

# NEUTRINO MASSES WITHOUT SEESAW MECHANISM IN A SUSY SU(5) MODEL WITH ADDITIONAL $\bar{5}'_L + 5'_L$

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A radiatively-induced neutrino mass matrix with a simple structure is proposed on the basis of an SU(5) SUSY GUT model with  $R$ -parity violation. The model has matter fields  $\bar{5}'_L + 5'_L$  in addition to the ordinary matter fields  $\bar{5}_L + 10_L$  and Higgs fields  $H_u + \bar{H}_d$ . The  $R$ -parity violating terms are given by  $\bar{5}_L \bar{5}_L 10_L$ , while the Yukawa interactions are given by  $\bar{H}_d \bar{5}'_L 10_L$ . Since the matter fields  $\bar{5}'_L$  and  $\bar{5}_L$  are different from each other at the unification scale, the  $R$ -parity violation effects at a low energy scale appear only through the  $\bar{5}'_L \leftrightarrow \bar{5}_L$  mixings. In order to make this  $R$ -parity violation effect harmless for proton decay, a discrete symmetry  $Z_3$  and a triplet-doublet splitting mechanism analogous to the Higgs sector are assumed.

## 1 Introduction

Why do the neutrinos have such tiny masses? There are typical two ideas of the origin of neutrino masses: One is the so-called “seesaw mechanism”<sup>1</sup>, and the other one is the “radiative mass-generation mechanism”<sup>2</sup>. The former can be embedded into a grand unification theory (GUT), but the latter is hard to be embedded into GUT. For example, a supersymmetric (SUSY) model with  $R$ -parity violation can provide radiative neutrino masses<sup>3</sup>, but the model inevitably induces unwelcome proton decay<sup>4</sup>. Therefore, as an origin of the neutrino masses, the idea of the seesaw mechanism is currently influential concerned with a GUT model. However, the unified description of quark and lepton mass matrices based on a GUT model is not still achieved even if we take the former standpoint. In the present talk, against the current opinion, I would like to investigate another possibility that the neutrino masses are radiatively generated.

The basic idea<sup>5,6</sup> is as follows: We introduce matter fields  $\bar{5}'_L + 5'_L$  in addition to the matter and Higgs fields  $\bar{5}_L + 10_L + \bar{H}_d + H_u$  in the conventional minimal SUSY SU(5) GUT model. The model has Yukawa interactions  $\bar{H}_d \bar{5}_L 10_L$  and  $R$ -parity violation-terms

$\bar{5}'_L \bar{5}'_L 10_L$ . Since the two  $\bar{5}$ -plet fields,  $\bar{5}_L$  and  $\bar{5}'_L$ , in the Yukawa interactions and  $R$ -parity violating terms, respectively, are different from each other, the  $R$ -parity violation-terms become visible only through  $\bar{5}_L \leftrightarrow \bar{5}'_L$  mixing. In order to make the  $R$ -parity violation harmless for proton decay, we will assume a mechanism analogous to a triplet-doublet splitting in the Higgs sector.

The explicit model is as follows: We introduce a discrete symmetry  $Z_3$  and assign the  $Z_3$  quantum numbers as follows:

$$\bar{H}_{d(-)} + H_{u(+)} + (\bar{5}_L + 10_L)_{(+)} + (\bar{5}'_L + 5'_L)_{(0)}, \quad (1)$$

where  $(+, 0, -)$  denote the  $Z_3$  transformation properties  $(\omega^{+1}, \omega^0, \omega^{-1})$  ( $\omega = e^{i2\pi/3}$ ). The  $Z_3$  invariant tri-linear terms are only three:

$$\begin{aligned} W_{tri} = & (Y_u)_{ij} H_{u(+)} 10_{L(+)} 10_{L(+)} \\ & + (Y_d)_{ij} \bar{H}_{d(-)} \bar{5}'_{L(0)} 10_{L(+)} \\ & + \lambda_{ijk} \bar{5}_{L(+)} \bar{5}'_{L(+)} 10_{L(+)} k. \end{aligned} \quad (2)$$

Note that  $\bar{5}'_L$  in the Yukawa interactions are different from  $\bar{5}_L$  in the  $R$ -parity violation-terms. On the other hand, the  $Z_3$  invariant bi-linear terms are only two. In order to give “triplet-doublet splitting”, we assume the following “effective” bi-linear terms:

$$W_{bi} = \bar{H}_{d(-)} (\mu + g_H \langle \Phi_{(0)} \rangle) H_{u(+)}$$

$$+ \bar{5}'_{L(0)i}(M_5 - g_5 \langle \Phi_{(0)} \rangle) \bar{5}'_{L(0)i}, \quad (3)$$

where  $\Phi$  is a 24-plet Higgs field and its vacuum expectation value (VEV) is  $\langle \Phi \rangle = v_{24} \text{diag}(2, 2, 2, -3, -3)$ . And we also assume a  $Z_3$  symmetry breaking term

$$W_{SB} = M_i^{SB} \bar{5}_{L(+i)} \bar{5}'_{L(0)i}, \quad (4)$$

which induces  $\bar{5}_L \leftrightarrow \bar{5}'_L$  mixing as follows:

$$\begin{aligned} \bar{5}'_{L(0)i} &= c_i \bar{5}_{Li}^{q\ell} + s_i \bar{5}_{Li}^{heavy}, \\ \bar{5}_{L(+i)} &= -s_i \bar{5}_{Li}^{q\ell} + c_i \bar{5}_{Li}^{heavy}, \end{aligned} \quad (5)$$

where

$$\begin{aligned} s_i^{(a)} &= \frac{M^{(a)}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}}, \\ c_i^{(a)} &= \frac{M_i^{SB}}{\sqrt{(M^{(a)})^2 + (M_i^{SB})^2}}, \end{aligned} \quad (6)$$

$M^{(2)} = M_5 + 3g_5 v_{24}$ , and  $M^{(3)} = M_5 - 2g_5 v_{24}$ . Therefore, we obtain the following effective  $R$ -parity violation-terms:

$$W_R^{eff} = s_i^{(a)} s_j^{(b)} \lambda_{ijk} \bar{5}_{Li}^{q\ell(a)} \bar{5}_{Lj}^{q\ell(b)} 10_{Lk}, \quad (7)$$

Hereafter, for simplicity, we denote  $\bar{5}_{Li}^{q\ell(a)}$  as  $\bar{5}_{Li}^{(a)}$ .

We take the parameter values as follows:

$$\begin{aligned} M^{(2)} &\sim M_{GUT}, \quad M^{(3)} \sim m_{SUSY}, \\ M_i^{SB} &\sim M_{GUT} \times 10^{-1}, \end{aligned} \quad (8)$$

so that we obtain values of the mixing parameters  $s_i^{(2)} \simeq 1$  and  $c_i^{(2)} \simeq M_i^{SB}/M^{(2)} \sim 10^{-1}$  for the doublet components, and  $s_i^{(3)} \simeq M^{(3)}/M_i^{SB} \sim 10^{-12}$  and  $c_i^{(3)} \simeq 1$  for the triplet components. Since the unwellcome  $R$ -parity violation-terms  $d_R^c d_R^c u_R^c$  and  $d_R^c (e_L u_L - \nu_L d_L)$  are suppressed by the factors  $s^{(3)} s^{(3)} \sim 10^{-24}$  and  $s^{(3)} s^{(2)} \sim 10^{-12}$ , respectively, the proton decay due to the  $R$ -parity violation-terms is suppressed by the factor of  $10^{-36}$ . On the other hand, the  $R$ -parity violation-terms  $(e_L \nu_L - \nu_L e_L) e_R^c$  are of the order of  $s^{(2)} s^{(2)} \sim 1$ .

Note that  $M_d \neq M_e^T$  in the present model, because

$$M_d^\dagger = C^{(3)} Y_d v_d, \quad M_e^* = C^{(2)} Y_d v_d, \quad (9)$$

where  $C^{(3)} = \mathbf{1} + O(10^{-24})$  and  $C^{(2)} \sim 10^{-1}$ .

## 2 Neutrino mass matrix

First, we calculate a radiative mass from the diagram Fig.1:

$$\begin{aligned} (M_{rad})_{ij} &\propto s_i s_j s_k s_n \lambda_{ikm}^* \lambda_{jnl}^* (M_e)_{kl}^* (\widetilde{M}_{eLR}^{2T})_{mn}^* \\ &+ (i \leftrightarrow j), \end{aligned} \quad (10)$$

where  $s_i = s_i^{(2)}$ .

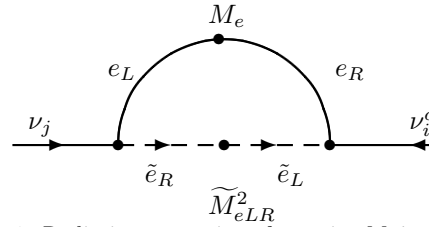


Figure 1. Radiative generation of neutrino Majorana mass

When we define

$$K = (S M_e L^T)^*, \quad (11)$$

$$\lambda_{ijk} = \varepsilon_{ijl} L_{lk}, \quad (12)$$

$$S = \text{diag}(s_1^{(2)}, s_2^{(2)}, s_3^{(2)}) \simeq \mathbf{1}, \quad (13)$$

we can express  $M_{rad}$  as

$$(M_{rad})_{ij} = m_0^{-1} s_i s_j \varepsilon_{ikm} \varepsilon_{jln} K_{ml} K_{nk}, \quad (14)$$

where

$$m_0^{-1} = \frac{2}{16\pi^2} (A + \mu^{(2)} \tan \beta) \frac{\ln(m_{\tilde{\nu}_R}^2/m_{\tilde{\nu}_L}^2)}{m_{\tilde{\nu}_R}^2 - m_{\tilde{\nu}_L}^2}. \quad (15)$$

Next, we calculate contributions from the non-vanishing sneutrino VEV  $\langle \tilde{\nu} \rangle \neq 0$ . In the present model, the VEV of sneutrino is exactly zero at tree level, because of the  $Z_3$  symmetry. However, only an effective  $m_{HLi}^2$ -term can appear via the loop diagram  $\overline{H}_d \rightarrow (\bar{5}_L^{q\ell})^c + (10_L)^c \rightarrow \bar{5}_L^{q\ell}$  (Fig. 2), which gives

$$(m_{HLi}^2)_{eff} \propto s_i s_j \lambda_{ijk} (M_e)_{jk} = s_i \varepsilon_{ijk} K_{jk}^*. \quad (16)$$

Since  $\langle \tilde{\nu}_i \rangle \propto (m_{HLi}^2)_{eff}^*$ , we obtain

$$(M_{VEV})_{ij} = \xi m_0^{-1} s_i s_j \varepsilon_{ikl} \varepsilon_{jmn} K_{kl} K_{mn}, \quad (17)$$

where  $\xi$  is a relative ratio of  $M_{VEV}$  to  $M_{rad}$ .

In conclusion, we obtain the following general form of the neutrino mass matrix<sup>6</sup>

$$(M_\nu)_{ij} = m_0^{-1} s_i s_j \varepsilon_{ikl} \varepsilon_{jmn} (K_{kn} K_{ml} + \xi K_{kl} K_{mn}), \quad (18)$$

i.e.

$$M_\nu = m_0^{-1} S [A(1 + \xi) + B] S \quad (19)$$

where

$$\begin{aligned} A &= (K - K^T)(K - K^T) - \mathbf{1} \text{Tr}(KK - KK^T), \\ B &= (K + K^T - \mathbf{1} \text{Tr}K) \text{Tr}K \\ &\quad - (KK + K^T K^T) + \mathbf{1} \text{Tr}(KK), \end{aligned} \quad (20)$$

Note that  $A$  is a rank-1 matrix which is independent of the diagonal elements of  $K$ ,  $K_{11}$ ,  $K_{22}$  and  $K_{33}$ .

### 3 A simple example

Hereafter, we discuss the quantities on the flavor basis where  $M_e$  is diagonal.

Let us consider a simple form of  $K$  which gives  $A \gg B$ . We assume the following form of  $K$ :

$$K/m_{0K} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} + \varepsilon \mathbf{1}, \quad (21)$$

where  $m_{0K}$  is a constant with a dimension of mass. The form (21) means that in the limit of  $\varepsilon = 0$ , the coefficients  $\lambda_{ijk}$  of the  $R$ -parity violation terms are given by  $\lambda_{ij1} = \text{const} \equiv \lambda$  and  $\lambda_{ij2} = \lambda_{ij3} = 0$ , i.e.  $\bar{5}_{Li} \bar{5}_{Lj}$

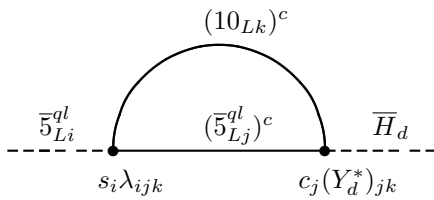


Figure 2. Effective  $\bar{5}_L^q \bar{5}_L^q \bar{H}_d^\dagger$  term

(i.e.  $\ell_{Li} \ell_{Lj}$ ) can couple only to  $10_{L1}$  (i.e.  $e_R^c$ ).

The assumption (21) leads to

$$M_\nu = (1 + \xi)A + B, \quad (22)$$

where

$$A = m_{0K}^2 \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{pmatrix}, \quad (23)$$

and

$$B = m_{0K}^2 \varepsilon \begin{pmatrix} 2\varepsilon & 1 & 1 \\ 1 & -(1 + \varepsilon) & 0 \\ 1 & 0 & -(1 + \varepsilon) \end{pmatrix}, \quad (24)$$

$m_0^\nu = m_0^{-1} m_{0K}^2$  and we have put  $S = \mathbf{1}$ . The mass matrix (22) gives the following eigenvalues and mixings:

$$\begin{aligned} m_{\nu 1} &= (\sqrt{3} - 1 - 2\varepsilon) \varepsilon m_0^\nu, \\ m_{\nu 2} &= -(\sqrt{3} + 1 + 2\varepsilon) \varepsilon m_0^\nu, \\ m_{\nu 3} &= 2(1 + \xi - \varepsilon - \varepsilon^2) m_0^\nu, \end{aligned} \quad (25)$$

$$U_\nu = \begin{pmatrix} \sqrt{\frac{\sqrt{3}+1}{2\sqrt{3}}} & -\sqrt{\frac{\sqrt{3}-1}{2\sqrt{3}}} & 0 \\ \frac{1}{\sqrt{6+2\sqrt{3}}} & \frac{1}{\sqrt{6-2\sqrt{3}}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{6+2\sqrt{3}}} & \frac{1}{\sqrt{6-2\sqrt{3}}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \quad (26)$$

Note that the structure of the mixing matrix  $U_\nu$ , (26), is independent of the parameters  $\xi$  and  $\varepsilon$ . Therefore, we obtain the neutrino mixing parameters

$$\tan^2 \theta_{solar} = \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 0.268, \quad (27)$$

together with  $\sin^2 2\theta_{atm} = 1$  and  $|U_{13}|^2 = 0$ , and the ratio of the neutrino mass squared

$$R \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} \simeq \frac{\sqrt{3}(1 + 2\varepsilon)}{(1 + \xi)(1 + \xi - 2\varepsilon)} \varepsilon^2. \quad (28)$$

Roughly speaking, these results are favorable to the recent neutrino data<sup>7,8</sup>. Although the predicted value of  $\tan^2 \theta_{solar}$  is somewhat smaller than the observed best fit value, the value can suitably be adjusted by a small deviation of  $S$  from  $S = \mathbf{1}$  and the renormalization group equation effects.

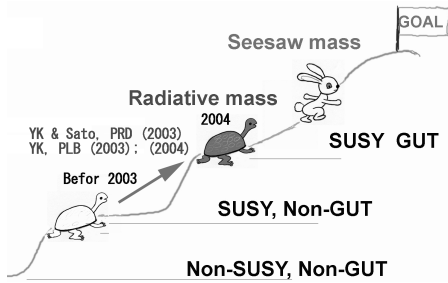


Figure 3. The status of the origin of the neutrino mass in 2004: the radiative neutrino mass hypothesis has considerably become plausible than before

#### 4 Conclusions

Based on a SUSY SU(5) GUT model with harmless  $R$ -parity violation, we have proposed a neutrino mass matrix with a simple form, which are given by sum of the radiative masses plus nonvanishing sneutrino VEV contributions. The model with a simple assumption (21) leads to reasonable results

$$\begin{aligned} \sin^2 2\theta_{23} &= 1, \quad |U_{13}| = 0, \\ \tan^2 \theta_{12} &= \frac{\sqrt{3} - 1}{\sqrt{3} + 1} = 0.268, \end{aligned} \quad (29)$$

independently of the parameters  $\varepsilon$  and  $\xi$ . However, at present, it is an open question why we should choose such the simple form of  $K$ .

Although the form (21) is only an example, we can, at least, say that, as the origin of the neutrino masses, we should seriously take a possibility of the radiative mass generation mechanism as well as that of the seesaw mechanism.

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