

Prospects of the Zee model

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The Zee model is one of promising models of neutrino mass generation mechanism. However, the original Zee model is not on the framework of the ground unification scenario, and moreover, it is recently pointed out that the predicted value of $\sin^2 2\theta_{solar}$ must be satisfied the relation $\sin^2 2\theta_{solar} > 0.99$. We discuss whether possible GUT versions of the Zee model can be free from the severe constraint $\sin^2 2\theta_{solar} > 0.99$ or not. We will conclude that the following two models are promising: an R -parity violating SUSY GUT model and an $SO(10)$ model with a 126-plet scalar.

1. INTRODUCTION

The Zee model [1] is one of the attractive models for neutrino mass matrix. The mass matrix form is given by

$$M_\nu = \begin{pmatrix} 0 & a & c \\ a & 0 & b \\ c & b & 0 \end{pmatrix}, \quad (1.1)$$

where

$$\begin{aligned} a &= f_{e\mu}(m_\mu^2 - m_e^2)K, \\ b &= f_{\mu\tau}(m_\tau^2 - m_\mu^2)K, \\ c &= f_{\tau e}(m_e^2 - m_\tau^2)K, \end{aligned} \quad (1.2)$$

and K is a common factor. The model has only 3 free parameters and it can naturally lead to a large neutrino mixing [2]. Especially, for the case $a = c \gg b$, it leads to a bi-maximal mixing [3]

$$U \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \quad (1.3)$$

with $\Delta m_{12}^2/\Delta m_{23}^2 \simeq \sqrt{2}b/a$.

However, from the standpoint of the unification of quarks and leptons, there are some problems to be overcome.

(1) The masses of the quarks and charged leptons are generated by the Higgs mechanism, so

that they depend on the Yukawa coupling constants y_{ij} , while only neutrino masses are generated radiatively, so that they depend on the Zee coupling constants f_{ij} . In general, f_{ij} are independent of y_{ij} . We must seek for a principle which relates f_{ij} to y_{ij} . (For such an attempt, for example, see Ref.[4],)

(2) The Zee model is not embedded in a Grand Unification Theory (GUT). We must seek for a room for the Zee scalar h^+ in a GUT scenario.

(3) The Zee model leads to a severe constraint [6] $\sin^2 2\theta_{solar} > 0.99$ under the observed ratio $\Delta m_{solar}^2/\Delta m_{atm}^2 \ll 1$. The present solar neutrino data [5] show $\sin^2 2\theta_{solar} \sim 0.8$. We must investigate a modified Zee model which can be free from the severe constraint $\sin^2 2\theta_{solar} > 0.99$.

The problem (3) has recently pointed out by the author[6]. A parameter independent investigation leads to a severe constraint on the value of $\sin^2 2\theta_{solar}$

$$\sin^2 2\theta_{solar} \geq 1 - \frac{1}{16} \left(\frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} \right)^2. \quad (1.4)$$

The conclusion cannot be loosened even if we take RGE effects into consideration: Also, we can show that the two-loop effect is negligibly small.

The simple ways to escape from the constraint (1.8) will be as follows: One is to consider [7] that the Yukawa vertices of the charged leptons can couple to both scalars ϕ_1 and ϕ_2 . Another one [8] is to introduce a single right-handed neutrino ν_R and a second singlet Zee scalar S^+ . Also, a

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model with a new doubly charged scalar k^{++} is interesting because the two loop effects in such a model can give non-negligible contributions to the neutrino masses [9]. As another attractive model, there is an idea [10,11] that in an R -parity violating SUSY model we identify Zee scalar h^+ as slepton \tilde{e}_R . Then, we can obtain additional contributions from d -quark loops to the neutrino masses. However, if we want to extend the model to a GUT scenario, we will meet a new trouble “proton decay” as stated in the next section.

Anyhow, these models are not connected to GUT scenarios. We want to embed the Zee model into a GUT scenario. For an extended Zee model based on a GUT scenario, there is, for example, the Haba-Matsuda-Tanimoto model [12]. They have regarded the Zee scalar h^+ as a member of the messenger field $M_{10} + \overline{M}_{10}$ of SUSY-breaking on the basis of an SU(5) SUSY GUT. However, their model cannot escape from the constraint (1.8) as we discuss in the next section.

2. EXTENDED ZEE MODEL WITH GUT

We identify the Zee scalar h^+ as a member of SU(5) 10-plet scalar (including a case of $\overline{5} + 10$ sfermion). Then, we must pay attention to the following items:

- (a) Proton decay is safely forbidden.
- (b) The model is free from the severe constraint $\sin^2 2\theta_{solar} > 0.99$ in the Zee model.

2.1. How to avoid proton decay

Generally, the $\overline{5}$ -plet and 10-plet scalars $\phi_{\overline{5}}$ and ϕ_{10} can couple to the $\overline{5} + 10$ -plet fermions as follows.

$$\begin{aligned} A_5 &\equiv (\overline{\psi}_{\overline{5}}^c)^A (\psi_{10})_{AB} (\phi_{\overline{5}})^B, \\ A_{10} &\equiv (\overline{\psi}_{\overline{5}}^c)^A (\psi_{\overline{5}})^B (\phi_{10})_{AB}, \\ B_5 &\equiv \varepsilon^{ABCDE} (\overline{\psi}_{10}^c)_{AB} (\psi_{10})_{CD} (\overline{\phi}_{\overline{5}})_E. \end{aligned} \quad (2.1)$$

The terms A_{10} can contribute to the radiative neutrino mass, while the terms A_5 and B_5 induce the proton decay. We want the terms A_{10} , but do not want terms A_5 and B_5 .

The conventional R -parity violating SUSY model [11] contains terms A_{10} , but also contains terms A_5 and B_5 , so that the model is ruled out

from the candidates of the extended Zee model based on a GUT scenario.

For example, in the Haba-Matsuda-Tanimoto mode [12], there is no SU(5) 5-plet scalar (except for the conventional Higgs scalars), so that there is no proton decay due to A_5 and B_5 terms. In their model, in principle, the down-quark loop diagrams can contribute to the neutrino masses in addition to the charged lepton loop diagrams. Such the down-quark loop diagrams can contribute to the non-diagonal elements of the neutrino mass matrix. However, the down-quark loop diagrams include a colored Higgs scalar (the triplet component of the SU(5) 5-plet Higgs scalar). We suppose that the colored Higgs scalar has a mass of the order of the grand unification scale in order to suppress the proton decay due to the colored Higgs scalar. Therefore, the contributions are negligibly small, so that the model cannot still be free from the severe constraint $\sin^2 2\theta_{solar} > 0.99$.

2.2. R -parity violating model

Note that if we assume that R -parity violating interactions are allowed only for special generations (families), then we can choose such the A_5 and B_5 interactions as they do not contribute to the proton decay without breaking the SU(5) GUT.

For example, we assume

$$\begin{aligned} A_5 &\equiv \lambda_{ij}^3 (\overline{\psi}_{\overline{5}}^c)_i^A (\psi_{10})_{3AB} (\tilde{\psi}_{\overline{5}})_j^B, \\ A_{10} &\equiv \lambda_{ij}^3 (\overline{\psi}_{\overline{5}}^c)_i^A (\psi_{\overline{5}})_j^B (\psi_{10})_{3AB}, \end{aligned} \quad (2.4)$$

i.e., only the interactions with the SU(5) 10-plet superfield of the third generation violate the R -parity. Besides, we must assume this expression (5.4) is true on the basis on which the up-quark mass matrix is diagonal, because if it is not so, the proton decay still occurs through the CKM mixing V_{13}^* . In order to forbid the u - t mixing at energy scale, we must assume additional flavor symmetries (a discrete symmetry, and so on).

Although the assumption (2.4) is somewhat unnatural, anyhow, the model can give nonvanishing diagonal elements in the neutrino mass matrix. The contributions give the form

$$M_{ij}^{e,d loop} = m_0 f_i f_j \quad \text{with } f_i = \lambda_{i3}^3. \quad (2.5)$$

This matrix form (2.5) corresponds to the Drees-Pakvasa-Tata-Veldhuis model [10].

2.3. Model with a SU(2) triplet scalar

Except for the “restricted” R-parity violating SUSY GUT model, we have failed to build a GUT scenario in which the original Zee scalar h^+ is embedded into an SU(5) 10 scalar (or SO(10) 120 scalar), as far as we adhere to the model-building without the constraint $\sin^2 2\theta_{solar} > 0.99$.

Alternatively, we consider an SU(2) triplet scalar $\phi = (\phi^{++}, \phi^+, \phi^0)$. Usually, such a Higgs scalar can directly give Majorana masses for the left-handed neutrinos ν_L . We assume that the potential for ϕ does not have a minimum. Then, we obtain the following radiative masses

$$M_{ij} = f_{ij} ((m_i^e)^2 + (m_j^e)^2) K , \quad (2.6)$$

instead of the Zee mass matrix (1.1) with (1.2).

We regard the triplet scalar ϕ as a member of the SO(10) 126-plet scalar

$$126 = 1_{-10} + \bar{5}_{-2} + 10_{-6} + \bar{15}_{+6} + 45_{+2} + \bar{50}_{-2} , \quad (2.7)$$

of $SU(5) \times U(1)$, where the triplet scalar ϕ belongs to the component $\bar{15}_{+6}$ and the 45_{+2} component plays a role in the unified description of quark and charged lepton mass matrices [13]. Of course, there are no dangerous terms for the proton decay in the coupling $\psi_{\bar{5}}^c \psi_{\bar{5}} \phi_{15}$.

However, the present model with a triplet scalar ϕ causes another problem, because the neutral component ϕ^0 generally takes non-vanishing value or the vacuum expectation value $\langle \phi^0 \rangle$.

3. SUMMARY

The embedding of the R-parity violating SUSY models into a GUT scenario, except for a special case, is ruled out because of the proton decay. The SU(5) 10+ $\bar{10}$ model [12], which is safe for the proton decay, is ruled out if the solar neutrino data establish $\sin^2 2\theta_{solar} \sim 0.8$. On the other hand, an SO(10) 126 model is promising if neutrino data show $\sin^2 2\theta_{solar} \sim 0.8$. However, such a model with a triplet scalar will cause another problems. More study is required. An

SUSY GUT model with restricted R-parity violating which was discussed in 2.2 is also interesting. The details will be given elsewhere.

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