Seesaw Mass Matrix Model of Quarks and Leptons with Flavor-Triplet Higgs Scalars

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Abstract

In a seesaw mass matrix model $M_f = m_L M_F^{-1} m_R^\dagger$ with a universal structure of $m_L \propto m_R$, as the origin of m_L (m_R) for quarks and leptons, flavor-triplet Higgs scalars whose vacuum expectation values v_i are proportional to the square roots of the charged lepton masses m_{ei}, i.e. $v_i \propto \sqrt{m_{ei}}$, are assumed. Then, it is investigated whether such a model can explain the observed neutrino masses and mixings (and also quark masses and mixings) or not.

1 Introduction

It is widely accepted that quarks and leptons are fundamental entities of the matter. If it is true, the masses and mixings of the quarks and leptons will obey a simple law of nature, and we will be able to find a beautiful relation among those values. If we can find such a relation, it will make a breakthrough in the unified understanding of the quarks and leptons. As one of such phenomenological mass relations, the following charged lepton mass formula [1, 2, 3]

$$m_e + m_\mu + m_\tau = \frac{2}{3}(\sqrt{m_e} + \sqrt{m_\nu} + \sqrt{m_\tau})^2 , \tag{1.1}$$

has been known. The formula (1.1) predicts the tau lepton mass value

$$m_\tau = 1776.97 \text{ MeV}, \tag{1.2}$$

from the observed electron and muon mass values [4], $m_e = 0.51099892$ MeV and $m_\mu = 105.658369$ MeV. The predicted value (1.2) is in excellent agreement with the observed value [4] $m_\tau = 1776.99^{+0.29}_{-0.26}$ MeV. This excellent agreement seems to be beyond a matter of accidental coincidence, so that we should consider the origin of the mass formula (1.1) seriously. Up to the present, the theoretical basis of the mass formula (1.1) is still not clear. However, although it is still important to pursue the origin of the relation (1.1), in the present paper, another approach will be taken: We assume the so-called universal seesaw mass matrix model [5] for an explanation of the charged lepton mass relation (1.1) and it is investigate whether the seesaw mass matrix model can also explain the observed quark and neutrino masses and mixings or not when the mass matrix parameters are settled by the observed charged lepton masses.

In order to obtain a clue to a unified description of the quark and lepton mass matrices, let us see the phenomenological features of the relation (1.1). The charged lepton mass formula (1.1) has the following peculiar features:
(a) The mass formula is described in terms of the root squared masse $\sqrt{m_{ei}}$.
(b) The mass formula is invariant under the exchanges $\sqrt{m_{ei}} \leftrightarrow \sqrt{m_{ej}}$.
(c) The formula gives a relation between mass ratios $\sqrt{m_e/m_\mu}$ and $\sqrt{m_\mu/m_\tau}$, whose behaviors under the renormalization group equation (RGE) effects are different from each other. Therefore,

the formula (1.1) is not invariant under the RGE effects. The formula is well satisfied at a low energy scale rather than at a high energy scale.

If we take the feature (c) seriously, we must abandon the idea that the mass spectrum originates in the structure of the Yukawa coupling constants Y_e, because, in general, the Yukawa coupling constants are influenced by the renormalization group equation (RGE) effects. Even if the mass spectrum satisfies the relation (1.1) at a unification energy scale $\mu = M_X$, the mass spectrum at a low energy scale will satisfy the relation (1.1) no longer. We should consider that the Yukawa coupling constant Y_e has a unit matrix form which is not affected by RGE effects. Instead, we consider that the mass spectrum originates the vacuum expectation values (VEVs) v_i of three Higgs scalars ϕ_i $(i = 1, 2, 3)$ at a low energy scale.

The feature (a) suggests that the charged lepton mass spectrum does not originate in the Yukawa coupling structure at the tree level, but it is given by a bilinear form on the basis of some mass-generation mechanism. For example, in Refs. [3, 6, 7, 8], a seesaw-like mechanism [5] has been assumed: $M_e = m M_E^{-1} m^\dagger$, where M_E is a mass matrix of hypothetical heavy leptons. As suggested from the feature (c), we consider that the matrix m is given by $m_{ij} = \delta_{ij} v_j$.

The feature (b) suggests that the theory is invariant under a permutation symmetry S_3 [9]. We will adopt an idea that what is essential is not a structure of the Yukawa coupling constants, but a structure of the vacuum expectation values (VEVs) of flavor-triplet (3-family) Higgs scalars [3, 7, 8]. In this idea, the VEVs v_i satisfies the relation

$$v_1^2 + v_2^2 + v_3^2 = \frac{2}{3}(v_1 + v_2 + v_3)^2 . \tag{1.3}$$

(For the derivation of the relation (1.3), for example, see Ref.[10].) Then, the charged lepton mass relation (1.1) is understood from a seesaw-like mechanism

$$M_e = m_L M_E^{-1} m_R^\dagger , \tag{1.3}$$

$$m_L = \frac{1}{\kappa} m_R = y_e \mathrm{diag}(v_1, v_2, v_3) , \tag{1.4}$$

$$M_E = \mu_E \mathbf{1} \equiv \mu_E \mathrm{diag}(1, 1, 1) , \tag{1.5}$$

where m_L and m_R are Dirac mass matrices for fermions $(\bar{e}_L, E_E)$ and $(\bar{E}_L, e_R)$, respectively, and M_E is a mass matrix of hypothetical heavy charged leptons E_i.

Stimulated by the successful derivation [10] of the VEV relation (1.3), in the present paper, we will investigate possible seesaw mass matrix structures of the quarks and leptons

$$M_f = m_L M_F^{-1} m_R^\dagger , \tag{1.6}$$

by introducing heavy fermions $10'_{Li} + \overline{10}'_{Li} + 1'_{Li}$ $(i = 1, 2, 3)$ of SU(5) in addition to the conventional quarks and leptons $\bar{5}_{Li} + 10_{Li}$ as shown in Fig. 1. Here, we assume that the VEVs of the flavor-triplet Higgs scalars $5_{Hi} + \bar{5}_{Hi} + 1_{Hi}$ have the same structures which satisfy the relations (1.3). We consider that a variety of the mass spectra and mixings of quarks and neutrinos is caused by a variety of the structures of the heavy fermion mass matrices M_F. As suggested by the feature (c), we want that the mass scale of M_F is as low as possible. We will build a seesaw mass matrix model with M_F of the order of 10 TeV for the quark sectors.

Note that although the relation (1.3) is a motivation for investigating the present model (Sec. 2), it is not essential in the present paper whether the relation (1.3) is a fundamental law or merely accidental. The purpose of the present paper is to demonstrate that we can explain the observed neutrino (and also quark) masses and mixings with the same values as the parameter values v_i which are fixed by

$$\frac{v_1}{\sqrt{m_e}} = \frac{v_2}{\sqrt{m_\mu}} = \frac{v_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}}, \tag{1.7}$$

in the charged lepton sector. In the present paper, we do not inquire the origin of the values of v_i. In the present standpoint, the relation (1.1) is a phenomenological fact, but it is not a theoretical result. When we accept a seesaw mass matrix form (1.6) which is motivated from the empirical relation (1.1), our interest is in how we can obtain the reasonable values of the quark and neutrino masses and mixings under the seesaw mass matrices (1.6) with a universal structure of m_L (and m_R), but with sector-dependent structures of M_F. In Secs. 3 and 4, we will assume a permutation symmetry S_3 for the structures of M_F.

2 Fundamental fermions and scalars

Suggested by the features (a), (b) and (c) discussed in Sec. 1, in this section, we discuss the seesaw mass matrix form (1.6) concretely by introducing some additional heavy fermions. For convenience, we use notations and conventions in an SU(5) GUT model for fermions and Higgs scalars, although we do not consider a gauge unification. If we consider the unification of the gauge coupling constants, it will be badly spoiled because there are many new particles in the present model. Nevertheless, we consider that the SU(5) scheme is useful for the description of the Yukawa interactions. In the present model, we have the following fermions and Higgs scalars:

$$(\overline{5}_L + 10_L)_{(+)} + (1'_L + \overline{10}'_L + 10'_L)_{(-)} + (1_H + \bar{5}_H + 5_H)_{(-)} + (1'_H)_{(+)}, \tag{2.1}$$

where the indices $(\pm)$ denote transformation properties of a discrete symmetry Z_2. [Here, $\overline{10}'_L$ and $\bar{5}_H$ are not Hermitian conjugates of $10'_L$ and 5_H, respectively, and $\overline{10}'_L$ and $10'_L$ ($\bar{5}_H$ and 5_H) are completely different particles each other.] Therefore, we have the following Yukawa interactions: $10'_L\bar{5}_L\bar{5}_H$, $10'_L 10_L 5_H$, $1'_L\bar{5}_L 5_H$, $10_L\overline{10}'_L 1_H$, $10'_L\overline{10}'_L 1'_H$, and $1'_L 1'_L 1'_H$. Here and hereafter, for convenience, we denote interaction terms as if those are superfields. However, if we take those SUSY partners into consideration at a low energy scale, the SU(3) color force cannot become asymptotically free. Therefore, we use those SUSY notations as an expedient. In other words, we assume the absence of the supersymmetric partners of the fields (2.1) at a low energy scale.

Our essential assumption is as follows: the Higgs potentials for the scalars 5_H, $\bar{5}_H$ and 1_H with the same Z_2 charges have the same structure. This suggests that the scalars $(\bar{5}_H + 5_H + 1_H)$ will belong to a same multiplet in a higher flavor symmetry. (However, in the present paper, we will not go into the investigation of such a higher symmetry.) As a result, the VEVs of the scalars 5_H, $\bar{5}_H$ and 1_H take the same structures of the VEVs

$$\langle 5_{Hi} \rangle = v_u z_i, \quad \langle \bar{5}_{Hi} \rangle = v_d z_i, \quad \langle 1_{Hi} \rangle = v_s z_i, \tag{2.2}$$

where z_i are normalized as $z_1^2 + z_2^2 + z_3^2 = 1$ and they satisfy the relation (1.3), i.e.

$$z_1^2 + z_2^2 + z_3^2 = \frac{2}{3} \left(z_1 + z_2 + z_3 \right)^2 , \tag{2.3}$$

at the low energy scale $\mu = M_Z$. Hereafter, for numerical estimates of the neutrino and quark mass matrices, we will use the values of z_i:

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_e + m_\mu + m_\tau}}, \tag{2.4}$$

i.e. $z_1 = 0.016473$, $z_2 = 0.23678$ and $z_3 = 0.97140$.

On the other hand, we assume that the couplings of those Higgs scalars with fermions are structure-less:

$$y_u \sum_i 10'_{Li} 10_{Li} 5_{Hi} + y_d \sum_i 10'_{Li} \bar{5}_{Li} \bar{5}_{Hi} + y_\nu \sum_i 1'_{Li} \bar{5}_{Li} 5_{Hi} + y_s \sum_i \overline{10}'_{Li} 10_{Li} 1_{Hi}. \tag{2.5}$$

For convenience, hereafter, we denote the fermion mass terms $\mu_{10} 10'_L \overline{10}'_L$ as

$$10'_L (\mu_{10}) \overline{10}'_L = U_{Li} (\mu_{10}^Q)_{ij} \bar{U}_{Lj} + D_{Li} (\mu_{10}^Q)_{ij} \bar{D}_{Lj} + U^c_{Ri} (\mu_{10}^U)_{ij} \bar{U}^c_{Rj} + E^c_{Ri} (\mu_{10}^E)_{ij} \bar{E}^c_{Rj}, \tag{2.6}$$

where we have denoted the heavy fermions as $10'_L = [(U_L, D_L), U^c_R, E^c_R]$ and $\overline{10}'_L = [(\bar{U}_L, \bar{D}_L), \bar{U}^c_R, \bar{E}^c_R]$. (Note that $\bar{U}_L$ is not the Hermitian conjugate of U_L, and so on.) Then, from the seesaw diagrams shown in Fig. 1, we obtain the following quark and lepton mass matrices:

$$(M_e)_{ij} = y_d y_s v_d v_s z_i (\mu_{10}^E)^{-1}_{ij} z_j, \tag{2.7}$$

$$(M_d)_{ij} = y_d y_s v_d v_s z_i (\mu_{10}^Q)^{-1}_{ij} z_j, \tag{2.8}$$

$$(M_u)_{ij} = y_u y_s v_u v_s z_i \left[(\mu_{10}^Q)^{-1} + (\mu_{10}^U)^{-1} \right]_{ij} z_j, \tag{2.9}$$

$$(M_\nu)_{ij} = y_\nu^2 v_u^2 z_i (y_S \langle 1'_H \rangle)^{-1}_{ij} z_j. \tag{2.10}$$

Here, we have supposed

$$\langle 1_H \rangle \sim 10^2 \text{ GeV}, \quad \mu_{10} \sim 10^4 \text{ GeV}, \quad \langle 1'_H \rangle \sim 10^{14} \text{ GeV}. \tag{2.11}$$

Although the scalar $1'_H$ can couple not only to $1'_L 1'_L$, but also to $\overline{10}'_L 10'_L$, the contributions $\langle 1'_H \rangle^{-1}$ in the mass matrices M_f ($f = e, u, d$) are negligibly small compared with $(\mu_{10})^{-1}$.

In order to explain the relation (1.1), we must take $(\mu_{10}^E)_{ij} = \mu_{10} \delta_{ij}$ so that

$$(M_e)_{ij} = \frac{y_d v_d y_s v_s}{\mu_{10}} z_i \delta_{ij} z_j. \tag{2.12}$$

At present, we do not inquire why the mass matrix μ_{10}^E is structure-less. We consider that the matrices M_F are, in general, not structure-less, and their structures are dependent on the sectors, so that mass spectra and mixings individual sectors appear.

The present model is based on a multi-Higgs model, because our Higgs scalars 5_H and $\bar{5}_H$ are flavor-triplets. In general, such a model leads to a serious trouble, i.e. the flavor-changing neutral current (FCNC) problem. However, since our Higgs scalars 5_H and $\bar{5}_H$ couple to the quarks and leptons not directly, but via $10'_L\bar{5}_L\bar{5}_H$ and $10'_L 10_L 5_H$, the FCNC problem in the present model can substantially be evaded. Roughly speaking, when we denote the mass matrices M_u and M_d given in Eqs. (2.8) and (2.9) as $M_q = m_L M_Q^{-1} m_s$ symbolically, the effective interactions of $\overline{q}q$ with the Higgs scalars ϕ (5_H or $\overline{5}_H$) are given by $\overline{q}\phi M_Q^{-1} m_s q$, so that the effective FCNC interactions through ϕ are suppressed by the order of $(M_Q^{-1} m_s)^2 \sim 10^{-4}$. Therefore, the FCNC effects practically become invisible.

3 Quark mass matrices

In order to obtain realistic quark mass matrices, we must consider that the quark sectors in the heavy fermion mass terms (2.6) have some structures differently from the lepton sector μ_{10}^E. The Yukawa interaction (2.6) is invariant under a permutation symmetry S_3. (The form (2.6) is not a general form of the S_3 invariant Yukawa interactions. The form is constrained more than the S_3 symmetry.) Therefore, we assume that the heavy fermion mass matrices μ_{10}^F ($F = Q, U$) are also S_3-invariant. We concretely assume that μ_{10}^F ($F = Q, U$) are diagonal on the $(F_\pi, F_\eta, F_\sigma)$ basis, i.e.

$$M_D\left(\overline{F}_\pi F_\pi + \overline{F}_\eta F_\eta\right) + M_S \overline{F}_\sigma F_\sigma, \tag{3.1}$$

after the SU(5) symmetry was broken, where F_σ and (F_π, F_η) are singlet and doublet of S_3, respectively, and those are define by

$$\begin{pmatrix} F_\pi \\ F_\eta \\ F_\sigma \end{pmatrix} = A \begin{pmatrix} F_1 \\ F_2 \\ F_3 \end{pmatrix} \tag{3.2}$$

$$A = \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{2}} & -\frac{2}{\sqrt{6}} \\ \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}. \tag{3.3}$$

Then, the inverse of the heavy fermion mass matrices, $(\mu_{10}^F)^{-1}$, are given by

$$(\mu_{10}^F)^{-1} = A^T \mathrm{diag}(M_D^{-1}, M_D^{-1}, M_S^{-1}) A$$

$$= \frac{1}{M_D}(\mathbf{1} - X) + \frac{1}{M_S} X, \tag{3.4}$$

where

$$\mathbf{1}=\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X=\frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{3.5}$$

In other words, the fermion mass matrices μ_{10}^F are given by the following S_3-invariant form

$$(\mu_{10}^F)_{ij} = \mu_{10}\,(\mathbf{1}_{ij} + 3b_F X_{ij})\,, \tag{3.6}$$

where $3b_F = M_D^F/M_S^F - 1$. The case in the charged lepton sector corresponds to a specific case with $M_D = M_S$, i.e. a case of $b_F = 0$.

The quark mass matrices M_u and M_d with the forms (3.6) have already investigated in Ref. [11] as the so-called "democratic universal seesaw mass matrix model", where it has been found that the values $b_u = -1/3$ and $b_d = -e^{i\beta}$ ($\beta \simeq 20^\circ$) can give reasonable quark masses and the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix parameters. In the present model, the parameters (b_d, b_u) in Ref. [11] correspond to (b_Q, b_U), so that we take

$$b_Q \simeq -e^{i\beta} \quad (\beta \simeq 20^\circ), \quad b_U \simeq -\frac{1}{3}. \tag{3.7}$$

As pointed out in Ref. [11], for the choice $b_U = -1/3$, there is no inverse matrix of μ_{10}^U (i.e. $\det(\mu_{10}^U) = 0$), so that one of the up-quark masses has a mass of the order of v_u, and we identify it as the top quark mass. Another prediction from $b_U = -1/3$ is [12, 11]

$$\frac{m_u}{m_c} \simeq \frac{3m_e}{4m_\mu}. \tag{3.8}$$

Also, the choice $b_Q \simeq -1$ leads to the relations [11]

$$\frac{m_c}{m_b} \simeq 4\frac{m_\mu}{m_\tau}, \quad \frac{m_d m_s}{m_b^2} \simeq 4\frac{m_e m_\mu}{m_\tau^2}, \quad \frac{m_u}{m_d} \simeq 3\frac{m_s}{m_c} \simeq 3\left|\sin\frac{\beta}{2}\right|. \tag{3.9}$$

Since the purpose of the present model is to give the outline of the model, we do not give numerical re-fitting of the values (b_Q, b_U). We also do not reject a possibility that there is another parameter set (b_Q, b_U) which leads to favorable quark masses and CKM parameters.

4 Neutrino mass matrix

In the present section, we investigate whether the model given in Sec. 2 in order to understand the charged lepton mass formula (1.1) can explain the observed neutrino masses and mixings or not. Obviously, if $\langle 1'_H \rangle$ in Eq. (2.10) is also structure-less, i.e. if $\langle 1'_H \rangle$ has a unit matrix form, the neutrino mass matrix M_ν becomes a diagonal mass matrix as well as the charged lepton mass matrix M_e, so that we can obtain neither neutrino mixings nor reasonable neutrino mass spectrum. Also, if we assume that $M_R = y_S \langle 1'_H \rangle$ has the same structure as M_F (3.6) in the quark sectors, we will find that such a model cannot explain the observed neutrino data [13]. The purpose of the present section is to investigate what additional assumptions are needed for the explanation of the observed neutrino data.

In the expression (2.10) of the neutrino mass matrix M_ν, we have already assumed that the heavy fermion mass terms $\mu_1 1'_L 1'_L$ are sufficiently large to be neglected compared with the contribution of $\langle 1'_H \rangle$. We consider the observed peculiar structure of the neutrino mass matrix comes from the interactions among the heavy particles, $1'_L 1'_L 1'_H$. We assume the following simple S_3-invariant form for the interactions $1'_L 1'_L 1'_H$:

$$(y_S)_{ijk} 1'_{Li} 1'_{Lj} 1'_{Hk} = y_S (1'_{L1}\ 1'_{L2}\ 1'_{L3}) \begin{pmatrix} 1'_{H1} & 1'_{H3} & 1'_{H2} \\ 1'_{H3} & 1'_{H2} & 1'_{H1} \\ 1'_{H2} & 1'_{H1} & 1'_{H3} \end{pmatrix} \begin{pmatrix} 1'_{L1} \\ 1'_{L2} \\ 1'_{L3} \end{pmatrix} . \tag{4.1}$$

(Of course, the form (4.1) is not a general form of the S_3-invariant cubic interactions. Only when we require both a cyclic permutation symmetry and the S_3 symmetry, the possible forms of the Yukawa interactions are confined in the two forms (2.5) and (4.1).) When we denote the VEVs of $1'_H$ as $\langle 1'_{Hi} \rangle = v_S Z_i$ with a normalization condition $Z_1^2 + Z_2^2 + Z_3^2 = 1$, from Eq. (2.10), we obtain the neutrino mass matrix

$$M_\nu = m_0 \begin{pmatrix} z_1^2 (Z_1^2 - Z_2 Z_3) & z_1 z_2 (Z_3^2 - Z_1 Z_2) & z_1 z_3 (Z_2^2 - Z_1 Z_3) \\ z_1 z_2 (Z_3^2 - Z_1 Z_2) & z_2^2 (Z_2^2 - Z_1 Z_3) & z_2 z_3 (Z_1^2 - Z_2 Z_3) \\ z_1 z_3 (Z_2^2 - Z_1 Z_3) & z_2 z_3 (Z_1^2 - Z_2 Z_3) & z_3^2 (Z_3^2 - Z_1 Z_2) \end{pmatrix} , \tag{4.2}$$

where $m_0 = (y_\nu^2 v_u^2 / y_S v_S)/(Z_3^3 + Z_2^3 + Z_1^3 - 3 Z_1 Z_2 Z_3)$. In order to obtain a nearly bimaximal mixing, we must be take $z_2 Z_2 \simeq z_3 Z_3$.

The parameters Z_i are free from the values z_i, because the VEVs $\langle 1'_H \rangle$ may have different values from the VEVs of $(\bar{5}_H + 5_H + 1_H)$. However, from an economical point of view of the parameters, we interest in a case that the parameters Z_i also satisfy the relation (1.3) as well as z_i. Considering the phenomenological requirement $z_2 Z_2 \simeq z_3 Z_3$, by way of trial, we assume

$$(Z_1, Z_2, Z_3) = (z_1, z_3, z_2). \tag{4.3}$$

Since the energy scales of $\langle 1'_H \rangle$ and $\langle 1_H \rangle$ are different from each other (i.e. $\langle 1'_H \rangle \sim 10^{14}$ GeV and $\langle 1_H \rangle \sim 10^2$ GeV), we do not consider that the relation (4.3) is exact at a low energy scale.

At present, we do not know the origin of such the inversion $2 \leftrightarrow 3$, and it is a pure phenomenological assumption. Although we can obtain favorable predictions of the neutrino masses and mixings for the trial choice (4.3) as we show below, we can also obtain favorable results for suitable parameter values of (Z_1, Z_2, Z_3) without the assumption (4.3). The choice (4.3) is merely one of the successful parameter values Z_i. The relation (4.3) is not an essential assumption in the present model.

For the trial choice (4.3), we find the following numerical results:

$$m_{\nu 1} = 0.00737 m_0, \quad m_{\nu 2} = 0.01651 m_0, \quad m_{\nu 3} = 0.09965 m_0, \tag{4.4}$$

$$U = \begin{pmatrix} 0.8011 & -0.5904 & 0.0985 \\ 0.4532 & 0.4907 & -0.7442 \\ 0.3911 & 0.6408 & 0.6607 \end{pmatrix}, \tag{4.5}$$

so that we obtain

$$R = \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = 0.023, \tag{4.6}$$

$$\sin^2 2\theta_{23} = 0.97, \tag{4.7}$$

$$\tan^2 \theta_{12} = 0.54, \tag{4.8}$$

which are in good agreement with the present observed best-fit values [14, 15] $R \simeq (7.9 \times 10^{-5})/(2.8 \times 10^{-3}) = 0.029$, $\sin^2 2\theta_{atm} = 1.0$ and $\tan^2 \theta_{solar} = 0.40^{+0.10}_{-0.07}$. It is worth noticing that we do not have any free parameter in the neutrino sector except for the postulation (4.3). Of course, if we take slight deviations from the assumption (4.3), we can obtain more excellent agreements with the observed values. Thus, at least, we can say that we are going in the right direction.

If we put $m_{\nu 3} = \sqrt{\Delta m^2_{atm}} = 0.053$ eV, then we obtain

$$m_{\nu 1} = 0.0039 \text{ eV}, \quad m_{\nu 2} = 0.0088 \text{ eV}, \quad m_{\nu 3} = 0.053 \text{ eV}. \tag{4.9}$$

5 Concluding remarks

The purpose of the present paper is not to explain the charged lepton mass formula (1.1). When we consider that the relation is remarkably satisfied at a low energy scale, we inevitably reach to the idea that the mass spectrum originates not in the Yukawa coupling structure at a unification energy scale, but in the VEV structure of a three-flavor Higgs structure at the low energy scale. The purpose of the present paper is also not to investigate the validity of the mass matrix (1.3) for the charged leptons. Our interest is in an extension of the mass matrix (1.3) to mass matrices of the quarks and neutrinos. The purpose of the present paper is to investigate whether such a model can also explain or not the observed quark and neutrino masses and mixings with the universal structure of m_L (m_R) which is fixed in the charged lepton sector.

In Sec. 2, we have assumed the additional fermions $10'_L + \overline{10}'_L + 1'_L$. If we consider another models, for example, with $5'_L + \overline{5}'_L$, we will encounter some troubles when we try to build a universal seesaw model for quarks and neutrinos. Only the choice $10'_L + \overline{10}'_L + 1'_L$ yields a natural extension of the seesaw mass matrix model for charged leptons (with $M_E \sim 10^4$ GeV) to a model for the quarks and leptons. The essential assumption in Sec. 2 is Eq. (2.2), i.e. the VEV structures of the scalars $5_H + \overline{5}_H + 1_H$ are universal. Then, our interest was whether such a model can also explain the observed quark and neutrino masses and mixings or not.

For quark sectors, we have assumed that the heavy fermion mass terms are invariant under the S_3 symmetry, i.e. the heavy fermion mass matrices $M_F = \mu^F_{10}$ take the form (3.1), which leads to the democratic seesaw mass matrix form (3.4). We can find the parameter values which can give reasonable quark masses and CKM mixing parameters.

For the neutrino sector, our essential assumption is the S_3-invariant interactions (4.1) of the heavy fermions $1'_L$. Then, the parameters $Z_i = \langle 1'_{Hi} \rangle$ can take values which can give reasonable values of the neutrino masses and mixings. Especially, it is interesting that the values also satisfy the relation (1.3) as well as $v_i = \langle \phi_i \rangle$ ($\phi_i = 5_{Hi}$, $\overline{5}_{Hi}$ and 1_{Hi}). The choice $(Z_1, Z_2, Z_3) = (z_1, z_3, z_2)$, Eq. (4.3), can give $\Delta m^2_{solar}/\Delta m^2_{atm} = 0.023$, $\sin^2 2\theta_{atm} = 0.97$, and $\tan^2 \theta_{solar} = 0.54$. (Of course, those are not inevitable predictions in the present model. The choice (4.3) is merely an example of the parameter choice.)

The present model, at present, can give neither gauge unification nor SUSY scenario. However, we may say that the three-flavor Higgs model with the VEVs $v_i \propto \sqrt{m_{ei}}$ has a possibility to explain quark and lepton mass matrices with the same parameter values of v_i. The investigation of a possibility that the fermion masses are closely related to VEVs of three-flavor Higgs scalars is just in the beginning.

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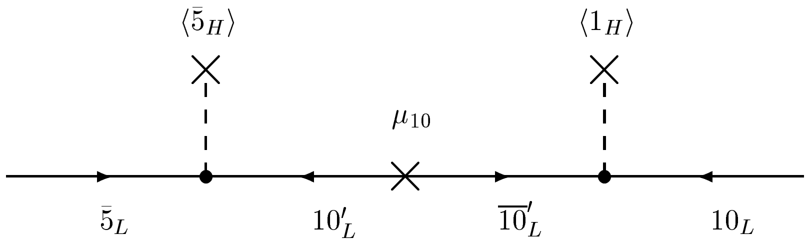
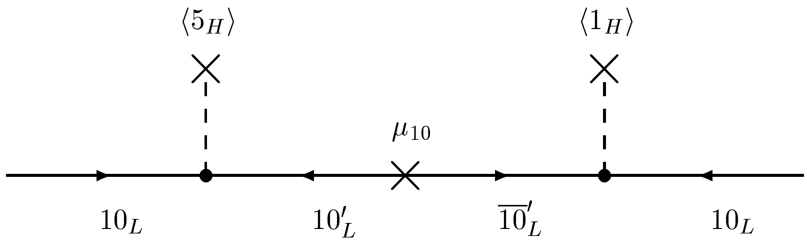
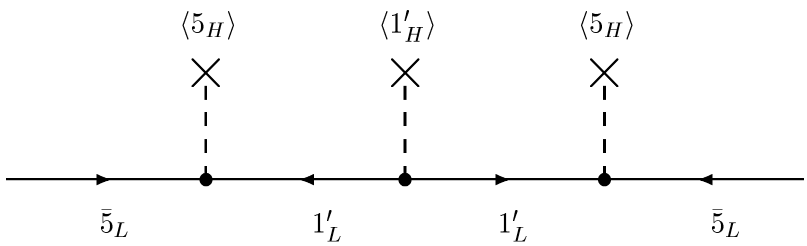


(a) M_e **and** M_d



(b) M_u



(c) M_ν

Fig. 1 Seesaw mass generation of the quark and leptons: (a) charged lepton and down-quark mass matrices M_e **and** M_d**, (b) up-quark mass matrix** M_u **and (c) neutrino mass matrix** M_ν**.**