Can the SO(10) Model with Two Higgs Doublets

Reproduce the Observed Fermion Masses?

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It is usually considered that the SO(10) model with one **10** and one **126** Higgs scalars cannot reproduce the observed quark and charged lepton masses. Against this conventional conjecture, we find solutions of the parameters which can give the observed fermion mass spectra. The SO(10) model with one **10** and one **120** Higgs scalars is also discussed.

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I. INTRODUCTION

The grand unification theory (GUT) is very attractive as a unified description of the fundamental forces in the nature. Especially, the SO(10) model is the most attractive to us when we take the unification of the quarks and leptons into consideration. However, in order to reproduce the observed quark and lepton masses and mixings, usually, a lot of Higgs scalars are brought into the model. We think that the nature is simple. What is of the greatest interest to us is to know the minimum number of the Higgs scalars which can give the observed fermion mass spectra. A model with one Higgs scalar is obviously ruled out for the description of the realistic quark and lepton mass spectra. Then, how is a model with two different types of Higgs scalars (e.g., **10** and **126** scalars)?

In the SO(10) GUT scenario, a model with one **10** and one **126** Higgs scalars leads to the relation [1]

$$M_e = c_u M_u + c_d M_d, \tag{1.1}$$

where M_e, M_u and M_d are charged lepton, up-quark and down-quark mass matrices, respectively. It is widely accepted that there will be no solution of c_u and c_d which give the observed fermion mass spectra. The reason is as follows : We take a basis on which the up-quark mass matrix M_u is diagonal ($M_u = D_u$). Then, the relation (1.1) is expressed as

$$\widetilde{M_e} = c_u D_u + c_d \widetilde{M_d}. \tag{1.2}$$

Considering that $\widetilde{M_d}$ is almost diagonal and the mass hierarchy of up-quark sector is much severe than that of down-quark sector, we observed that the contribution to the first and the second generation part of $\widetilde{M_e}$ from the up-quark part D_u is negligible so that it is proportional to that of $\widetilde{M_d}$. Thus, the relation (1.1) which predicts $m_e/m_\mu \simeq m_d/m_s$ does not reproduce the observed hierarchical structure of the down-quark and charged lepton masses [2] such as predicted by Georgi-Jarskog mass relations $m_b = m_\tau$, $m_s = m_\mu/3$ and $m_d = 3m_e$ at the GUT scale [3]. However, the above conclusion is somewhat impatient one. (i) It is too simplified to regard $\widetilde{M_d}$ as almost diagonal. (ii) We must check a possibility that the mass relations are satisfied with the opposite signs, i.e., $m_b = \pm m_\tau$, $m_s = \pm m_\mu/3$ and $m_d = \pm 3m_e$. (iii) The mass values at the GUT scale which are evaluated from the observed values by using the renormalization group equations show sizable deviations from the Georgi-Jarskog relations. The purpose of the present paper is to investigate systematically whether there are solutions of c_u and c_d which give the realistic quark and lepton masses or not.

II. OUTLINE OF THE INVESTIGATION

In the SO(10) GUT model with one **10** and one **126** Higgs scalars, the down-quark and down-lepton mass matrices M_d and M_e are given by

$$M_d = M_0 + M_1, \quad M_e = M_0 - 3M_1, \tag{2.1}$$

where M_0 and M_1 are mass matrices which are generated by the **10** and **126** Higgs scalars ϕ_{10} and ϕ_{126}, respectively. Inversely, we obtain

$$M_0 = \frac{1}{4}(3M_d + M_e), \quad M_1 = \frac{1}{4}(M_d - M_e). \tag{2.2}$$

On the other hand, the up-quark mass matrix M_u is given by

$$M_u = c_0 M_0 + c_1 M_1, \tag{2.3}$$

where

$$\begin{aligned} c_0 &= v_0^u/v_0^d = \langle\phi_{10}^{u0}\rangle/\langle\phi_{10}^{d0}\rangle, \\ c_1 &= v_1^u/v_1^d = \langle\phi_{126}^{u0}\rangle/\langle\phi_{126}^{d0}\rangle, \end{aligned} \tag{2.4}$$

and ϕ^u and ϕ^d denote Higgs scalar components which couple with up- and down-quark sectors, respectively. Therefore, by using the relations Eq.(2.2), we obtain the relation

$$M_e = c_d M_d + c_u M_u, \tag{2.5}$$

where

$$c_d = -\frac{3c_0 + c_1}{c_0 - c_1} \ , \quad c_u = \frac{4}{c_0 - c_1} \ . \tag{2.6}$$

For convenience, first, we investigate the case that the matrices M_u, M_d and M_e are symmetrical matrices at the unification scale because we assume that they are generated by the **10** and **126** Higgs. Then, we can diagonalize those by unitary matrices U_u, U_d and U_e, respectively, as

$$U_u^T M_u U_u = D_u \ , \quad U_d^T M_d U_d = D_d \ , \ U_e^T M_e U_e = D_e \ , \tag{2.7}$$

where D_u, D_d and D_e are diagonal matrices. Since the Cabibbo-Kobayashi-Maskawa (CKM) matrix V is given by

$$V = U_u^T U_d^* \ , \tag{2.8}$$

the relation (2.5) is re-written as follows:

$$(U_e^\dagger U_u)^T D_e (U_e^\dagger U_u) = c_d V D_d V^T + c_u D_u. \tag{2.9}$$

At present, we have almost known the experimental values of D_e, D_u and $V D_d V^\dagger$. Therefore, we obtain the independent three equations:

$$\mathrm{Tr} D_e D_e^\dagger = |c_d|^2 \, \mathrm{Tr}\Big[(V D_d V^T + \kappa D_u)(V D_d V^T + \kappa D_u)^\dagger\Big], \tag{2.10}$$

$$\mathrm{Tr} (D_e D_e^\dagger)^2 = |c_d|^4 \, \mathrm{Tr}\Big[((V D_d V^T + \kappa D_u)(V D_d V^T + \kappa D_u)^\dagger)^2\Big], \tag{2.11}$$

$$\det D_e D_e^\dagger = |c_d|^6 \, \det\Big[(V D_d V^T + \kappa D_u)(V D_d V^T + \kappa D_u)^\dagger\Big], \tag{2.12}$$

where $\kappa = c_u / c_d$. Removing the parameter c_d, we have two equations for the parameter κ:

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^3}{m_e^2 m_\mu^2 m_\tau^2} = \frac{(2.10)^3}{(2.12)}, \tag{2.13}$$

$$\frac{(m_e^2 + m_\mu^2 + m_\tau^2)^2}{(m_e^2 m_\mu^2 + m_\mu^2 m_\tau^2 + m_\tau^2 m_e^2)} = \frac{(2.10)^2}{(2.10)^2 - (2.11)}, \tag{2.14}$$

where $(2.10)^3$, for instance, means the right handside of (2.10) to the third power. Let us denote the parameter values of κ evaluated from (2.13) and (2.14) as κ_A and κ_B, respectively. If κ_A and κ_B coincide with each other, then we have a possibility that the SO(10) GUT model can reproduce the observed quark and lepton mass spectra. If κ_A and κ_B do not so, the SO(10) model with one **10** and one **126** Higgs scalars is ruled out, and we must bring more Higgs scalars into the model. Of course, in the numerical evaluation, the values κ_A and κ_B will have sizable errors, because the observed values D_e, D_u, D_d and V have experimental errors, and the values at the GUT scale also have errors. Therefore, we will first evaluate κ_A and κ_B by using the center values at $\mu = m_Z$. The values κ_A and κ_B are not so sensitive to the renormalization group equation effect (evolution effect), because those are almost determined only by the mass ratios. For reference, we also evaluate the parameters κ_A and κ_B from the input values at the unification scale $\mu = \Lambda_X$ for the two special cases (non-SUSY and SUSY models). If we find $\kappa_A \simeq \kappa_B$, we will give further detailed numerical study only for the case.

III. NUMERICAL RESULTS

First, we investigate at the energy scale $\mu = m_Z$. Eqs.(2.13) and (2.14) are realized by GUT scale because Eq.(2.7) is broken at $\mu = m_Z$. However, even if $\mu = m_Z$, the ratio of each matrix element in Yukawa coupling scarcely changes, as a rule. Therefore, we assume that each Yukawa coupling at $\mu = m_Z$ is a symmetrical matrix in approximation. For the fermion masses at $\mu = m_Z$, we use the following values: [4]

$$\begin{aligned} &m_t = 181 \pm 13 \text{ GeV}, && m_b = 3.00 \pm 0.11 \text{ GeV}, \\ &m_c = 677^{+56}_{-61} \text{ MeV}, && m_s = 93.4^{+11.8}_{-13.0} \text{ MeV}, \\ &m_u = 2.33^{+0.42}_{-0.45} \text{ MeV}, && m_d = 4.69^{+0.60}_{-0.66} \text{ MeV}, \\ &m_\tau = 1746.7 \pm 0.3 \text{ MeV}, \\ &m_\mu = 102.75138 \pm 0.00033 \text{ MeV}, \\ &m_e = 0.48684727 \pm 0.00000014 \text{ MeV}. \end{aligned} \tag{3.1}$$

The input values for the CKM matrix parameters have been taken as [5]

$$\begin{aligned} &\theta_{12} = 0.219 - 0.226, && \theta_{23} = 0.037 - 0.043, \\ &\theta_{13} = 0.002 - 0.005, \end{aligned} \tag{3.2}$$

where

$$V = \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta} & -s_{23}c_{12} - c_{23}s_{12}s_{12}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \tag{3.3}$$

with $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$. The calculation has been performed allowing all the combinations of the quark mass signatures. Here it should be noted that, since m_u is much smaller than m_c and m_t, the difference of the sign of m_u scarcely makes a change of allowed regions. In this calculation, we have selected θ_{23} and δ as input parameters and m_s, c_d and κ as output parameters because the calculation is sensitive to these parameters. We give the numerical results in Fig 1. Here, except for m_s, θ_{23} and δ, we have adopted the center values of Eq.(3.1) as input values. Moving θ_{23} at intervals of 0.0005 rad and fixing $\delta = 60°$, we search the solutions where κ_A and κ_B become coincident. Our numerical analysis shows that the solutions exist in the combinations of Table I. In a table II, we show the nearest solution of m_s, θ_{23} and δ to the center values of Eq.(3.1).

In the following we perform data fitting for the case of top line of Table II. Eqs. (2.10)-(2.12) can constrain only the absolute value of c_d. The argument of the parameter c_d may be decided by taking neutrino sector into consideration in the future. For the time being, we set $c_d \equiv |c_d|e^{i\sigma} = e^{0.107i}$ so that c_0 becomes a real number:

$$c_0 = \frac{1 - c_d}{c_u} = 34.7, \tag{3.4}$$

$$c_1 = -\frac{3 + c_d}{c_u} = 101.8 - 10.8i. \tag{3.5}$$

In this case, the mass matrices in MeV are

$$M_0 = \frac{3VD_dV^T + c_d(\kappa D_u + VD_dV^T)}{4} = \begin{pmatrix} -12.4 - 0.7i & -23.0 - 1.8i & 9.6 - 13.2i \\ -23.0 - 1.8i & -91.5 - 3.9i & 194.0 + 10.5i \\ 9.6 - 13.2i & 194.0 + 10.5i & 1874.9 - 180.0i \end{pmatrix}, \tag{3.6}$$

$$M_1 = \frac{VD_dV^T - c_d(\kappa D_u + VD_dV^T)}{4} = \begin{pmatrix} 4.19 + 0.69i & 7.68 + 1.43i & -3.72 + 4.09i \\ 7.68 + 1.43i & 24.14 + 3.88i & -65.05 - 10.48i \\ -3.72 + 4.09i & -65.05 - 10.48i & 1119.67 + 179.98i \end{pmatrix}. \tag{3.7}$$

Here, using the condition $\sqrt{|v_0^u|^2 + |v_0^d|^2 + |v_1^u|^2 + |v_1^d|^2} =$ 246GeV, we can get VEV's as

$$v_0^d = \frac{246\,[\text{GeV}]}{\sqrt{(|c_0|^2 + 1) + (|c_1|^2 + 1)|\rho|^2}} \tag{3.8}$$

with $\rho \equiv v_1^d/v_0^d$. Then, the Yukawa couplings about **10** and **126** become

$$Y_{10} = \frac{M_0}{v_0^d}, \quad Y_{126} = \frac{M_1}{v_1^d}. \tag{3.9}$$

We consider that the model should be calculable perturbativly. We can see that every element of the Yukawa coupling constants (3.9) is smaller than one if we take a suitable value of $|\rho|$.

In order to see the evolution effects, we have also considered the following two cases :

$$\begin{aligned} &m_t = 84^{+18}_{-13}.\ \text{GeV}, \qquad m_b = 1.07 \pm 0.04\ \text{GeV}, \\ &m_c = 272.^{+22}_{-24}\ \text{MeV}, \qquad m_s = 38.7^{+4.9}_{-5.4}\ \text{MeV}, \\ &m_u = 0.94^{+0.17}_{-0.18}\ \text{MeV}, \qquad m_d = 1.94^{+0.25}_{-0.28}\ \text{MeV}, \qquad (3.10) \\ &m_\tau = 1770.6 \pm 0.3\ \text{GeV}, \\ &m_\mu = 104.15246 \pm 0.00033\ \text{MeV}, \\ &m_e = 0.49348567 \pm 0.00000014\ \text{MeV}, \\ &\theta_{12} = 0.222, \quad \theta_{23} = 0.0433, \quad \theta_{13} = 0.0035 \qquad (3.11) \end{aligned}$$

which are values at $\mu = \Lambda_X = 2 \times 10^{16}$GeV with the Higgs boson mass $m_H = 246.2$GeV in Non-SUSY model [4], and

$$\begin{aligned} &m_t = 129.^{+196}_{-40}\ \text{GeV}, \qquad m_b = 1.00 \pm 0.04\ \text{GeV}, \\ &m_c = 302.^{+25}_{-27}\ \text{MeV}, \qquad m_s = 26.5^{+3.3}_{-3.7}\ \text{MeV}, \\ &m_u = 1.04^{+0.19}_{-0.20}\ \text{MeV}, \qquad m_d = 1.33^{+0.17}_{-0.19}\ \text{MeV}, \qquad (3.12) \\ &m_\tau = 1171.4 \pm 0.2\ \text{MeV}, \\ &m_\mu = 68.59813 \pm 0.00022\ \text{MeV}, \\ &m_e = 0.32502032 \pm 0.00000009\ \text{MeV}, \\ &\theta_{12} = 0.222, \quad \theta_{23} = 0.0318, \quad \theta_{13} = 0.0026 \qquad (3.13) \end{aligned}$$

which are values at $\mu = \Lambda_X = 2 \times 10^{16}$GeV in the minimal SUSY model with $\tan\beta = 10$ [4]. Each solution is shown in Tables III and IV, respectively. The solutions give the same trends; m_s is smaller and θ_{23} is larger than the value in Eqs.(3.10)-(3.13) as the case of $\mu = m_Z$. In other words, the evolution effects scarcely make change of the previous results.

IV. 10 AND 120

In the SO(10) GUT scenario, we can also discuss the model with one **10** and one **120** by the same method.

The Yukawa couplings of **10** and **120** are symmetric and antisymmetric, respectively. If we consider a case that the Yukawa coupling constants of **10** are real and **120** pure imaginary, we can make them Hermitian, i.e., $Y_{10}^\dagger = Y_{10}$ and $Y_{120}^\dagger = Y_{120}$. Therefore, by considering the real vacuum expectation values v_{10} and v_{120}, we can obtain the Hermitian mass matrices M_u, M_d and M_e:

$$\begin{aligned} M_d &= M_0 + M_2, \quad M_e = M_0 - 3M_2, \\ M_u &= c_0 M_0 + c_2 M_2. \end{aligned} \tag{4.1}$$

Then, we can diagonalize those by unitary matrices U_u, U_d and U_e as

$$U_u^\dagger M_u U_u = D_u, \quad U_d^\dagger M_d U_d = D_d, \quad U_e^\dagger M_e U_e = D_e. \tag{4.2}$$

Since the CKM matrix V is given by

$$V = U_u^\dagger U_d \ , \tag{4.3}$$

the relation (4.1) is re-written as follows:

$$(U_u^\dagger U_e) D_e (U_u^\dagger U_e)^\dagger = c_d V D_d V^\dagger + c_u D_u. \tag{4.4}$$

As stated previously, we have almost known the experimental values of D_e, D_u and $V D_d V^\dagger$. Therefore, we obtain the independent three equations:

$$\mathrm{Tr} D_e = c_d [\mathrm{Tr} D_d + \kappa \mathrm{Tr} D_u], \tag{4.5}$$

$$\mathrm{Tr} D_e^2 = c_d^2 [\mathrm{Tr} D_d^2 + 2\kappa \mathrm{Tr}(D_u V D_d V^\dagger) + \kappa^2 \mathrm{Tr} D_u^2], \tag{4.6}$$

$$\det D_e = c_d^3 \det(V D_d V^\dagger + \kappa D_u), \tag{4.7}$$

where $\kappa = c_u/c_d$. For the parameter κ, we have two equations:

$$\begin{aligned} &\frac{m_e^2 + m_\mu^2 + m_\tau^2}{(m_e + m_\mu + m_\tau)^2} \\ &\qquad = \frac{\mathrm{Tr} D_d^2 + 2\kappa \mathrm{Tr}(D_u V D_d V^\dagger) + \kappa^2 \mathrm{Tr} D_u^2}{(\mathrm{Tr} D_d + \kappa \mathrm{Tr} D_u)^2}, \end{aligned} \tag{4.8}$$

$$\frac{m_e m_\mu m_\tau}{(m_e + m_\mu + m_\tau)^3} = \frac{\det(V D_d V^\dagger + \kappa D_u)}{(\mathrm{Tr} D_d + \kappa \mathrm{Tr} D_u)^3}. \tag{4.9}$$

Eqs. (4.8) and (4.9) are more simple than Eqs. (2.13) and (2.14). c_d and κ are real since we have assumed the M_u, M_d and M_e to be Hermitian. So the calculation is easier than the case for **10** and **126**. The neumerical results are listed in Table V-VIII.

V. SUMMARY AND DISCUSSION

In conclusion, we have investigated whether an SO(10) model with two Higgs scalars can reproduce the observed mass spectra of the up- and down-quark sectors and charged lepton sector or not. What is of great interest is to see whether we can find reasonable values of the parameters c_u and c_d which satisfy the SO(10) relation (2.5) or not. For the case with one **10** and one **126** scalars, in a parameter $\kappa = c_u/c_d$, we have obtained two equations (2.13) and (2.14) which hold at the unification scale $\mu = \Lambda_X$ and which are described in terms of the observable quantities (the fermion masses and CKM matrix parameters). We have sought for the solution of κ approximately by using the observed fermion masses and CKM matrix parameters at $\mu = m_Z$ instead of the observable quantities at $\mu = \Lambda_X$. Although we have found no solution for real κ, we have found four solutions for complex κ which satisfy Eqs. (2.13) and (2.14) within the experimental errors. Similarly, we have found four solutions for a model with one **10** and one **120** scalars. It should be worth while noting that the solutions in the latter model are real. The latter model is very attractive because the origin of the CP violation attributes only to the **120** scalar. In the both models, we can make the magnitudes of all the Yukawa coupling constants smaller than one, so that the models are safely calculable under the perturbation theory.

By the way, note that the numerical results are very sensitive to the values of m_s and θ_{23}. For numerical fittings, it is favor that the strange quark mass m_s is somewhat smaller than the center value $m_s = 93.4$ MeV which is quoted in Ref. [4].

Also note that the relative sign of m_d to m_s in each solution is positive, i.e, $m_d/m_s > 0$ as seen in Tables I and V. It is well known that a model with a texture $(M_d)_{11} = 0$ on the nearly diagonal basis of the up-quark mass matrix M_u leads to the relation $|V_{us}| = \sqrt{-m_d/m_s}$ [6], where the relative sign is negative, i.e., $m_d/m_s < 0$. On the contrary, we can conclude that in the SO(10) model with two Higgs scalars, we cannot adopt a model with the texture $(M_d)_{11} = 0$.

In the present paper, we have demonstrated that the unified description of the quark and charged lepton masses in the SO(10) model with two Higgs scalars is possible. However, we have not referred to the neutrino masses. Concerning this problem, Brahmachari and Mohapatra have recently showed that one **10** and one **126** model is incompatible with large ν_μ-ν_τ mixing angle [7]. Since there are many possibilities for neutrino mass generation mechanism, we are optimistic about this problem, too. Investigating for a question whether an SO(10) model with two Higgs scalars can give a unified description of quark and lepton masses including neutrino masses and mixings or not is our next big task.

[1] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **70** 2845

(1993); D-G Lee and R.N. Mohapatra, Phys. Rev. **D51** 1353 (1995).

[2] K. Oda,E. Takasugi,M. Tanaka and M. Yoshimura, Phys. Rev. **D59** 055001 (1999).

[3] H. Georgi and C. Jarlskog, Phys. Rev. **B86** 297 (1979).

[4] H. Fusaoka and Y. Koide, Phys. Rev. **D57**, 3986 (1998).

[5] D.E. Groom et al, The European Physical Journal **C15**, 1 (2000).

[6] S. Weinberg, Ann. N. Y, Acad. Sci. **38**, 185 (1977); H. Fritzsch, Phys. Lett. **73B**, 317 (1978); Nucl. Phys. **B155**, 189 (1979); H. Georgi and D. V. Nanopoulos, *ibid.* **B155**, 52 (1979).

[7] B. Brahmachari and R.N. Mohapatra, Phys. Rev. **D58** 015001 (1998).

num.	(m_t, m_c, m_u)	(m_b, m_s, m_d)	(m_τ, m_μ, m_e)
(a)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ \pm\ \pm)$
(b)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ \pm\ \pm)$

TABLE I. The combinations of the signs of (m_t, m_c, m_u), (m_b, m_s, m_d) and (m_τ, m_μ, m_e). The notation (m_t, m_c, m_u) $= (+\ -\ +)$ denotes $m_t > 0$, $m_c < 0$ and $m_u > 0$. Eqs. (2.13) and (2.14) are not affected by the signs of charged leptons.

	Input		Output		
	$\|\theta_{23}\|$[rad]	$\delta[^\circ]$	m_s[MeV]	$\|c_d\|$	κ
(a)	0.0420	60.	76.3	3.15698	$-0.01928 - 0.00089i$
	0.0420	60.	76.3	3.03577	$-0.01937 - 0.00101i$
(b)	0.0420	60.	76.3	3.13307	$-0.01929 - 0.00092i$
	0.0420	60.	76.3	3.00558	$-0.01939 - 0.00105i$

TABLE II. Four sets of parameters giving good data fitting at $\mu = m_Z$ for one **10** and one **126** Higgs scalars. (a) and (b) correspond to the mass signatures in Table I, and the upper and lower lines do to the two intersections in Fig.1

	Input		Output		
	$\|\theta_{23}\|$[rad]	$\delta[^\circ]$	m_s[MeV]	$\|c_d\|$	κ
(a)	0.0460	60.	29.3	8.80715	$-0.01486 - 0.00058i$
	0.0460	60.	29.3	8.53853	$-0.01493 - 0.00064i$
(b)	0.0465	60.	29.6	8.63097	$-0.01490 - 0.00061i$
	0.0465	60.	29.6	8.34147	$-0.01498 - 0.00068i$

TABLE III. Same as Table II but to incorporate mass evolution in non-SUSY frame.

	Input		Output		
	$\|\theta_{23}\|$[rad]	$\delta[^\circ]$	m_s[MeV]	$\|c_d\|$	κ
(a)	0.0340	60.	19.6	8.76011	$-0.00867 - 0.000241i$
	0.0340	60.	19.6	8.50744	$-0.00870 - 0.000258i$
(b)	0.0340	60.	19.6	8.72425	$-0.00867 - 0.000244i$
	0.0340	60.	19.6	8.45025	$-0.00871 - 0.000262i$

TABLE IV. Same as Table II but to incorporate mass evolution in SUSY frame.

num.	(m_t, m_c, m_u)	(m_b, m_s, m_d)	(m_τ, m_μ, m_e)
(a-1)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ +\ +)$
(a-2)	$(+\ -\ +)$	$(+\ -\ -)$	$(+\ +\ -)$
(b-1)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ +\ +)$
(b-2)	$(+\ -\ -)$	$(+\ -\ -)$	$(+\ +\ -)$

TABLE V. The combinations of the signs of (m_t, m_c, m_u), (m_b, m_s, m_d) and (m_τ, m_μ, m_e) for one **10** and one **120** Higgs scalars.

	Input		Output		
	$\|\theta_{23}\|$[rad]	$\delta[^\circ]$	m_s[MeV]	c_d	κ
(a-1)	0.0415	60.	79.551	0.05905	-0.01957
(a-2)	0.0415	60.	79.238	0.06124	-0.01942
(b-1)	0.0415	60.	79.673	0.05855	-0.01960
(b-2)	0.0415	60.	79.316	0.06080	-0.01945

TABLE VI. Four sets of parameters giving good data fitting at $\mu = m_Z$ for one **10** and one **120** Higgs scalars. (a-i) and (b-i) correspond to the mass signatures in Table V.

	Input		Output		
	$\|\theta_{23}\|$[rad]	$\delta[^\circ]$	m_s[MeV]	c_d	κ
(a-1)	0.0455	60.	30.283	0.12800	-0.01502
(a-2)	0.0455	60.	30.269	0.13166	-0.01493
(b-1)	0.0455	60.	30.298	0.12731	-0.01504
(b-2)	0.0455	60.	30.278	0.13105	-0.01495

TABLE VII. Same as Table VI but to incorporate mass evolution in non-SUSY frame.

	Input		Output		
	$\|\theta_{23}\|$[rad]	$\delta[^\circ]$	m_s[MeV]	c_d	κ
(a-1)	0.0335	60.	20.175	0.07457	-0.00873
(a-2)	0.0335	60.	20.191	0.07660	-0.00870
(b-1)	0.0335	60.	20.179	0.07421	-0.00874
(b-2)	0.0335	60.	20.194	0.07628	-0.00870

TABLE VIII. Same as Table VI but to incorporate mass evolution in SUSY frame.

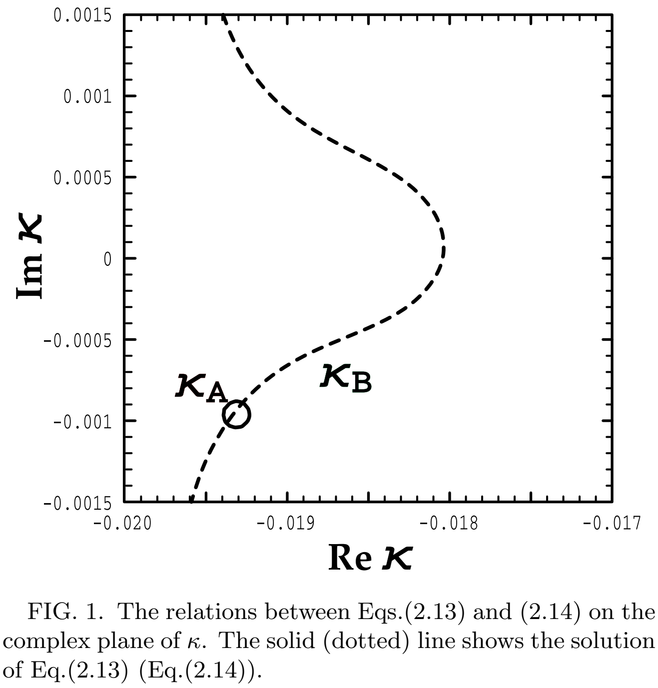

FIG. 1. The relations between Eqs.(2.13) and (2.14) on the complex plane of κ. The solid (dotted) line shows the solution of Eq.(2.13) (Eq.(2.14)).