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Democratic Universal Seesaw Model with Three Harmless Sterile Neutrinos

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Abstract

Based on the "democratic" universal seesaw model, where mass matrices M_f of quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$) are given by a seesaw form $M_f \simeq -m_L M_F^{-1} m_R$, and m_L and m_R are universal for all the fermion sectors, and the mass matrices M_F of hypothetical heavy fermions F_i have a democratic structure, a possible neutrino mass matrix is investigated. In the model, there are three sterile neutrinos ν_{iR} which mix with the active neutrinos ν_{iL} with $\theta \sim 10^{-2}$ and which are harmless for cosmological constraints. The atmospheric, solar and LSND neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$, $\nu_{eL} \leftrightarrow \nu_{eR}$ and $\nu_{eL} \leftrightarrow \nu_{\mu L}$, respectively. The model predicts that $\Delta m^2_{solar}/\Delta m^2_{atm} \simeq m_e/\sqrt{m_\mu m_\tau}$ and $\Delta m^2_{LSND}/\Delta m^2_{atm} \simeq (1/4)\sqrt{m_\mu/m_e}$ ($\Delta m^2_{solar} \simeq 3 \times 10^{-6}$ eV^2 and $\Delta m^2_{LSND} \simeq 0.5$ eV^2 for $\Delta m^2_{atm} \simeq 2.2 \times 10^{-3}$ eV^2), and $\sin^2 2\theta_{atm} \simeq 1$ and $\sin^2 2\theta_{LSND} \simeq 4 m_e/m_\mu \simeq 0.02$.

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1 Introduction

In order to seek for a clue to the unified understanding of quarks and leptons, many attempts to give a unified description of the quark and lepton mass matrices have been proposed. The universal seesaw mass matrix model [1] is one of the promising attempts to view the unified description, where the mass matrices M_f for the conventional quarks and leptons f_i ($f = u, d, \nu, e$; $i = 1, 2, 3$) are given by

$$(\overline{f}_L \ \overline{F}_L) \begin{pmatrix} 0 & m_L \\ m_R & M_F \end{pmatrix} \begin{pmatrix} f_R \\ F_R \end{pmatrix}, \tag{1.1}$$

and m_L and m_R are universal for all fermion sectors f. For $O(M_F) \gg O(m_R) \gg O(m_L)$, the mass matrix (1.1) leads to the well-known seesaw expression

$$M_f \simeq -m_L M_F^{-1} m_R. \tag{1.2}$$

As a specific version of such universal seesaw model, Fusaoka and one of the authors (Y.K.) have proposed a so-called "democratic" seesaw model [2]: The heavy fermion matrices M_F have a simple structure [(unit matrix)+(democratic matrix)], i.e.,

$$M_F = m_0 \lambda_f (\mathbf{1} + 3 b_f X), \tag{1.3}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{1.4}$$

on the basis on which the matrices m_L and m_R are diagonal:

$$m_L = \frac{1}{\kappa} m_R = m_0 Z = m_0 \begin{pmatrix} z_1 & 0 & 0 \\ 0 & z_2 & 0 \\ 0 & 0 & z_3 \end{pmatrix}, \tag{1.5}$$

where the parameters z_1, z_2 and z_3 are normalized as $z_1^2 + z_2^2 + z_3^2 = 1$, and m_0 is of the order of the electroweak symmetry breaking scale, i.e., $m_0 \sim 10^2$ GeV. Since the parameter b_f in the charged lepton sector is taken as $b_e = 0$, the parameters z_i are fixed as

$$\frac{z_1}{\sqrt{m_e}} = \frac{z_2}{\sqrt{m_\mu}} = \frac{z_3}{\sqrt{m_\tau}} = \frac{1}{\sqrt{m_\tau + m_\mu + m_e}}. \tag{1.6}$$

For the up-quark sector, the parameter b_f is taken as $b_u = -1/3$, which leads to $\det M_U = 0$, and the seesaw mechanism does not work for one of the three families, and hence we obtain the mass $m_t \simeq m_0/\sqrt{3}$ without the seesaw suppression factor κ/λ_u (we identify it as the top quark mass). Furthermore, we also obtain a relation $m_u/m_c \simeq 3m_e/m_\mu$, which is in good agreement with the observed values. Moreover, when we take $b_d \simeq -1$ ($b_d = -e^{i\beta_d}$ with $\beta_d = 18°$) for the down-quark sector, we can obtain reasonable quark mass ratios and the Cabbibo-Kobayashi-Maskawa [3] (CKM) matrix.

The neutrino mass matrix in the universal seesaw mass matrix model is given as follows:

$$\left(\overline{\nu}_L \ \ \overline{\nu}^c_R \ \ \overline{N}_L \ \ \overline{N}^c_R \right) \left(\begin{array}{cccc} 0 & 0 & 0 & m_L \\ 0 & 0 & m_R^T & 0 \\ 0 & m_R & M_{NL} & M_D \\ m_L^T & 0 & M_D^T & M_{NR} \end{array} \right) \left(\begin{array}{c} \nu^c_L \\ \nu_R \\ N^c_L \\ N_R \end{array} \right), \tag{1.7}$$

where $\psi^c_L \equiv (\psi_L)^c = C\psi_L^T$. [We consider a SO(10)$_L\times$SO(10)$_R$ model [4], where fermions $(f_L + F^c_R)$ and $(f_R + F^c_L)$ are assigned to (16,1) and (1,16) under SO(10)$_L\times$SO(10)$_R$, respectively. Hereafter, we will denote the Majorana mass matrices M_{NL} and M_{NR} of the neutral heavy leptons N_L and N_R as $M_R = M_{NL}$ and $M_L = M_{NR}$, respectively.]

For $O(m_L) \ll O(m_R) \ll O(M_D), O(M_L), O(M_R)$, we obtain the following 6×6 seesaw mass matrix for (ν^c_L, ν_R)

$$M^{(6\times6)} \simeq -\left(\begin{array}{cc} 0 & m_L \\ m_R^T & 0 \end{array} \right) \left(\begin{array}{cc} M_R & M_D \\ M_D^T & M_L \end{array} \right)^{-1} \left(\begin{array}{cc} 0 & m_R \\ m_L^T & 0 \end{array} \right), \tag{1.8}$$

which leads to the 3×3 seesaw matrices for ν_L and ν_R

$$M(\nu_L) \simeq -m_L M_L^{-1} m_L^T, \tag{1.9}$$

$$M(\nu_R) \simeq -m_R M_R^{-1} m_R^T. \tag{1.10}$$

The scenario corresponding to $O(m_L M_L^{-1} m_L^T) \ll O(m_R M_R^{-1} m_R^T)$ has already been investigated by one of the authors (Y.K.) [5]. He has concluded that although either the atmospheric [6] or solar [7] neutrino data can be explained by the mixings $\nu_\mu \leftrightarrow \nu_\tau$ or $\nu_e \leftrightarrow \nu_\mu$, however, simultaneous explanation of the both data cannot be obtained in this model.

In the present paper, we consider another possibility $O(m_L M_L^{-1} m_L^T) \sim O(m_R M_R^{-1} m_R^T)$. In this case, mixings between ν_{iL} and ν_{iR} are induced. The solar neutrino data [7] are understood from a small mixing between ν_{eL} and ν_{eR}. The atmospheric [6] and the LSND [8] neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$ and $\nu_{eL} \leftrightarrow \nu_{\mu L}$, respectively. The vantage point of the democratic seesaw model [2] is that parameters z_i in the mass matrices m_L and m_R are given in terms of the charged lepton masses and thereby the mass spectrum and mixings of ν_{iL} and ν_{iR} can also be predicted in terms of the charged lepton masses.

2 Parameter b_ν

In the present paper, for simplicity, we assume that all the neutral heavy fermion mass matrices M_D, M_L and M_R have the same flavor structure

$$\frac{1}{\lambda_D} M_D = \frac{1}{\lambda_L} M_L = \frac{1}{\lambda_R} M_R = m_0(\mathbf{1} + 3b_\nu X), \tag{2.1}$$

and we will investigate only the case $b_\nu = -1/2$.

The excuse for considering only the case $b_\nu = -1/2$ is as follows. The choices of b_f ($b_e = 0, b_u = -1/3, b_d \simeq -1$) have given the successful description of the quark masses and mixings in terms of the charged lepton masses. When, instead of the expression (1.3), we denote M_F as

$$M_F = m_0 \lambda_F (\cos\phi_f E - \sin\phi_f S), \tag{2.2}$$

$$E = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad S = \frac{1}{\sqrt{6}} \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \tag{2.3}$$

where E and S have been normalized as $\mathrm{Tr}E^2 = \mathrm{Tr}S^2 = 1$, the cases $b_e = 0$, $b_u = -1/3$ and $b_d = -1$ correspond to $(\cos\phi_f, \sin\phi_f) = (1,0)$, $(\sqrt{2/3}, \sqrt{1/3})$ and $(0,1)$, respectively. Considering an empirical relation $\phi_d = \pi/2 - \phi_e$ for $(\cos\phi_e, \sin\phi_e) = (1,0)$ and $(\cos\phi_d, \sin\phi_d) = (0,1)$, we consider that the value of b_ν is also given by $\phi_\nu = \pi/2 - \phi_u$ for $(\cos\phi_u, \sin\phi_u) = (\sqrt{2/3}, \sqrt{1/3})$, i.e., we assume

$$(\cos\phi_\nu, \sin\phi_\nu) = (\sqrt{1/3}, \sqrt{2/3}), \tag{2.4}$$

which corresponds to the case $b_\nu = -1/2$.

Besides, from the phenomenological point of view, the case $b_\nu = -1/2$ is also interesting. The inverse matrix of the M_L with $b_\nu = -1/2$

$$M_L = m_0 \lambda_L (\mathbf{1} - \frac{1}{2} \cdot 3X) = \frac{1}{2} m_0 \lambda_L \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix}, \tag{2.5}$$

is given by

$$M_L^{-1} = -\frac{1}{m_0\lambda_L}\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \tag{2.6}$$

so that the seesaw matrix $M_\nu \simeq -m_L M_L^{-1} m_L^T$ is expressed as

$$M_\nu \simeq m_0 \frac{1}{\lambda_L}\begin{pmatrix} 0 & z_1 z_2 & z_1 z_3 \\ z_1 z_2 & 0 & z_2 z_3 \\ z_1 z_3 & z_2 z_3 & 0 \end{pmatrix}. \tag{2.7}$$

The form (2.7) is just a Zee-type mass matrix [9], which has recently been revived [10] as a promising neutrino mass matrix form.

3 Mass spectrum and mixing

For the specific form (2.1) with $b_\nu = -1/2$, the 6×6 seesaw matrix $M^{(6\times6)}$ given by Eq. (1.8) becomes

$$M^{(6\times6)} \simeq -m_0 \begin{pmatrix} 0 & Z \\ \kappa Z & 0 \end{pmatrix}\begin{pmatrix} \lambda_R Y & \lambda_D Y \\ \lambda_D Y & \lambda_L Y \end{pmatrix}^{-1}\begin{pmatrix} 0 & \kappa Z \\ Z & 0 \end{pmatrix}$$

$$= -m_0 \frac{1}{\lambda_R\lambda_L - \lambda_D^2}\begin{pmatrix} \lambda_R Z Y^{-1} Z & -\kappa\lambda_D Z Y^{-1} Z \\ -\kappa\lambda_D Z Y^{-1} Z & \kappa^2 \lambda_L Z Y^{-1} Z \end{pmatrix}, \tag{3.1}$$

where

$$Y = \mathbf{1} + 3b_\nu X, \quad Y^{-1} = \mathbf{1} + 3a_\nu X, \tag{3.2}$$

$$a_\nu = -b_\nu/(1+3b_\nu). \tag{3.3}$$

Therefore, the matrix $M^{(6\times6)}$ is diagonalized by the 6×6 unitary matrix $U^{(6\times6)}$

$$U^{(6\times6)} = \begin{pmatrix} \cos\theta\cdot U & -\sin\theta\cdot U \\ \sin\theta\cdot U & \cos\theta\cdot U \end{pmatrix}, \tag{3.5}$$

as

$$U^{(6\times6)\dagger} M^{(6\times6)} U^{(6\times6)} = \mathrm{diag}(m_{\nu_{1L}}, m_{\nu_{2L}}, m_{\nu_{3L}}, m_{\nu_{1R}}, m_{\nu_{2R}}, m_{\nu_{3R}})$$

$$= m_0 \mathrm{diag}(\xi_L\rho_1, \xi_L\rho_2, \xi_L\rho_3, \xi_R\rho_1, \xi_R\rho_2, \xi_R\rho_3), \tag{3.6}$$

where

$$U^\dagger Z Y^{-1} Z U = \mathrm{diag}(\rho_1, \rho_2, \rho_3), \tag{3.7}$$

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \begin{pmatrix} \lambda_R & -\kappa\lambda_D \\ -\kappa\lambda_D & \kappa^2\lambda_L \end{pmatrix} \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} = \begin{pmatrix} \lambda'_L & 0 \\ 0 & \lambda'_R \end{pmatrix}, \tag{3.8}$$

$$\xi_L = \frac{\lambda'_L}{\lambda_R\lambda_L - \lambda_D^2}\ , \quad \xi_R = \frac{\lambda'_R}{\lambda_R\lambda_L - \lambda_D^2}\ , \tag{3.9}$$

$$\begin{pmatrix} \lambda'_L \\ \lambda'_R \end{pmatrix} = \frac{1}{2}(\lambda_R + \kappa^2\lambda_L) \pm \frac{1}{2}(\lambda_R - \kappa^2\lambda_L)\sqrt{1 + \tan^2 2\theta}\ . \tag{3.10}$$

The mixing angle θ between ν_{iL} and ν_{iR} is given by

$$\tan 2\theta = \frac{2\kappa\lambda_D}{\lambda_R - \kappa^2\lambda_L}. \tag{3.11}$$

The light neutrino masses $m(\nu_{iL})$ and $m(\nu_{iR})$ are given by

$$m(\nu_{iL}) = m_0\xi_L\rho_i\ , \quad m(\nu_{iR}) = m_0\xi_R\rho_i\ . \tag{3.12}$$

For the case of $b_\nu = -1/2$, the eigenvalues ρ_i of the matrix $ZY^{-1}Z$ are given by

$$\rho_1 \simeq -2z_1^2, \quad \rho_2 \simeq -\left(z_2 + \frac{z_1^2}{2z_2} - z_1^2\right), \quad \rho_3 \simeq z_2 + \frac{z_1^2}{2z_2} + z_1^2, \tag{3.13}$$

so that

$$\rho_3^2 - \rho_2^2 \simeq 4z_2z_1^2, \quad \rho_2^2 - \rho_1^2 \simeq z_2^2. \tag{3.14}$$

The 3×3 mixing matrix U for the case $b_\nu = -1/2$ is given by

$$U \simeq \begin{pmatrix} -1 & -\frac{1}{\sqrt{2}}\frac{z_1}{z_2}(1 - z_2) & \frac{1}{\sqrt{2}}\frac{z_1}{z_2}(1 + z_2) \\ \frac{z_1}{z_2} & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ z_1 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{3.15}$$

4 Explanations of the neutrino data

The atmospheric [6] and solar [7] neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$ and $\nu_{eL} \leftrightarrow \nu_{eR}$, respectively. As seen in the mixing matrix (3.15), the neutrinos $\nu_{\mu L}$ and $\nu_{\tau L}$ are maximally mixed. On the other hand, the mixing between ν_{eL} and ν_{eR} is given by Eq. (3.11). Since the solar neutrino data are in disfavor [11] with sterile neutrinos, we take the small angle solution in the Mikheyev-Smirnov-Wolfenstein(MSW) mechanism [12],

$$\Delta m^2_{solar} \simeq 4.0 \times 10^{-6}\ \mathrm{eV}^2, \quad \sin^2 2\theta_{solar} \simeq 6.9 \times 10^{-3}. \tag{4.1}$$

Here, the values in Eq. (4.1) have been quoted from the recent analysis for $\nu_e \to \nu_s$ by Bahcall, Krastev and Smirnov [13]. The value $\sin^2 2\theta_{solar} \simeq 7 \times 10^{-3}$ will be fitted by adjusting the parameters λ_L, λ_R/κ^2 and λ_D/κ as we discuss later.

As seen from Eqs. (3.6) and (3.14), the ratio of $\Delta m^2_{solar} = (m_{\nu_{1L}})^2 - (m_{\nu_{1R}})^2$ to $\Delta m^2_{atm} = (m_{\nu_{3L}})^2 - (m_{\nu_{2L}})^2$ is given by

$$\frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} \simeq \frac{\lambda'^2_L - \lambda'^2_R}{\lambda'^2_L} \frac{4z_1^4}{4z_2 z_1^2} \simeq (1-R^2)\frac{m_e}{\sqrt{m_\mu m_\tau}}, \tag{4.2}$$

where

$$R = \frac{\lambda'_R}{\lambda'_L} = \frac{\xi_R}{\xi_L} = \frac{m(\nu_{iR})}{m(\nu_{iL})}. \tag{4.3}$$

If we consider $R^2 \ll 1$, then we obtain the prediction

$$\frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} \simeq \frac{m_e}{\sqrt{m_\mu m_\tau}} \simeq 1.2 \times 10^{-3}, \tag{4.4}$$

which is in agreement with the observed value

$$\frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} \simeq \frac{4.0 \times 10^{-6}\mathrm{eV}^2}{2.2 \times 10^{-3}\mathrm{eV}^2} \simeq 1.8 \times 10^{-3}. \tag{4.5}$$

Here, as the best-fit value of Δm^2_{atm}, we have used the value reported by Super-Kamiokande collaboration [14]. If we use the very recent updated global analysis of the atmospheric neutrino data by Fornengo, Gonzalez-Garcia and Valle [15], $\Delta m^2_{atm} = 3.0 \times 10^{-3}$ eV^2, we can obtain a more favorable value $\Delta m^2_{solar}/\Delta m^2_{atm} \simeq 1.3 \times 10^{-3}$ to the prediction (4.4).

If we consider $R > 1$, we can obtain a more suitable fitting of the ratio $\Delta m^2_{solar}/\Delta m^2_{atm}$ to the data (4.5) by adjusting the parameter R. However, we will choose the case $R^2 \ll 1$ as we discuss later.

The LSND data [8] is explained by the mixing $\nu_{eL} \leftrightarrow \nu_{eR}$. The mass-squared difference $\Delta m^2_{LSND} = (m_{\nu_{2L}})^2 - (m_{\nu_{1L}})^2$ is given by the ratio

$$\frac{\Delta m^2_{LSND}}{\Delta m^2_{atm}} \simeq \frac{z_2}{4z_1} \simeq \frac{1}{4}\sqrt{\frac{m_\mu}{m_e}} = 2.2 \times 10^2, \tag{4.6}$$

which leads to

$$\Delta m^2_{LSND} \simeq 0.48 \text{ eV}^2, \tag{4.7}$$

for $\Delta m^2_{atm} \simeq 2.2 \times 10^{-3}$ eV^2. On the other hand, the mixing angle θ_{LSND} is given by

$$\sin^2 2\theta_{LSND} \simeq -4U_{e1}U_{\mu 1} \simeq 4\left(\frac{z_1}{z_2}\right)^2 \simeq 4\frac{m_e}{m_\mu} \simeq 0.019. \tag{4.8}$$

The solution ($\Delta m^2 \simeq 0.5$ eV$^2, \sin^2 2\theta \simeq 0.02$) is just fitted in the allowed narrow region given by the LSND experiment [8].

The mixing between ν_{eL} and $\nu_{\tau L}$ is given by

$$U_{e3} \simeq \frac{1}{\sqrt{2}}\frac{z_1}{z_2}(1+z_2) \simeq \sqrt{\frac{m_e}{2m_\mu}}\left(1+\sqrt{\frac{m_\mu}{m_\tau}}\right) \simeq 0.061, \tag{4.9}$$

which safely satisfies the constraint $|U_{e3}| \leq (0.22 - 0.14)$ obtained from the CHOOZ reactor neutrino experiment [16].

Since $m^2(\nu_{3L}) \simeq m^2(\nu_{2L}) \gg m^2(\nu_{1L})$, from Eq. (4.7), we obtain

$$m(\nu_{3L}) \simeq m(\nu_{2L}) \simeq 0.69 \text{ eV}, \quad m(\nu_{1L}) \simeq 0.016 \text{ eV}. \tag{4.10}$$

For the case $R^2 \ll 1$, the sterile neutrinos have negligibly small masses $m(\nu_{iR}) = Rm(\nu_{iL})$ $(i = 1, 2, 3)$. In order to obtain a best-fitting value of $\Delta m^2_{solar}/\Delta m^2_{atm}$ given in Eq. (4.2), if we consider a case $R > 1$, we obtain $R = 1.6$ from the relation (4.2) and the experimental value (4.5), so that we obtain the sterile neutrino masses

$$m(\nu_{3R}) \simeq m(\nu_{2R}) \simeq 1.11 \text{ eV}, \quad m(\nu_{1R}) \simeq 0.026 \text{ eV}. \tag{4.11}$$

In the present scenario, there are three light sterile neutrinos ν_{iR} $(i = 1, 2, 3)$. However, those neutrinos do not spoil the big bang nucleosynthesis (BBN) scenario, which puts the following constraint [17] for a mixing between the active neutrino ν_α $(\alpha = e, \mu, \tau)$ and a sterile neutrino ν_s,

$$(\sin^2 2\theta_{\alpha s})^2 \Delta m^2_{\alpha s} < 3.6 \times 10^{-4} \text{ eV}^2. \tag{4.12}$$

Even for the case $R = 1.6$, the value of $(\sin^2 2\theta)^2 \Delta m^2$ in our model is less than 10^{-4} eV^2, because the mixing angle θ in the present model is sufficiently small, i.e., $(\sin^2 2\theta)^2 = (6.9 \times 10^{-3})^2 = 4.8 \times 10^{-5}$.

However, we must note that there is a more severe constraint on the neutrino masses from the cosmic structure formation in a low-matter-density universe [18]

$$N_\nu m_\nu < 1.8 \text{ eV} \quad (1.5 \text{ eV}), \tag{4.13}$$

for flat universe (for open universes), where N_ν is the number of almost degenerate neutrinos with the highest mass. The case $R^2 \ll 1$ gives $N_\nu m_\nu \simeq 1.4$ eV, so that the case satisfies the constraint (4.13), while the case $R \simeq 1.6$ dose not satisfies the constraint (4.13), because the case gives $N_\nu m_\nu \simeq 3.6$ eV. Therefore, if the constrain (3.13) is taken stringently, the case $R \simeq 1.6$ is ruled out.

5 Conclusion and discussion

In conclusion, we have investigated a neutrino mass matrix in the framework of the "democratic" universal seesaw model. Although the model has three light sterile neutrinos ν_{iR} $(i = 1, 2, 3)$, they do not spoil the BBN scenario, because the mixing angle θ between the active and sterile neutrinos is taken as $\sin^2 2\theta \simeq 7 \times 10^{-3}$. The atmospheric, solar and LSND neutrino data are explained by the mixings $\nu_{\mu L} \leftrightarrow \nu_{\tau L}$, $\nu_{eL} \leftrightarrow \nu_{eR}$ and $\nu_{eL} \leftrightarrow \nu_{\mu L}$, respectively. The model with the parameter $b_\nu = -1/2$ gives the predictions in terms of the charged lepton masses,

$$\frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} \simeq \frac{m_e}{\sqrt{m_\mu m_\tau}} \ , \quad \frac{\Delta m^2_{LSND}}{\Delta m^2_{atm}} \simeq \frac{1}{4}\sqrt{\frac{m_\mu}{m_e}} \ , \tag{5.1}$$

$$\sin^2 2\theta_{atm} \simeq 1 \ , \quad \sin^2 2\theta_{LSND} \simeq 4\frac{m_e}{m_\mu} \ , \tag{5.2}$$

which are well satisfied by the observed data.

In the present scenario, the following intermediate energy scales have been considered: The neutral leptons N_L and N_R acquire large Majorana masses M_R and M_L at $\mu = \Lambda_{NL} = m_0\lambda_R$ and $\mu = \Lambda_{NR} = m_0\lambda_L$, respectively. The fermions N and F $(F = U, D, E)$ acquire large Dirac masses M_D and M_F at $\mu = \Lambda_D = m_0\lambda_D$ and $\mu = \Lambda_F = m_0\lambda_F$, respectively. The gauge symmetries SU(2)$_R$ and SU(2)$_L$ are broken at $\mu = \Lambda_R = m_0\kappa$ and $\mu = \Lambda_L = m_0$, respectively. We have known that

$$\frac{\Lambda_R}{\Lambda_F} = \frac{\kappa}{\lambda_F} \sim 10^{-2} \tag{5.3}$$

from the study of the quark mass spectrum [2]. Then, let us see what constraints the present neutrino mass matrix phenomenology put on these intermediate energy scales.

From Eqs. (3.9), (3.10) and (3.11), the values of $\xi_L$and R are given by

$$\xi_L = \frac{(x-1)\tan 2\theta}{\lambda_D/\kappa} \frac{x + 1 + (x-1)\sqrt{1+\tan^2 2\theta}}{4x - (x-1)^2 \tan^2 2\theta}, \tag{5.4}$$

$$R = \frac{\xi_R}{\xi_L} = \frac{x+1-(x-1)\sqrt{1+\tan^2 2\theta}}{x+1+(x-1)\sqrt{1+\tan^2 2\theta}}, \tag{5.5}$$

where $x = \lambda_R/\kappa^2\lambda_L$. In the case $R^2 \ll 1$, for example, the bound $R^2 \leq 0.1$ corresponds to $x \geq 3$, which gives $\xi_L \geq 0.08\,\kappa/\lambda_D$. The constraint $\xi_L \geq 0.08\kappa/\lambda_D$ with $\xi_L m_0 = m(\nu_{3L})/\rho_3 \simeq 3$ eV and $m_0 \sim 10^2$ GeV put a constraint on the value of κ/λ_D

$$\kappa/\lambda_D \leq 10^{-9}. \tag{5.6}$$

Therefore, as seen in the conditions (5.3) and (5.6), we cannot take the idea that Dirac masses M_D and M_F are generated at the same energy scale $\Lambda_D = \Lambda_F$. It is likely that the energy scale Λ_D is smaller than the Planck mass $M_P \sim 10^{19}$ GeV, so that we obtain a constraint on the parameter κ

$$\kappa < 10^8. \tag{5.7}$$

On the other hand, the condition $R^2 \leq 0.1$ means $x \geq 3$, i.e.,

$$\lambda_R \geq 3\kappa^2\lambda_L\ . \tag{5.8}$$

If we consider $\Lambda_{NL} \leq M_P$ and $\Lambda_{NR} \geq \Lambda_F$, then we obtain

$$10^2\kappa \leq \lambda_L \leq 10^{16}\kappa^{-2}, \tag{5.9}$$

so that we obtain a more severe constraint on κ, $\kappa < 10^5$.

Note that in the conventional universal seesaw model, the neutrino masses are of the order of $\Lambda_L^2/\Lambda_{NR} = m_0/\lambda_L$, because of $M(\nu_L) \simeq m_L M_L^{-1} m_L^T$, so that we consider $\lambda_L \sim 10^9$. In contrast with the conventional model, in the present model, the neutrino masses are of the order of $m_0\xi_L \sim (\kappa/\lambda_K)x\tan 2\theta$ [for $(\tan^2 2\theta)^{-1} \gg x \gg 1$]. Therefore, for example, the conclusion on the intermediate energy scales based on the SO(10)$_L\times$ SO(10)$_R$ model in Ref. [19] is not applicable to the present model, because in Ref. [19] the solutions have been investigated under the condition $\lambda_L \sim 10^9$. It is a future task to seek for a unification model which satisfies these constraints on the intermediate energy scales, (5.3) and (5.6)-(5.9).

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