New Trends in the Zee Model

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Abstract

Recent trends in the Zee model are reviewed. Especially, the importance of a serious constraint in the Zee model, $\sin^2 2\theta_{solar} = 1.0$, is pointed out.

Key words: neutrino mass matrix, Zee model, lepton flavor violating decays
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1 introduction

The Zee model [1] is very attractive to us. The model has only three parameters and it can naturally give a large neutrino mixing. The Zee mass matrix is given by the form

$$M_\nu = m_0 \begin{pmatrix} 0 & a & c \\ a & 0 & b \\ c & b & 0 \end{pmatrix}, \tag{1.1}$$

where

$$\begin{aligned} a &= f_{e\mu}(m_\mu^2 - m_e^2), \\ b &= f_{\mu\tau}(m_\tau^2 - m_\mu^2), \\ c &= f_{\tau e}(m_e^2 - m_\tau^2), \end{aligned} \tag{1.2}$$

and f_{ij} are lepton flavor violating Yukawa coupling constants of the Zee scalar. Especially, for the case $a = c \gg b$, it leads to a bi-maximal mixing [2]

$$U \simeq \begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & \frac{1}{2} & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1.3}$$

with $\Delta m_{12}^2/\Delta m_{23}^2 \simeq \sqrt{2} b/a$. Besides the model can provide rich new physics, especially, in lepton flavor violating processes.

2 Present and future in the Zee model

Can the model explain the observed Δa_μ *?* The Zee model can provide

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us rich phenomenology: radiative decays of the charged leptons $e_i^- \to e_j^- \gamma$, two gamma decay of the CP even neutral Higgs scalar $h^0 \to \gamma\gamma$, lepton flavor changing Z decays $Z \to e_i^\pm e_j^\mp$, and so on. (For example, see the references in Ref. [3].) Especially, we are interested in the observed excess of muon anomalous magnetic moment Δa_μ. Here, We give a short review for typical two models. One is a case with $|f_{e\mu}| \gg |f_{\tau e}| \gg |f_{\mu\tau}|$ proposed by Jarlskog *et al.* [2], and another is a case with $|f_{\mu\tau}| \gg |f_{e\mu}| \gg |f_{\tau e}|$ proposed by Smirnov and Tanimoto [4], The former leads to a nearly bimaximal mixing (1.3), so that we can explain both the solar data, but we cannot explain the value of Δa_μ because of $|f_{\mu\tau}| \ll |f_{\tau e}| \ll |f_{\mu\tau}|$. On the other hand, the latter can give the value of Δa_μ because of $|f_{\mu\tau}| \gg |f_{e\mu}| \gg |f_{\tau\mu}|$, but it leads to $\sin^2 2\theta_{12} = 0$ and $\sin^2 2\theta_{23} = 1$, and cannot explain the solar neutrino data. In order to explain the solar neutrino data, we must consider, for example, a sterile neutrino ν_s which mixes with ν_e [4],

Can the model be related to the charged lepton masses ? The Zee coupling constants f_{ij} are free parameters which are irrelevant to Yukawa coupling constants $y_i^{(f)}$ ($f = e, u, d$), we must seek for a further ansatz for f_{ij} in order to relate f_{ij} with the charged lepton masses and so on. One of such attempts has been proposed by Koide and Ghosal [5] They have put a simple ansatz on the transition matrix elements in the infinite momentum frame (not on the mass matrix), and they have obtained the relations

$$f_{ij} = \varepsilon_{ijk}[m_k^e/(m_i^e + m_j^e)]f, \quad (2.2)$$

where $m_i^e = (m_e, m_\mu, m_\tau)$, which leads to the prediction

$$R \equiv \frac{\Delta m_{12}^2}{\Delta m_{23}^2} \simeq \sqrt{2}\frac{m_e}{m_\mu} = 6.9 \times 10^{-3}. \quad (2.3)$$

The predicted value (2.3) is in excellent agreement with the observed value (best fit values) [6,7]

$$R_{exp} \simeq \frac{2.2 \times 10^{-5}\mathrm{eV}^2}{3.2 \times 10^{-3}\mathrm{eV}^2} = 6.9 \times 10^{-3}. \quad (2.4)$$

However, the meaning of the ansatz for matrix elements on the infinite momentum frame is still unclear. Further study will be required.

What experimental value is serious for the Zee model ? Recently, it has been pointed out that the Zee model cannot give the observed sizable deviation from $\sin^2 2\theta_{solar} = 1$, i.e., $\sin^2 2\theta_{observ}^{solar} \sim 0.8$ under the condition $\Delta m_{solar}^2/\Delta m_{atm}^2 \ll 1$: A parameter independent investigation leads to a severe constraint [8] on the value of $\sin^2 2\theta_{solar}$

$$\sin^2 2\theta_{solar} \geq 1 - \frac{1}{16}\left(\frac{\Delta m_{solar}^2}{\Delta m_{atm}^2}\right)^2 . \quad (2.4)$$

The conclusion cannot be loosened even if we take RGE effect into consideration.

How can the model be embedded into a GUT scenario ? Another problem in the Zee model is that the original Zee model is not on a GUT scenario. Where is a room of the Zee scalar h^+ in a GUT scenario?

The scalar h^+ belongs to $(1.1)_{Y=2}$ of $SU(3)_c \times SU(2)_L \times U(1)_Y$. The candidates are as follows: (i) slepton $\tilde{e}_r$, i.e., a member of $10_{\tilde{f}}$ of SU(5) in a R-parity breaking SUSY model; (ii) a member of messengers $10_M + \overline{10}_M$ of SUSY-breaking in a R-parity conserving SUSY model; (iii) a member of new hypothetical 10-plet scalar of SU(5). These extensions can bring new additional contributions into the neutrino masses. Therefore, we will be free from the severe constraint $\sin^2 2\theta_{solar} = 1.0$.

3 Summary

The Zee model is very attractive to us, because the model can provide us rich phenomenology. The effort to relate the Zee coupling constants with the Yukawa coupling constants of the charged leptons will be important. An attempt has been reviewed: $f_{ij} = \varepsilon_{ijk}[m^e_k/(m^e_i + m^e_j)]f$. On the other hand, recently, a serious constraint in the Zee model has been reported. It is very interesting whether the experiments rule out the value $\sin^2 2\theta_{solar} = 1.0$ or not. Attempts to embed the Zee model into a GUT scenario will become more important in order to be free from the severe constraint $\sin^2 2\theta_{solar} = 1.0$ on the original Zee model.

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