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Parameter-Independent Quark Mass Relation in the U(3)×U(3)′ Model

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Abstract

Recently, we have proposed a quark mass matrix model based on U(3)×U(3)′ family symmetry, in which up- and down-quark mass matrices M_u and M_d are described only by complex parameters a_u and a_d, respectively. When we use charged lepton masses as additional input values, we can successfully obtain predictions for quark masses and Cabibbo-Kobayashi-Maskawa mixing. Since we have only one complex parameter a_q for each mass matrix M_q, we can obtain a parameter-independent mass relation by using three equations for $\mathrm{Tr}[H_q]$, $\mathrm{Tr}[H_q H_q]$ and $\det H_q$, where $H_q \equiv M_q M_q^\dagger$ $(q = u, d)$. In this paper, we investigate its parameter-independent feature of the quark mass relation in the model.

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1 Introduction

Recently, we have proposed a quark mass matrix model[1] based on U(3)×U(3)′ symmetry, in which mass matrices for up, down quarks, charged leptons, and neutrinos, M_f $(f = u, d, e, \nu)$, are described respectively with only one complex parameters a_f by

$$(M_f)_i^{\ j} = m_{0f} (\Phi_f)_i^{\ \alpha} (S_f^{-1})_\alpha^{\ \beta} (\bar{\Phi}_f)_\beta^{\ j}. \tag{1.1}$$

Here Φ_f and S_f are vacuum expectation values (VEVs) matrices. The $i, j = 1, 2, 3$ are indexes of U(3) family and $\alpha, \beta = 1, 2, 3$ are indexes of U(3)′ family. Although Φ_f and S_f have a dimension of "mass", we put the factor m_{0f} with a dimension of mass in Eq.(1.1)., since we treat those as dimensionless quantities as seen in (1.2), (1.3) and (1.5) later.

In (1.1), M_ν is a Dirac neutrino mass matrix. Although, we consider that the observed neutrinos are Majorana neutrinos and the Majorana neutrino mass matrix is given by a similar mechanism [1, **?**] to the so-called neutrino seesaw mechanism [**?**], we do not discuss the structure of M_ν in the present paper, because the purpose of the present paper is to discuss the quark mass relation.

We define structure of the matrix Φ_f as an dimensionless expression

$$\Phi_f = \Phi_0 P_f, \tag{1.2}$$

where

$$\Phi_0 = \mathrm{diag}(z_1, z_2, z_3), \tag{1.3}$$

$$P_f = \mathrm{diag}(e^{i\phi_{f1}}, e^{i\phi_{f2}}, e^{i\phi_{f3}}). \tag{1.4}$$

Since we consider that the U(3)$'$ is broken into a discrete symmetry S$_3$, the matrix (S_f^{-1}) is given by

$$(S_f)^{-1} = (\mathbf{1} + a_f X) = (\mathbf{1} + b_f X)^{-1}, \tag{1.5}$$

where

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{1.6}$$

and a_f is a complex parameter:

$$a_f = -\frac{b_f}{1+b_f}. \tag{1.7}$$

Only for the charged lepton mass matrix M_e, the parameter a_e is given by $a_e = 0$, so that the mass matrix M_e is given by

$$M_e = m_{0e}\Phi_e\Phi_e^\dagger = m_{0e}\Phi_0\Phi_0, \tag{1.8}$$

where we can put $P_e = \mathbf{1}$ without losing a generality. Also, we take $S_e = \mathbf{1}$ only for $f = e$. Therefore, the parameters z_i ($i = 1, 2, 3$) are given by

$$z_i = \sqrt{\frac{m_{e_i}}{m_{e1} + m_{e2} + m_{e3}}}, \tag{1.9}$$

where $(m_{e1}, m_{e2}, m_{e3}) = (m_e, m_\mu, m_\tau)$. Here, as the input values (m_e, m_μ, m_τ), the running mass values at a scale $\mu = m_Z$, $(m_e, m_\mu . m_\tau) = (0.000486849 \text{ GeV}, 0.102751 \text{ GeV}, 1.7467 \text{ GeV})$, are used, not the pole masses, because the predicted quark mass values are calculated at the scale $\mu = m_Z$. (The study of the quark mass matrix (1.1) with the form (1.5) have been substantially done in Ref.[3] although the model has been based on U(3)-family symmetry, not U(3)$\times$U(3)$'$.)

In this model, when we choose suitable values of the complex parameters a_q ($q = u, d$) together with additional input values, (m_e, m_μ, m_τ), we can successfully obtain [1] predictions for quark masses and Cabibbo-Kobayashi-Maskawa (CKM)mixing [2]. However, so far, it is not clear whether the successful parameter fitting is unique or not, and that there are another good parameter solutions or not.

In order to settle these questions, parameter-independent mass relations are usefull, which can be obtained in this model ; We have three independent equations for the each mass matrix M_q ($q = u, d$), while we have only one complex parameter a_q, therefore we can obtain one mass relation. In this paper, we investigate such the parameter-independent mass relation in the U(3)×U(3)′ model. At present, the observed quark mass values, especially, for the first generation quarks have considerably large error, i.e. $m_u = 1.27^{+0.50}_{-0.42}$ MeV and $m_d = 2.90^{+1.24}_{-1.19}$ MeV at $\mu = m_Z$ [5]. By obtaining such a parameter-independent quark mass relation, we can check whether the U(3)×U(3)′ model is reasonable or not and what values of m_u and m_d are acceptable to the U(3)×U(3)′ model .

2 Brief review of the U(3)×U(3)′ model

In our model based on U(3)×U(3)′ symmetry, we consider hypothetical fermions F_α ($\alpha = 1, 2, 3$), which belong to $(\mathbf{1}, \mathbf{3})$ of U(3)×U(3)′, in addition to quarks and leptons f_i ($i = 1, 2, 3$) which belong to $(\mathbf{3}, \mathbf{1})$.

We assume that the VEV form (1.1) originates from the following 6×6 mass matrix model:

$$(\bar{f}_L^i \ \ \bar{F}_L^\alpha) \left(\begin{array}{cc} (0)_i^{\ j} & (\Phi_f)_i^{\ \beta} \\ (\bar{\Phi}_f)_\alpha^{\ j} & -(S_f)_\alpha^{\ \beta} \end{array} \right) \left(\begin{array}{c} f_{Rj} \\ F_{R\beta} \end{array} \right). \tag{2.1}$$

Here, $F_{L(R)}$ are heavy fermions with $(\mathbf{1}, \mathbf{1}, \mathbf{3})$ of SU(2)$_L$×U(3)×U(3)′. On the other hand, f_R are right-handed quarks and leptons, $f_R = (u, d, \nu, e^-)_R$, while f_L are not physical fields. They are given by the following combinations:

$$f_L \equiv (f_u, f_d, f_\nu, f_e)_L \equiv \left(\frac{1}{\Lambda_H} H_u^\dagger q_L, \frac{1}{\Lambda_H} H_d^\dagger q_L, \frac{1}{\Lambda_H} H_u^\dagger \ell_L, \frac{1}{\Lambda_H} H_d^\dagger \ell_L \right) \tag{2.2}$$

where

$$q_L = \left(\begin{array}{c} u_L \\ d_L \end{array} \right), \quad \ell_L = \left(\begin{array}{c} \nu_L \\ e_L^- \end{array} \right), \quad H_u = \left(\begin{array}{c} H_u^0 \\ H_u^- \end{array} \right), \quad H_d = \left(\begin{array}{c} H_d^+ \\ H_d^0 \end{array} \right). \tag{2.3}$$

In other words, the matrix given in Eq.(2.1) denotes would-be Yukawa coupling constants.

After the U(3) and U(3)′ have been completely broken, the quarks and leptons are described by the effective Hamiltonian

$$\mathcal{H}_Y = (\bar{\nu}_L)^i (M_\nu)_i^{\ j} (\nu_R)_j + (\bar{e}_L^i (M_e)_i^{\ j} (e_R)_j + y_R (\bar{\nu}_R)^i (Y_R)_{ij} (\nu_R^c)^j$$
$$+ (\bar{u}_L)^i (M_u)_i^{\ j} (u_R)_j + (\bar{d}_L)^i (M_d)_i^{\ j} (d_R)_j. \tag{2.4}$$

Note that the quarks and leptons f_i are not U(3) family triplet any more in the exact meaning, but they are mixing states between the fermions f and F. However, for convenience, we will still use the index of U(3) family for these fermion states.

By performing a seesaw-like approximation with $\Lambda_2 = O(\langle\Phi_f\rangle) \ll \Lambda_1 = O(\langle S_f\rangle)$, the mass matrix (2.1) leads to the following Dirac mass matrices of quarks and leptons:

$$(M_f)_i^{\ j} \simeq \frac{\langle H_{u/d}\rangle}{\Lambda_H} \langle\Phi_f\rangle_i^{\ \alpha} \langle (S_f)^{-1}\rangle_\alpha^{\ \beta} \langle\bar{\Phi}_f\rangle_\beta^{\ j}. \tag{2.5}$$

However, in this paper, since we interest only in the relative ratios of the quark masses in the same sector ($q = u$ or $q = d$), the factor $\langle H_{u/d}\rangle/\Lambda_H$ takes a common value, so that the factor $\langle H_{u/d}\rangle/\Lambda_H$ do not play any essential role in our study.

As seen in this section, Φ_f and S_f have a dimension of mass. However, for convenience, hereafter, we use a dimensionless expressions (1.2) and (1.5), and define the parameter m_{0f} with a mass dimension by Eq.(1.1).

3 Three equations for mass relations

When we define

$$H_q = M_q M_q^\dagger, \tag{3.1}$$

the explicit form of H_q is given by

$$\begin{aligned} H_q = M_q M_q^\dagger &= m_{0q}^2 \Phi_q S_q^{-1} \Phi_q^\dagger \Phi_q (S_q^\dagger)^{-1} \Phi_q^\dagger \\ &= k_q^{\ 2} P_q D_e^{1/2} (\mathbf{1} + a_q X) D_e (\mathbf{1} + a_q X) D_e^{1/2} P_q^{\ \dagger}, \end{aligned} \tag{3.2}$$

where

$$k_q = \frac{m_{0q}}{m_{0e}}. \tag{3.3}$$

The Hermitian matrix H_q is diagonalized as

$$U_q H_q U_q^\dagger = D_q^{\ 2} \equiv \mathrm{diag}(m_{q1}^2, m_{q2}^2, m_{q3}^2). \tag{3.4}$$

Hereafter, for convenience, we define

$$(m_1, m_2, m_3) = \frac{1}{k_q}(m_{q1}, m_{q2}, m_{q3}), \tag{3.5}$$

and

$$\tilde{D}_q \equiv \mathrm{diag}(m_1, m_2, m_3) = \frac{1}{k_q} D_q. \tag{3.6}$$

In general, we have the following three equations for the matrix H_q:

$$c_1 \equiv {m_1}^2 + {m_2}^2 + {m_3}^2 = \mathrm{Tr}[{\tilde{D}_q}^2] = \frac{1}{{k_q}^2}\mathrm{Tr}[H_q], \tag{3.7}$$

$$\begin{aligned} c_2 \equiv {m_1}^2{m_2}^2 + {m_2}^2{m_3}^2 + {m_3}^2{m_1}^2 &= \frac{1}{2}\left(\mathrm{Tr}[{\tilde{D}_q}^2{\tilde{D}_q}^2] - (\mathrm{Tr}[{\tilde{D}_q}^2])^2\right) \\ &= \frac{1}{{k_q}^4}\frac{1}{2}\left\{\mathrm{Tr}[H_qH_q] - (\mathrm{Tr}[H_q])^2\right\} \end{aligned} \tag{3.8}$$

and

$$c_3 \equiv {m_1}^2{m_2}^2{m_3}^2 = \mathrm{det}{\tilde{D}_q}^2 = \frac{1}{{k_q}^6}\mathrm{det}H_q. \tag{3.9}$$

For convenience, hereafter, we denote $\mathrm{Tr}[A]$ as $[A]$ simply. By using the explicit form (3.2), we obtain c_1, c_2 and c_3 as follows:

$$c_1 = \left|1 + \frac{1}{3}a_q\right|^2 [D_e^2] + \frac{1}{9}|a_q|^2([D_e]^2 - [D_e^2]), \tag{3.10}$$

$$c_2 = \frac{1}{2}\left|1 + \frac{2}{3}a_q\right|^2 ([D_e^2]^2 - [D_e^4]) + \frac{2}{9}|a_q|^2[D_e]\mathrm{det}D_e, \tag{3.11}$$

$$c_3 = |1 + a_q|^2(\mathrm{det}D_e)^2 = |1 + a_q|^2\mathrm{det}D_e^2. \tag{3.12}$$

Let us define

$$Q_1 \equiv \frac{[D_q^2]}{[D_e^2]}, \quad Q_2 \equiv \frac{[D_q^2]^2 - [D_q^4]}{[D_e^2]^2 - [D_e^4]}, \quad Q_3 \equiv \frac{\det D_q^2}{\det D_e^2}, \tag{3.13}$$

and

$$L_1 \equiv \frac{[D_e]^2 - [{D_e}^2]}{[{D_e}^2]}, \qquad L_2 \equiv \frac{[D_e]\det D_e}{[{D_e}^2]^2 - [{D_e}^4]}. \tag{3.14}$$

then, we obtain the following relations:

$$Q_1 = \left|1 + \frac{1}{3}a_q\right|^2 + \frac{1}{9}|a_q|^2 L_1, \tag{3.15}$$

$$Q_2 = \left|1 + \frac{2}{3}a_q\right|^2 + \frac{4}{9}|a_q|^2 L_2, \tag{3.16}$$

$$Q_3 = |1 + a_q|^2. \tag{3.17}$$

Note that L_1 and L_2 are given only by the charged lepton masses, and Q_1, Q_2 and Q_3 are expressed by quark masses (m_1, m_2, m_3) after (m_e, m_μ, m_τ) are substituted.

Finally, by eliminating the parameter a_q from Eqs.(3.15) -(3.17), we obtain the mass relation

$$-b_0 + b_1({m_1}^2 + {m_2}^2 + {m_3}^2) - b_2({m_1}^2{m_2}^2 + {m_2}^2{m_3}^3 + {m_3}^2{m_1}^2) + b_3\, {m_1}^2{m_2}^2{m_3}^2 = 0. \tag{3.18}$$

Here the coeffients b_0, b_1, b_2, and b_3 are defined by

$$b_0 = \left(1 + \frac{1}{2}L_1 - 4L_2\right), \tag{3.19}$$

$$b_1 = 3(1 - 2L_2)\frac{1}{[{D_e}^2]}, \tag{3.20}$$

$$b_2 = 6\left(1 + \frac{1}{2}L_1\right)\frac{1}{[{D_e}^2] - [{D_e}^4]}, \tag{3.21}$$

$$b_3 = (1 - L_1 + 2L_2)\frac{1}{\det {D_e}^2}, \tag{3.20}$$

which are expressed only by the charged lepton masses.

4 Behavior of m_1/m_2 versus m_2/m_3

In order to investigate the behavior of m_1/m_2 versus m_2/m_3, we define parameters

$$x \equiv \frac{m_2}{m_3} = \frac{m_{q2}}{m_{q3}}, \qquad y \equiv \frac{m_1}{m_2} = \frac{m_{q1}}{m_{q2}}. \tag{4.1}$$

Note that the parameters x and y are independent of the value of k_q defined in (3.3). Then, since (m_1, m_2, m_3) are expressed as

$$(m_1, m_2, m_3) = (xy, x, 1)m_3, \tag{4.2}$$

the relation (3.18) becomes

$$-b_0 + b_1{m_3}^2(1 + x^2 + x^2y^2) - b_2{m_3}^4(x^2 + x^2y^2 + x^4y^2)x + b_3\, {m_3}^6 x^4y^2. \tag{4.3}$$

Therefore, we get a relation $y = f(x)$:

$$y = \frac{1}{x}\sqrt{\frac{(b_1{m_3}^2 - b_2{m_3}^4)x^2 - (b_0 - b_1{m_3}^2)}{(b_2{m_3}^4 - b_3{m_3}^6)x^2 - (b_1{m_3}^2 - b_2{m_3}^4)}}. \tag{4.4}$$

As seen in (4.4), the function $y = f(x)$ has poles at

$$x = 0, \quad \text{and} \quad x = \pm\sqrt{\frac{b_1 {m_3}^2 - b_2 {m_3}^4}{b_2 {m_3}^2 - b_3 {m_3}^6}}, \tag{4.5}$$

and a zero point at

$$x = \pm\sqrt{\frac{b_0 - b_1 {m_3}^2}{b_1 {m_3}^2 - b_2 {m_3}^4}}. \tag{4.6}$$

The explicit values b_0, b_1, b_2 and b_3 are given by

$$\begin{aligned} b_0 &= 1.04888, \\ b_1 &= 0.97489 \ (\text{GeV})^{-2}, \\ b_2 &= 87.6457 \ (\text{GeV})^{-4}, \\ b_3 &= 1.16204 \times 10^8 \ (\text{GeV})^{-6}. \end{aligned} \tag{4.7}$$

The behavior of $y = f(x)$ is illustrated in Fig.1. The behavior depends on the input value of m_3. Note that since

$$\frac{b_0}{b_1} = 1.0758 \ (\text{GeV})^2, \tag{4.8}$$

the factor $(b_0 - b_1 {m_3}^2)$ in (4.4) changes the sign according as $m_3 > m_{30}$ or $m_3 < m_{30}$, where

$$m_{30} \equiv \sqrt{\frac{b_0}{b_1}} = 1.0373 \ \text{GeV}. \tag{4.9}$$

Hereafter, we call the behavior in the case $m_3 > m_{30}$ as normal type, and the behavior in the case $m_3 < m_{30}$ as non-normal type.

Now, let us compare our parameter-independent results with the observed quark mass values in detail. The observed quark mass values at at $\mu = m_Z$ [5] are as follows:

$$\begin{array}{lll} m_u = 0.00127^{+0.00050}_{-0.00042} \ \text{GeV}, & m_c = 0.619 \pm 0.084 \ \text{GeV}, & m_t = 171.7 \pm 3.0 \ \text{GeV}, \\ m_d = 0.00290^{+0.00124}_{-0.00119} \ \text{GeV}, & m_s = 0.055^{+0.016}_{-0.015} \ \text{GeV}, & m_b = 2.89 \pm 0.09 \ \text{GeV}. \end{array} \tag{4.10}$$

Hereafter, we use the following values as the mass ratios m_1/m_2 and m_2/m_3:

$$\frac{m_d}{m_s} = 0.0527^{+0.0508}_{-0.0286}, \qquad \frac{m_s}{m_b} = 0.0190^{+0.00063}_{-0.00056}, \tag{4.11}$$

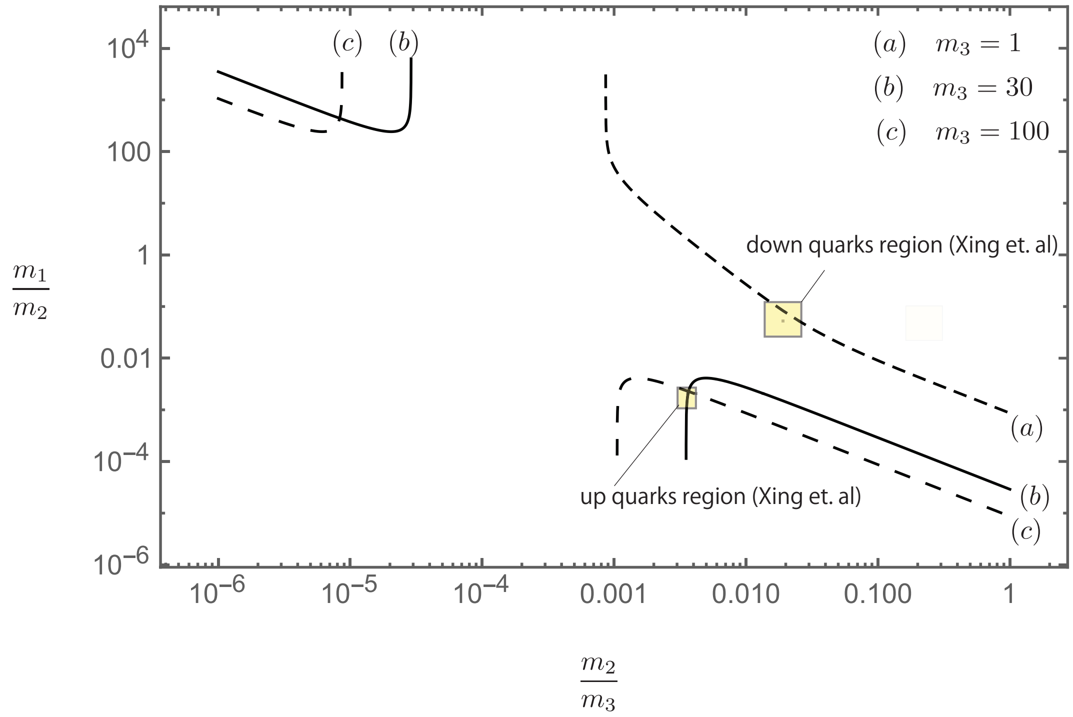

Figure 1: m_3 dependence of the behavior of m_1/m_2 versus m_2/m_3. The curves of the mass relation $y = f(x)$ given in Eq.(4.4) are drown in the $(m_2/m_3, m_1/m_2)$ plane for the cases (a) $m_3 = 1$ GeV, (b) $m_3 = 30$ GeV, and (c) $m_3 = 100$ GeV. The shaded square regions are correspond to the observed mass ratios in (4.11) and (4.12) for up quarks sector and down quarks sector respectively obtained by Xing et. al.

$$\frac{m_u}{m_c} = 0.00205^{+0.00126}_{-0.00084}, \qquad \frac{m_c}{m_t} = 0.00361^{+0.00056}_{-0.00054}. \tag{4.12}$$

As seen in Fig.1, the behavior of m_1/m_2 in the normal type has a maximum whose value is smaller than $\sim 10^{-2}$. On the other hand, as seen in Eq.(4.11), the observe value of m_d/m_s is $m_d/m_s \simeq 0.05$. Therefore, the mass ratios for down-quark sector cannot be described by the behavior of the normal type. Thus we have the solution for Eqs.(4.11) and (4.12) by the behavior of the normal type for the up-quark sector, and by the behavior of the non-normal type for the down-quark sector.

Down-quark sector

First, let us see behaviors of the mass ratios $(m_1/m_2, m_2/m_3)$ in the down-quark sector.

As seen in Fig.2, we can determine a value m_3 from the observed center values in (4.11) as $m_3 = 1.03$ GeV. If we take $m_3 > 1.04$ GeV, the behavior of the mass ratios becomes the normal type from non-normal type as seen in the curve (c) of Fig.2. Furthermore, if we take a larger

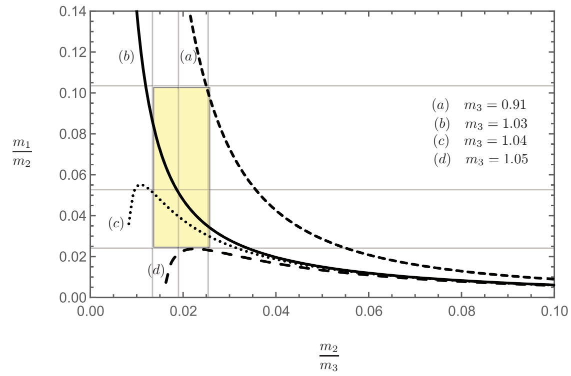

Figure 2: The m_3 value in the mass relation consistent with the observed values m_d/m_s and m_s/m_b in the down quarks sector. The curves of the mass relation $y = f(x)$ in Eq.(4.4) are drown in the $(m_2/m_3, m_1/m_2)$ plane for the cases with (a) $m_3 = 0.91$ GeV, (b) $m_3 = 1.03$ GeV, (c) $m_3 = 1.04$ GeV, and (d) $m_3 = 1.05$ GeV. The shaded square region is correspond to the observed mass ratios in (4.11) for down quarks sector obtained by Xing et. al.

value $m_3 > 1.05$, then the curve is out of the error vregion as seen in the curve (d) of Fig.2. Similarly, there is no solution of m_3 for $m_3 < 091$ GeV as seen in the curve (a) of Fig.2. Thus we obtain

$$m_3 = 1.03^{+0.02}_{-0.12} \text{ GeV}, \tag{4.13}$$

from the consistency between and the mass relation (4.4) and the observed mass ratios (4.11).

If we take $m_3 = 1.03$,GeV we obtain

$$k_d = \frac{m_b}{m_3} = 2.89, \tag{4.14}$$

from the definition (3.5). In this choice of the value of k_d, we have

$$m_b = 2.98^{+0.06}_{-0.35} \text{ GeV}, \tag{4.15}$$

which has smaller errar bar for upper limit than that of observed m_b in (4.10).

Up-quark sector

In order to get a reasonable value of m_3 in the mass relation, we illustrate curves of the mass relation for several values of m_3 in Fig. 3 and Fig 4. We find that there are two solutions of the m_3 which are consistent with the observed up-quark mass ratios (4.12) as seen in Figs.3 and 4:

$$m_3 = 30.5^{+8.5}_{-4.8} \text{ GeV}, \qquad m_3 = 114^{+114}_{-\ 57} \text{ GeV}. \tag{4.16}$$

Both solutions can give reasonable quark mass ratios $(m_u/m_c, m_c/m_t) \simeq (2.4, 3.6) \times 10^{-3}$. However, those two center values $m_3 = 30.5$ GeV and $m_3 = 114$ GeV give

$$k_u = 5.63 \quad \text{and} \quad k_u = 1.51, \tag{4.17}$$

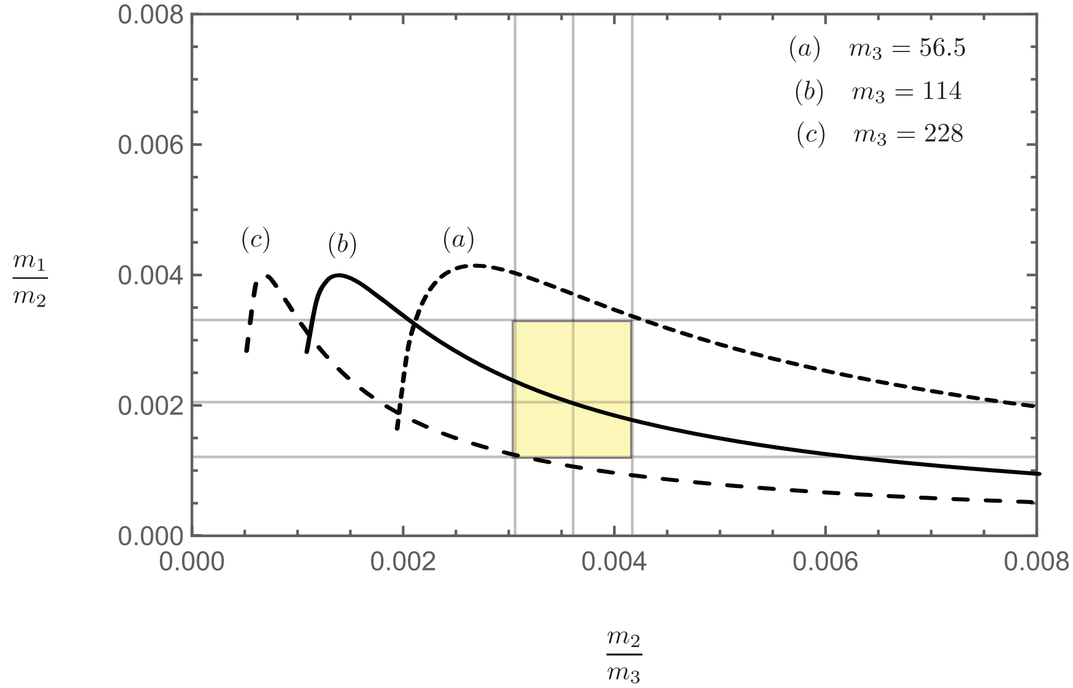

Figure 3: The m_3 value in the mass relation consistent with the observed values m_u/m_c and m_c/m_t in the up quarks sector. The curves of the mass relation $y = f(x)$ in Eq.(4.4) are drown in the $(m_2/m_3, m_1/m_2)$ plane for the cases with (a) $m_3 = 56.5$ GeV, (b) $m_3 = 114$ GeV, and (c) $m_3 = 228$ GeV. The shaded square region is correspond to the observed mass ratios in (4.12) for up quarks sector obtained by Xing et. al.

respectively. On the other hand, we have obtained $k_d = 2.9$ in the down-quark sector as seen in Eq.(4.14). Therefore, whichever we take the value in (4.17), the value is poor agreement with $k_d = 3$. It is natural to consider that the relations of quark mass matrices M_u and M_d to the charged lepton mass matrixM_e take the same weight between the up- and down-quark sectors, i.e. $k_u = k_d$, except for the parameters a_u anda_d. If we want to consider $k_u = k_d = 3$, we have to choose $m_3 = 57$ GeV from $m_t^{obs} = 172$ GeV. Only when we chose the lowest value $m_3 = 57$ GeV in the solution $m_3 = 114^{+114}_{-\ 57}$ GeV, the value can give $k_u = 3$, so that we can realize the relation $k_u = k_d = 3$. (If we require $k_u = k_d = 2.98$, the case leads to $m_3 = 59.4$ GeV, which is within the lower limit value $m_3 = 57$ GeV.)

First generation quark masses

Next, we see the constraints on the first generation quark masses m_u and m_d. From the curves (a) $m_3 = 0.91$ GeV, (b) $m_3 = 1.03$ GeV and (c) $m_3 = 1.04$ GeV in Fig.2, we obtain

$$m_d = 2.9^{+3.7}_{-1.8} \text{ MeV}, \tag{4.18}$$

where we have used the input value [5] $(m_s)^{obs} = 55$ MeV. Our result (4.18) has wide error

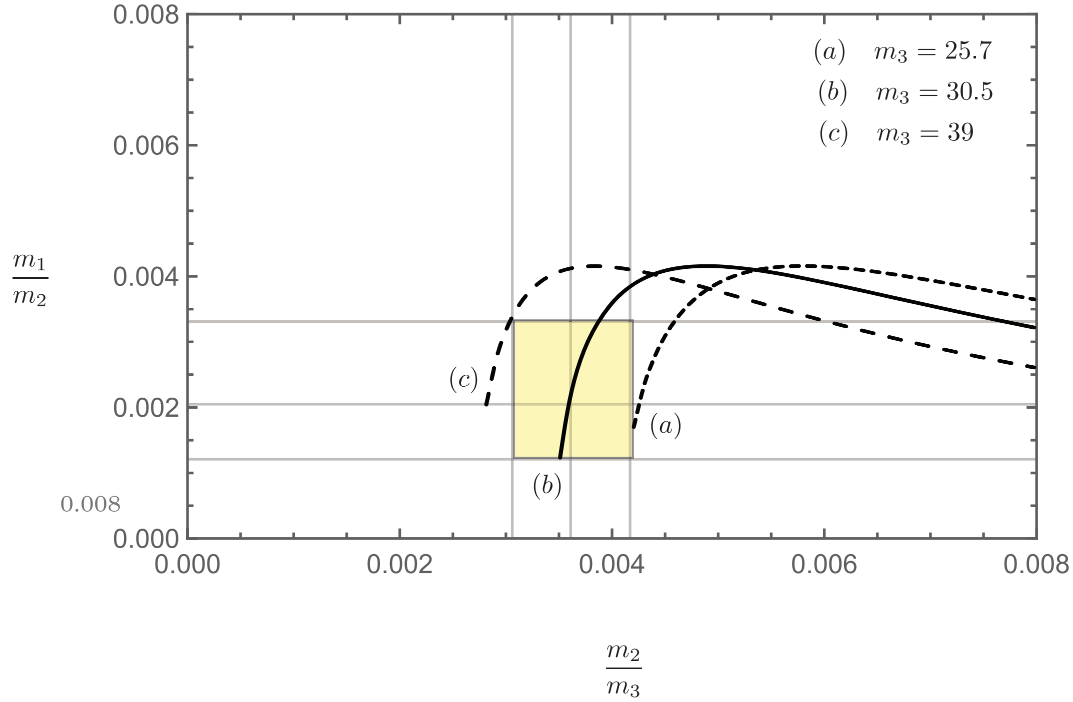

Figure 4: Another m_3 value in the mass relation consistent with the observed values m_u/m_c and m_c/m_t in the up quarks sector. The curves of the mass relation $y = f(x)$ in Eq.(4.4) are drown in the $(m_2/m_3, m_1/m_2)$ plane for the cases with (a) $m_3 = 25.7$ GeV, (b) $m_3 = 30.5$ GeV, and (c) $m_3 = 39$ GeV. The shaded squre region is correspond to the observed mass ratios in (4.12) for up quarks sector obtained by Xing et. al.

compared with the observed value [5] $(m_s)^{obs} = 2.90^{+1.24}_{-1.19}$ MeV. Similarly, from Figs.3 and 4, we obtain the following two solutions of m_u,

$$m_u = 1.49^{+0.45}_{-0.75}\ \text{MeV}, \quad \text{and} \quad m_u = 1.3^{+0.7}_{-\infty}\ \text{MeV}, \tag{4.19}$$

respectively, where we have taken $(m_c)^{obs} = 0.619$ GeV. We also see that our results (4.19) cannot put any severe constraint on the observed value $(m_u)^{obs} = 1.27^{+0.50}_{-0.42}$ MeV.

5 Concluding remarks

In conclusion, we have investigate a pameter-independent quark mass relation in the U(3)×U(3)′ model. Considering our results with the observed quark mass values [5], we conclude that the choice $k_u = k_d = 3$ in the previous work [1] of the explicit parameter fitting of a_u and a_d was reasonable. However, we have found that there are two solutions in the up-quark sector as we have shown in Figs.3 and 4. This is not so serious problem when we take the error range of the observed quark mass values into consideration.

We are convinced that our parameter-independent analysis is useful for model checking in future study of the U(3)×U(3)′ model.

We did not investigate a similar parameter-indepedent study for the CKM mixing. The similar study of the CKM matrix elements V_{ij} cannot been obtained unless the results include quark masses. We are interested in the values V_{us}, V_{cb}, V_{td} and so on, while those values will be disturbed by existence of large elements V_{us}, V_{cs}, V_{tb} and so on. It is our future task to obtain a relation without such large contribution terms.

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