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Quark and Lepton Mass Matrix Structures Suggested by the Observed Unitary Triangle Shape

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Abstract

Under the hypothesis that the CP violating phase parameter δ in the CKM matrix V takes own value so that the radius $R(\delta)$ of the circle circumscribed about the unitary triangle takes its minimum value, possible phase conventions of the CKM matrix are investigated. We find that two of the 9 phase conventions can give favorable predictions for the observed shape of the unitary triangle. One of the successful two suggests phenomenologically interesting structures of the quark and lepton mass matrices, which lead to $|V_{us}| \simeq \sqrt{m_d/m_s} = 0.22$, $|V_{ub}| \simeq \sqrt{m_u/m_t} = 0.0036$ and $|V_{cb}| \simeq \sqrt{m_c/2m_t} = 0.043$ for the CKM matrix V, and to $\sin^2 2\theta_{atm} = 1$, $\tan^2 \theta_{solar} \simeq |m_{\nu 1}/m_{\nu 2}|$ and $|U_{13}| \simeq \sqrt{m_e/2m_\tau}$ for the lepton mixing matrix U under simple requirements for the textures.

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1 Introduction

Recent remarkable progress of the experimental B physics [1] has put the shape of the unitary triangle in the quark sector within our reach. The world average value of the angle β [2] which has been obtained from B_d decays is

$$\sin 2\beta = 0.736 \pm 0.049 \quad \left(\beta = 23.7^{\circ}{}^{+2.2^\circ}_{-2.0^\circ}\right), \tag{1.1}$$

and the best fit [2] for the Cabibbo-Kobayasi-Maskawa (CKM) matrix [3, 4] V also gives

$$\gamma = 60^\circ \pm 14^\circ \ , \quad \beta = 23.4^\circ \pm 2^\circ \ , \tag{1.2}$$

where the angles α, β and γ are defined by

$$\alpha = \mathrm{Arg}\left[-\frac{V_{31}V_{33}^*}{V_{11}V_{13}^*}\right], \quad \beta = \mathrm{Arg}\left[-\frac{V_{21}V_{23}^*}{V_{31}V_{33}^*}\right], \quad \gamma = \mathrm{Arg}\left[-\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*}\right]. \tag{1.3}$$

We are interested what logic can give the observed magnitude of the CP violation.

Usually, we assume a peculiar form of the quark mass matrices at the start, and thereby, we predict a magnitude of the CP violation and a shape of the unitary triangle. However, recently, the author [5] has investigated a quark mass matrix model on the basis of an inverse procedure: by noticing that predictions based on the maximal CP violation hypothesis [6] depend on the phase convention, the author has, at the start, investigated what phase conventions can give favorable predictions of the unitary triangle, and then he has investigated what quark mass matrices can give such a phase convention of the CKM matrix. Here, we have assumed that the three rotation angles in the CKM matrix V are fixed by the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$, and only the CP violating phase parameter δ is free.

There are, in general, 9 cases [7] for the phase convention of the CKM matrix. When we define the expression of the CKM matrix V as

$$V = V(i,k) \equiv R_i^T P_j R_j R_k \qquad (i \neq j \neq k), \tag{1.4}$$

where

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_3(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.5}$$

($s = \sin\theta$ and $c = \cos\theta$) and $P_1 = \mathrm{diag}(e^{i\delta},\ 1,\ 1)$, $P_2 = \mathrm{diag}(1,\ e^{i\delta},\ 1)$, and $P_3 = \mathrm{diag}(1,\ 1,\ e^{i\delta})$, then the rephasing invariant quantity [8] J is given by

$$J = \frac{|V_{i1}||V_{i2}||V_{i3}||V_{1k}||V_{2k}||V_{3k}|}{(1-|V_{ik}|^2)|V_{ik}|} \sin\delta \ . \tag{1.6}$$

And also, angles ϕ_ℓ ($\ell = 1,2,3$; $\phi_1 = \beta$, $\phi_2 = \alpha$ and $\phi_3 = \gamma$ in the conventional angle notations) of the triangle $\triangle^{(31)}$ for the unitary condition

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0. \tag{1.7}$$

are given by the formula

$$\sin\phi_\ell = \frac{|V_{i1}||V_{i2}||V_{i3}||V_{1k}||V_{2k}||V_{3k}| \sin\delta}{|V_{m1}||V_{m3}||V_{n1}||V_{n3}|(1-|V_{ik}|^2)|V_{ik}|}, \tag{1.8}$$

where (ℓ, m, n) is a cyclic permutation of (1,2,3). (Note that the 5 quantities $|V_{i1}|$, $|V_{i2}|$, $|V_{i3}|$, $|V_{1k}|$, $|V_{2k}|$ and $|V_{3k}|$ in the expression $V(i,k)$ are independent of the phase parameter δ. In other words, only the remaining 4 quantities are dependent of δ.) The author [5] has found that phase conventions which lead to successful predictions under the maximal CP violation hypothesis [6] are only two: the original Kobayasi-Maskawa phase convention [4] $V(1,1)$ and the Fritzsch-Xing phase convention [9].

The author [5] has also pointed out that the phenomenological success of the expression $V(3,3)$ suggests a quark mass matrix form [9, 10]

$$M_q = P_1^q R_1^q R_3^q D_q R_3^{qT} R_1^{qT} P_1^{q\dagger} \quad (q = u, d), \tag{1.9}$$

where

$$D_q = \text{diag}(m_{q1}, m_{q2}, m_{q3}). \tag{1.10}$$

The quark mass matrix form (1.9) leads to the well-known successful prediction [11]

$$|V_{us}| \simeq \sqrt{m_d/m_s} \tag{1.11}$$

under the texture-zero requirement $(M_d)_{11} = 0$, while the texture-zero requirement $(M_u)_{11} = 0$ predicts $|V_{ub}|/|V_{cb}| \simeq \sqrt{m_u/m_c} \simeq 0.059$ (we have used values [12] at $\mu = m_Z$ as quark mass values), which is in poor agreement with the observed value [2] $|V_{ub}|/|V_{cb}| = 0.089^{+0.015}_{-0.014}$.

Therefore, in the present paper, we will investigate another possibility instead of the maximal CP violation hypothesis. In Sec. 2, by assuming that the CP violating phase parameter δ in the CKM matrix V takes own value so that the radius $R(\delta)$ of the circle circumscribed about the unitary triangle takes its minimum value, we will find that only two types $V(2,3)$ and $V(2,1)$ can give favorable predictions for the observed shape of the unitary triangle. Stimulated by this result, in Secs. 3 and 4, we will assume that the quark mixing matrix $V = U_u^\dagger U_d$ and lepton mixing matrix $U = U_e^\dagger U_\nu$ are given by the type $V(2,3)$, and we will obtain successful predictions $|V_{us}| \simeq \sqrt{m_d/m_s} = 0.22$, $|V_{ub}| \simeq \sqrt{m_u/m_t} = 0.0036$ and $|V_{cb}| \simeq \sqrt{m_c/2m_t} = 0.043$ for the CKM matrix V, and $\sin^2 2\theta_{23} = 1$, $\tan^2\theta_{12} \simeq |m_{\nu 1}/m_{\nu 2}|$ and $|U_{13}| \simeq \sqrt{m_e/2m_\tau}$ for the lepton mixing matrix [13] U under simple requirements for mass matrix textures. Finally, Sec. 5 will be devoted to concluding remarks.

2 Ansatz for the unitary triangle

Of the three unitary triangles $\triangle^{(ij)}$ [$(ij) = (12)$, (23), (31)] which denote the unitary conditions

$$\sum_k V_{ki}^* V_{kj} = \delta_{ij} \ , \tag{2.1}$$

we usually discuss the triangle $\triangle^{(31)}$, i.e.

$$V_{ud}^* V_{ub} + V_{cd}^* V_{cb} + V_{td}^* V_{tb} = 0, \tag{2.2}$$

because the triangle $\triangle^{(31)}$ is the most useful one for the experimental studies. Seeing from the geometrical point of view, the triangle $\triangle^{(31)}$ has the plumpest shape compared with other triangles $\triangle^{(12)}$ and $\triangle^{(23)}$, so that the triangle $\triangle^{(31)}$ has the shortest radius $(R_{(31)})_{mini}$ of the circumscribed circle compared with the other cases $\triangle^{(12)}$ and $\triangle^{(12)}$.

Therefore, let us put the following assumption: the phase parameter δ takes the value so that the circumscribed circle $R_{(31)}(\delta)$ takes its minimum value.

The radius $R_{(31)}(\delta)$ is given by the sine rule

$$\frac{r_1}{\sin\phi_1} = \frac{r_2}{\sin\phi_2} = \frac{r_3}{\sin\phi_3} = 2R_{(31)}, \tag{2.3}$$

where

$$r_1 = |V_{13}||V_{11}|, \quad r_2 = |V_{23}||V_{21}|, \quad r_3 = |V_{33}||V_{31}|, \tag{2.4}$$

and the angles $(\phi_1, \phi_2, \phi_3) \equiv (\beta, \alpha, \gamma)$ are defined by Eq. (1.3). Note that in the expression $V(i,k)$ the side r_i is independent of the parameter δ. Therefore, the minimum of the radius $R_{(31)}(\delta)$ means the maximum of $\sin\phi_i(\delta)$. We put a further assumption: the phase parameter δ takes own value so that $\sin\phi_i(\delta)$ takes its maximal value, i.e. $\sin\phi_i = 1$.

Then, we find that 6 cases of the 9 cases $V(i,j)$ except for $V(3,3)$, $V(2,3)$ and $V(2,1)$ cannot give $\sin\phi_i = 1$ (i.e. $|\sin\phi_i| < 1$) under the observed values [2] of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$

$$|V_{us}| = 0.2200 \pm 0.0026, \quad |V_{cb}| = 0.0413 \pm 0.0015, \quad |V_{ub}| = 0.00367 \pm 0.00047. \tag{2.5}$$

Of the three candidates, the case $V(3,3)$ ruled out, because the requirement $\sin\phi_3 = 1$ (i.e. $\gamma/2 = \pi$) disagrees with the observed value (1.2). Therefore, the possible candidates for the requirement $\sin\phi_i = 1$ are only $V(2,3)$ and $V(2,1)$. The requirement $\sin\phi_i = 1$ means the requirement $\sin\alpha = 1$ in $V(2,3)$ and $V(2,1)$. From the relation (2.3), we obtain

$$\sin\beta = \frac{r_1}{r_2}\sin\alpha = \frac{|V_{13}||V_{11}|}{|V_{23}||V_{21}|}, \quad \sin\gamma = \frac{r_3}{r_2}\sin\alpha = \frac{|V_{33}||V_{31}|}{|V_{23}||V_{21}|}. \tag{2.6}$$

For example, the case $V(2,3)$ predicts

$$|V_{td}| = \left(8.36^{+0.24}_{-0.27}\right) \times 10^{-3}, \quad \beta = 23.2^\circ \pm 0.1^\circ, \quad \gamma = 66.8^{\circ}{}^{+3.8^\circ}_{-4.3^\circ}, \tag{2.7}$$

with $\delta = 113.2^{\circ}{}^{-3.8^\circ}_{+4.3^\circ}$ from the requirement $\alpha = 90^\circ$ and the observed values (2.5). The predictions (2.7) are in good agreement with the observed values (1.1) and (1.2). For the case $V(2,1)$, we obtain the same numerical results (2.7) (but with a different value of δ). As far as the phenomenology of the unitary triangle is concerned,

we cannot determine which case is favor.

However, as we see in the next section, the case $V(2,3)$ suggests an interesting texture of the quark mass matrices, which leads to predictions $|V_{us}| \simeq \sqrt{m_d/m_s}$, $|V_{ub}| \simeq \sqrt{m_u/m_t}$ and $|V_{cb}| \simeq \sqrt{m_c/2m_t}$ under a simple ansatz. For the case $V(2,1)$, we cannot obtain such an interesting texture.

3 Speculation on the quark mass matrix form

As we have assumed in the previous paper, the successful expression $V(i,k)$ suggests the following situation: The phase factors in the quark mass matrices M_f $(f=u,d)$ are factorized by the phase matrices P_f as

$$M_f = P_{fL}^\dagger \widetilde{M}_f P_{fR} \ , \tag{3.1}$$

where P_f are phase matrices and $\widetilde{M}_f$ are real matrices. The real matrices $\widetilde{M}_f$ are diagonalized by rotation (orthogonal) matrices R_f as

$$R_f^\dagger \widetilde{M}_f R_f = D_f \equiv \mathrm{diag}(m_{f1},\ m_{f2},\ m_{f3}), \tag{3.2}$$

[for simplicity, we have assumed that M_f are Hermitian (or symmetric) matrix, i.e. $P_{fR}=P_{fL}$ (or $P_{fR}=P_{fL}^*$)], so that the CKM matrix V is given by

$$V = R_u^T P R_d \ , \tag{3.3}$$

where $P = P_{uL}^\dagger P_{dL}$. The quark masses m_{fi} are only determined by $\widetilde{M}_f$. In other words, the rotation parameters are given only in terms of the quark mass ratios, and independent of the CP violating phases. In such a scenario, the CP violation parameter δ can be adjusted without changing the quark mass values.

For example, the case $V(2,3)$ suggests the quark mass matrix structures

$$\begin{aligned} \widetilde{M}_u &= R_1(\theta_{23}^u) R_2(\theta_{13}^u) D_u R_2^T(\theta_{13}^u) R_1^T(\theta_{23}^u) \ , \\ \widetilde{M}_d &= R_1(\theta_{23}^d) R_3(\theta_{12}^d) D_d R_3^T(\theta_{12}^d) R_1^T(\theta_{23}^d) \ , \end{aligned} \tag{3.4}$$

with $\theta_{23} = \theta_{23}^d - \theta_{23}^u$. The explicit forms of $V(2,3)$, $\widetilde{M}_u$ and $\widetilde{M}_f$ are given as follows:

$$V(2,3) = R_2^T(\theta_{13}^u) P_1(\delta) R_1(\theta_{23}) R_3(\theta_{12}^d)$$

$$= \begin{pmatrix} e^{i\delta} c_{13}^u c_{12}^d - s_{23} s_{13}^u s_{12}^d & e^{i\delta} c_{13}^u s_{12}^d + s_{23} s_{13}^u c_{12}^d & -c_{23} s_{13}^u \\ -c_{23} s_{12}^d & c_{23} c_{12}^d & s_{23} \\ e^{i\delta} s_{13}^u c_{12}^d + s_{23} c_{13}^u s_{12}^d & e^{i\delta} s_{13}^u s_{12}^d - s_{23} c_{13}^u c_{12}^d & c_{23} c_{13}^u \end{pmatrix}, \tag{3.5}$$

$$\widetilde{M}_u = \left(\begin{matrix} m_{u1}(c_{13}^u)^2 + m_{u3}(s_{13}^u)^2 & (m_{u3}-m_{u1}) c_{13}^u s_{13}^u s_{23}^u \\ (m_{u3}-m_{u1}) c_{13}^u s_{13}^u s_{23}^u & [m_{u1}(s_{13}^u)^2 + m_{u3}(c_{13}^u)^2](s_{23}^u)^2 + m_{u2}(c_{23}^u)^2 \\ (m_{u3}-m_{u1}) c_{13}^u s_{13}^u c_{23}^u & [m_{u1}(s_{13}^u)^2 + m_{u3}(c_{13}^u)^2 - m_{u2}] c_{23}^u s_{23}^u \end{matrix} \right.$$

$$\left. \begin{matrix} (m_{u3}-m_{u1}) c_{13}^u s_{13}^u c_{23}^u \\ \left[m_{u1}(s_{13}^u)^2 + m_{u3}(c_{13}^u)^2 - m_{u2}\right] c_{23}^u s_{23}^u \\ \left[m_{u1}(s_{13}^u)^2 + m_{u3}(c_{13}^u)^2\right] (c_{23}^u)^2 + m_{u2}(s_{23}^u)^2 \end{matrix} \right), \tag{3.6}$$

$$\widetilde{M}_d = \begin{pmatrix} m_{d1}(c^d_{12})^2 + m_{d2}(s^d_{12})^2 & (m_{d2} - m_{d1})c^d_{12}s^d_{12}c^d_{23} \\ (m_{d2} - m_{d1})c^d_{12}s^d_{12}c^d_{23} & [m_{d1}(s^d_{12})^2 + m_{d2}(c^d_{12})^2](c^d_{23})^2 + m_{d3}(s^d_{23})^2 \\ -(m_{d2} - m_{d1})c^d_{12}s^d_{12}s^d_{23} & [m_{d3} - m_{d2}(c^d_{12})^2 - m_{d1}(s^d_{12})^2]c^d_{23}s^d_{23} \end{pmatrix.$$

$$\begin{matrix} -(m_{d2} - m_{d1})c^d_{12}s^d_{12}s^d_{23} \\ \left[m_{d3} - m_{d2}(c^d_{12})^2 - m_{d1}(s^d_{12})^2\right] c^d_{23}s^d_{23} \\ \left[m_{d1}(s^d_{12})^2 + m_{d2}(c^d_{12})^2\right] (s^d_{23})^2 + m_{d3}(c^d_{23})^2 \end{pmatrix} . \quad (3.7)$$

In the mass matrix (3.7), the ansatz $(\widetilde{M}_d)_{11} = 0$ leads to the well–known relation (1.11). On the other hand, for the up-quark mass matrix (3.6), the constraint $(\widetilde{M}_u)_{11} = 0$ leads to the relation

$$|V_{ub}| \simeq s^u_{13} \simeq \sqrt{\frac{m_u}{m_t}} = 0.0036, \quad (3.8)$$

which is in excellent agreement with the experimental value (2.5). For the case $V(2,1)$, we cannot obtain such a simple relation. Therefore, we will concentrate further investigation on the $V(2,3)$ model with the quark mass matrices (3.6) and (3.7).

In order to fix the value of θ_{23}, we put an ansatz

$$\frac{(\widetilde{M}_u)_{23}}{(\widetilde{M}_u)_{22}} = -\frac{(\widetilde{M}_u)_{13}}{(\widetilde{M}_u)_{12}}. \quad (3.9)$$

At present, there is no theoretical reason for the constraint (3.9). It is pure phenomenological ansatz. The requirement (3.9) leads to

$$s^u_{23} = \sqrt{\frac{-m_{u2}/2}{m_{u3} + m_{u1} - m_{u2}}} \simeq \sqrt{\frac{m_c}{2m_t}} = 0.043, \quad (3.10)$$

which is in good agreement of the observed value of $|V_{cb}|$
in (2.5). If we assume a constraint

$$(\widetilde{M}_d)_{23} = (\widetilde{M}_d)_{13} = 0, \quad (3.11)$$

which corresponds to a special case in a requirement analogous to (3.9), we obtain $s^d_{23} = 0$, so that we can obtain a successful prediction

$$|V_{cb}| = s_{23} = |s^u_{23}| \simeq \sqrt{\frac{m_c}{2m_t}} = 0.043. \quad (3.12)$$

Although, at present, the origins of the constraints (3.9) and (3.11) are unknown, in the next section, we will find that similar requirements for the lepton sector also lead to successful predictions.

4 Application to the lepton sector

If we suppose the correspondence $M_u \leftrightarrow M_\nu$ and $M_d \leftrightarrow M_e$ for the lepton mass matrices (M_ν, M_e), the lepton mixing matrix [13] $U = U_e^\dagger U_\nu$ will be given by the type $V(3,2)$. However, the case gives a wrong prediction $|U_{12}| = |c_{23} s_{12}^e| < \sqrt{m_e/m_\mu}$ under the constraint $(M_e)_{11} = 0$. Although it does not need to adhere the constraint $(M_e)_{11} = 0$, phenomenologically, it is more interesting to assume that the lepton mixing matrix U is also described by the type $V(2,3)$, and not by the type $V(3,2)$.

In the case $U = V(2,3)$, correspondingly to the expression

$$U = R_2^T(\theta_{13}^e) P_1(\delta) R_1(\theta_{23}) R_3(\theta_{12}^\nu), \tag{4.1}$$

the lepton mass matrices are given by the structures

$$\begin{aligned} \widetilde{M}_e &= R_1(\theta_{23}^e) R_2(\theta_{13}^e) D_e R_2^T(\theta_{13}^e) R_1^T(\theta_{23}^e) \ , \\ \widetilde{M}_\nu &= R_1(\theta_{23}^\nu) R_3(\theta_{12}^\nu) D_\nu R_3^T(\theta_{12}^\nu) R_1^T(\theta_{23}^\nu) \ , \end{aligned} \tag{4.2}$$

with $\theta_{23} = \theta_{23}^\nu - \theta_{23}^e$.

Similarly to the quark sector, we put the following constraints:

$$(M_e)_{11} = 0, \tag{4.3}$$

$$(M_\nu)_{11} = 0, \tag{4.4}$$

$$\frac{(\widetilde{M}_\nu)_{23}}{(\widetilde{M}_\nu)_{22}} = -\frac{(\widetilde{M}_\nu)_{13}}{(\widetilde{M}_\nu)_{12}}, \tag{4.5}$$

$$(\widetilde{M}_e)_{23} = (\widetilde{M}_e)_{13} = 0. \tag{4.6}$$

The requirements (4.3) and (4.4) lead to familiar relations

$$\frac{s_{13}^e}{c_{13}^e} = \sqrt{\frac{m_e}{m_\tau}} = 0.0167, \quad \frac{s_{12}^\nu}{c_{12}^\nu} = \sqrt{\frac{-m_{\nu 1}}{m_{\nu 2}}}, \tag{4.7}$$

respectively. The requirement (4.6) also leads to a similar result

$$c_{23}^e = 0 \quad \left(\theta_{23}^e = \frac{\pi}{2}\right), \tag{4.8}$$

(but note that the result is different from the result $s_{23}^d = 0$ in the down-quark mass matrix). On the other hand, correspondingly to Eq. (3.10), the requirement (4.5) lead to a relation

$$c_{23}^\nu = \sqrt{\frac{m_{\nu 3}/2}{m_{\nu 3} - m_{\nu 2} - m_{\nu 1}}}. \tag{4.9}$$

If we suppose $m_{\nu 3}^2 \gg m_{\nu 2}^2 > m_{\nu 1}^2$, we obtain

$$\theta_{23}^\nu = \frac{\pi}{4} - \varepsilon, \tag{4.10}$$

where

$$\varepsilon \simeq \frac{m_{\nu 2} + m_{\nu 1}}{2 m_{\nu 3}}. \tag{4.11}$$

By using the observed values $\Delta m_{32}^2 \simeq 2.8 \times 10^{-3}$ eV^2 [14], $\Delta m_{21}^2 = (7.1_{-0.6}^{+1.2}) \times 10^{-5}$ eV^2 and $\theta_{solar} = 32.5_{-2.3}^{+2.4}$ degrees [15], we estimate

$$\begin{aligned} m_{\nu 3} &\simeq \sqrt{\Delta m_{32}^2} = 0.053 \text{ eV}, \\ m_{\nu 2} &= \sqrt{\Delta m_{21}^2/(1 - \tan^2\theta_{21})} = 0.011 \text{ eV}, \\ -m_{\nu 1} &= m_{\nu 2}\sqrt{\tan^2\theta_{21}} = 0.0069 \text{ eV}. \end{aligned} \tag{4.12}$$

Therefore, the deviation ε from $\theta_{23}^\nu = \pi/4$ is

$$\varepsilon = \frac{m_{\nu 2} - |m_{\nu 1}|}{2 m_{\nu 3}} = 0.038 \quad (2.2^\circ), \tag{4.13}$$

so that the deviation ε does not visibly affect the prediction $\sin^2 2\theta_{23} = 1$. That is, the relation (4.9) naturally leads to the prediction $\sin^2 2\theta_{atm} = 1$. Also the present model gives the prediction

$$|U_{13}| = c_{23} s_{13}^e \simeq \sqrt{\frac{m_e}{2 m_\tau}} = 0.012. \tag{4.14}$$

5 Concluding remarks

By assuming that three rotation angles in the CKM matrix V are fixed by the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$ and only a CP violating phase parameter δ is free, and by putting an ansatz that the phase parameter δ takes own value so that the radius $R_{(31)}(\delta)$ of the circumscribed circle about the unitary triangle takes its minimal value, we have found that only the phase conventions $V(2,3)$ and $V(2,1)$ can predict the observed shape of the unitary triangle.

Of the two successful phase conventions, we have noticed the case $V(2,3)$, which suggests the quark mass matrix structures

$$\begin{aligned} \widetilde{M}_u &= R_1(\theta^u_{23}) R_2(\theta^u_{13}) D_u R_2^T(\theta^u_{13}) R_1^T(\theta^u_{23}) \ , \\ \widetilde{M}_d &= R_1(\theta^d_{23}) R_3(\theta^d_{12}) D_d R_3^T(\theta^d_{12}) R_1^T(\theta^d_{23}) \ , \end{aligned} \tag{5.1}$$

where $\widetilde{M}_f$ is defined by Eq. (3.1). Under the phenomenological constraints, $(M_f)_{11} = 0$ and Eqs. (3.9) and (3.11), we have obtained successful results $|V_{us}| \simeq \sqrt{m_d/m_s} = 0.22$ [Eq. (1.11)], $|V_{ub}| \simeq \sqrt{m_u/m_t} = 0.0036$ [Eq. (3.8)] and $|V_{cb}| \simeq \sqrt{m_c/2m_t} = 0.043$ [Eq. (3.12)].

Since we have assumed that the lepton mixing matrix $U = U_e^\dagger U_\nu$ is also given by the type $V(2,3)$, we have speculated that the lepton mass matrix structures are given by

$$\begin{aligned} \widetilde{M}_e &= R_1(\theta^e_{23}) R_2(\theta^e_{13}) D_e R_2^T(\theta^e_{13}) R_1^T(\theta^e_{23}) \ , \\ \widetilde{M}_\nu &= R_1(\theta^\nu_{23}) R_3(\theta^\nu_{12}) D_d R_3^T(\theta^\nu_{12}) R_1^T(\theta^\nu_{23}) \ , \end{aligned} \tag{5.2}$$

and we have naturally derived the relation $\sin^2 2\theta_{atm} = 1$ under the requirements (4.3) – (4.6) similar to the quark sector and the neutrino mass hierarchy $m^2_{\nu 3} \gg m^2_{\nu 2} > m^2_{\nu 1}$. Note that the result $\theta_{23} \simeq \pi/2$ can be obtained only for the case of the normal neutrino mass hierarchy, and it can never be done for the inverse hierarchy, as seen in Eq. (4.9).

By the way, the structures (5.1) and (5.2) ostensibly looks like the quark-to-lepton correspondence $(M_u, M_d) \leftrightarrow (M_e, M_\nu)$. However, we have put the constraints

$$\frac{(\widetilde{M}_f)_{23}}{(\widetilde{M}_f)_{22}} = -\frac{(\widetilde{M}_f)_{13}}{(\widetilde{M}_f)_{12}}, \tag{5.3}$$

on M_u and M_ν, and

$$(\widetilde{M}_f)_{23} = (\widetilde{M}_f)_{13} = 0, \tag{5.4}$$

on M_d and M_e, respectively. As the results of these constraints together with the constraint $(M_f)_{11} = 0$, we obtain the final forms of the quark and lepton mass matrices

$$\widetilde{M}_f = \begin{pmatrix} 0 & a & a\lambda \\ a & -b & b\lambda \\ a\lambda & b\lambda & b(\lambda^2 - 2) \end{pmatrix}, \tag{5.5}$$

for M_u and M_ν, and

$$\widetilde{M}_f = \begin{pmatrix} 0 & \sqrt{-m_{f1} m_{f2}} & 0 \\ \sqrt{-m_{f1} m_{f2}} & m_{f2} + m_{f1} & 0 \\ 0 & 0 & m_{f3} \end{pmatrix}, \tag{5.6}$$

for M_d and M_e, respectively. Here, the expression (5.5) is taken as $\lambda = c_{23}/s_{23} > 1$ and $b > 0$ for M_u, and as $-\lambda = s_{23}/c_{23} < 1$ and $b < 0$ for M_ν. The expression (5.6) is taken as $(m_{f1}, m_{f2}, m_{f3}) = (-m_d, m_s, m_b)$ for M_d and $(m_{f1}, m_{f2}, m_{f3}) = (-m_e, m_\tau, m_\mu)$ for M_e. Thus, in the final expressions (5.5) and (5.6), the quark-to-lepton correspondence $(M_u, M_d) \leftrightarrow (M_\nu, M_e)$ is recovered.

If we attach great importance to the $(M_u, M_d) \leftrightarrow (M_\nu, M_e)$ correspondence, we may take $(m_{e1}, m_{e2}, m_{e3}) = (-m_e, m_\mu, m_\tau)$ instead of $(m_{e1}, m_{e2}, m_{e3}) = (-m_e, m_\tau, m_\mu)$ in the charged lepton mass matrix (5.6). Then, the rotation matrix $R_1(\theta_{23}^e)$ with $\theta_{12}^e = \pi/2$ is replaced with the unit matrix $R_1(0)$. For this case, the prediction $\sin^2 2\theta_{23} = 1$ is still unchanged, but the prediction (4.14) will be replaced with

$$|U_{13}| = c_{23} s_{13}^e \simeq \sqrt{\frac{m_e}{2m_\mu}} = 0.049. \tag{5.7}$$

So far, we have not discussed a renormalization group equation (RGE) effects on the mass matrices. Since we know that the mass ratios m_d/m_s and m_u/m_c are insensitive to the RGE effects, the textures $\widehat{M}_d$ and $\widehat{M}_e$ given by Eq. (5.6) are almost unchanged under the RGE effects. On the other hand, although the mass ratios m_c/m_t and m_s/m_b are, in general, sensitive to the RGE effects, in the texture (5.5), the effects can be almost absorbed into the factor λ for the case $|y_t|^2 \gg |y_b|^2$. Therefore, the texture (5.5) is also almost insensitive to the RGE effects.

In conclusion, suggested by the observed shape of the unitary triangle, we have found a unified structures of the quark and lepton mass matrices, (5.5) and (5.6). In other words, if we assume the structures (5.5) and (5.6) for the quark and lepton mass matrices, we can obtain the successful relations (1.11), (3.8), (3.12), (4.7), (4.10) and (4.14) between the fermion mixing matrices and fermion mass ratios. Moreover, if we put the minimal $R_{(31)}(\delta)$ hypothesis, we can obtain the successful predictions (2.7) for the shape of the unitary triangle. However, at present, it is an open question what symmetries can explain the forms (5.5) and (5.6). And, the meaning of the minimal radius hypothesis is also an open question.

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