How Far Can the SO(10) Two Higgs Model Describe the Observed Neutrino Masses and Mixings ?

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Can the SO(10) model with one **10** and one **126** Higgs scalars give the observed masses and mixings of quarks and leptons without any other additional Higgs scalars? Recently, at least, for quarks and charged leptons, it has been demonstrated that it is possible. However, for the neutrinos, it is usually said that parameters which are determined from the quark and charged lepton masses cannot give the observed large neutrino mixings. This problem is systematically investigated, and it is concluded that the present data cannot exclude SO(10) model with two Higgs scalars although it cannot give the best fit values of the data.

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I. INTRODUCTION

SO(10) GUT model seems to us the most attractive model when we take the unification of the quarks and leptons into consideration. However, in order to reproduce the observed quark and lepton masses and mixings, usually, a lot of Higgs scalars are brought into the model. So it is the very crucial problem to know the minimum number of the Higgs scalars which can give the observed fermion mass spectra and mixings. A model with one Higgs scalar is obviously ruled out for the description of the realistic quark and lepton mass spectra. Two Higgs models were initially discussed by Mohapatra et.al [2].

In the previous paper [1], we discussed 2 Higgs scalars, {**10** and **126**} case and {**10** and **120**} case, and showed that they reproduce quark-lepton mass matrices unlike the conventional results [3]. One of new points of our approach is that we adopt general forms of Yukawa couplings allowable in the SO(10) framework. However, we did not argue there about the neutrino mass matrix since it may incorporate additional assumptions like the seesaw mechanism etc.

One of the merits of the SO(10) model is that it includes a right-handed Majorana neutrinos in the fundamental representation and naturally leads to the seesaw mechanism. Also Brahmachari-Mohapatra claimed that two Higgs model ({**10** and **126**+$\overline{\mathbf{126}}$}) does not reproduce the large mixing angle of the atmospheric neutrino deficit [4]. So in this paper we apply our method developed in the previous paper to the neutrino mass matrix, fitting the other parameters of the quark-lepton mass matrices. Our model has the two Higgs scalars {**10** and **126**} both of which are symmetric with respect to the family index. Therefore mass matrices are symmetric whose entries are complex valued. We do not adopt another choice {**10** and **120**}. For it does not involve the mass term of the right-handed Majorana neutrinos which are the ingredients of the seesaw mechanism.

We begin with the short review of our previous work [1]. In the case where two Higgs scalars, ϕ_{10} and ϕ_{126}, are incorporated in the SO(10) model, the mass matrices of quarks and charged leptons have the following forms

$$M_u = c_0 M_0 + c_1 M_1, \ M_d = M_0 + M_1, \ M_e = M_0 - 3M_1. \tag{1.1}$$

Here M_0 and M_1 are the mass matrices generated by the Higgs scalars ϕ_{10} and ϕ_{126}, respectively. Also c_0 and c_1 are the ratios of VEV's,

$$\begin{aligned} c_0 &= v_0^u/v_0^d = \langle\phi_{10}^{u0}\rangle/\langle\phi_{10}^{d0}\rangle, \\ c_1 &= v_1^u/v_1^d = \langle\phi_{126}^{u0}\rangle/\langle\phi_{126}^{d0}\rangle, \end{aligned} \tag{1.2}$$

and ϕ^u and ϕ^d denote Higgs scalar components which couple with up- and down-quarks, respectively. Eliminating M_0 and M_1 from Eq.(1.1), we obtain

$$M_e = c_d M_d + c_u M_u, \tag{1.3}$$

where

$$c_d = -\frac{3c_0 + c_1}{c_0 - c_1}\ , \quad c_u = \frac{4}{c_0 - c_1}\ . \tag{1.4}$$

Since M_u, M_d, and M_e are complex symmetric matrices, they are diagonalized by unitary matrices U_u, U_d, and U_e, respectively, as

$$U_u^T M_u U_u = D_u\ , \quad U_d^T M_d U_d = D_d\ , \quad U_e^T M_e U_e = D_e\ , \tag{1.5}$$

where D_u, D_d, and D_e are diagonal matrices given by

$$\begin{aligned} &D_u \equiv \mathrm{diag}(m_u, m_c, m_t)\ , \quad D_d \equiv \mathrm{diag}(m_d, m_s, m_b)\ , \\ &D_e \equiv \mathrm{diag}(m_e, m_\mu, m_\tau)\ , \end{aligned} \tag{1.6}$$

Since the Cabibbo-Kobayashi-Maskawa (CKM) matrix V_q is given by

$$V_q = U_u^T U_d^* \ , \tag{1.7}$$

the relation (1.3) is re-written as follows:

$$(U_e^\dagger U_u)^T D_e (U_e^\dagger U_u) = c_d V_q D_d V_q^T + c_u D_u . \tag{1.8}$$

Therefore, we obtain the independent three equations:

$$\mathrm{Tr} D_e D_e^\dagger = |c_d|^2 \,\mathrm{Tr}\Big[(V_q D_d V_q^T + \kappa D_u)(V_q D_d V_q^T + \kappa D_u)^\dagger\Big], \tag{1.9}$$

$$\mathrm{Tr}(D_e D_e^\dagger)^2 = |c_d|^4 \,\mathrm{Tr}\Big[((V_q D_d V_q^T + \kappa D_u)(V_q D_d V_q^T + \kappa D_u)^\dagger)^2\Big], \tag{1.10}$$

$$\det D_e D_e^\dagger = |c_d|^6 \det\Big[(V_q D_d V_q^T + \kappa D_u)(V_q D_d V_q^T + \kappa D_u)^\dagger\Big], \tag{1.11}$$

where $\kappa = c_u/c_d$. By eliminating the parameter c_d, we have two equations for the parameter κ:

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^3}{m_e^2 m_\mu^2 m_\tau^2} = \frac{(1.9)^3}{(1.11)}, \tag{1.12}$$

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^2}{2(m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_\tau^2 m_e^2)} = \frac{(1.9)^2}{(1.9)^2 - (1.10)}, \tag{1.13}$$

where $(1.9)^3$, for instance, means the right-hand side of Eq.(1.9) to the third power. Let us denote the parameter values of κ evaluated from Eqs.(1.12) and (1.13) as κ_A and κ_B, respectively. If κ_A and κ_B coincide with each other, then we have a possibility that the SO(10) GUT model can reproduce the observed quark and charged lepton mass spectra. If κ_A and κ_B do not so, the SO(10) model with one **10** and one **126** Higgs scalars is ruled out, and we must bring more Higgs scalars into the model.

Note that Eqs. (1.9)-(1.11) can constrain only the absolute value of $c_d \equiv |c_d| e^{i\sigma}$. The argument of the parameter c_d can be determined by taking neutrino sector into consideration. In the previous paper [1], we have found that only for the signs of the masses

$$(m_t, m_c, m_u; m_b, m_s, m_d; m_\tau, m_\mu, m_e) = (+,-,+;+,-,-;+,\pm,\pm) \quad \text{(a)}, \tag{1.14}$$

and

$$= (+,-,-;+,-,-;+,\pm,\pm) \quad \text{(b)}, \tag{1.15}$$

there are solutions which gives $\kappa_A = \kappa_B$, and the corresponding parameter values $(|c_d|, \kappa)$ are

$$(|c_d|, \kappa) = (3.15698, -0.019296 e^{2.64172^\circ i}), \tag{1.16}$$

$$(3.03577, -0.019398 e^{2.99570^\circ i}) \quad \text{for (a)}, \tag{1.17}$$

and

$$= (3.13307, -0.019314 e^{2.71464^\circ i}), \tag{1.18}$$

$$(3.00558, -0.019420 e^{3.10014^\circ i}) \quad \text{for (b)} \tag{1.19}$$

and $m_s = 76.3$ [MeV] for input $\theta_{23} = 0.0420$ [rad] and $\delta = 60^\circ$ at $\mu = m_Z$ (m_Z is the neutral weak boson mass). For the relation between the values at $\mu = m_Z$ and those at $\mu = \Lambda_X$ (Λ_X is a unification scale), see Ref. [1]. The purpose of the present paper is to investigate whether these solutions can give reasonable values for observed neutrino masses and mixings or not.

II. THE NUMBER OF PARAMETERS IN THE SO(10) MODEL WITH TWO HIGGS SCALARS

As we have discussed in the previous section, among four freedoms of complex $\{c_0,\ c_1\}$ or $\{c_d,\ \kappa\}$, we have been able to fix the three of them, κ and $|c_d|$. This is not accidental. Let us discuss the situation in detail in the SO(10) two Higgs model.

In the previous paper, by using the relation (1.8), we have investigated whether there is a set of parameters which can give the 13 observable quantities D_e, D_u, D_d, and V_q or not. We can rewrite Eq.(1.8) as

$$A_e^T D_e A_e = c_d (V_q D_d V_q^T + \kappa D_u), \tag{2.1}$$

where

$$A_e = U_e^\dagger U_u, \tag{2.2}$$

$$c_d = |c_d| e^{i\sigma}. \tag{2.3}$$

The quantities D_e, D_u, D_d, and V_q are inputs, and the quantities $|c_d|$, κ, and A_e are the parameters which should be fixed from those observed quantities. In general, an $n \times n$ unitary matrix for n generations has n^2 parameters. Therefore, the number of the parameters is

$$N(\mathrm{pmt}) = N(A_e) + N(c_d) + N(\kappa) = n^2 + 2 + 2. \tag{2.4}$$

On the other hand, the number of equations is

$$N(\mathrm{eqs}) = n(n+1), \tag{2.5}$$

because Eq.(2.1) is symmetric. Therefore, the number of the unfixed parameters is given by

$$N_{\mathrm{free}} = N(\mathrm{pmt}) - N(\mathrm{eqs}) = 4 - n = 1, \tag{2.6}$$

for $n = 3$, i.e., the 13 observed quantities fix the parameters $|c_d|$, κ, and A_e, but 1 parameter σ remains as an unknown parameter.

In the present paper, we will try to predict neutrino masses

$$D_\nu = U_\nu^T M_\nu U_\nu, \tag{2.7}$$

and mixing matrix

$$V_\ell = U_e^T U_\nu^*, \tag{2.8}$$

by using the observed quantities D_e, D_u, D_d, and V_q and the parameter values $|c_d|$, κ, and A_e fixed by Eq.(2.1).

SO(10) GUT asserts that the Dirac neutrino mass matrix M_D is given by the form

$$M_D = c_0 M_0 - 3c_1 M_1, \tag{2.9}$$

and Majorana mass matrices of the left-handed and right-handed neutrinos, M_L and M_R, are proportional to the matrix M_1:

$$M_L = c_L M_1, \quad M_R = c_R M_1, \tag{2.10}$$

where M_0 and M_1 are related to the quark and charged lepton mass matrices M_u, M_d, and M_e as follows:

$$M_0 = \frac{3M_d + M_e}{4}, \tag{2.11}$$

$$M_1 = \frac{M_d - M_e}{4}. \tag{2.12}$$

Then the neutrino mass matrix derived form the seesaw mechanism becomes

$$\begin{aligned} M_\nu &= M_L - M_D M_R^{-1} M_D^T \\ &= c_L M_1 \\ &\quad - c_R^{-1}(c_0 M_0 - 3c_1 M_1) M_1^{-1} (c_0 M_0 - 3c_1 M_1)^T. \end{aligned} \tag{2.13}$$

In the present paper we adopt $c_L = 0$. Also we may ignore the phase of c_R which does not affect the observed values. Therefore, we can rewrite Eq.(2.13) as

$$|c_R| A_\nu^T D_\nu A_\nu = \widetilde{M}_D \widetilde{M}_1^{-1} \widetilde{M}_D^T, \tag{2.14}$$

similarly to Eq.(2.1), where

$$\widetilde{M}_D = c_0 \widetilde{M}_0 - 3c_1 \widetilde{M}_1, \tag{2.15}$$

$$\widetilde{M}_0 = \frac{1}{4}(3\widetilde{M}_d + \widetilde{M}_e), \tag{2.16}$$

$$\widetilde{M}_1 = \frac{1}{4}(\widetilde{M}_d - \widetilde{M}_e), \tag{2.17}$$

with

$$\widetilde{M}_d = U_u^T M_d U_u = V_q D_d V_q^T, \tag{2.18}$$

$$\begin{aligned} \widetilde{M}_e &= U_u^T M_e U_u = A_e^T D_e A_e \\ &= c_d (V_q D_d V_q^T + \kappa D_u). \end{aligned} \tag{2.19}$$

Differently from the previous work, the quantities D_ν and V_ℓ are unknown parameters at the present stage. Since

$$V_\ell = A_e^* A_\nu^T, \tag{2.20}$$

and A_e is fixed from Eq.(2.1), the number of the unknown parameters in Eq.(2.20) is

$$N(A_\nu) = N(V_\ell) = n^2. \tag{2.21}$$

Of course, the unknown parameters in A_ν contain the n unphysical parameters which cannot be determined because of the rephasing in the fields e_L. Therefore, the number of the unknown parameters is

$$\begin{aligned} N(\text{pmt}) &= N(D_\nu) + N(A_\nu) + N(|c_R|) + N(\sigma) \\ &= n + n^2 + 1 + 1 = n^2 + n + 2 \end{aligned} \tag{2.22}$$

and from the number of equations $N(\text{eqs}) = n(n+1)$ in Eq.(2.14), we obtain the number of the unfixed parameters as

$$\begin{aligned} N_{\text{free}} &= N(\text{pmt}) - N(\text{eqs}) \\ &= (n^2 + n + 2) - n(n+1) = 2. \end{aligned} \tag{2.23}$$

This means that we can predict neutrino masses and mixing completely if we give the two values $|c_R|$ and σ. The numerical predictions will be investigated in the next section.

III. NUMERICAL RESULTS

Here we discuss the numerical results of the neutrino mass spectrum and neutrino mass matrix. For example, we use the set in Eq.(1.18). Even if the other sets are used, our results are scarcely changed. The allowed values of neutrino mass square differences and lepton flavor mixing angles depict complicated tracks with moving $\sigma \equiv \arg c_d$ (Fig. 2). This figure shows a general tendency that the lepton flavor mixing angles θ_{12} and θ_{23} get larger as σ approaches to $3\pi/2$. For an illustration we take $\sigma = 149\pi/100$, then these values become

$$\begin{aligned} &\frac{\Delta m_{12}^2}{\Delta m_{13}^2} = 0.15, \qquad \frac{\Delta m_{23}^2}{\Delta m_{13}^2} = 0.85, \\ &\sin^2(2\theta_{12}) = 0.76, \quad \sin^2(2\theta_{23}) = 0.75, \\ &\sin^2(2\theta_{13}) = 0.16. \end{aligned} \tag{3.1}$$

There still remain a little bit discrepancies between our results and experiments. However our results are much improved in comparison with those by Babu-Mohapatra [2] in which they obtained $\sin\theta_{12} = 0 - 0.3$, $\sin\theta_{13} = 0.05$, and $\sin\theta_{23} = 0.12 - 0.16$. The purpose of the present paper is to study the general tendency of the fittings and not to pursuit the precise data fitting, for the data themselves are not affirmative and we have theoretical ambiguities not incorporated in the present data fitting like the renormalization group effect.

In the choice of Eq.(3.1), we have

$$|c_d| = 3.16 \tag{3.2}$$

$$c_0 = \frac{1 - c_d}{c_u} = 54.84 e^{-20.24^\circ i}, \tag{3.3}$$

$$c_1 = -\frac{3 + c_d}{c_u} = 70.54 e^{+41.90^\circ i}. \tag{3.4}$$

In this case, Eq.(2.11) - Eq.(2.13) are re-written in the basis of $M_u = D_u$ (see Eq.(1.8)) as

$$M_0 = \frac{3V_qD_dV_q^T + c_d(\kappa D_u + V_qD_dV_q^T)}{4}$$
$$= 2.1646 \times 10^3 e^{+10.48^\circ i} \begin{pmatrix} -0.00405e^{-57.29^\circ i} & -0.00753e^{-56.24^\circ i} & -0.00533e^{+65.46^\circ i} \\ -0.00753e^{-56.24^\circ i} & -0.02986e^{-51.59^\circ i} & +0.06358e^{-57.64^\circ i} \\ -0.00533e^{+65.46^\circ i} & +0.06358e^{-57.64^\circ i} & +1.00000 \end{pmatrix}, \tag{3.5}$$
$$M_1 = \frac{V_qD_dV_q^T - c_d(\kappa D_u + V_qD_dV_q^T)}{4}$$
$$= 9.5127 \times 10^2 e^{-24.44^\circ i} \begin{pmatrix} -0.00715e^{+95.23^\circ i} & -0.01333e^{+96.54^\circ i} & +0.00944e^{+38.23^\circ i} \\ -0.01333e^{+96.54^\circ i} & -0.04878e^{+90.73^\circ i} & +0.11247e^{+95.13^\circ i} \\ +0.00944e^{+38.23^\circ i} & +0.11247e^{+95.13^\circ i} & +1.00000 \end{pmatrix}, \tag{3.6}$$
$$|c_R|M_\nu = (c_0M_0 - 3c_1M_1)M_1^{-1}(c_0M_0 - 3c_1M_1)^T$$
$$= -4.6628 \times 10^6 e^{-52.17^\circ i} \begin{pmatrix} +0.1163e^{+26.89^\circ i} & +0.2165e^{+28.06^\circ i} & -0.1536e^{-30.53^\circ i} \\ +0.2165e^{+28.06^\circ i} & +0.8193e^{+28.00^\circ i} & -1.9276e^{+29.52^\circ i} \\ -0.1536e^{-30.53^\circ i} & -1.9276e^{+29.52^\circ i} & +1.0000 \end{pmatrix}. \tag{3.7}$$

Let us choose the free parameter $|c_R|$ so as to result in small neutrino masses, for example when $|c_R| = 3.2 \times 10^8$, we have $\Delta m_{23}^2 = 1.5 \times 10^{-3}[\mathrm{eV}^2]$ (QVO).

Here there arises a question what makes the two flavor mixing angles large. We need to investigate the mixing matrices U_e and U_ν which diagonalize M_e and M_ν, respectively. Those are obtained as

$$U_e = \begin{pmatrix} +0.863 & +0.504e^{+9.46^\circ i} & -0.022e^{+56.66^\circ i} \\ -0.493e^{-9.82^\circ i} & +0.834 & -0.248e^{+16.63^\circ i} \\ -0.110e^{-21.40^\circ i} & +0.223e^{-18.10^\circ i} & +0.969 \end{pmatrix}, \tag{3.8}$$
$$U_\nu = \begin{pmatrix} +0.992 & -0.092e^{-15.94^\circ i} & -0.088e^{+12.86^\circ i} \\ +0.049e^{+76.86^\circ i} & +0.724 & -0.688e^{-16.08^\circ i} \\ +0.117e^{+9.80^\circ i} & +0.683e^{+16.74^\circ i} & +0.721 \end{pmatrix}. \tag{3.9}$$

Here, $|U_{e11}|$, $|U_{e12}|$, $|U_{e21}|$, $|U_{e22}| \gtrsim 0.5$ for the charged lepton mass matrix and $|U_{\nu22}|$, $|U_{\nu23}|$, $|U_{\nu32}|$, $|U_{\nu33}| \gtrsim 0.7$ for the neutrino mass matrix. Therefore the components of the lepton flavor mixing matrix become $|V_{l11}|$, $|V_{l12}|$, $|V_{l21}|$, $|V_{l22}|$, $|V_{l23}|$, $|V_{l32}|$, $|V_{l33}| \gtrsim 0.5$:

$$V_l = \begin{pmatrix} +0.844e^{+2.10^\circ i} & -0.494e^{-9.95^\circ i} & +0.206e^{+23.61^\circ i} \\ +0.527e^{+3.26^\circ i} & +0.696e^{-8.84^\circ i} & -0.488e^{+24.97^\circ i} \\ +0.098e^{-15.78^\circ i} & +0.521e^{-27.43^\circ i} & +0.848e^{+6.32^\circ i} \end{pmatrix}. \tag{3.10}$$

The mixing angle θ_{23} becomes larger, while the mixing angle θ_{12} smaller, if we take the smaller value of $|m_t|$ or $|m_d|$, or larger $|m_c|$, or $|m_b|$, $|m_s|$ than their center values.

As a simple example, the shift of $|m_d|$ and $|m_s|$ causes the change of mixing angles and neutrino mass square differences as depicted in Fig.2. Fig.2 shows that the θ_{23} and θ_{13} can approach the 99%C.L. of SK [7]+CHOOZ [8] but θ_{12} and Δm_{12}^2 are out of the range of $99\% - 99.9\%$C.L. of SOLAR [9]+CHOOZ.

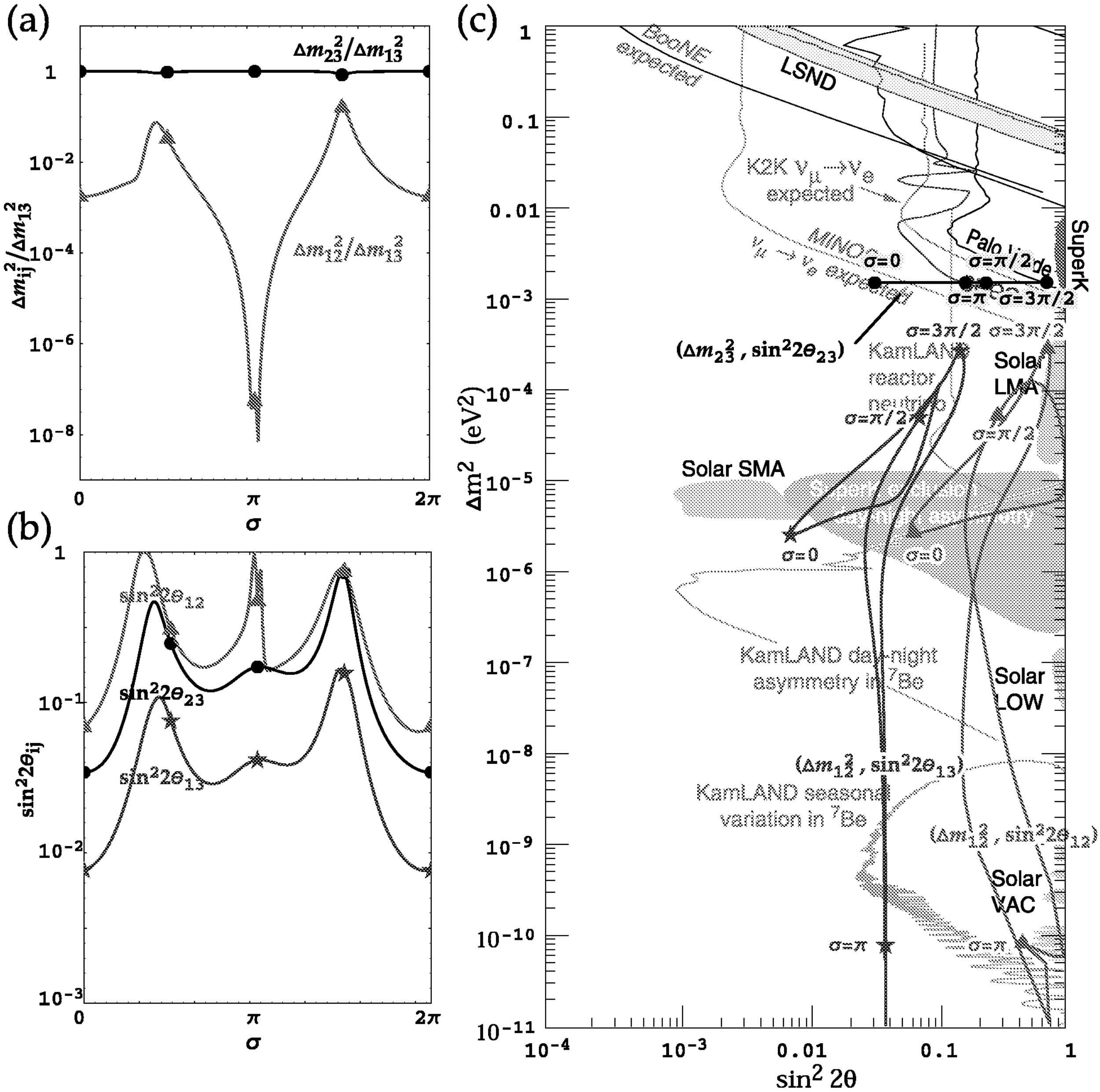

FIG. 1. The relation between our results and the two-flavor oscillation analysis [6] when σ is moved. (a) The circles and triangles indicate the values of $\Delta m_{23}^2/\Delta m_{13}^2$ and $\Delta m_{12}^2/\Delta m_{13}^2$ at every $\pi/2$ of σ. (b) The circles, triangles, and stars indicate the values of $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, and $\sin^2 2\theta_{13}$ at every $\pi/2$ of σ, respectively. (c) The circles, triangles, and stars indicate the values of $(\Delta m_{23}^2, \sin^2 2\theta_{23})$, $(\Delta m_{12}^2, \sin^2 2\theta_{12})$, and $(\Delta m_{12}^2, \sin^2 2\theta_{13})$ at every $\pi/2$ of σ. Here we have set $\Delta m_{23}^2 = 1.5 \times 10^{-3}$[eV^2] in every case.

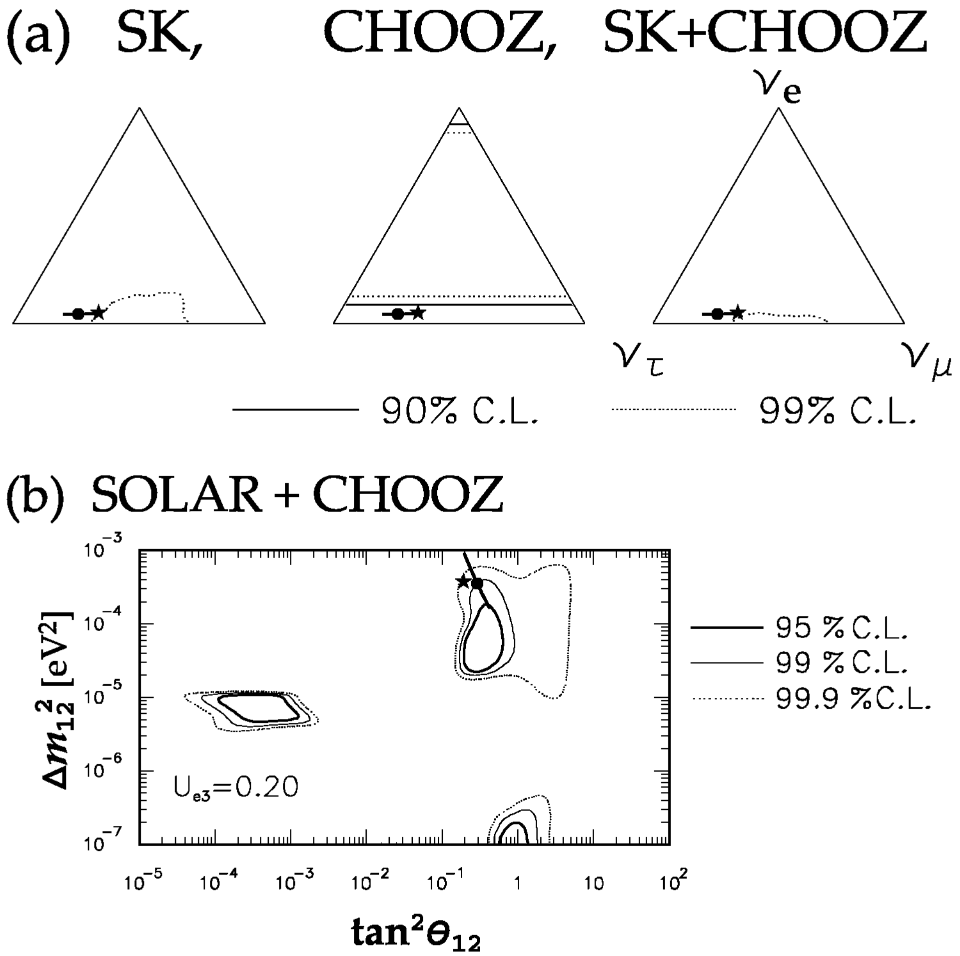

FIG. 2. The relation between 3ν Oscillation analyses by G.L.Fogli et al. [10] and by us for $\Delta m^2_{23} = 1.5 \times 10^{-3}[\mathrm{eV}^2]$ (QVO). (a) For SK+CHOOZ. (b) For SOLAR+CHOOZ. The circles indicate our solutions for Eq.(3.1). The solid line through them is the track as m_d is varied. It goes from the experimental limit that $|m_d|$ moves over the range, 4.03 - 5.29 MeV [5]. At that time, $|m_s|$ simultaneously changes over the range, 76.3 - 76.2 MeV so as to satisfy the relations (1.12) and (1.13). If we take the smaller $|m_d|$ with the fixed σ, the solution in (a) moves rightward and the solution in (b) does left-upward (Table I (i)). Since the minimum $|m_d|$ for (b) gives bad fitting, we have changed σ from $149\pi/100$ to $146\pi/100$, which is denoted by star (Table I (ii)). Thus our result approaches the 99%C.L. of SK+CHOOZ and 99.9%C.L. of SOLAR+CHOOZ.

IV. DISCUSSION

Since there are only two basic matrix M_0 and M_1 in this model, the number of parameters in Eq.(2.1) and (2.14) is

$$\begin{array}{lr} D_u,\, D_d,\, D_e,\, D_\nu & 3{\times}4 = 12 \\ c_d,\, |c_R|,\, \kappa & 2{+}1{+}2 = \;\;5 \\ \underline{V_q,\, A_e,\, A_\nu} & \underline{4{+}9{+}9 = 22} \\ \mathrm{sum.} & 39 \end{array} \tag{4.1}$$

and the number of equations is $N(\mathrm{eqs}) = 12 \times 2 = 24$. Therefore the number of free parameters is $N(\mathrm{pmt}) - N(\mathrm{eqs}) = 39 - 24 = 15$. On the other hand, the number of the physical parameters which can be determined by experiments is

$$\begin{array}{lr} m_u,\, m_c,\, m_t, & 3 \\ m_d,\, m_s,\, m_b, & 3 \\ \mathrm{CKM}\!:\, \theta_{12},\, \theta_{23},\, \theta_{13},\, \delta, & 4 \\ m_e,\, m_\mu,\, m_\tau, & 3 \\ m_{\nu_e},\, m_{\nu_\mu},\, m_{\nu_\tau}, & 3 \\ \underline{\mathrm{MNS}\!:\, \theta_{12},\, \theta_{23},\, \theta_{13},\, \delta,\, \beta,\, \rho} & \underline{6} \\ \mathrm{sum.} & 22 \end{array} \tag{4.2}$$

where β and ρ are Majorana phases because of no rephasing in the neutrino fields ν_L. To sum up the matter, we discuss the consistency test about 22 physical parameters by using only 15 free parameters. The consistency test in the quark sector is good, as shown in our previous paper. In the lepton sector, the test is not so bad when we adopt the QVO solution of solar neutrino deficit, and this model favors the normal hierarchy of neutrino mass spectrum.

Also we can predict the yet unobserved values such as the averaged neutrino masses $\langle m\rangle_{\alpha\beta}$ and Jarlskog parameter in the lepton part. The averaged neutrino masses appear in the reactions where the Majorana neutrinos propagate in the intermediate states. They are

$$\langle m_\nu\rangle_{\alpha\beta} \equiv \left|\sum_{j=1}^{3} U_{\alpha j}U_{\beta j}m_j\right| , \tag{4.3}$$

where α and β are $(e,\ \mu,\ \tau)$. They correspond to neutrinoless double beta decay [11] for $\alpha=\beta=e$, $\mu-e$ conversion $(\mu^-+(A,Z)\to e^+ +(A,Z-2)$ for $\alpha=\mu,\ \beta=e$, and K decay $(K^-\to\pi^+\mu^-\mu^-)$ for $\alpha=\beta=\mu$ [12] etc. In Fig.3 we have depicted σ dependence of $\langle m_\nu\rangle_{\alpha\beta}/\sqrt{\Delta m_{23}^2}$. In the case of Eq.(3.1), these values become as follows.

$$\frac{\langle m\rangle_{\alpha\beta}}{\sqrt{\Delta m_{23}^2}} \simeq \begin{pmatrix} 0.87 & 0.35 & 0.048 \\ & 0.50 & 0.20 \\ & & 0.14 \end{pmatrix} . \tag{4.4}$$

For instance, if we input $\Delta m_{23}^2 = 1.5\times10^{-3}$ [eV^2], $\langle m\rangle_{ee}$ becomes 0.034[eV]. This value is sufficiently sensitive to the next generation experiments such as GENIUS [13], CUORE [14], and MOON [15]. Jarlskog parameter [16] appears in three generations

$$P(\nu_e\to\nu_\mu)-P(\nu_\mu\to\nu_e) = J\frac{\Delta E_{21}\Delta E_{32}\Delta E_{31}}{\Delta E_{21}^M\Delta E_{32}^M\Delta E_{31}^M}$$
$$\times\sin\left(\frac{\Delta E_{21}^M L}{2}\right)\sin\left(\frac{\Delta E_{32}^M L}{2}\right)\sin\left(\frac{\Delta E_{31}^M L}{2}\right) \tag{4.5}$$

with

$$J\equiv \mathrm{Im}(V_{l12}V_{l22}^*V_{l13}^*V_{l23}). \tag{4.6}$$

Here we have adopted the notation

$$\Delta E_{jk}\equiv E_j-E_k=\frac{\Delta m_{jk}^2}{2E}$$
$$\Delta E_{jk}^M\equiv E_j^M-E_k^M \tag{4.7}$$

with

$$U\mathrm{diag}(E_1,E_2,E_3)U^{-1}+\mathrm{diag}(a,0,0)$$
$$\equiv U^M\mathrm{diag}(E_1^M,E_2^M,E_3^M)(U^M)^{-1} \tag{4.8}$$

The σ dependence of J is depicted in Fig.4. For Eq.(3.1), it takes

$$J\simeq 0.00015. \tag{4.9}$$

However, it needs careful consideration that J drastically changes at $\sigma\simeq 3\pi/2$. $\langle m\rangle_{\alpha\beta}$ and J in the cases of Table I (i) and (ii) discussed in Fig.2 are also listed in Table II (i) and (ii). In this paper we have discussed how far the SO(10) two Higgs scalar model describes the quark-lepton masses and mixing parameters. We can conclude that this model cannot be rejected within the existing data. It should be remarked that the whole parameters can be decided from the existing data in principle.

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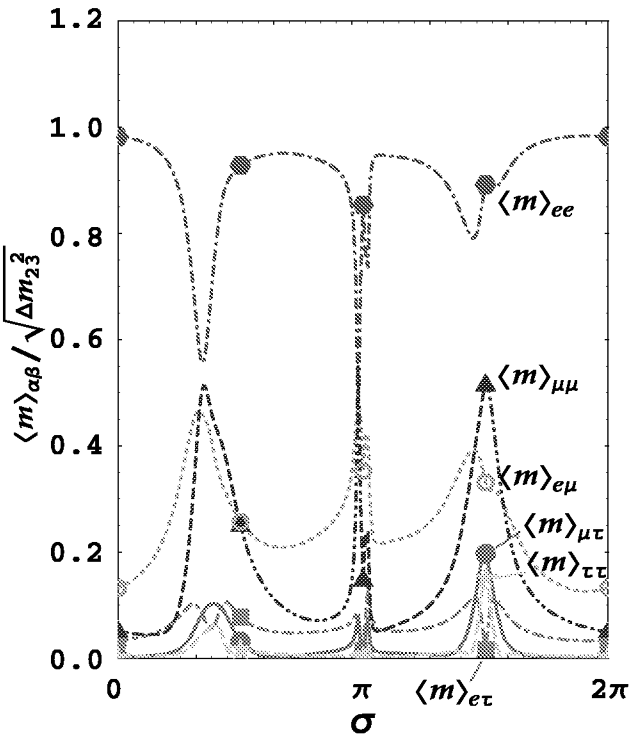

FIG. 3. The relations between the averaged neutrino masses of lepton number violation process and σ. The hexagons, white circles, boxes, triangles, black circles, and stars indicate the values of $\langle m\rangle_{ee}/\sqrt{\Delta m_{23}^2}$, $\langle m\rangle_{e\mu}/\sqrt{\Delta m_{23}^2}$, $\langle m\rangle_{e\tau}/\sqrt{\Delta m_{23}^2}$, $\langle m\rangle_{\mu\mu}/\sqrt{\Delta m_{23}^2}$, $\langle m\rangle_{\mu\tau}/\sqrt{\Delta m_{23}^2}$, and $\langle m\rangle_{\tau\tau}/\sqrt{\Delta m_{23}^2}$ at every $\pi/2$ of σ, respectively.

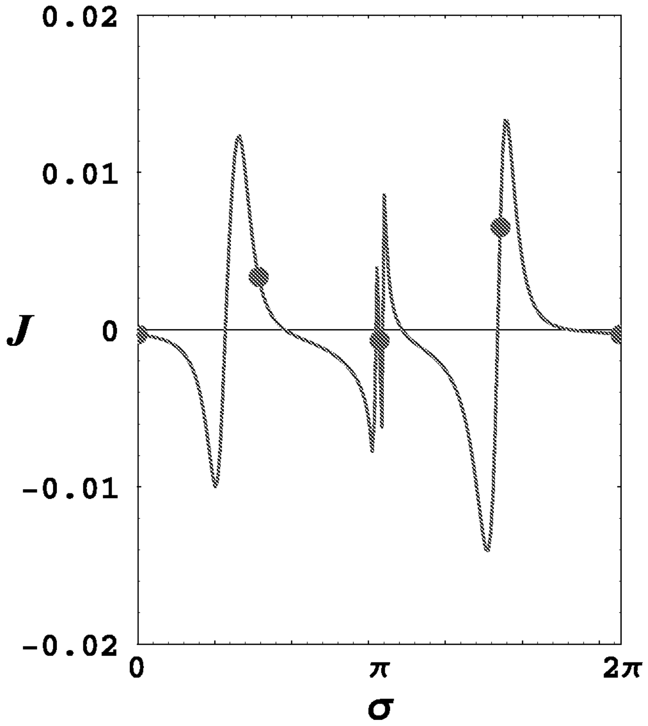

FIG. 4. The relation between Jarlskog parameter J and σ. The circles indicate the values of J at every $\pi/2$ of σ.

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(i)	$\lvert m_d\rvert = 4.03$[eV], $\lvert m_s\rvert = 76.3$[eV], $\sigma = 149\pi/100$, $(\Delta m^2_{12})/(\Delta m^2_{13}) = 0.43$, $(\Delta m^2_{23})/(\Delta m^2_{13}) = 0.57$, $\sin^2(2\theta_{12}) = 0.52$, $\sin^2(2\theta_{23}) = 0.91$, $\sin^2(2\theta_{13}) = 0.17$
(ii)	$\lvert m_d\rvert = 4.03$[eV], $\lvert m_s\rvert = 76.3$[eV], $\sigma = 146\pi/100$, $(\Delta m^2_{12})/(\Delta m^2_{13}) = 0.20$, $(\Delta m^2_{23})/(\Delta m^2_{13}) = 0.80$, $\sin^2(2\theta_{12}) = 0.54$, $\sin^2(2\theta_{23}) = 0.88$, $\sin^2(2\theta_{13}) = 0.20$

TABLE I. Our solution (the second and third lines) from the input parameters (the first line). The result of (i) is obtained when we move $|m_d|$ from 4.69[eV] to 4.03[eV]. (ii) is the result when we move $|m_d|$ as (i) and, furthermore, σ from $149\pi/100$ to $146\pi/100$. These data fitting corresponds to Fig.2.

(i)	$\langle m\rangle_{ee}/\sqrt{\Delta m^2_{23}} = 1.16$, $\langle m\rangle_{e\mu}/\sqrt{\Delta m^2_{23}} = 0.32$, $\langle m\rangle_{e\tau}/\sqrt{\Delta m^2_{23}} = 0.09$, $\langle m\rangle_{\mu\mu}/\sqrt{\Delta m^2_{23}} = 0.65$, $\langle m\rangle_{\mu\tau}/\sqrt{\Delta m^2_{23}} = 0.40$, $\langle m\rangle_{\tau\tau}/\sqrt{\Delta m^2_{23}} = 0.36$, $J = 0.0091$
(ii)	$\langle m\rangle_{ee}/\sqrt{\Delta m^2_{23}} = 0.94$, $\langle m\rangle_{e\mu}/\sqrt{\Delta m^2_{23}} = 0.36$, $\langle m\rangle_{e\tau}/\sqrt{\Delta m^2_{23}} = 0.16$, $\langle m\rangle_{\mu\mu}/\sqrt{\Delta m^2_{23}} = 0.44$, $\langle m\rangle_{\mu\tau}/\sqrt{\Delta m^2_{23}} = 0.23$, $\langle m\rangle_{\tau\tau}/\sqrt{\Delta m^2_{23}} = 0.15$, $J = -0.014$

TABLE II. The values of averaged neutrino masses and the Jarlskog parameter for the case (i) and (ii) in Table I.