rangle = \langle\phi_i^d\rangle$. Then, the mass eigenvalues (2.7) lead to

$$\begin{aligned} m_\pi &= \left(\tfrac{1}{\sqrt{6}} y_0 + \tfrac{1}{2\sqrt{3}} y_1 \pm \tfrac{1}{2}|y_2|\right) v_u, \\ m_\eta &= \left(\tfrac{1}{\sqrt{6}} y_0 + \tfrac{1}{2\sqrt{3}} y_1 \mp \tfrac{1}{2}|y_2|\right) v_u, \\ m_\sigma &= \left(\tfrac{1}{\sqrt{6}} y_0 - \tfrac{1}{\sqrt{3}} y_1\right) v_u. \end{aligned} \tag{2.10}$$

Note that the mass spectrum is independent of the parameters v_π^u/v_σ^u and v_η^u/v_σ^u, and only depends on the parameters y_1/y_0 and $|y_2|/y_0$. On the other hand, as seen in Eq.(2.5), the mixing angle $\theta_{\pi\eta}$ is independent of the parameters y_i and only depends on the parameter v_π^u/v_η^u.

As we discussed in Sec.1, the observed tribimaximal mixing suggests that the neutrino mass eigenstates are $(\nu_\eta, \nu_\sigma, \nu_\pi)$. If the mass hierarchy is a normal type, it demands $m_\eta^2 < m_\sigma^2 \ll m_\pi^2$, and if it is an inverse type, it demands $m_\pi^2 \ll m_\eta^2 < m_\sigma^2$. In order to check those cases, we estimate the differences among those masses as follows:

$$m_\pi^2 - m_\eta^2 = \pm\frac{1}{\sqrt{3}}|y_2|(\sqrt{2}y_0 + y_1)v_u^2, \tag{2.11}$$

$$m_\pi^2 - m_\sigma^2 = \frac{1}{4\sqrt{3}}(\sqrt{3}y_1 \pm |y_2|)(2\sqrt{2}y_0 - y_1 \pm \sqrt{3}|y_2|)v_u^2, \tag{2.12}$$

$$m_\eta^2 - m_\sigma^2 = \frac{1}{4\sqrt{3}}(\sqrt{3}y_1 \mp |y_2|)(2\sqrt{2}y_0 - y_1 \mp \sqrt{3}|y_2|)v_u^2. \tag{2.13}$$

Since we have defined the factor $(\sqrt{2}y_0+y_1)$ as positive in Eq.(2.8), Eq.(2.11) means that, for the case of the normal hierarchy with $m_\pi^2 > m_\eta^2$, we must take the upper signs in Eqs.(2.12)-(2.13), i.e.

$$m_\pi^2 - m_\sigma^2 = \frac{1}{4\sqrt{3}}(\sqrt{3}y_1 + |y_2|)(2\sqrt{2}y_0 - y_1 + \sqrt{3}|y_2|)v_u^2 > 0, \tag{2.14}$$

$$m_\eta^2 - m_\sigma^2 = \frac{1}{4\sqrt{3}}(\sqrt{3}y_1 - |y_2|)(2\sqrt{2}y_0 - y_1 - \sqrt{3}|y_2|)v_u^2 < 0, \tag{2.15}$$

and, for the case of the inverse hierarchy with $m_\pi^2 < m_\eta^2$, we must take the lower signs in Eqs.(2.12)-(2.13), i.e.

$$m_\pi^2 - m_\sigma^2 = \frac{1}{4\sqrt{3}}(\sqrt{3}y_1 - |y_2|)(2\sqrt{2}y_0 - y_1 - \sqrt{3}|y_2|)v_u^2 < 0, \tag{2.16}$$

$$m_\eta^2 - m_\sigma^2 = \frac{1}{4\sqrt{3}}(\sqrt{3}y_1 + |y_2|)(2\sqrt{2}y_0 - y_1 + \sqrt{3}|y_2|)v_u^2 < 0. \tag{2.17}$$

In conclusion, the model gives $m_\pi^2 > m_\eta^2$ or $m_\eta^2 > m_\pi^2$ according as $y_2 v_\eta > 0$ or $y_2 v_\eta < 0$. Since the observed neutrino mixing is approximately by the tribimaximal mixing, if we want to build a model with a normal mass hierarchy (or an inverse mass hierarchy), we must seek for a model with $m_\eta^2 < m_\sigma^2 < m_\pi^2$ (or $m_\pi^2 < m_\eta^2 < m_\sigma^2$). The conditions for $m_\eta^2 < m_\sigma^2 < m_\pi^2$ and $m_\pi^2 < m_\eta^2 < m_\sigma^2$ are given by Eqs.(2.14)-(2.15) and Eqs.(2.16)-(2.17), respectively.

By the way, we have still two adjustable parameters y_1/y_0 and y_2/y_0 to predict the neutrino mass spectrum. In the following sections, we will investigate two typical cases by putting assumptions for the coupling constants y_0, y_1 and y_2. Of course, the assumptions must also be applicable to the charged lepton coupling constants (2.2).

3 Case with $y_0^2 = y_1^2 + y_2^2$

In the mass matrix (2.3), the y_1- and y_2-terms are traceless, while the trace of the y_0-term is not zero. This suggests that the y_0-term may be distinguished from the other terms under a higher symmetry. Therefore, by way of trial, we put the following normalization condition for the coupling constants

$$y_0^2 = y_1^2 + y_2^2 + \frac{1}{2}(y_3^2 + y_4^2), \tag{3.1}$$

which is satisfied by the coupling constants (2.2) in the charged lepton sector. Since we have assumed that $y_3 = y_4 = 0$ in the neutrino sector, we can explicitly write the condition (3.1) as

$$y_1 = y_0 \sin\alpha, \quad y_2 = y_0 \cos\alpha. \tag{3.2}$$

Then, we can rewrite Eqs.(2.10) as

$$\begin{aligned} m_\pi &= \left[\tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}} \sin\left(\alpha \mp \tfrac{2}{3}\pi\right)\right] y_0 v_u, \\ m_\eta &= \left[\tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}} \sin\left(\alpha \pm \tfrac{2}{3}\pi\right)\right] y_0 v_u, \\ m_\sigma &= \left[\tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}} \sin\alpha\right] y_0 v_u, \end{aligned} \tag{3.3}$$

where $-\frac{\pi}{2} \leq \alpha \leq \frac{\pi}{2}$ $(\cos\alpha > 0)$, and, for $\frac{\pi}{2} \leq \alpha < \frac{3}{2}\pi$, we substitute $\pi - \alpha$ for α in Eq.(3.3).

Note that the case with the condition (3.1) which leads to Eq.(3.3) gives the relation

$$m_\pi^2 + m_\eta^2 + m_\sigma^2 = \frac{2}{3}(m_\pi + m_\eta + m_\sigma)^2. \tag{3.4}$$

Since these masses $(m_\pi, m_\eta, m_\sigma)$ are Dirac masses in the neutrino seesaw mass matrix $M_\nu = m_L^\nu M_N^{-1} (m_L^\nu)^T$, if we take the heavy Majorana mass matrix M_N with the unit matrix form, we obtain the neutrino masses which are proportional to m_π^2, m_η^2 and m_σ^2, respectively. Therefore, the neutrino masses will satisfy the relation (1.7) which has speculated by Brannen [10].

The expressions (2.11)-(2.13) become

$$m_\pi^2 - m_\eta^2 = \pm\frac{1}{\sqrt{3}} \cos\alpha(\sqrt{2} + \sin\alpha) y_0^2 v_u^2, \tag{3.5}$$

$$m_\pi^2 - m_\sigma^2 = \pm\frac{1}{\sqrt{3}} \cos\left(\alpha \mp \frac{\pi}{3}\right) \left[\sqrt{2} - \sin\left(\alpha \mp \frac{\pi}{\sqrt{3}}\right)\right] y_0^2 v_u^2, \tag{3.6}$$

$$m_\eta^2 - m_\sigma^2 = \mp\frac{1}{\sqrt{3}} \cos\left(\alpha \pm \frac{\pi}{3}\right) \left[\sqrt{2} - \sin\left(\alpha \pm \frac{\pi}{3}\right)\right] y_0^2 v_u^2, \tag{3.7}$$

where $|\alpha| < \pi/2$. For a case with a normal hierarchy, as seen in Eq.(2.22), we should read the upper signs in Eqs.(3.5)-(3.7), so that we obtain

$$m_\eta^2 < m_\sigma^2 < m_\pi^2 \quad \text{for} \quad -\frac{\pi}{6} < \alpha < \frac{\pi}{6}. \tag{3.8}$$

For a case with an inverse hierarchy, since we should read the lower signs in Eqs.(3.5)-(3.7), we obtain

$$m_\pi^2 < m_\eta^2 < m_\sigma^2 \quad \text{for} \quad -\frac{\pi}{2} < \alpha < -\frac{\pi}{6}. \tag{3.9}$$

Next, let us seek for the numerical value of α which gives the ratio of the observed values $\Delta m^2_{solar} = (7.9^{+0.6}_{-0.5}) \times 10^{-5}$ eV^2 [12] to $\Delta m^2_{atm} = (2.72^{+0.38}_{-0.25}) \times 10^{-3}$ eV^2 [13],

$$R_{obs} \equiv \frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} = (2.9 \pm 0.5) \times 10^{-2}. \tag{3.10}$$

From the numerical study of

$$R(\alpha) \equiv \frac{m_\sigma^4(\alpha) - m_\eta^4(\alpha)}{m_\pi^4(\alpha) - m_\sigma^4(\alpha)}, \tag{3.11}$$

we find

$$\alpha = \left(3.0^{-1.2}_{+1.4}\right)^\circ, \tag{3.12}$$

where the sign $\mp$ corresponds to the sign $\pm$ of the experimental error in Eq.(3.10), as a normal hierarchical solution. However, we could not find a solution with an inverse hierarchy.

The solution $\alpha = (3.0^{-1.2}_{+1.4})^\circ$ gives

$$\begin{aligned} m_\eta &= -(0.076^{+0.006}_{-0.008}) y_0 v_u, \\ m_\sigma &= (0.38 \pm 0.01) y_0 v_u, \\ m_\pi &= (0.923 \mp 0.006) y_0 v_u. \end{aligned} \tag{3.13}$$

The result $m_\eta < 0$ leads to the change of sign $\sqrt{m_{\nu 1}} \to -\sqrt{m_{\nu 1}}$ in Eq.(1.13) which has been speculated by Brannen [10]. The values (3.13) predicts the following neutrino masses

$$\begin{aligned} m_{\nu 1} &= (3.5 \pm 0.5) \times 10^{-4} \text{ eV}, \\ m_{\nu 2} &= (8.7 \pm 0.2) \times 10^{-3} \text{ eV}, \\ m_{\nu 3} &= (5.22^{-0.25}_{+0.35}) \times 10^{-2} \text{ eV}. \end{aligned} \tag{3.14}$$

Generally, when masses $m_{fi} = z_{fi}^2 m_{f0}$ $(i = 1, 2, 3)$ satisfy the relation (1.7) [or (1.13)], the parameters z_{fi} can always be expressed by the form

$$\begin{aligned} z_{f1} &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}} \sin \xi_f, \\ z_{f2} &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}} \sin(\xi_f + \tfrac{2}{3}\pi), \\ z_{f3} &= \tfrac{1}{\sqrt{6}} - \tfrac{1}{\sqrt{3}} \sin(\xi_f + \tfrac{4}{3}\pi), \end{aligned} \tag{3.15}$$

where we have taken $z_{f1}^2 < z_{f2}^2 < z_{f3}^2$. From the observed charged lepton mass values, we obtain the numerical value of ξ_e

$$\xi_e = \frac{\pi}{4} - \varepsilon = 42.7324^\circ \quad (\varepsilon = 2.2676^\circ). \tag{3.16}$$

Note that, in the limit of $\varepsilon \to 0$, the electron mass becomes zero. We consider that the parameter ε is a fundamental parameter which governs the charged lepton mass spectrum.

Recently, Brannen [10] has speculated that the value of ξ_ν is given by

$$\xi_\nu = \xi_e + \frac{\pi}{12}, \tag{3.17}$$

which predicts

$$\xi_\nu = 57.7324^\circ. \tag{3.18}$$

Comparing the expression (3.3) (with the upper signs) with the expression (3.15), we find that the parameter α is connected to ξ_ν by the relation

$$\alpha = \frac{\pi}{3} - \xi_\nu. \tag{3.19}$$

Therefore, we obtain

$$\xi_\nu - \xi_e = \left(\frac{\pi}{3} - \alpha\right) - \left(\frac{\pi}{4} - \varepsilon\right) = \frac{\pi}{12} + \varepsilon - \alpha. \tag{3.20}$$

Since the value of α, (3.12), which is a solution of $R(\alpha) = R_{obs}$, is very close to the value $\varepsilon = 2.27^\circ$ from the observed charged lepton masses, if we regard α as $\alpha = \varepsilon$, we obtain the relation (3.17). Thus, the speculation (3.17) is acceptable phenomenologically.

However, this does not mean that the present model gives a basis for Brannen's speculation (3.17). In the present model with $y_3 = y_4$, as discussed in Sec.2, the mass spectrum of the neutrinos is independent of the VEVs v_i^u, and it depends only on the values y_0, y_1 and y_2. On the other hand, the charged lepton mass spectrum depends only on the VEVs v_i^d, because we have assumed the universality of the Yukawa coupling constants. The parameter ξ_e (therefore, ε) is one which characterizes the VEV spectrum (v_1^d, v_2^d, v_3^d), while the parameters ξ_ν (therefore, α) in the present model is one which characterizes the structure of the neutrino Yukawa coupling constants. Therefore, the parameter ξ_ν is different in kind from the parameter ξ_e. At present, it is an open question whether the coincidence $\alpha \simeq \varepsilon$ is accidental or not.

4 Case with $y_0^2 + y_1^2 = y_2^2$

In the previous section, we have assumed a constraint (3.1) on the Yukawa coupling constants y_0, y_1 and y_2. However, the theoretical basis of the constraint is not clear. In the present section, instead of the constraint (3.1), we assume another constraint

$$y_0^2 + y_1^2 = y_2^2 + \frac{1}{2}(y_3^2 + y_4^2), \tag{4.1}$$

which is again satisfied by the Yukawa coupling constants (2.2) in the charged lepton sector. The condition (4.1) means a requirement of the universality of the coupling constants in an extended meaning: individually normalized coupling constants of scalars ϕ_σ and ϕ_π (ϕ_η) are equal to each other.

In the neutrino sector, since we have assumed $y_3 = y_4 = 0$, we can denote the condition (4.1) as

$$y_0 = y_2 \cos\beta, \quad y_1 = y_2 \sin\beta. \tag{4.2}$$

Then, the mass eigenvalues (2.10) are expressed as follows :

$$\begin{aligned} m_\pi &= \tfrac{1}{2}\left[\sin(\beta+\phi_0) \pm 1\right] |y_2| v_u, \\ m_\eta &= \tfrac{1}{2}\left[\sin(\beta+\phi_0) \mp 1\right] |y_2| v_u, \\ m_\sigma &= \tfrac{1}{\sqrt{2}} \cos(\beta+\phi_0) y_2 v_u, \end{aligned} \tag{4.3}$$

where

$$\sin\phi_0 = \sqrt{\frac{2}{3}}, \quad \cos\phi_0 = \frac{1}{\sqrt{3}}, \quad (\phi_0 = 54.74^\circ), \tag{4.4}$$

we have again took the condition (2.8), i.e.

$$\sin(\beta+\phi_0) > 0 \ \ (-\phi_0 < \beta < \pi - \phi_0), \tag{4.5}$$

and the upper and lower signs in Eq.(4.3) correspond to the cases $y_2 v_\eta > 0$ (a normal hierarchy case) and $y_2 v_\eta < 0$ (an inverse hierarchy case), respectively.

From the expression (4.3), we find

$$m_\pi^2 + m_\eta^2 + m_\sigma^2 = y_2^2 v_u^2, \tag{4.6}$$

$$m_\eta + m_\sigma + m_\pi = \sqrt{\frac{3}{2}} y_2 v_u \cos\beta. \tag{4.7}$$

Therefore, we obtain

$$\frac{\frac{2}{3}(m_\pi + m_\eta + m_\sigma)^2}{m_\pi^2 + m_\eta^2 + m_\sigma^2} = \cos^2\beta = 1 - \sin^2\beta. \tag{4.8}$$

Thus, the parameter β in the present model denotes a deviation from the mass formula (1.13) [(1.7)].

Note that if we find a solution $\beta = \beta_1$ which gives $R(\beta) = R_{obs}$ [$R(\beta)$ is given by Eq.(3.11) with $\alpha \to \beta$, and R_{obs} is given by Eq.(3.10)], the value $\beta_2 = 2\phi_0 - \beta_1$ [ϕ_0 is defined by Eq.(4.4)] is also a solution of $R(\beta) = R_{obs}$. From the expression (4.3), it is obvious that the solutions β_1 and β_2 give the same values for m_π and m_η, but they give the values with the opposite signs to each other for m_σ. We list those solutions of $R(\beta) = R_{obs}$ in Table 1, together with the values of m_η, m_σ and m_π.

In Table 1, we also list the predicted values of the neutrino masses $m_{\nu 1} = m_\eta^2/M_N$, $m_{\nu 2} = m_\sigma^2/M_N$ and $m_{\nu 3} = m_\pi^2/M_N$ (M_N is a Majorana mass $M_N \equiv M_{N1} = M_{N2} = M_{N3}$ of the heavy neutrinos N_i). Here, as the input value, we have used $m_{\nu 3} = \sqrt{\Delta m^2_{atm}} = 0.0522$ eV for the normal hierarchy case, and $m_{\nu 2} = \sqrt{\Delta m^2_{atm}} = 0.0522$ eV for the inverse hierarchy case. At present, the numerical values of $m_{\nu i}$ should not be taken rigidly. Therefore, we have omitted the error values from Table 1.

5 Neutrino mixing matrix

As we discussed in Sec.2, the additional rotation $R(\theta_{\pi\eta})$ from the tribimaximal mixing, (2.4), depends only on the value v^u_π/v^u_η, and it is independent of the values of y_0, y_1 and y_2, i.e. of the mass eigenvalues. If we want that the neutrino mixing is purely the tribimaximal mixing, we must take $v^u_\pi/v^u_\eta = 0$, i.e. $v^u_\eta = v^u_\sigma$. However, such a model in which the VEVs of the up-scalars ϕ^u_i can completely be unrelated to those of the down-scalars ϕ^d_i is not so attractive to us. We expect that $\langle\phi^u_i\rangle$ will have some relation to $\langle\phi^d_i\rangle$.

In order to see the effects of the additional rotation $R(\theta_{\pi\eta})$ defined by Eq.(2.6), we change from the basis $(\nu_\pi, \nu_\eta, \nu_\sigma)$ defined by Eq.(1.4) into the basis $(\nu_\eta, \nu_\sigma, \nu_\pi)$ given by

$$\begin{pmatrix} \nu_\eta \\ \nu_\sigma \\ \nu_\pi \end{pmatrix} = U^T \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix}, \tag{5.1}$$

where U is the tribimaximal mixing matrix defined by Eq.(1.1). If $v_\pi/\nu_\eta \neq 0$, i.e. $R(\theta_{\pi\eta}) \neq \mathbf{1}$, the neutrino mixing matrix U_ν is given by

$$U_\nu = U \begin{pmatrix} c_{\pi\eta} & 0 & s_{\pi\eta} \\ 0 & 1 & 0 \\ -s_{\pi\eta} & 0 & c_{\pi\eta} \end{pmatrix} = \begin{pmatrix} -\frac{2}{\sqrt6}c_{\pi\eta} & \frac{1}{\sqrt3} & -\frac{2}{\sqrt6}s_{\pi\eta} \\ \frac{1}{\sqrt6}c_{\pi\eta} + \frac{1}{\sqrt2}s_{\pi\eta} & \frac{1}{\sqrt3} & \frac{1}{\sqrt6}s_{\pi\eta} - \frac{1}{\sqrt2}c_{\pi\eta} \\ \frac{1}{\sqrt6}c_{\pi\eta} - \frac{1}{\sqrt2}s_{\pi\eta} & \frac{1}{\sqrt3} & \frac{1}{\sqrt6}s_{\pi\eta} + \frac{1}{\sqrt2}c_{\pi\eta} \end{pmatrix}, \tag{5.2}$$

i.e.

$$\tan^2\theta_{solar} = \frac{1}{2c^2_{\pi\eta}}, \tag{5.3}$$

$$\sin^2 2\theta_{atm} = \left(1 - \frac{4}{3}s^2_{\pi\eta}\right)^2, \tag{5.4}$$

$$(U_\nu)^2_{13} = \frac{2}{3}s^2_{\pi\eta}. \tag{5.5}$$

For convenience, we define the following z_i-parameters

$$\langle\phi^u_i\rangle = z^u_i v_u, \quad \langle\phi^d_i\rangle = z^d_i v_d, \tag{5.6}$$

with the normalizations $\sum_i (z^u_i)^2 = \sum_i (z^d_i)^2 = 1$. For the z^d_i-parameters, from the relation (1.7), we obtain

$$\frac{z^d_1}{\sqrt{m_e}} = \frac{z^d_2}{\sqrt{m_\mu}} = \frac{z^d_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}}, \tag{5.7}$$

i.e.

$$z^d_1 = 0.016473, \quad z^d_2 = 0.23678, \quad z^d_3 = 0.97140. \tag{5.8}$$

If we assume $z^u_i = z^d_i (i = 1, 2, 3)$, we obtain $z^u_\pi = 0.51939$, $z^u_\eta = 0.47982$ and $z^u_\sigma = 1/\sqrt2$ from the definition (1.4). Then, the rotation angle $\theta_{\pi\eta} = -(1/2)\tan^{-1}(v^u_\pi/v^u_\eta) = -23.63^\circ$ is too large to explain the observed neutrino mixings [see Eqs.(5.3)-(5.5)], so that the case is ruled out.

As we discussed in Sec.1, the S_3 bases A and B are dual each other. Although we have investigated the neutrino mass matrix form on the basis B, as the second best idea instead of the case $(z^u_\pi, z^u_\eta, z^u_\sigma) = (z^d_\pi, z^d_\eta, z^d_\sigma)$, it is likely that the VEVs $(v^u_\pi, v^u_\eta, v^u_\sigma)_B$ on the basis B is given by

$$\begin{pmatrix} z^u_\pi \\ z^u_\eta \\ z^u_\sigma \end{pmatrix}_B = A \begin{pmatrix} z^d_1 \\ z^d_2 \\ z^d_3 \end{pmatrix} = B \begin{pmatrix} z^d_3 \\ z^d_2 \\ z^d_1 \end{pmatrix}, \tag{5.9}$$

contrast to the VEVs of the down-type scalars ϕ^d on the basis B, i.e. $(z^d_\pi, z^d_\eta, z^d_\sigma)^T_B = B(z^d_1, z^d_2, z^d_3)^T$. When we assume the relation (5.9), we obtain

$$z^u_\pi = 0.15584, \quad z^u_\eta = -0.68972, \quad z^u_\sigma = 1/\sqrt{2}, \tag{5.10}$$

$$\tan 2\theta_{\pi\eta} = 0.2260, \quad \theta_{\pi\eta} = 6.366^\circ, \tag{5.11}$$

so that we obtain the predictions

$$\tan^2\theta_{solar} = 0.5062, \tag{5.12}$$

$$\sin^2 2\theta_{atm} = 0.9675, \tag{5.13}$$

$$(U_\nu)^2_{13} = 0.00820. \tag{5.14}$$

At present, the values (5.12)–(5.14) cannot be ruled out by the observed neutrino oscillation data [12, 13], so that the possibility (5.9) is acceptable. However, of course, the relation (5.9) is only a speculation at present.

6 Concluding remarks

In conclusion, based on a universal seesaw mass matrix model with three scalars ϕ_i, and by assuming an S_3 flavor symmetry for Yukawa interactions, we have investigated the neutrino masses and mixings. For the VEV values of ϕ^f_i ($f = u, d$), stimulated from a Higgs potential model [9] for ϕ_i, we have assumed the constraint

$$\langle\phi^f_\pi\rangle^2 + \langle\phi^f_\eta\rangle^2 = \langle\phi^f_\sigma\rangle^2, \tag{6.1}$$

where $(\phi_\pi, \phi_\eta, \phi_\sigma)$ are defined by Eq.(1.4). Since the observed neutrino mixing is approximately given by the tribimaximal mixing (1.1), we have investigated only a simple case with ν_π-ν_η mixing, where (ν_π, ν_η) is a doublet of S_3. In the case, the mass eigenvalues depends only on the values of the coupling constants y_0, y_1 and y_2, while the ν_π-ν_η mixing angle $\theta_{\pi\eta}$ depends only on the value of $\langle\phi^u_\pi\rangle/\langle\phi^u_\eta\rangle$.

From the economical point of view of the parameter number, we have investigate two typical cases with the constraints $y_0^2 = y_1^2 + y_2^2$ and $y_0^2 + y_1^2 = y_2^2$. The former case leads to a case which satisfies Brannen's relation (1.13) for the neutrino masses. Although it is very interesting, the theoretical basis of the constraint $y_0^2 = y_1^2 + y_2^2$ is not so clear. On the other hand, the later case cannot satisfy the relation (1.13). However, for a small value of the parameter β, the deviation from the relation (1.13) is negligibly small. For example, for the solution $\beta = 2.94^\circ$ given in

Table 1, the deviation from the relation (1.13) is very small, $\sin^2\beta = 0.003$, as seen in Eq.(4.8), so that the relation (1.13) is approximately satisfied.

In any case, in order to fit the present data from the neutrino oscillation experiments, the existence of the y_1-term with a small coupling constant. Although we have given the "prediction" of the neutrino masses in Secs.3 and 4, exactly speaking, those are not predictions. Those are results by adjusting the parameter α (or β).

Since we consider that the mass matrix structures in the leptons should be as simple as possible, we may speculate a concise structure [14] of the Yukawa interaction in the neutrino sector:

$$H_\nu = y_\nu \left(\frac{\overline{\ell}_\pi N_\pi + \overline{\ell}_\eta N_\eta + \overline{\ell}_\sigma N_\sigma}{\sqrt{3}} \phi_\sigma^u + \frac{\overline{\ell}_\pi N_\eta + \overline{\ell}_\eta N_\pi}{\sqrt{2}} \phi_\pi^u + \frac{\overline{\ell}_\pi N_\pi - \overline{\ell}_\eta N_\eta}{\sqrt{2}} \phi_\eta^u \right), \tag{6.2}$$

where we have required the universality of the coupling constants as well as in the charged lepton sector [however, not in the basis (ψ_1, ψ_2, ψ_3), but in the basis $(\psi_\pi, \psi_\eta, \psi_\sigma)$]. Then, the neutrino masses are predicted as

$$\begin{aligned} m_{\nu 1} &= \left(\tfrac{1}{\sqrt{6}} - \tfrac{1}{2} \right)^2 m_0^\nu, \\ m_{\nu 2} &= \tfrac{1}{6} m_0^\nu, \\ m_{\nu 3} &= \left(\tfrac{1}{\sqrt{6}} + \tfrac{1}{2} \right)^2 m_0^\nu, \end{aligned} \tag{6.3}$$

without an adjustable parameter. The case predicts

$$R = \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{4\sqrt{6} - 9}{4\sqrt{6} + 9} = 0.0423. \tag{6.4}$$

The value (6.4) is somewhat large comparing with the observed value (3.10), but, at present, the case is not ruled out within three sigma. Again, regarding $m_{\nu 3}$ as $m_{\nu 3} = \sqrt{\Delta m^2_{atm}}$, we predict the explicit neutrino mass values as follows:

$$\begin{aligned} m_{\nu 1} &= (5.3^{+0.4}_{-0.3}) \times 10^{-4} \text{ eV}, \\ m_{\nu 2} &= (1.05^{+0.07}_{-0.05}) \times 10^{-2} \text{ eV}, \\ m_{\nu 3} &= (5.22^{+0.35}_{-0.25}) \times 10^{-2} \text{ eV}. \end{aligned} \tag{6.5}$$

The case (6.2) is also interesting because of the simpleness of its structure.

By the way, in the neutrino Yukawa interaction (6.2), by assuming the universality of the coupling constants, we have taken $y_0 = y_2 = y_\nu$. However, as seen in Eq.(5.10), the case gives $v_\eta^u < 0$ for the choice (5.9) on the basis B defined by Eq.(1.4). From the discussion in Sec.2, it seems that the case $y_2 = y_0 > 0$ gives $y_2 v_\eta < 0$, so that the case leads not to a normal hierarchy case ($m_\pi^2 > m_\eta^2$), but to an inverse hierarchy case ($m_\pi^2 < m_\eta^2$). Therefore, it seems that we must choose not the case $y_2 = y_0 > 0$, but the case $y_2 = -y_0 < 0$. However, this is not serious problem. When we redefine the bases A and B by the alternative matrices

$$A' = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \tag{6.6}$$

$$B' = \begin{pmatrix} 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{2}{\sqrt{6}} & -\frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}, \tag{6.7}$$

instead of A and B defined by Eqs.(1.3) and (1.5), respectively, the signs of the fields ψ_η $(\phi_\eta, \nu_\eta, \cdots)$ are changed, so that we obtain $v_\eta > 0$. Then, we also obtain the trimaximal mixing

$$U = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{6.8}$$

instead of the form (1.1). This phase change $v_\eta \to -v_\eta$ does not affect to the results (5.3)-(5.5), because the sign of $\theta_{\pi\eta}$ is also changed together with $(\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}) \to (-\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}})$.

Although the case with $v_\pi^u = 0$ which exactly leads to the tribimaximal mixing (1.1) is attractive to us, at present, there is no reason for $v_\pi^u = 0$. Rather, the case (5.9) is attractive although it is still speculative. Then, we can predict small deviations (5.12), (5.13) and (5.14) from the ideal tribimaximal mixing (1.1). At present, there is no reason for such a permuted VEV relation (5.9) between (v_1^u, v_2^u, v_3^u) and (v_1^d, v_2^d, v_3^d). However, from the phenomenological point of view, it will be worth while investigating quark mass matrices with such a permuted VEV relation. The study will be given elsewhere.

In conclusion, the present model (a lepton mass matrix model with a bilinear form) based on the S_3 symmetry has given many interesting features for the mass spectra and mixings. However, the model still includes some adjustable parameters. Further investigation based on another symmetry which gives stronger constraints on the parameters than those in the S_3 symmetry will be desired.

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Table 1 Solution of $R(\beta) = R_{obs}$

β	$m_\eta/y_2 v_u$	$m_\sigma/y_2 v_u$	$m_\pi/y_2 v_u$	$m_{\nu 1}$ [eV]	$m_{\nu 2}$ [eV]	$m_{\nu 3}$ [eV]
$2.94°$	-0.0775	0.3781	0.9225	0.000368	0.00877	0.0522
$67.59°$	-0.0775	-0.3781	0.9225	0.000368	0.00877	0.0522
$-35.64°$	0.6636	0.6682	-0.3364	0.0515	0.0522	0.0132
$106.17°$	0.6636	-0.6682	-0.3364	0.0515	0.0522	0.0132