

Challenge to the Mystery of the Charged Lepton Mass Formula ¹

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Abstract

Why the charged lepton mass formula $m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$ is mysterious is reviewed, and guiding principles to solve the mystery are presented. According to the principles, an example of such a mass generation mechanism is proposed, where the origin of the mass spectrum is attributed not to the structure of the Yukawa coupling constants, but to a structure of vacuum expectation values of flavor-triplet scalars under $Z_4 \times S_3$ symmetries.

1. Introduction

It is widely accepted that quarks and leptons are fundamental entities of the matter. If it is true, the masses and mixings of the quarks and leptons will obey a simple law of nature, and we will be able to find a beautiful relation among those values. Nowadays, for all of the 3 family quarks, we know their mass values, but the accuracy of those values is still somewhat unsatisfactory for testing the validity of the model rigorously. The experimental situation in the neutrino masses and mixings is also not in the satisfactory accuracy. In contrast to quarks and neutrinos, for the charged leptons, we know their mass values with sufficient accuracy. If we can find a beautiful mass (and mixing) relation, it will make a breakthrough in the unified understanding of the quarks and leptons.

In 1992, the observed tau lepton mass value was revised by new experiments [1] as

$$m_\tau^{old} = 1784 \pm 4 \text{ MeV} \implies m_\tau^{new} = 1776.99_{-0.26}^{+0.29} \text{ MeV}. \quad (1.1)$$

(The new value has been quoted from Ref. [2].) Since the new value $m_\tau = 1777 \text{ MeV}$ has already been predicted by a charged lepton mass formula [3, 4, 5]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2, \quad (1.2)$$

the mass formula had received considerable attention at one time. Indeed, the formula (1.2) predicts the tau lepton mass value

$$m_\tau = 1776.97 \text{ MeV}, \quad (1.3)$$

from the observed electron and muon mass values [2] $m_e = 0.51099892 \text{ MeV}$ and $m_\mu = 105.658369 \text{ MeV}$. The predicted value (1.3) is in excellent agreement with the observed value (1.1) [2]. The

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excellent agreement seems to be beyond a matter of accidental coincidence, so that we should consider the origin of the mass formula (1.2) seriously. However, up to the present, the theoretical basis of the mass formula (1.2) is still not clear.

The formula was first found [3] in 1982 on the basis of a composite model of quarks and leptons. Here, we have assumed that the charged lepton masses m_{ei} are described as

$$m_{ei} = m_0(z_i + z_0)^2, \quad (1.4)$$

where

$$z_1 + z_2 + z_3 = 0, \quad (1.5)$$

$$z_0 = \frac{1}{\sqrt{3}} \sqrt{z_1^2 + z_2^2 + z_3^2}. \quad (1.6)$$

However, such the scenario based on a composite model has not been justified from the field theoretical point of view. The explicit expression (1.2) was given in Ref. [5]. Here, a mixing between octet and singlet states in the U(3) family symmetry has been assumed. Since 1993, several authors [6, 7, 8] have challenged to give an explanation of the mass formula (1.2), but, at present, there is no convinced one.

2. How the formula is mysterious

The charged lepton mass formula (1.2) has the following peculiar features:

- (a) The mass formula is described in terms of the root squared masse $\sqrt{m_{ei}}$.
- (b) The mass formula is invariant under the exchanges $\sqrt{m_{ei}} \leftrightarrow \sqrt{m_{ej}}$. We know that the electron mass m_e is negligibly small compared with other charged lepton masses. If we put $m_e = 0$ in the formula (1.2), we will obtain a wrong prediction $m_\tau = [(\sqrt{3} + 1)/(\sqrt{3} - 1)]^2 m_\mu = 1471.63$ MeV instead of (1.3). Thus, the non-zero value of m_e is essential in the formula (1.2).
- (c) The formula gives a relation between mass ratios $\sqrt{m_e/m_\mu}$ and $\sqrt{m_\mu/m_\tau}$, whose behaviors under the renormalization group equation (RGE) effects are different from each other. Therefore, the formula (1.2) is not invariant under the RGE effects. The formula is well satisfied at a low energy scale rather than at a high energy scale.

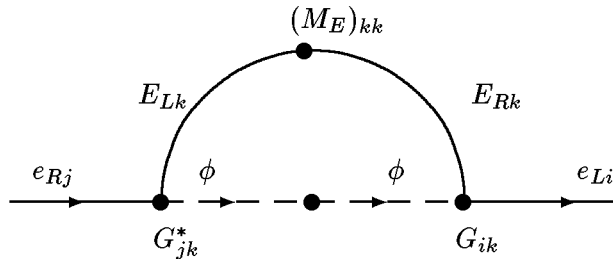


Fig. 1 Radiative mass generation of the charged leptons

Suggestion (A): The feature (a) suggests that the charged lepton mass spectrum is not originated in the Yukawa coupling structure at the tree level, but it is given by a bilinear form

on the basis of a some mass generation mechanism. For example, in Refs. [3, 4], a radiative-mass-generation-like mechanism shown in Fig. 1 has been assumed:

$$(M_e)_{ij} = m_0 \sum_k G_{ik} G_{jk}^* , \quad (2.1)$$

where G_{ij} are coupling constants of the interactions $\bar{e}_i E_j \phi$. On the other hand, in Refs. [5, 9, 10, 11], a seesaw-like mechanism [12] shown in Fig. 2 has been assumed:

$$M_e = m M_E^{-1} m^\dagger . \quad (2.2)$$

In any cases, we need hypothetical heavy charged leptons E .

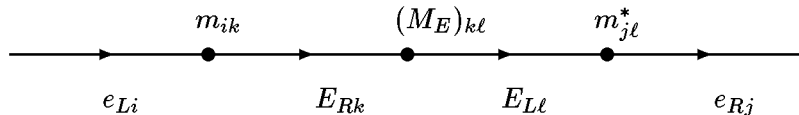


Fig. 2 Seesaw-like mass generation of m_{ei}

Suggestion (B): The feature (b) suggests that the theory is invariant under the permutation symmetry S_3 . As an example of the S_3 invariant mass matrix, the so-called democratic mass matrix [13] is well known. The derivation of (1.2) in Ref. [14] is based on a democratic mass matrix model. However, in the present paper, as we review in the next section, we will adopt another idea, where what is essential is not a structure of the Yukawa coupling constants, but a structure of the vacuum expectation values (VEVs) of flavor-triplet Higgs scalars.

Suggestion (C): The feature (c) suggests that the mechanism which leads to the relation (1.2) must work at a low energy scale. In the conventional model, the mass matrix structure is due to the Yukawa coupling structure, which is given at the unification energy scale $\mu = M_{GUT}$. The mass spectrum at a low energy scale must be evaluated by taking the RGE effects into consideration. Against such the conventional models, the idea that mass spectrum is due to the VEV structure of Higgs scalars at a low energy scale is very attractive as an explanation of the non-RGE-invariant mass formula.

3. S_3 symmetry and VEV of flavor-triplet-scalars

The idea to relate the VEVs structure to a mass matrix model has first been proposed in 1990 [5] (and also see [10]) although the model was based not on the S_3 symmetry, but on a $U(3)$ symmetry. A model based on the S_3 symmetry has been investigated in 1999 [11].

The basic idea is as follows: We consider the following S_3 invariant Higgs potential

$$V = \mu^2 \sum_i (\bar{\phi}_i \phi_i) + \frac{1}{2} \lambda \left[\sum_i (\bar{\phi}_i \phi_i) \right]^2 + \eta (\bar{\phi}_\sigma \phi_\sigma) (\bar{\phi}_\pi \phi_\pi + \bar{\phi}_\eta \phi_\eta), \quad (3.1)$$

where $(\bar{\phi}_i \phi_i) = \phi_i^- \phi_i^+ + \bar{\phi}_i^0 \phi_i^0$, and

$$\begin{aligned}\phi_\pi &= \frac{1}{\sqrt{2}}(\phi_1 - \phi_2) , \\ \phi_\eta &= \frac{1}{\sqrt{6}}(\phi_1 + \phi_2 - 2\phi_3) , \\ \phi_\sigma &= \frac{1}{\sqrt{3}}(\phi_1 + \phi_2 + \phi_3) .\end{aligned}\tag{3.2}$$

(For more general S_3 -invariant Higgs potential, see Ref. [11]. Even under a more general S_3 -invariant potential (but with some constraints), the relation (3.7) given below is unchanged.) The conditions for the VEVs $v_i \equiv \langle \phi_i^0 \rangle$ at which the potential (3.2) takes the minimum are

$$\mu^2 + \lambda \sum_i |v_i|^2 + \eta (|v_\pi|^2 + |v_\eta|^2) = 0 ,\tag{3.3}$$

$$\mu^2 + \lambda \sum_i |v_i|^2 + \eta |v_\sigma|^2 = 0 ,\tag{3.4}$$

so that we obtain

$$|v_\sigma|^2 = |v_\pi|^2 + |v_\eta|^2 = \frac{-\mu^2}{2\lambda + \eta} .\tag{3.5}$$

Therefore, from the relation

$$\bar{\phi}_1 \phi_1 + \bar{\phi}_2 \phi_2 + \bar{\phi}_3 \phi_3 = \bar{\phi}_\pi \phi_\pi + \bar{\phi}_\eta \phi_\eta + \bar{\phi}_\sigma \phi_\sigma ,\tag{3.6}$$

we obtain

$$|v_1|^2 + |v_2|^2 + |v_3|^2 = |v_\pi|^2 + |v_\eta|^2 + |v_\sigma|^2 = 2|v_\sigma|^2 = 2 \left(\frac{v_1 + v_2 + v_3}{\sqrt{3}} \right)^2 .\tag{3.7}$$

If we consider a model in which the charged lepton masses are given by

$$\sum_i \bar{e}_i \langle \bar{\phi}_i^0 \rangle \langle \phi_i^0 \rangle e_i ,\tag{3.8}$$

we can obtain the charged lepton mass formula (1.2). Note that the formula (1.2) can be derived independently of the values of λ and η . The relation (3.7) holds at the $SU(2)_L$ symmetry breaking energy scale M_W , the formula (1.2) is also valid at $\mu = M_W$.

However, this scenario has some troubles. We know that the mass terms $m_{ei}(\bar{e}_{Li} e_{Ri} + \bar{e}_{Ri} e_{Li})$ are $\Delta I = 1/2$, while Eq. (3.8) will be come from $\Delta I = 1$ terms. Besides, in this model, there are 3-family Higgs scalars, so that they cause, in principle, flavor changing neutral currents (FCNC). Moreover, if we wish to build a GUT model, the 3 family scalars affect on the RGE effects dangerously, so that the beautiful unification of the gauge coupling constants, $g_1 = g_2 = g_3$, at $\mu = M_{GUT}$ in the minimal SUSY GUT model will be spoiled.

A most straightforward improvement of this model will be to change the 3-family $SU(2)_L$ -doublet scalars ϕ_i into 3-family $SU(2)_L$ -singlet scalars ϕ_i^0 . When we introduce additional heavy fermions $(5'_L + 10'_L)_{(+)}$ and flavor-triplet $SU(5)$ -singlet scalars $\phi_{(-)}$ in addition to the quarks and leptons $(\bar{5}_L + 10_L)_{(-)}$ and Higgs fields $\bar{H}_{d(-)} + H_{u(+)}$, where $(+)$ and $(-)$ denote the Z_2 charges, the charged lepton masses are given by a seesaw form

$$(M_e)_{ij} \simeq \delta_{ij} \langle \phi_i^0 \rangle \langle H_u^0 \rangle^{-1} \langle \phi_j^0 \rangle \quad (3.9)$$

as shown in Fig. 3. (Here, we have assumed the Z_2 symmetry in order to forbid the direct coupling of \bar{H}_d to $\bar{5}_L 10_L$, so that the leading terms are the seesaw mass terms given in (3.9).) However, if the additional fermions are indeed only $(5'_L + \bar{10}'_L)$, the fermions cannot be heavy, so that the 3-family fermions bring serious troubles into the theory. (For example, the color $SU(3)$ does not become asymptotically free.) Therefore, we must introduce further additional fermions $(\bar{5}'_L + 10'_L)$. However, then, the seesaw form (3.9) will be spoiled because of the dominant mass terms $(\mu_5 \bar{5}'_L 5_L + \mu_{10} \bar{10}'_L 10_L)$.

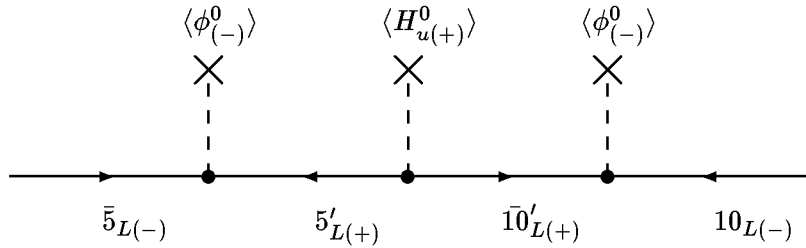


Fig. 3 Seesaw mass M_e in a model with fermions $3(\bar{5}_{L(-)} + 10_{L(-)} + 5'_{L(+)} + \bar{10}'_{L(+)})$ and scalars $(H_{u(+)} + 3\phi_{(-)}^0)$ under Z_2 symmetry

In the next section, we will propose a model under consideration of these problems.

4. Model

According to the guiding principles (A), (B) and (C) suggested in Sec. 2 and the idea reviewed in Sec. 3, in this section, let us try to build a model which gives the formula (1.2) reasonably. In this section, we will concentrate our attention on the charged lepton masses, so that we will not touch the quark and neutrino masses. For convenience, we use notations and conventions in an $SU(5)$ SUSY GUT model, but we do not always assume the SUSY GUT.

The basic idea in the present model is as follows: we assume 3-family $SU(5)$ singlet scalars instead of 3-family $SU(2)_L$ doublet Higgs scalars ϕ_i in Sec. 3. The $SU(5)$ singlet fields do not cause FCNC, and do not affect the RGE effects of the gauge coupling constants.

We assume the following flavor-triplet matter fields and flavor-singlet Higgs fields,

$$3(1_L + \bar{5}_L + 10_L)_{(+1)} + 3(1'_L + \bar{5}'_L + \bar{5}'_L + \bar{10}'_L + 10'_L)_{(+2)} + \bar{H}_{d(0)} + H_{u(+2)}, \quad (4.1)$$

where \bar{H}_d and H_u denote $SU(5)$ $\bar{5}$ and 5 Higgs fields, respectively, and the number (n) in $\psi_{L(n)}$ denotes the Z_4 charge, i.e. $\psi_{L(n)} \rightarrow e^{i(\pi/2)n} \psi_{L(n)}$ under a discrete symmetry Z_4 . The Z_4 invariant

superpotential W is given by

$$\begin{aligned}
W = & (10_{Li} \ 10'_{Li} \ \bar{10}'_{Li}) \begin{pmatrix} Y_{ij}^u H_u & 0 & \lambda_{ijk}^u 1_{Lk} \\ 0 & 0 & \mu_{10} \delta_{ij} \\ \lambda_{ijk}^u 1_{Lk} & \mu_{10} \delta_{ij} & Y_{ij}^{u'} \bar{H}_d \end{pmatrix} \begin{pmatrix} 10_{Lj} \\ 10'_{Lj} \\ \bar{10}'_{Lj} \end{pmatrix} \\
& + (\bar{5}_{Li} \ \bar{10}'_{Li} \ \bar{5}'_{Li}) \begin{pmatrix} 0 & \lambda_{ijk}^{(5,5')} 1_{Lk} & 0 \\ \lambda_{ijk}^{(10',10)} 1_{Lk} & 0 & \mu_{10} \delta_{ij} \\ 0 & \mu_5 \delta_{ij} & Y_{ij}^{d'} \bar{H}_d \end{pmatrix} \begin{pmatrix} 10_{Lj} \\ \bar{5}'_{Lj} \\ \bar{10}'_{Lj} \end{pmatrix} \\
& + \lambda_{ijk}^{(10,10,5')} 10_{Li} 10_{Lj} \bar{5}'_{Lk} + \lambda_{ijk}^{(5,5,10')} \bar{5}_{Li} \bar{5}_{Lj} 10'_{Lk} \\
& + \lambda_{ijk}^{(5',5,10)} \bar{5}'_{Li} \bar{5}_{Lj} 10_{Lk} + \lambda_{ijk}^{d'} 1_{Li} 1_{Lj} 1'_{Lk} \\
& + Y_{ij}^{d'} 1_{Li} \bar{5}_{Lj} H_u + Y_{ij}^{d'} 1'_{Li} \bar{5}'_{Lj} \bar{H}_d + Y_i^{(H)} 1'_{Li} \bar{H}_d H_u + \mu'_{5i} \bar{5}'_{Li} H_u .
\end{aligned} \tag{4.2}$$

Although the up-quark masses are given by the Yukawa interactions $Y_{ij}^u 10_{Li} 10_{Lk} \langle H_u^0 \rangle$, the down-quarks and charged leptons do not have such Yukawa interactions at tree level, so that the mass matrices are given by the seesaw form

$$(M_{d,e})_{ij} \simeq \frac{1}{\mu_5 \mu_{10}} \lambda_{ii'k}^{(5,5')} \lambda_{jj'k'}^{(10',10)} Y_{i'j'}^d v_{S_k} v_{S_{k'}} \tag{4.3}$$

as shown in Fig. 4, where $v_{S_i} = \langle 1_{Li} \rangle$. We assume universality of the coupling constants

$$\lambda_{ijk}^{(5,5')} = \lambda_{ijk}^{(10',10)} \equiv \lambda \delta_{ij} \delta_{jk} , \tag{4.4}$$

$$Y_{ij}^d = y_d \delta_{ij} . \tag{4.5}$$

Then, we obtain a simple form

$$(M_{d,e})_{ij} = \delta_{ij} \lambda^2 y_d \frac{v_{S_i}^2 v_d}{\mu_5 \mu_{10}} . \tag{4.6}$$

Of course, for the scalar parts of the fields 1_{Li} , we assume a similar mechanism to ϕ_i in the Higgs potential (3.3) as discussed in Sec. 3. Then, we obtain the relation (1.2). (At present, we unwillingly obtain the same mass matrix form for the down-quarks, i.e. $M_d = M_e^T$.)

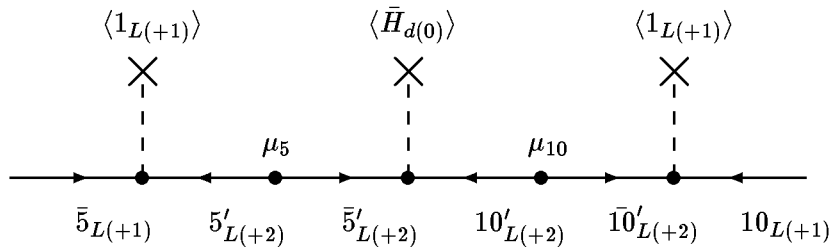


Fig. 4 Charged lepton mass generation

For the neutrino mass matrix M_ν , we also obtain a seesaw form

$$(M_\nu)_{ij} \simeq Y_{ii}^\nu v_u (\lambda_{i'j'k}^\nu v'_{S_k})^{-1} Y_{jj'}^\nu v_u, \quad (4.7)$$

through the diagram given in Fig. 5, where $v'_{S_i} = \langle 1'_{Li} \rangle$. Since $v_u \sim 10^2$ GeV, we suppose $v'_S \sim 10^{14}$ GeV as well as in the conventional seesaw model.

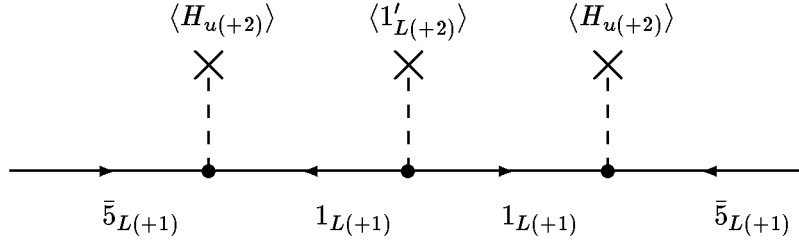


Fig. 5 Neutrino mass generation

On the other hand, in order to obtain $M_e \sim 1$ GeV, we must suppose

$$v_S^2 / \mu_5 \mu_{10} \sim 10^{-2}, \quad (4.8)$$

for $v_d \sim 10^2$ GeV. If we suppose $v_S \sim 10$ GeV, we have to take $\sqrt{\mu_5 \mu_{10}} \sim 10^2$ GeV. However, such a small value means that the additional matter fields of 6 families survive until $\mu = \sqrt{\mu_5 \mu_{10}} \sim 10^{-2}$ GeV, so that the asymptotic freedom of the color SU(3) is destroyed. Therefore, for example, we put $\mu_{10} \sim 10^{16}$ GeV and $\mu_5 \sim 10^{14}$ GeV, so that we take $v_S \sim 10^{14}$ GeV. Then, the matter fields affect the RGE effects as follows: 3 + 6 families for $\mu > M_{GUT} \sim 10^{16}$ GeV; 3 families of $(\bar{5} + 10)_L$ and 6 families of $(\bar{5}' + 5')$ for $M_{GUT} > \mu \geq \mu_5 \sim 10^{14}$ GeV; 3 families of $(\bar{5} + 10)_L$ for $\mu_5 > \mu \geq M_{weak}$. Here, our parameter values are summarized as

$$\begin{aligned} M_{weak} &\sim v_u \sim v_d \sim 10^2 \text{ GeV}, \\ \mu_5 &\sim v_S \sim v'_S \sim 10^{14} \text{ GeV}, \\ M_{GUT} &\sim \mu_{10} \sim 10^{16} \text{ GeV}. \end{aligned} \quad (4.9)$$

For these parameter values, the gauge unification at $\mu = M_{GUT}$ is still kept as shown in Fig. 6.

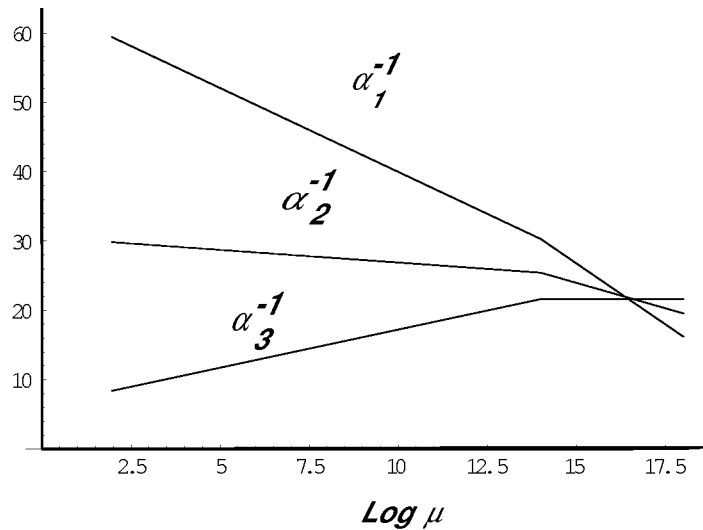


Fig. 6 Evolution of the gauge coupling constants

5. Concluding remarks

In conclusion, we have proposed a model which gives the charged lepton mass formula (1.2), where the origin of the mass spectrum is attributed not to the structure of the Yukawa coupling constants, but to a structure of VEVs of flavor-triplet scalars under $Z_4 \times S_3$ symmetries. The model can be described within a framework of the SUSY GUT. (Of course, at present, the form of the scalar potential (3.1) has been given by hand, it is not a logical consequence from the SUSY GUT model.)

However, the choice $v_S \sim 10^{14}$ GeV does not satisfy the suggestion (C) that the formula (1.2) is valid at the low energy scale. If we adhere to the idea, we must give up the gauge unification. If we assume that only leptonic part of $(\bar{5}' + 5' + \bar{10}' + 10')_L$ survive until $\mu \sim \mu_5 = \mu_5 \sim 10^2$ GeV (we assume a triplet-doublet splitting [15] mechanism similar to that for the Higgs fields), we can choose $v_S \sim 10$ GeV without destroying the asymptotic freedom of the color SU(3). In order to get the mass formula (1.2) at a low energy scale, rather, we should abandon the SUSY GUT scenario.

So far, we have not discussed quark mass matrices (and also neutrino mass matrix). In the present model, the up-quark and neutrino mass matrices are generated by the diagrams given in Figs. 4 and 5, respectively. If we assume the universal couplings similar to (4.4) and (4.5), we will obtain diagonal forms of those mass matrices as well as in the charged lepton mass matrix, so that the Cabibbo-Kobayashi-Maskawa (CKM) and Maki-Nakagawa-Sakata (MNS) matrices will become unit matrices. Except for the charged lepton sector, we will have to consider a more general form of the coupling constants which is S_3 invariant. For example, we must consider that Yukawa couplings Y_{ij}^u are given by the form

$$y_{(1)}^u \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} + y_{(2)}^u \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad (5.1)$$

and the coupling constants $\lambda_{ijk}^\nu v'_{Sk}$ which come from the interactions $1_L 1_L 1'_L$ are given by

$$\lambda_{(1)}^\nu \begin{pmatrix} v'_{S1} & 0 & 0 \\ 0 & v'_{S2} & 0 \\ 0 & 0 & v'_{S3} \end{pmatrix} + \lambda_{(2)}^\nu \begin{pmatrix} 0 & v'_{S3} & v'_{S2} \\ v'_{S3} & 0 & v'_{S1} \\ v'_{S2} & v'_{S1} & 0 \end{pmatrix} \quad (5.2)$$

We must also consider a mechanism which yields $M_d \neq M_e^T$. Possibly, the mechanism will be related to the triplet-doublet splitting of the SU(5) $\bar{5}$ (and/or 5) fields.

Thus, the present model has many problems, but the formula (1.2) is too beautiful to be accidental coincidence. (Some of the problems will be solved by abandoning the GUT scenario.) In the present paper, we have investigated a possible model within the framework of an extended seesaw mechanism, but, on the other hand, the radiative mass generation hypothesis is also promising. The idea that the origin of the mass spectrum is attributed not to the structure of the Yukawa coupling constants, but to the structure of the VEVs of flavor-triplet scalars will be worthwhile noticing. It is a future task to seek for a more elegant and simple model which can lead to the mass formula (1.2).

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