

# Sumino Model and My Personal View<sup>1</sup>.

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## Abstract

There are two formulas for charged lepton mass relation: One is a formula (formula A) which was proposed based on a U(3) family model on 1982. The formula A will be satisfied only masses switched off all interactions except for U(3) family interactions. Other one (formula B) is an empirical formula which we have recognized after a report of the precise measurement of tau lepton mass, 1992. The formula B is excellently satisfied by pole masses of the charged leptons. However, this excellent agreement may be an accidental coincidence. Nevertheless, 2009, Sumino has paid attention to the formula B. He has proposed a family gauge boson model and thereby he has tried to understand why the formula B is so well satisfied with pole masses. In this talk, the following views are given: (i) What direction of flavor physics research is suggested by the formula A; (ii) How the Sumino model is misunderstood by people and what we should learn from his model; (iii) What is strategy of my recent work, U(3)×U(3)' model.

## 1 Two formulas for charged lepton masses

Prior to discussing the Sumino model [1], let us review a charged lepton mass relation, We know two formulas for the charged lepton masses. One is a formula (formula A) which was proposed based on a U(3) family model on 1982 [2]:

$$K(m_{ei}) \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}. \quad (1)$$

The formula A will be satisfied only masses which are given in the world switched off all interactions except for the U(3) family interactions. Other one (formula B) is an empirical formula which we have recognized since precise observation of tau lepton mass [3], 1992:

$$K(m_{ei}^{pole}) = \frac{2}{3} \times (0.999989 \pm 0.000014). \quad (2)$$

The formula B is excellently satisfied with pole masses of the charged leptons. However, this excellent agreement may be an accidental coincidence.

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I regret that some people simple-honestly tried to search mathematical quantities which leads to the form  $K = 2/3$ . They did not understand that  $m_{ei}$  in the formula B are pole masses, and besides, that the mass spectra cannot discuss independently of the flavor mixing. Most of their attempts could not left any physical result. Their attempts are nothing but playing of a mathematical puzzle, not physics.

Independently of whether the formula A can give well numerical agreement or not, if we accept the formula A, we will also accept the following points of view:

(i) We know the quark mixing and neutrino mixing. Therefore, the formula A holds only in the charged lepton sector, so that a similar relation never hold in other sectors (up-quark, down-quark and neutrino sectors). In other words, we should discuss flavor physics on the diagonal bases of the charged lepton mass matrix  $M_e$ .

(ii) Masses and mixings should be investigated based on  $M_e^{1/2}$ , not  $M_e$ .

(iii) The observed hierarchical mass spectra in quarks and leptons suggest that those cannot be understood from a conventional symmetry approach (symmetry + a small breaking), and it has to be understood from vacuum expectation values (VEVs) of scalars with Higgs-like mechanism [4, 5].

(iv) If we put  $m_e = 0$  in the formula A, we obtain unwelcome ratio of  $m_\mu/m_\tau = 1/(2+\sqrt{3})$ . This suggests that we have to seek for mass matrix model in which the electron mass should be given by a non-zero value from the beginning even if it is very small. The mass spectrum  $(m_e, m_\mu, m_\tau)$  has to be understood simultaneously, that is, without considering a mass generation model with two or three steps.

## 2 Impact of the Sumino model

However, against such my personal view given in the previous section, 2009, Sumino [1] has paid attention to the formula B. He has proposed a family gauge boson model and thereby, he has tried to understand why the formula B is so well satisfied with pole masses.

The formula A is invariant under a transformation

$$m_{ei} \rightarrow m_{ei}(1 + \varepsilon_0), \quad (3)$$

where  $\varepsilon_0$  is a constant which is independent of the family-number. The deviation from the formula A due to QED correction comes from  $\log m_{ei}$ :

$$\delta m_{ei} = m_{ei} \left( 1 + c_1^{QED} \log m_{ei} + c_0^{QED} \right), \quad (4)$$

at the level of the one-loop correction [6]. Therefore, Sumino has assumed an existence of family gauge bosons (FGBs)  $A_i^j$  with the masses  $M_{ij}^2 = k(m_{ei} + m_{ej})$  and thereby, he has proposed a cancellation mechanism between  $\log m_{ei}$  in the QED correction and  $\log M_{ii}$  in the FGB one-loop correction:

$$\delta m_{ei} = m_{ei} \left( c_1^{QED} \log m_{ei} + c_1^{FGB} \log M_{ii} + const \right) = m_{ei} \times const. \quad (5)$$

His model is based on  $U(3) \times O(3)$  symmetry. In his model, in order to obtain a minus sign for the cancellation, the quarks and leptons  $f$  are assigned to  $(f_L, f_R) = (\mathbf{3}, \mathbf{3}^*)$  of  $U(3)$ , so that the model is not anomaly free. Besides, effective interactions with  $\Delta N_{fam} = 2$  ( $N_{fam}$  is a family number) appears. Therefore, in order to remove those shortcomings, an extended Sumino model has been proposed based on  $U(3) \times U(3)'$  symmetry and with an inverted mass hierarchy of FGBs [7]. However, the purpose of this talk is not to review those details.

The big objection is that there are many diagrams which we should take into consideration, so that the Sumino cancellation mechanism cannot work effectively. However, it is misunderstanding for the Sumino mechanism. Sumino has already taken those effects into consideration. The Sumino cancellation mechanism does not mean complete cancellation, but it means practical cancellation at a level of the present experimental accuracy. In fact, Sumino says that if accuracy in the present tau lepton mass measurement can be improved to one order, the deviation from the formula B will be observed. Also, he has said that the upper bound in which the cancellation mechanism is effective is  $10^3$ - $10^4$  TeV. We hope that the soon coming tau mass measurement will verify Sumino's conjecture.

In his model, the masses  $M_{ij}$  are related to the charged lepton masses  $m_{ei}$ , and the family gauge coupling constant  $g_F$  is related to QED gauge coupling constant  $e$ . Therefore, the FGB model has highly predictability.

The most notable point of Sumino FGB model is that there is a upper limit of the FGB mass scale, which comes from applicability of the Sumino mechanism. Therefore, when Sumino FGBs cannot be discovered at the expected scale, we cannot excuse the undiscovered fact by extending the scale to one order. In such a case, we have to abandon the Sumino model.

Even apart from the Sumino cancellation mechanism, his FGB model has many notable characteristics. In his model, the FGB masses  $M_{ij}$  and the charged lepton masses  $m_{ei}$  are generated by the same scalar  $\Phi = (\mathbf{3}, \mathbf{3})$  of  $U(3) \times O(3)$ , so that, when the charged lepton mass matrix  $M_e$  is diagonal, the FGB mass matrix is also diagonal. Therefore, family-number violation does not occur in the lepton sector. Family-number violation appears only in the quark sector only via quark mixing. Therefore, in the limit of zero quark mixing, family-number violation disappears in the quark sector, too. Thus, the Sumino family FGB model offers us FGBs with considerably low scale without constraining the conventional view from the observed  $K^0$ - $\bar{K}^0$  mixing and so on [8]. Now we may expect observations of FGBs in terrestrial experiments. We will have fruitful new physics related to Sumino FGB model.

### 3 Strategy of the $U(3) \times U(3)'$ model

Stimulating by the Sumino model, I have recently investigated a unified description of quarks and leptons based on  $U(3) \times U(3)'$  symmetry [9, 10]. Here, quarks and leptons are assigned to  $(\mathbf{3}, \mathbf{1})$  of  $U(3) \times U(3)'$ . Nevertheless, we need additional symmetry  $U(3)'$  with considerably higher scale. Why? The reason will see an example in the following mass matrix model (although

our interest is not only in masses and mixing).

In order to give a review of the  $U(3) \times U(3)'$  model concretely, let us take a mass matrix model based on the  $U(3) \times U(3)'$  symmetry. In this model, heavy fermions  $F_\alpha$  ( $\alpha = 1, 2, 3$ ) are introduced in addition to quarks and leptons  $f_i$  ( $i = 1, 2, 3$ ).  $F_\alpha$  and  $f_i$  belong to  $(\mathbf{1}, \mathbf{3})$  and  $(\mathbf{3}, \mathbf{1})$  of  $U(3) \times U(3)'$ , respectively. We consider a seesaw-like mass matrix:

$$(\bar{f}_L^i \quad \bar{F}_L^\alpha) \begin{pmatrix} (0)_i^j & (\Phi_f)_i^\beta \\ (\bar{\Phi}_f)_\alpha^j & -(S_f)_\alpha^\beta \end{pmatrix} \begin{pmatrix} f_{Rj} \\ F_{R\beta} \end{pmatrix}. \quad (6)$$

Since we consider that  $U(3)'$  is broken into a discrete symmetry  $S_3$ , a VEV form of  $\hat{S}_f$ , in general, takes a form (unit matrix + democratic matrix):

$$\langle \hat{S}_f \rangle = v_S (\mathbf{1} + b_f X_3), \quad (7)$$

where  $\mathbf{1}$  and  $X_3$  are defined as

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (8)$$

and  $b_f$  are complex parameters. On the other hand, we consider that  $U(3)$  is broken by VEVs of  $\Phi_f$  with  $(\mathbf{3}, \mathbf{3}^*)$  of  $U(3) \times U(3)'$ , not by  $(\mathbf{R}, \mathbf{1})$  of  $U(3) \times U(3)'$ . Here, the VEV forms are diagonal and given by

$$\langle \Phi_f \rangle = v_\Phi \text{diag}(z_1 e^{i\phi_1^f}, z_2 e^{i\phi_2^f}, z_3 e^{i\phi_3^f}). \quad (9)$$

Since we consider  $|\langle \hat{S}_f \rangle| \gg |\langle \Phi_f \rangle|$ , we obtain a seesaw-like Dirac mass matrix for  $f$  [11]

$$(\hat{M}_f)_i^j = \langle \Phi_f \rangle_i^\alpha \langle \hat{S}_f^{-1} \rangle_\alpha^\beta \langle \bar{\Phi}_f \rangle_\beta^j. \quad (10)$$

Since our model gives  $b_e = 0$  for the charged lepton sector, so that the charged lepton mass matrix is diagonal, the parameters  $z_i$  given in Eq.(9) can be expressed as

$$z_i = \frac{\sqrt{m_{ei}}}{\sqrt{m_e + m_\mu + m_\tau}}. \quad (11)$$

As a result, masses and mixings of quarks and neutrinos are only the family-number independent parameters  $b_f$ . (We will take the phase factors  $\phi_i^f$  as  $\phi_i^f = 0$  except for  $f = u$ .) Even for the family-number dependent parameters  $\phi_i^u$ , we can express those by the parameters  $(z_1, z_2, z_3)$  and two family-number independent parameters [12].) Thus, masses and mixings of quarks and leptons are governed by rules in  $U(3) \times U(3)'$ , not in  $U(3)$ .

Note that we have used the observed values of charged lepton masses for the parameters  $z_i$  given in Eq.(11). We never ask any origin of the charged lepton mass spectrum  $(m_e, m_\mu, m_\tau)$ . Our strategy is as follows: our aim is to describe quarks and neutrino masses and mixings only by using the observed values  $(m_e, m_\mu, m_\tau)$  and without using any family-number dependent parameters. We consider that it is too early to investigate the origin of  $(m_e, m_\mu, m_\tau)$ , that is, U(3) symmetry breaking mechanism. It is a future task to us.

However, there are still many remaining tasks in the  $U(3)\times U(3)'$  model. We have to improved this model into more simple and reliable model. (For a recent work in the  $U(3)\times U(3)'$  model, for example, see Ref.[13].)

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