UNIVERSAL TEXTURE OF QUARK AND LEPTON MASS MATRICES

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Against the conventional picture that the mass matrix forms in the quark sectors will take somewhat different structures from those in the lepton sectors, a possibility that all the mass matrices of quarks and leptons have the same form as in the neutrinos is investigated. For the lepton sectors, the model leads to a nearly bimaximal mixing with the prediction $|U_{e3}|^2 = m_e/2m_\mu = 0.0024$ and $\tan^2\theta_{sol} \simeq m_{\nu 1}/m_{\nu 2}$, and so on. For the quark sectors, it can lead to reasonable values of the CKM mixing matrix and masses: $|V_{us}| \simeq \sqrt{m_d/m_s}$, $|V_{ub}| \simeq |V_{cb}|\sqrt{m_u/m_c}$, $|V_{td}| \simeq |V_{cb}| \cdot |V_{us}|$, and so on.

1. Model

Recent neutrino oscillation experiments[1,2,3,4] have highly suggested a nearly bimaximal mixing ($\sin^2 2\theta_{12} \sim 1$, $\sin^2 2\theta_{23} \simeq 1$) together with a small ratio $R \equiv \Delta m^2_{12}/\Delta m^2_{23} \sim 10^{-2}$. On the other hand, we know that the observed quark mixing matrix V_{CKM} is characterized by small mixing angles. Thus, the mixing matrices of quarks and leptons are very different from each other. Therefore, usually, the following picture is accepted: the mass matrix forms in the quark sectors will take somewhat different structures from those in the lepton sectors.

Against such a conventional picture, we investigate a possibility that all the mass matrices of quarks and leptons have the same forms as in the neutrino sector:

$$M_f = P_{Lf}^\dagger \widehat{M}_f P_{Rf} \ , \tag{1}$$

$$\widehat{M}_f = \begin{pmatrix} 0 & a_f & a_f \\ a_f & b_f & c_f \\ a_f & c_f & b_f \end{pmatrix} \ (f = u, d, \nu, e) \,, \tag{2}$$

where $P_f = \mathrm{diag}(e^{i\delta_1^f}, e^{i\delta_2^f}, e^{i\delta_3^f})$, and the mass matrix $\widehat{M}_f$ is invariant under a permutation symmetry between second and third generations. The mass matrix parameters a_f, b_f, and c_f can be expressed in terms of the mass eigenvalues as $a_f = \sqrt{m_{f2} m_{f1}/2}$, $b_f = (m_{f3}/2)\,[1 + (m_{f2} - m_{f1})/m_{f3}]$, and $c_f = -(m_{f3}/2)\,[1 - (m_{f2} - m_{f1})/m_{f3}]$.

The mass matrix form (2) was suggested from the neutrino mass matrix form [5] which leads to a nearly bimaximal mixing

$$U_\nu \equiv \begin{pmatrix} c_\nu & s_\nu & 0 \\ -\frac{s_\nu}{\sqrt{2}} & \frac{c_\nu}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_\nu}{\sqrt{2}} & \frac{c_\nu}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} \ , \tag{3}$$

where $U_\nu^T M_\nu U_\nu = \mathrm{diag}(-m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$ and

$$c_\nu = \cos\theta_\nu = \sqrt{\frac{m_{\mu 2}}{m_{\nu 2} + m_{\nu 1}}} \ , \quad s_\nu = \sin\theta_\nu = \sqrt{\frac{m_{\nu 1}}{m_{\nu 2} + m_{\nu 1}}} \ . \tag{4}$$

Note that the matrix form (1) with (2) is almost invariant under the renormalization group equation (RGE) effects, so that we can use the expression (1) with (2) for the predictions of the physical quantities in the low-energy region, as well as those at the unification scale. The zeros in this mass matrix are constrained by a discrete symmetry Z_3 that is discussed in Ref.[6], defined at a unification scale (the scale does not always mean "grand unification scale"). This discrete symmetry Z_3 is broken below $\mu = M_R$, at which the right-handed neutrinos acquire heavy Majorana masses. Therefore, the matrix form (1) will, in general, be changed by the RGE effects. Nevertheless, we can use the expression (1) with (2) for the predictions of the physical quantities in the low-energy region, as discussed in Ref. [6].

2. Quark mixing matrix

The quark mass matrices with the form (1) are diagonalized by the bi-unitary transformation $D_f = U_{Lf}^\dagger M_f U_{Rf}$, where $U_{Lf} \equiv P_{Lf}^\dagger O_f$, $U_{Rf} \equiv P_{Rf}^\dagger O_f$, and O_d (O_u) is given by

$$O_f \equiv \begin{pmatrix} c_f & s_f & 0 \\ -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ -\frac{s_f}{\sqrt{2}} & \frac{c_f}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix} . \tag{5}$$

(For simplicity, hereafter, we will take $P_R = P_L^\dagger$, so that the matrix M_f becomes a symmetric matrix. However, this assumption is not essential for the results in the present model.) Then, the Cabibbo–Kobayashi–Maskawa (CKM) quark mixing matrix V is given by

$$\begin{aligned} V &= U_{Lu}^\dagger U_{Ld} = O_u^T P_u P_d^\dagger O_d \\ &= \begin{pmatrix} c_u c_d + \rho s_u s_d & c_u s_d - \rho s_u c_d & -\sigma s_u \\ s_u c_d - \rho c_u s_d & s_u s_d + \rho c_u c_d & \sigma c_u \\ -\sigma s_d & \sigma c_d & \rho \end{pmatrix} , \end{aligned} \tag{6}$$

where ρ and σ are defined by

$$\rho = \frac{1}{2}(e^{i\delta_3} + e^{i\delta_2}) \ , \quad \sigma = \frac{1}{2}(e^{i\delta_3} - e^{i\delta_2}) \ . \tag{7}$$

Here we have put $P \equiv P_u P_d^\dagger \equiv \mathrm{diag}(e^{i\delta_1}, e^{i\delta_2}, e^{i\delta_3})$, and we have taken $\delta_1 = 0$ without loss of generality.

From the expression (6), we obtain the phase-parameter independent predictions (the 3rd generation quark-mass independent predictions [7])

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{s_u}{c_u} = \sqrt{\frac{m_u}{m_c}} \ , \quad \frac{|V_{td}|}{|V_{ts}|} = \frac{s_d}{c_d} = \sqrt{\frac{m_d}{m_s}} \ , \tag{8}$$

which are almost independent of the RGE effects.

Next let us fix the parameters δ_3 and δ_2. From the relation

$$|V_{cb}| = \frac{1}{\sqrt{1 + m_u/m_c}} \sin\frac{\delta_3 - \delta_2}{2} \ , \tag{9}$$

and the observed value [8] $|V_{cb}| = 0.0412 \pm 0.0020$, we obtain $\delta_3 - \delta_2 = 4.59^\circ \pm 0.21^\circ$. Also, from the expression of $|V_{us}|$, we can obtain the value $\delta_3 + \delta_2 = 93^\circ \pm 22^\circ$. Because of the small value $\sin(\delta_3 - \delta_2)/2 \simeq 0.04$, we obtain the following approximate relations

$$|V_{us}| \simeq \sqrt{\frac{m_d}{m_s}} \ , \quad |V_{cd}| \simeq \sqrt{\frac{m_d}{m_s}} \ , \quad |V_{td}| \simeq |V_{cb}| \cdot |V_{us}| \ , \tag{10}$$

which are consistent with the present experimental data [8].

In the present model, the rephasing invariant Jarlskog parameter J is given by

$$J = |\sigma|^2 |\rho| c_u s_u c_d s_d \sin\frac{\delta_3+\delta_2}{2} \simeq |V_{ub}||V_{cb}||V_{us}| \sin\frac{\delta_3+\delta_2}{2} . \tag{11}$$

Therefore, the phase factor δ in the standard expression of V corresponds to $\delta \simeq \delta_3 + \delta_2/2$ in the present model. We predict $|J| = (1.91 \pm 0.38) \times 10^{-5}$.

3. Lepton mixing matrix

We assume that the neutrino masses are generated via the seesaw mechanism $M_\nu = -M_D M_R^{-1} M_D^T$. Here M_D and M_R are the Dirac neutrino and the right-handed Majorana neutrino mass matrices. Note that when we assume the same matrix forms (1) for M_D and M_R, the effective neutrino mass matrix $M_\nu = -M_D M_R^{-1} M_D$ is again given by the same texture (1):

$$M_\nu = -P_\nu^\dagger \widehat{M}_D \widehat{M}_R^{-1} \widehat{M}_D^T P_\nu^\dagger = P_\nu^\dagger \widehat{M}_\nu P_\nu^\dagger \ . \tag{12}$$

Therefore, we obtain the lepton mixing matrix U

$$U = O_e^T P O_\nu = \begin{pmatrix} c_e c_\nu + \rho_\nu s_e s_\nu & c_e s_\nu - \rho_\nu s_e c_\nu & -\sigma_\nu s_e \\ s_e c_\nu - \rho_\nu c_e s_\nu & s_e s_\nu + \rho_\nu c_e c_\nu & \sigma_\nu c_e \\ -\sigma_\nu s_\nu & \sigma_\nu c_\nu & \rho_\nu \end{pmatrix} , \tag{13}$$

where $P \equiv P_e P_\nu^\dagger \equiv \mathrm{diag}(e^{i\delta_{\nu 1}}, e^{i\delta_{\nu 2}}, e^{i\delta_{\nu 3}})$. Hereafter we will again take $\delta_{\nu 1} = 0$ without loss of generality. Note that $V = O_u^T P O_d$, while $U = O_e^T P O_\nu$, so that all the mixing formulae in the lepton sectors are given by the replacement $(m_u, m_c, m_t) \to (m_e, m_\mu, m_\tau)$ and $(m_d, m_s, m_b) \to (m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$ in those in the quark sectors. However, this does not means that the physics in the up-quark (down-quark) sector corresponds to the physics in the charged lepton (neutrino) sector. In fact, as we see in Table 1, the parameter values in each sector are different from the other.

We obtain the phase-parameter independent predictions

$$\frac{|U_{13}|}{|U_{23}|} = \frac{s_e}{c_e} = \sqrt{\frac{m_e}{m_\mu}} \ , \quad \frac{|U_{31}|}{|U_{32}|} = \frac{s_\nu}{c_\nu} = \sqrt{\frac{m_{\nu 1}}{m_{\nu 2}}} \ . \tag{14}$$

The neutrino mixing angle θ_{atm} under the constraint $|\Delta m_{23}^2| \gg |\Delta m_{12}^2|$ is given by

$$\sin^2 2\theta_{\rm atm} \equiv 4\left|U_{23}\right|^2 \left|U_{33}\right|^2 = 4\left|\rho_\nu\right|^2 \left|\sigma_\nu\right|^2 c_e^2 = \frac{m_\mu}{m_\mu + m_e} \sin^2(\delta_{\nu 3} - \delta_{\nu 2}) \ . \tag{15}$$

Table 1. Input values m_i and output values a_f, b_f and c_f. The numerical values are given in unit of GeV except for the neutrino sector (in unit of eV). The input values are those [9] at $\mu = m_Z$ except for the neutrino sector (for the neutrino sector, see the text).

	Inputs			Outputs		
f	m_{f1}	m_{f2}	m_{f3}	a_f	b_f	$-c_f$
u	0.00233	0.677	181	0.0280	90.8	90.2
d	0.00469	0.0934	3.00	0.0148	1.54	1.46
e	0.000487	0.103	1.75	0.00500	0.924	0.822
ν	0.0030	0.0088	0.050	0.0036	0.028	0.022

We assume the maximal mixing between ν_μ and ν_τ, so that we take $\delta_{\nu 3} - \delta_{\nu 2} = \pi/2$. Then, the model predicts

$$|U_{13}|^2 = \frac{1}{2}\frac{m_e}{m_\mu + m_e} = 0.00236 \ , \tag{16}$$

which is consistent with the constraint $|U_{13}|^2_{\rm exp} < 0.03$ from the CHOOZ data [3]. The mixing angle θ_{solar} is given by

$$\sin^2 2\theta_{solar} \equiv 4\left|U_{11}\right|^2\left|U_{12}\right|^2 \simeq \frac{4 m_{\nu 1}/m_{\nu 2}}{(1+m_{\nu 1}/m_{\nu 2})^2} \ , \tag{17}$$

which leads to the relation $m_{\nu 1}/m_{\nu 2} \simeq \tan^2\theta_{solar}$. Therefore, the best fit value [2] $\tan^2\theta_{solar} = 0.34$ predicts the neutrino mass ratio $m_{\nu 1}/m_{\nu 2} \simeq 0.34$, so that we can obtain the neutrino masses

$$m_{\nu 1} = 0.0030\,{\rm eV} \ , \quad m_{\nu 2} = 0.0088\,{\rm eV} \ , \quad m_{\nu 3} = 0.050\,{\rm eV} \ , \tag{18}$$

where we have used the best fit values [4,1] of $\Delta m^2_{solar} = 6.9 \times 10^{-5}$ eV^2 and $\Delta m^2_{atm} = 2.5 \times 10^{-3}$ eV^2.

We also obtain the averaged neutrino mass $\langle m_\nu \rangle \sim (10^{-3} - 10^{-4})$ eV, but the explicit value is highly dependent on the value of $\delta_\nu \equiv (\delta_{\nu 3} + \delta_{\nu 2})/2$. At present, we cannot fix the value of δ_ν.

4. Conclusion

In conclusion, stimulated by recent neutrino data, which suggest a nearly bimaximal mixing, we have investigated a possibility that all the mass matrices of quarks and leptons have the same texture as the neutrino mass matrix. In spite of the assumption of the universal texture for all the fermion mass matrices, we can obtain the differences between V_{quark} and V_{lepton} as follows: (i) the mixing between 1st and 2nd generations is given by $\tan\theta_{12} = \sqrt{m_1/m_2}$, so that the well-known empirical relation $|V_{us}| \simeq \sqrt{m_d/m_s}$ is due to the observed mass rations $m_u/m_c \ll m_d/m_s \ll 1$, and the nearly maximal mixing $|V_{e2}| \sim 1/\sqrt{2}$ is due to the approximate degeneracy $m_{\nu 1} \sim m_{\nu 2}$ and the observed mass ratio $m_e/m_\mu \ll 1$; (ii) the mixing between 2nd and 3rd generations is given by the relation (9) (and the corresponding relation in the lepton sector), so that the small value $|V_{cb}| \simeq 0.04$ means $(\delta_3 - \delta_2)/2 \simeq 0.04$ and $m_u/m_c \ll 1$, and the maximal mixing $V_{\mu 3} \simeq 1/\sqrt{2}$ means $\delta_3 - \delta_2 \simeq \pi/2$ together with $m_e/m_\mu \ll 1$.

The present data in the quark sectors have already fixed the CP violating phase parameters δ_3 and δ_2, while the present neutrino data have yet not fixed the parameter $(\delta_{\nu 3} + \delta_{\nu 2})/2$, although they have fixed the value of $(\delta_{\nu 3} - \delta_{\nu 2})/2$. We hope that future experiments on the CP violation will fix our remaining parameter δ_ν. Then, we will be able to obtain a clue to the origin of our phase parameters δ_i $(\delta_{\nu i})$.

Since, in the present model, each mass matrix M_f (i.e. the Yukawa coupling Y_f) takes different values of the parameters a_f, b_f, and so on, the present model cannot be embedded into a GUT scenario. In spite of such a demerit, however, it is worth while noting that the present model can give a unified description of quark and lepton mass matrices with the same texture.

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