Flavon VEV Scales in U(3)×U(3)′ Model

Yoshio Koide[a] **and Hiroyuki Nishiura**[b]

[a] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
E-mail address: koide@kuno-g.phys.sci.osaka-u.ac.jp

[b] *Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata, Osaka 573-0196, Japan*
E-mail address: hiroyuki.nishiura@oit.ac.jp

Abstract

We have already proposed a quark and lepton mass matrix model based on U(3)×U(3)′ family symmetry as the so-called Yukawaon model, in which the U(3) symmetry is broken by VEVs of flavons $(\Phi_f)_i{}^\alpha$ which are $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′. The model has successfully provided the unified description of quark and lepton masses and mixings by using the observed charged lepton masses as only family-number dependent input parameters. Our next concern is scales of VEVs of the flavons. In the present paper, we estimate the magnitudes of the VEV scales of flavons of the model which is newly reconstructed without changing the previous phenomenological success of parameter fitting for masses and mixings of quarks and leptons. We estimate that VEVs of flavons with $(\mathbf{8}+\mathbf{1}, \mathbf{1})$, $(\mathbf{3}, \mathbf{3}^*)$, and $(\mathbf{1}, \mathbf{8}+\mathbf{1})$ are of the orders of 10 TeV, 10^4 TeV, and 10^7 TeV, respectively.

PCAC numbers: 11.30.Hv, 12.60.-i, 12.15.Ff, 14.60.Pq,

1 Introduction

The greatest concern in the flavor physics is how to understand the origin of the observed hierarchical structures of masses and mixings of quarks and leptons. Recently, we have proposed a quark and lepton mass matrix model based on U(3)×U(3)′ flavor symmetry [1, 2]. The model is an extended version of the so-called "Yukawaon" model [3]. The model is a kind of flavon model [4] to understand origin of masses (mass matrices) by vacuum expectation values (VEVs) of flavons.

In the U(3)×U(3)′ model, our quarks and leptons are triplets of U(3)×U(3)′ family symmetry. The U(3) symmetry is broken not by scalars in U(3), but by scalars $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′, Φ_f, whose VEVs are given by a universal form $\langle\Phi_f\rangle = v_\Phi \mathrm{diag}(z_1, z_2, z_3)$. (Only for up-quark sector $f = u$, $\langle\Phi_u\rangle$ takes phase factors as seen in Eq.(2.13).) Family-number dependent parameters are only z_i, and we will take $(z_1, z_2, z_3) \propto (\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ as seen in Eq.(1.6). However, in this model, we do not ask the origin of the values of z_i, This is a future task to us. On the other hand, the difference among sectors $f = (u, d, \nu, e)$ seen in the observed masses and mixings is brought by scalars $S_f = (\mathbf{1}, \mathbf{8}+\mathbf{1})$ of U(3)×U(3)′ by assuming that the U(3)′ symmetry is broken into a discrete symmetry S_3. The VEV forms of S_f are given by Eq.(1.2), which are described only by one family-number-independent parameter b_f. Thus, masses and mixings of quarks and leptons are described only by this parameter b_f.

Let us review our model more concretely. In the previous model [1, 2], quark and lepton mass matrices $\hat{M}_f$ $(f = u, d, \nu, e)$ have been given by a common form

$$(\hat{M}_f)_i^{\ j} = \langle \Phi_f \rangle_i^{\ \alpha} \langle \hat{S}_f^{-1} \rangle_\alpha^{\ \beta} \langle \bar{\Phi}_f \rangle_\beta^{\ j}, \tag{1.1}$$

where i, j and α, β are indexes of U(3) and U(3)$'$, respectively.[1] In this model, all of scalars (except for Higgs scalars H) are singlets of SU(3)$_c$×SU(2)×U(1)$_Y$. Such a unified description of quark and lepton mass matrices based on the form (1.1) with matrix forms (1.2) and (1.3) given later has first been proposed in Ref.[5] in order to get top quark mass enhancement. (However, their work was not based on the symmetry U(3)×U(3)$'$.) We consider that the form (1.1) comes from a seesaw-like mechanism for quarks and leptons f_i and hypothetical heavy fermions F_α. (The detail formulation is discussed in Sec.2.)

We assume that the symmetry U(3)$'$ is broken into a discrete symmetry S$_3$ at $\mu = \Lambda'$, so that VEV forms of $\hat{S}_f$ are given by a form (unit matrix + democratic matrix) [6]:

$$\langle \hat{S}_f \rangle = v_S(\mathbf{1} + b_f X_3), \tag{1.2}$$

where $\mathbf{1}$ and X_3 are defined by

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{1.3}$$

and b_f are complex parameters.

On the other hand, we consider that U(3) is broken by VEVs of Φ_f with $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)$'$, the VEV forms of which are assumed to be diagonal as given by

$$\langle \Phi_f \rangle = v_\Phi \, \mathrm{diag}(z_1 e^{i\phi_1^f}, z_2 e^{i\phi_2^f}, z_3 e^{i\phi_3^f}). \tag{1.4}$$

We shall see that $\phi_i^f = 0$ for $f = d, e, \nu$ in later discussions. Therefore, we hereafter denote $(\phi_1^u, \phi_2^u, \phi_3^u)$ as (ϕ_1, ϕ_2, ϕ_3) simply.

We will give $b_e = 0$, i.e. $\hat{S}_e = v_S \mathbf{1}$ as seen in Eq.(2.9) in the next section. Thereby the charged lepton mass matrix is given by

$$\hat{M}_e \propto \mathrm{diag}(z_1^2, z_2^2, z_3^2), \tag{1.5}$$

[1] In this paper, we denote flavons with $(\mathbf{8}+\mathbf{1}, \mathbf{1})$ and $(\mathbf{1}, \mathbf{8}+\mathbf{1})$ of U(3)×U(3)$'$ as $\hat{A}_i^{\ j}$ and $\hat{A}_\alpha^{\ \beta}$ with "hat", respectively. We also denote a flavon $(\mathbf{3}^*, \mathbf{3}^*)$ as $\bar{A}^{\alpha i}$ in contrast to $A_{i\alpha}$ of $(\mathbf{3}, \mathbf{3})$. However, for flavons with $(\mathbf{3}, \mathbf{3}^*)$ and $(\mathbf{3}^*, \mathbf{3})$, we denote those as $A_i^{\ \alpha}$ and $\bar{A}_\alpha^{\ i}$, respectively, giving priority to U(3) index.

from which we get a relation

$$z_i = \frac{\sqrt{m_{ei}}}{\sqrt{m_e + m_\mu + m_\tau}}. \tag{1.6}$$

Here $m_{ei} = (m_e, m_\mu, m_\tau)$ are charged lepton masses, and the real parameters z_i are normalized as $z_1^2 + z_2^2 + z_3^2 = 1$. Note that the parameter values (z_1, z_2, z_3) are universal for every sector $f = u, d, e, \nu$. On the other hand, the parameters b_f are sector-dependent (f-dependent but family-independent (f-independent). Family-dependent parameter ϕ_i are described in terms of z_i [7] as seen in later discussions. Therefore, family dependent parameters are only z_i. In this model, parameters which we can adjust are only the family-independent parameters b_f. Note that the parameters b_f determine not only mass eigenvalues (m_{f1}, m_{f2}, m_{f3}) but also family mixing matrix U_f.

The origin of the parameter values (1.6) has not been given in the Yukawaon model. The relation (1.6) is nothing but an ad hoc assumption. Here, we would like to emphasize the strategy of the Yukawaon model based on the U(3)×U(3)$'$ symmetry. In most mass matrix models, a symmetry for quarks and leptons is given, and thereby the masses and mixings are derived. (It is investigated how the family symmetry breaks into a suitable sub-symmetry or discrete symmetry.) In the Yukawaon model, we do not propose an explicit symmetry breaking mechanism for U(3) symmetry. We only use the observed values of the charged lepton masses for the relation (1.5). The mechanism which gives charged lepton masses is left for a future task, and, for the moment, we do not ask the origin. Instead, we try to give unified description of all mass ratios and mixings in terms of the charged lepton mass values, and without using any family-number dependent parameters at all. The purpose of the Yukawaon model is to show that all the quark and lepton mass ratios and mixings can be described when we accept the observed charged lepton mass values as only family-dependent parameters. Anyhow, we could successfully obtain [1] a unified description of quark and lepton masses and mixing by using the observed charged lepton masses as only family-number dependent input parameters. The aim has been almost accomplished in the phenomenological level.

Here, let us comment on our U(3)×U(3)$'$ gauge model. The symmetry is an extended version of Sumino's U(3)×O(3) model [8]. The model is easily extended into a U(3)×U(3)$'$ model [9]. The Sumino's family gauge boson (FGB) model has the following characteristics: (i) Family symmetry U(3) is broken at a low energy scale of $\mathcal{O}(10^3)$ TeV by a scalar Φ_i^α which is $(\mathbf{3}, \mathbf{3})$ of U(3)×O(3). (ii) The FGB mass matrix is diagonal in the flavor basis in which the charged lepton mass matrix is diagonal, so that lepton family-number violation does not occur. (iii) FGB masses and gauge coupling g_F are not free parameters. FGB masses and g_F are related to the charged lepton masses and to the electroweak gauge coupling, respectively. Hence the predictions in the model are less ambiguous. (iv) Family-number violation in quark sector can appear only via quark mixing. Therefore, the family-number violation is forbidden in the limit of $V_{CKM} = \mathbf{1}$ (V_{CKM} is the Cabibbo-Kobayashi-Maskawa matrix [10]). At present, the most sever constraint [11] on the Sumino's FGB masses comes from the observed P^0-$\bar{P}^0$ mixings ($P = K, D, B, B_s$).

The original intention in the Sumino model [8] was to give an interpretation for why the charge lepton mass relation [12] holds for pole mass excellently. (A brief review is given in Appendix A.) Sumino has proposed a cancellation mechanism between QED and family gauge boson diagrams. However, the cancellation is only for $\log m_{ei}$ term, and besides, since there are other contributions to the running masses, the Sumino cancellation holds only within a limited energy scale range. Sumino has speculated that the scale of the FGB mass is 10^3 TeV [8]. If we accept the Sumino's speculation, too large VEV scale of the flavon Φ_f will be ruled out, while too small VEV scale is contrary to the constraint from the observed P^0-$\bar{P}^0$ mixings.

Our concern in this paper is a VEV scale of the flavon Φ_f. In the past Yukawaon models, VEV scales of flavons have not been discussed. In this paper, quark and lepton masses and mixings are re-investigated by taking the energy scales of VEV of flavons into consideration. In order to discuss the VEV scales consistently, we rebuild the model. As a result, the basic formulation is drastically changed, although the phenomenological study inherits from the previous works [1, 2],

The new model in this paper is characterized as follows: (i) There are no Yukawaons, although the flavons Φ_f and $\hat{S}_f$ still play an essential role in the new model, too. (ii) There are three type of VEV scales. We denote these scales as

$$\langle \hat{S}_f \rangle_{\bullet}{}^{\bullet} \sim \Lambda_1, \quad \langle \Phi_f \rangle_{\circ}{}^{\bullet} \sim \Lambda_2, \quad \langle E_{\circ\circ} \rangle \sim \Lambda_3. \tag{1.7}$$

More exactly speaking, we define Λ_1 as $\Lambda_1 = v_S$ in Eq.(1.2), and Λ_2 as $\Lambda_2 = v_\Phi$ (except for $f = \nu$) in Eq.(1.4), respectively. We have also denote a scale Λ_3 as $v_E = \Lambda_3$ which appears in Eq.(2.14) later. Here and hereafter, in order to be easy to see a scale of flavons, we will sometimes show the indexes of U(3) and U(3)$'$, by "$\circ$" and "$\bullet$", instead of $i, j, k, \cdots$" and "$\alpha, \beta, \gamma, \cdots$", respectively. We consider $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$. The purpose of the present paper is to fix those scales by the observed data. (iii) The most troublesome problem is VEV scales in neutrino sector. In our model, in oder to make the model anomaly free, the right-handed neutrinos ν_R belong to triplet of U(3) as well as ν_L, so that the right-handed neutrino Majorana mass matrix $(M_R)^{ij}$ are given by VEV of a flavon $\bar{Y}_R$, $\langle \bar{Y}_R \rangle^{ij}$. We demand that Majorana mass matrix of the left-handed neutrinos is given by a seesaw mechanism

$$(M_\nu^{Maj})_{\circ\circ} = (\hat{M}_\nu)_{\circ}{}^{\circ} \langle \bar{Y}_R^{-1} \rangle_{\circ\circ} (\hat{M}_\nu^T)^{\circ}{}_{\circ}. \tag{1.8}$$

However, in the present model, a scale of the right-handed neutrino Majorana mass matrix is considerably small differently from the conventional neutrino seesaw [13], because our flavon seems to have $\langle \bar{Y}_R \rangle^{\circ\circ} \sim \Lambda_3$. This is somewhat troublesome. We will squarely grapple with this problem.

In Sec.2, we discuss our Dirac mass matrix forms which are given in Eqs.(1.1) - (1.3). In Sec.3, we discuss relations between phase parameters ϕ_i given in Eq.(1.4) and the charged lepton masses m_{ei}. That is, we show that the apparently "family-number dependent parameters ϕ_i can be described in terms of charged lepton mass parameters (m_e, m_μ, m_τ). This was first pointed out in Ref.[7]. In our present model, since the VEV relations are completely revised, the relations between ϕ_i and m_{ei} are also completely different from previous ones. The new relations lead us not only to that the relations give an understanding of the values ϕ_i on the basis of the charged lepton masses m_{ei}, but also to that we can predict the scales Λ_1, Λ_2 and Λ_3 by using the result of data-fitting in the ϕ_i-m_{ei} relations. In Sec.4, we discuss slight VEV deviations among flavons with the same transformation of U(3)×U(3)′. This is not essential to predict Λ_2/Λ_1 and Λ_3/Λ_2. However, in order to obtain more consistency in our scenario, we will discuss somewhat detailed VEV relations. In Sec.5, we discuss a Mojorana mass matrix form of the right-handed neutrinos ν_R as a preparation for predicting the scales Λ_1, Λ_2 and Λ_3. In Sec.6, we discuss the VEV scales Λ_1, Λ_2 and Λ_3. Finally, Sec.7 is devoted to concluding remarks.

2 Dirac mass matrices

2.1 Seesaw-type mechanism in the Dirac mass matrix

In our model based on U(3)×U(3)′ symmetry, we consider hypothetical fermions F_α ($\alpha = 1, 2, 3$), which belong to $(\mathbf{1}, \mathbf{3})$ of U(3)×U(3)′, in addition to quarks and leptons f_i ($i = 1, 2, 3$) which belong to $(\mathbf{3}, \mathbf{1})$. In this model, differently from the Yukawaon formulation, we assume that the VEV form (1.1) originates from the following 6×6 mass matrix model:

$$(\bar{f}_L^i \;\; \bar{F}_L^\alpha) \left(\begin{array}{cc} (0)_i^{\ j} & (\Phi_f)_i^{\ \beta} \\ (\bar{\Phi}_f)_\alpha^{\ j} & -(S_f)_\alpha^{\ \beta} \end{array} \right) \left(\begin{array}{c} f_{Rj} \\ F_{R\beta} \end{array} \right). \tag{2.1}$$

Here, $F_{L(R)}$ are heavy fermions with $(\mathbf{1}, \mathbf{1}, \mathbf{3})$ of SU(2)$_L$×U(3)×U(3)′. On the other hand, f_R are right-handed quarks and leptons, $f_R = (u, d, \nu, e^-)_R$, while f_L are not physical fields. They are given by the following combinations:

$$f_L \equiv (f_u, f_d, f_\nu, f_e)_L \equiv \left(\frac{1}{\Lambda_H} H_u^\dagger q_L, \frac{1}{\Lambda_H} H_d^\dagger q_L, \frac{1}{\Lambda_H} H_u^\dagger \ell_L, \frac{1}{\Lambda_H} H_d^\dagger \ell_L \right) \tag{2.2}$$

where

$$q_L = \left(\begin{array}{c} u_L \\ d_L \end{array} \right), \quad \ell_L = \left(\begin{array}{c} \nu_L \\ e_L^- \end{array} \right), \quad H_u = \left(\begin{array}{c} H_u^0 \\ H_u^- \end{array} \right), \quad H_d = \left(\begin{array}{c} H_d^+ \\ H_d^0 \end{array} \right). \tag{2.3}$$

In other words, the matrix given in Eq.(2.1) denotes would-be Yukawa coupling constants.

After the U(3) and U(3)′ have been completely broken, the quarks and leptons are described by the effective Hamiltonian

$$\mathcal{H}_Y = (\bar{\nu}_L)^i (\hat{M}_\nu)_i{}^j (\nu_R)_j + (\bar{e}_L^i (\hat{M}_e)_i{}^j (e_R)_j + y_R (\bar{\nu}_R)^i (Y_R)_{ij} (\nu_R^c)^j$$
$$+ (\bar{u}_L)^i (\hat{M}_u)_i{}^j (u_R)_j + (\bar{d}_L)^i (\hat{M}_d)_i{}^j (d_R)_j. \tag{2.4}$$

Of course, $(\hat{M}_f)_i{}^j$ do not mean flavons although we have called those "Yukawaons" in the past Yukawaon model [3]. Note that the quarks and leptons f_i are not U(3) family triplet any more in the exact meaning, but they are mixing states between f and F. However, we will still use the index of U(3) family for these fermion states. Also, note that there are no Yukawaons in the present model. By performing a seesaw-like approximation with $\Lambda_2 \ll \Lambda_1$, the mass matrix (2.1) leads to the following Dirac mass matrices of quarks and leptons:

$$(\hat{M}_f)_i{}^j \simeq \frac{\langle H_{u/d} \rangle}{\Lambda_H} \langle \Phi_f \rangle_i{}^\alpha \langle (\hat{S}_f)^{-1} \rangle_\alpha{}^\beta \langle \bar{\Phi}_f \rangle_\beta{}^j. \tag{2.5}$$

In this paper, we search VEV scales of flavons by assuming that our flavon VEV scale is given by one of Λ_1, Λ_2 and Λ_3, except for $H_{u/d}$. As we stated in Sec.2, we put $1/(\Lambda_1)^n$ for superpotential term with dimension $(3+n)$. However, we assume that these rules are exceptional for the factor $H_{u/d}/\Lambda_H$. The VEV of H_u and H_d are fixed as

$$v_{Hu} = v_{Hd} \equiv v_H \equiv \frac{1}{\sqrt{2}} \times 246 \text{ GeV}. \tag{2.6}$$

(The reason of $v_{Hu} = v_{Hd}$ will be given in Eq.(4.6) later.) We will take Λ_H as a different value from Λ_1, Λ_2 and Λ_3. That is, we consider that the additional factor $H_{u/d}/\Lambda_H$ in Eq.(2.5) is factor at the electroweak scale after our family symmetries U(3)×U(3)′ are completely broken. For the time being, we do not discuss the mechanism which gives the factor $H_{u/d}/\Lambda_H$.

2.2 R-charge assignment of Φ_f

We assume that R charges are still conserved under the block diagonalization of (2.1), so that we consider an R charge relation

$$R(\hat{S}_f) = R(\Phi_f) \equiv r_f. \tag{2.7}$$

Here we denote R charge of the flavon $\hat{S}_f$ as $R(\hat{S}_f)$ and so on. We have taken R charge of the Higgs fields as $R(H_{u/d}) = 0$. Then, the Dirac mass matrix $\hat{M}_f$ has effectively a R charge r_f (although $\hat{M}_f$ is not a flavon). Note that if there is a flavon combination $(\Phi_{f'} \bar{\Phi}_{f''})_i{}^j/\Lambda_1$ with $r_{f'} + r_{f''} = r_f$, the combination can couple to $\bar{f}_L^j f_{Ri}$ as well as $(\hat{M}_f)_i{}^j$ with the same energy scale $(\Lambda_2)^2/\Lambda_1$. Besides, if $(\bar{\Phi}_f \Phi_{f'})$ and another $(\bar{\Phi}_{f"} \Phi_{f"'})$ have the same R charge, those can have the same VEV structure. This is an unwelcome situation, because we demand that Φ_u and Φ_ν have somewhat different VEV structures from Φ_e and Φ_d as we state later. A simple way to avoid appearance of such unwelcome combinations is to take R charge difference among Φ_f's completely different from each other: $r_e - r_u = 2/3$, $r_d - r_e = 3/3$, $r_\nu - r_d = 4/3$, i.e.

$$(r_u, r_e, r_d, r_\nu) = \left(\frac{1}{3}, 1, 2, \frac{10}{3}\right). \tag{2.8}$$

The reason $r_u < r_e$ will also be discussed later.

The reason why we choose $r_e = 1$ is as follows: Only when $r_e = 1$, we can write a superpotential for $\hat{S}_e$,

$$W_{Se} = \lambda_1^S [(\hat{S}_e)_\alpha{}^\beta (\hat{S}_e)_\beta{}^\alpha] + \lambda_2^S [(\hat{S}_e)_\alpha{}^\alpha][(\hat{S}_e)_\beta{}^\beta]. \tag{2.9}$$

Supersymmetric vacuum condition for this W_{Se} leads to $\hat{S}_e = v_S \mathbf{1}$, so that we obtain $b_e = 0$. This means that the charged lepton mass matrix $\hat{M}_e$ is proportional to $\mathrm{diag}(z_1^2, z_2^2, z_3^2)$. Therefore, our parameters z_i are given by (1.6). The phase parameters ϕ_i^e do not have physical meaning because $\langle \Phi_e \rangle$ is commutable with $\langle \hat{S}_e \rangle = v_S \mathbf{1}$, so that we can hereafter put $\phi_i^e = 0$, i.e. $\langle \Phi_e \rangle = v_e\, \mathrm{diag}(z_1, z_2, z_3)$.

In Table.1 we list R charges of leading flavons.

2.3 VEV relation among Φ_f

In this paper, we will often give effective superpotential terms. We write down those effective superpotentials according to the following rules: (i) For effective superpotential terms with a higher mass-dimension $(n+3)$, we put a factor $(1/\Lambda_1)^n$ in order to adjust the term to the dimension 3. (ii) In order to avoid unwelcome terms with higher dimension, we assume R charge conservation and positive R charge assignment (except for special flavons Θ as we show later), so that superpotential terms with $R > 2$ are forbidden.

A VEV of Φ_d is obtained by the following superpotential

$$W_d = \left\{ (\Phi_d)_i{}^\beta (\hat{E})_\beta{}^\alpha + \lambda_d \frac{1}{\Lambda_1} (\Phi_e)_i{}^\beta (\hat{S}_e)_\beta{}^\gamma (\hat{E})_\gamma{}^\alpha \right\} (\bar{\Theta}_d)_\alpha^{\ i}. \tag{2.10}$$

Here and hereafter, we assume that the Θ fields always take $\langle \Theta \rangle = 0$. Therefore, from $\partial W_d / \partial \Theta_d = 0$, we obtain

$$(\Phi_d)_i{}^\beta (\hat{E})_\beta{}^\alpha = -\lambda_d \frac{1}{\Lambda_1} (\Phi_e)_i{}^\beta (\hat{S}_e)_\beta{}^\gamma (\hat{E})_\gamma{}^\alpha. \tag{2.11}$$

Therefore, when the scale of $(\hat{E})_\gamma{}^\alpha$ is Λ_1, we obtain the VEV $\langle \Phi_d \rangle$ with the same order as $\langle \Phi_e \rangle$, i.e. $\langle \Phi_d \rangle = v_d\, \mathrm{diag}(z_1, z_2, z_3)$ with $\phi_i^d = 0$.

However, for Φ_u with $R = 1/3$, we have to consider another type of superpotential,

$$W_u = \left\{ \frac{\lambda_{u1}}{\Lambda_1} (\bar{\Phi}_u)_\alpha^{\ k} (\Phi_u)_k^{\ \gamma} (\hat{E})_\gamma^{\ \beta} + \lambda_{u2} (\bar{\Phi}_d)_\alpha^{\ k} (\Phi_d)_k^{\ \beta} \right\} (\hat{\Theta}_u)_\beta^{\ \alpha}. \tag{2.12}$$

Here, we have introduced a new flavon $\hat{E}_\bullet^{\ \bullet}$ with $R = 5/3$ and a VEV form $\langle \hat{E} \rangle = v_E \mathbf{1}$. Since this potential gives a relation between $\Phi_u \bar{\Phi}_u$ and $\Phi_d \bar{\Phi}_d$, $\langle \Phi_u \rangle$ can have phase factors differently from real VEV matrix $\langle \Phi_d \rangle$. Hereafter, we simply denote phase parameters ϕ_i^u as ϕ_i:

$$\langle \Phi_u \rangle = v_u \, \mathrm{diag}(z_1 e^{i\phi_1}, z_2 e^{i\phi_2}, z_3 e^{i\phi_3}). \tag{2.13}$$

Meanwhile, in this model, flavons E with VEV matrix form $v_E \mathrm{diag}(1,1,1)$ frequently appear. For E with the VEV scale Λ_3 and $R = 2/3$, we consider a superpotential given by

$$W_E = \lambda_1 \mathrm{Tr}[\hat{E}_\circ^{\ \circ} E_{\circ\circ} \bar{E}^{\circ\circ}] + \lambda_2 \mathrm{Tr}[\hat{E}_\circ^{\ \circ}] \mathrm{Tr}[E_{\circ\circ} \bar{E}^{\circ\circ}]. \tag{2.14}$$

SUSY vacuum condition leads to

$$\hat{E}_\circ^{\ \circ} = v_E \mathbf{1}, \qquad E_{\circ\circ} \bar{E}^{\circ\circ} = (v_E)^2 \mathbf{1}. \tag{2.15}$$

For $E_{\circ\circ}$, we take a special solution $E_{\circ\circ} = v_E \mathbf{1}$ in the relation $E_{\circ\circ} \bar{E}^{\circ\circ} = (v_E)^2 \mathbf{1}$. For $\hat{E}_\bullet^{\ \bullet}$ and $E_{\bullet\bullet}$, there is not such a simple superpotential. We have to assume the following superpotential

$$W_E = \lambda_1 \mathrm{Tr}[\hat{E}_\bullet^{\ \bullet} \hat{E}_\bullet^{\ \bullet} (\hat{\Theta}_{-4/3})_\bullet^{\ \bullet}] + \lambda_2 \mathrm{Tr}[E_{\bullet\bullet} \bar{E}^{\bullet\bullet} (\hat{\Theta}_{-4/3})_\bullet^{\ \bullet}] + \ldots, \tag{2.16}$$

where $(\hat{\Theta}_{-4/3})_\bullet^{\ \bullet}$ has $R = -4/3$. We also assume

$$W_E = \mathrm{Tr}[\hat{E}_\circ^{\ \circ} E_{\circ\bullet} \bar{E}^{\bullet\circ} (\hat{\Theta}_{2/3})_\circ^{\ \circ}] + \cdots, \tag{2.17}$$

where $(\hat{\Theta}_{2/3})_\circ^{\ \circ}$ has $R = 2/3$.

Finally, let us discuss a VEV relation of $\langle \Phi_\nu \rangle$. We cannot consider the similar mechanism as $\langle \Phi_f \rangle$ using $\hat{E}$, since we need to give a small scale compared with $\langle \Phi_e \rangle \sim \Lambda_2$ in order to satisfy the seesaw approximation (1.8). (Since the scale Λ_2 means the maximal scale of a flavon with $A_\circ^{\ \bullet}$, the requirement $\langle \Phi_\nu \rangle < \Lambda_2$ has no problem.) Therefore, we assume the following superpotential term:

$$W_\nu = \left\{ \mu_\nu (\Phi_\nu)_\circ^{\ \bullet} + \lambda_\nu \frac{1}{(\Lambda_1)^2} (\Phi_e)_\circ^{\ \bullet} \bar{E}^{\bullet\bullet} E_{\bullet\circ} \bar{E}^{\circ\bullet} \right\} (\bar{\Theta}_\nu)_\bullet^{\ \circ}, \tag{2.18}$$

where new flavons $E_{\circ\circ}$ and $E_{\bullet\bullet}$ with VEV form $v_E \mathbf{1}$ has R charges

$$R(E_{\circ\circ}) = \frac{2}{3}, \qquad R(E_{\bullet\bullet}) = \frac{5}{3}, \qquad R(E_{\circ\bullet}) = \frac{1}{3}. \tag{2.19}$$

Table 1: R charges and VEV scales of leading flavons. Transformation property under U(3)×U(3)′ is indicated by "∘" and "•", respectively.

flavon	$(\Phi_u)_\circ{}^\bullet$	$(\Phi_e)_\circ{}^\bullet$	$(\Phi_d)_\circ{}^\bullet$	$(\Phi_\nu)_\circ{}^\bullet$	$(\hat{S}_u)_\bullet{}^\bullet$	$(\hat{S}_e)_\bullet{}^\bullet$	$(\hat{S}_d)_\bullet{}^\bullet$	$(\hat{S}_\nu)_\bullet{}^\bullet$
R	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{6}{3}$	$\frac{10}{3}$	$\frac{1}{3}$	$\frac{3}{3}$	$\frac{6}{3}$	$\frac{10}{3}$
Scale	Λ_2	Λ_2	Λ_2	Λ_3	Λ_1	Λ_1	Λ_1	Λ_2

$E_{\circ\circ}$	$\hat{E}_\circ{}^\circ$	$E_{\circ\bullet}$	$E_{\bullet\bullet}$	$\hat{E}_\bullet{}^\bullet$	$(\hat{Y}_{eu})_\bullet{}^\bullet$	$(\bar{S}'_u)^{\circ\bullet}$	$(\bar{Y}_R)^{\circ\circ}$
$\frac{2}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{5}{3}$	$\frac{5}{3}$	$\frac{4}{3}$	$\frac{5}{3}$	$\frac{13}{3}$
Λ_3	Λ_3	Λ_2	Λ_1	Λ_1	Λ_1	Λ_2	Λ_3

$(\bar{\Theta}_d)_\bullet{}^\circ$	$\bar{\Theta}_u)_\bullet{}^\bullet$	$(\bar{\Theta}_\nu)_\bullet{}^\circ$	$(\hat{\Theta}_\nu)_\bullet{}^\bullet$	$(\hat{\Theta}_\phi)_\bullet{}^\bullet$	$(\Theta_R)_{\circ\circ}$	$(\hat{\Theta}_{eu})_\bullet{}^\bullet$	$(\Theta'_{Su})_{\bullet\circ}$	$(\hat{\Theta}_E)_\bullet{}^\bullet$	$(\hat{\Theta}_E)_\circ{}^\circ$
$-\frac{5}{3}$	$-\frac{2}{3}$	$-\frac{4}{3}$	$-\frac{4}{3}$	$\frac{2}{3}$	-3	$\frac{2}{3}$	-1	$-\frac{4}{3}$	$\frac{2}{3}$
0	0	0	0	0	0	0	0	0	0

Therefore, a VEV scale of $(\Phi_\nu)_\circ{}^\bullet$ with $R = 10/3$ is given by

$$\|\Phi_\nu\| = \frac{(\Lambda_2)^3}{\mu_\nu \Lambda_1}, \tag{2.20}$$

differently from other $(\Phi_f)_\circ{}^\bullet$. Hereafter, for convenience, we denote a VEV scale of a flavon A as a notation $\|A\|$. Also, in the estimation of VEV scales, for simplicity, we put $\lambda = 1$ for all dimensionless coefficients λ in superpotentials (2.9), (2.10), and so on. We denote Eq.(2.20) as

$$\langle \Phi_\nu \rangle = \xi_\nu \langle \Phi_e \rangle, \qquad \xi_\nu = \frac{(\Lambda_2)^2}{\mu_\nu \Lambda_1}. \tag{2.21}$$

We have obtained $b_e = 0$, i.e. $\langle \hat{S}_e \rangle = v_S \mathbf{1}$ by assigning $r_e = 1$ as shown in Eq.(2.9). However, since $\hat{S}_\nu$ has $R = 10/3$, we cannot assert $b_\nu = 0$ by means of a similar way to Eq.(2.9). We want a similar VEV structure of $\hat{M}_\nu$ to $\hat{M}_e$ except for its VEV scale. Therefore, we assume a superpotential similar to (2.18):

$$W_{S\nu} = \left\{ \mu_{S\nu} (\hat{S}_\nu)_\bullet{}^\bullet + \lambda_\nu \frac{1}{(\Lambda_1)^2} (\hat{S}_e)_\bullet{}^\bullet \bar{E}^{\bullet\bullet} E_{\bullet\circ} \bar{E}^{\circ\bullet} \right\} (\hat{\Theta}_\nu)_\bullet{}^\bullet. \tag{2.22}$$

Then, we obtain a result similar to (2.21):

$$\langle \hat{S}_\nu \rangle = \xi_{S\nu} \langle \hat{S}_e \rangle, \qquad \xi_{S\nu} = \frac{(\Lambda_2)^2}{\mu_{S\nu} \Lambda_1}, \tag{2.23}$$

so that we can obtain $\hat{M}_\nu$ with the same form as $\hat{M}_e$:

$$\hat{M}_\nu = \xi_M \hat{M}_e, \quad \xi_M = (\xi_\nu)^2 (\xi_{S\nu})^{-1} = \frac{\mu_{S\nu}(\Lambda_2)^2}{(\mu_\nu)^2 \Lambda_1}. \tag{2.24}$$

2.4 Parameter values in the quark sector

As far as quark sector is concerned, the VEV matrices are exactly the same as those in the previous paper, although the model is completely different from the previous one.

We choose parameter values[2] as

$$b_u = -1.011, \quad b_d = -3.522\, e^{i\ 17.7^\circ}, \tag{2.25}$$

and

$$(\tilde{\phi}_1, \tilde{\phi}_2) = (-176.05^\circ,\ -167.91^\circ), \tag{2.26}$$

where we have fitted (ϕ_1, ϕ_2, ϕ_3) as $(\tilde{\phi}_1, \tilde{\phi}_2,\ 0)$ without losing generality. Then, we can obtain reasonable quark masses and CKM mixing as shown in Appendix B. Note that the parameter ϕ_0 cannot be fixed in the CKM fitting, i.e, it is an unobservable parameter as we discuss in the next section.

3 Relation between phase parameters ϕ_i and charged lepton masses m_{ei}

The parameters (ϕ_1, ϕ_2, ϕ_3) were typical family-number dependent parameters. Now we try to describe (ϕ_1, ϕ_2, ϕ_3) in terms of charged lepton masses m_{ei} and two family-number independent parameters. This has first been pointed out by the authors [7]. However, since the R-charge assignment in the present model is completely different from that in the previous model, the relations between the phase parameters ϕ_i and charged lepton masses m_{ei} is also changed. New relations will be simply denoted with fractional coefficients and thereby it will make possible to estimate the scales of U(3)×U(3)′.

We assume the following superpotential

$$W_\phi = \left\{ \frac{\lambda_1^\phi}{\Lambda_1} \left[(\bar{\Phi}_u)_\bullet{}^\circ E_{\circ\circ} (\bar{E})^{\circ\bullet} + h.c. \right] + \frac{\lambda_2^\phi}{(\Lambda_1)^2} (\bar{\Phi}_u)_\bullet{}^\circ (E)_{\circ\bullet} (\bar{E})^{\bullet\circ} (\Phi_u)_\circ{}^\bullet \right.$$

$$\left. + \frac{\lambda_2^\phi}{(\Lambda_1)^3} (\bar{\Phi}_u)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet (\bar{\Phi}_u)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet \right\} (\hat{\Theta}_\phi)_\bullet{}^\bullet, \tag{3.1}$$

where $R(\Theta_\phi) = 2/3$. Note that in addition to the second term $(\bar{\Phi}_u)_\bullet{}^\circ (E)_{\circ\bullet} (\bar{E})^{\bullet\circ} (\Phi_u)_\circ{}^\bullet$ in Eq.(3.1), the following additional terms are allowed: $(\bar{\Phi}_u)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet (E)_{\bullet\circ} (\bar{E})^{\circ\bullet}$, $(\bar{\Phi}_u)_\bullet{}^\circ (E)_{\circ\bullet} (\Phi_u^T)^\bullet{}_\circ (\bar{E})^{\circ\bullet}$,

[2] In the parameter fitting, we have used running masses at $\mu = m_Z$ as input quark and lepton masses.

$(E)_{\bullet\circ}(\bar{\Phi}_u^T)^{\circ}{}_{\bullet}\bar{E}^{\bullet\circ}(\Phi_u)_{\circ}{}^{\bullet}$, and $(E)_{\bullet\circ}(\bar{E})^{\circ\bullet}(\bar{\Phi}_u)_{\bullet}{}^{\circ}(\Phi_u)_{\circ}{}^{\bullet}$. Here, some remarks are in order: (i) we regard those additional terms as "substantially same terms", (ii) but, we count a flavon $A^\dagger$ as a different field from A, and (iii) the coefficient λ is defined as follows: the λ is a coefficient with a factor $1/n$ for sum of n substantially same terms. For example, in the second term in (3.1), the factor λ_2^ϕ is defined as one for sum of the five terms with 1/5. However, for simplicity, we denote only representative one even if there are many equivalent terms, and give the coefficient λ instead of λ/n. Also note that the $h.c$ term in the first term is different from the original one according to our counting rule.

The flavon $\hat{\Theta}_\phi$ in (3.1) has VEV value of zero. The SUSY vacuum condition $\partial W_\phi/\partial\Theta_\phi = 0$ leads to a condition

$$2c_1 z_i \cos\phi_i = c_2 z_i^2 + c_3 z_i^4, \tag{3.2}$$

where parameters c_1, c_2 and c_3 are family-number independent parameters and they have scales

$$c_1 = \lambda_1 \frac{(\Lambda_2)^2}{(\Lambda_1)^2}\Lambda_1\Lambda_3, \qquad c_2 = \lambda_2 \frac{(\Lambda_2)^4}{(\Lambda_1)^2}, \qquad c_3 = \lambda_3 \frac{(\Lambda_2)^4}{(\Lambda_1)^2}. \tag{3.3}$$

When we denote these parameters ϕ_i as

$$\begin{aligned} \phi_1 &= \phi_0 + \tilde{\phi}_1, \\ \phi_2 &= \phi_0 + \tilde{\phi}_2, \\ \phi_3 &= \phi_0, \end{aligned} \tag{3.4}$$

the parameter ϕ_0 is unobservable in the CKM parameter fitting, while it is not unobservable in the U(3)×U(3)′ model. From the input values $(\tilde{\phi}_1,\, \tilde{\phi}_2) = (-176.05^\circ,\ -167.91^\circ)$ given in (2.26), we obtain

$$\phi_0 = 86.69^\circ, \quad \frac{c_2}{c_1} = 1.368, \quad \frac{c_3}{c_2} = -0.967, \tag{3.5}$$

which leads to

$$(\phi_1,\, \phi_2,\, \phi_3) = (-89.36^\circ,\ -87.25^\circ,\ 86.69^\circ). \tag{3.6}$$

Thus, the family-number dependent parameters (ϕ_1, ϕ_2, ϕ_3) can be reduced into family-number independent parameters $(c_2/c_1, c_3/c_2)$. (Note that the parameter ϕ_0 is not unobservable any longer in this model.)

Note that the numerical results (3.5) suggests $|c_3/c_2| = 1$ if we take $\lambda_2 = \lambda_3$. Then, it is natural that we consider $\lambda_1 = \lambda_2 = \lambda_3$. Only the ratio c_2/c_1 is not one. When we compare (3.3) with the numerical result $c_2/c_1 \equiv \rho \simeq 1.37$ given in (3.5), we obtain

$$R_\Lambda \equiv \frac{\Lambda_2}{\Lambda_1} = \rho\frac{\Lambda_3}{\Lambda_2}. \tag{3.7}$$

We will see $R_\Lambda \sim 10^{-3}$ in Sec.5.

4 Slight deviation of the scales in the lepton sector

4.1 $\hat{M}_{lepton}$ versus $\hat{M}_{quark}$

Let us comment on the ratio v_{Hu}/v_{Hd}. Usually, it is understood that the value of $\tan\beta \equiv v_{Hu}/v_{Hd}$ is $\tan\beta \sim 10-40$. However, from (2.5), our quark masses are predicted as

$$\begin{aligned} (m_u, m_c, m_t) &= (0.0003964,\ 0.106411,\ 29.743) m_{0u}, \\ (m_d, m_s, m_b) &= (0.0007249,\ 0.01467,\ 0.5365) m_{0d}, \end{aligned} \tag{4.1}$$

where m_{0f} are defined by

$$m_{0f} = \frac{v_{Hf}}{\Lambda_H}\frac{(v_f)^2}{v_{Sf}} = \frac{v_{Hf}}{\Lambda_H}\rho\Lambda_3. \tag{4.2}$$

In the last term in Eq.(4.2), we have used the relation (3.7). The numerical values in (4.1) are eigenvalues of the dimensionless matrix

$$Z(\mathbf{1} + b_f X_3)^{-1} Z, \tag{4.3}$$

where

$$Z = \mathrm{diag}(z_1, z_2, z_3), \tag{4.4}$$

and we have used input values $b_u = -1.011$ and $b_d = -3.3522 e^{i\,17.7^\circ}$ [1].

The prediction (4.1) can give reasonable values not only for mass ratios m_u/m_c, m_c/m_t, m_d/m_s, and m_s/m_b, but also for the mass ratio of up-quark mass to down-quark mass

$$\frac{m_t}{m_b} = 55.44, \tag{4.5}$$

if we take $m_{0u} = m_{0d}$. The value (4.5) roughly agrees with the observed value [14] $(m_t/m_b)^{obs} \simeq 59.41$. Therefore, in the present model, we can regard

$$\frac{m_{0u}}{m_{0d}} = 1, \quad i.e \quad v_{Hu} = v_{Hd}. \tag{4.6}$$

Therefore, we have already taken $v_{Hu} = v_{Hd}$ in Eq.(2.6).

Also, since $m_\tau = (z_3)^2 m_{0e}$ ($z_3 = 0.97170$), we obtain

$$m_{0e} = 1.8499 \text{ GeV}, \tag{4.7}$$

so that we get

$$\frac{m_{0u}}{m_{0e}} = 3.121. \tag{4.8}$$

This suggests

$$\frac{m_{0u}}{m_{0e}} = 3, \tag{4.9}$$

in contrast to Eq.(4.6). We consider that such the factor 3 originates in a slight difference between flavon VEV scales in the lepton sector and the quark sector. In the next subsection, we will discuss such an additional VEV scale difference.

4.2 Slight deviation of the scale $\|\Phi_{lepton}\|$ from $\|\Phi_{quark}\|$

So far, we have not mentioned the origin of the parameter value of ρ defined by Eq.(3.7) and the factor 3 in Eq.(4.9). In order to understand those numerical factors, in this subsection. we define additional scale deviation factors of Φ_ℓ and $\hat{S}_\ell$ ($\ell = e, \nu$ and $q = u, d$) from Φ_q and $\hat{S}_q$, respectively. However, since the deviations are very small (smaller than $O(1)$), arguments in this subsection will not give an essential influence on our main purpose which is to estimate the scales of U(3)×U(3)$'$.

We consider the following VEV deviation factors, η_Φ and η_S, of the lepton sector from the quark sector:

$$\|(\Phi_\ell)_\circ{}^\bullet\| = \eta_\Phi \Lambda_2, \qquad \|(\hat{S}_\ell)_\bullet{}^\bullet\| = \eta_S \Lambda_1, \tag{4.10}$$

which is slightly different from $\|(\Phi_q)_\circ{}^\bullet\| = \Lambda_2$ and $\|(\hat{S}_q)_\bullet{}^\bullet\| = \Lambda_1$. Here, we have consider that factors η_Φ and η_S are orders of one in contrast to factors ξ_ν and $\xi_{S\nu}$ with orders of 10^{-3} as shown later.

The modification (4.10) gives

$$\|(\hat{M}_\ell)_\circ{}^\circ\| = (\eta_\Phi)^2 (\eta_S)^{-1} \|(\hat{M}_q)_\circ{}^\circ\|, \tag{4.11}$$

so that (4.9) demands

$$\frac{(\eta_\Phi)^2}{\eta_S} = \frac{1}{3}, \quad i.e. \quad 3(\eta_\Phi)^2 = \eta_S. \tag{4.12}$$

On the other hand, the VEV relation (2.11) does not hold unless $\|(\Phi_\ell)_\circ{}^\bullet\| \cdot \|(\hat{S}_\ell)_\bullet{}^\bullet\| = \Lambda_2\Lambda_1$. Therefore, we put additional relation

$$\eta_\Phi \eta_S = 1, \tag{4.13}$$

so that we obtain

$$3(\eta_\Phi)^3 = 1 \quad \Rightarrow \quad \eta_\Phi = \frac{1}{\eta_S} = \frac{1}{3^{1/3}}. \tag{4.14}$$

As a result, we can give the parameter value ρ defined in Eq.(3.7) as $3^{1/3} = 1.44$ as seen in Eq.(5.30).

5 Flavons in the neutrino sector

We discuss Majorana mass matrix, Y_R, of the right-handed neutrinos ν_R. Note that the mass matrix $\bar{Y}_R$ is $(\mathbf{6}^*, \mathbf{1})$ of U(3)×U(3)′, i.e. $(\bar{Y}_R)^{\circ\circ}$.

In the present neutrino mass matrix model, the Dirac neutrino mass matrix $\hat{M}_\nu$ is given by Eq.(2.5), i.e.

$$(\hat{M}_\nu)_\circ{}^\circ = \frac{\langle H_u \rangle}{\Lambda_H} (\Phi_\nu)_\circ{}^\bullet (\hat{S}_\nu^{-1})_\bullet{}^\bullet (\bar{\Phi}_\nu)_\bullet{}^\circ. \tag{5.1}$$

In this section, since we pay attention to VEV scales of flavons, it is important whether those flavons belong to U(3) or U(3)′. From Eq.(5.1), we find that the scale of $(\hat{M}_\nu)_\circ{}^\circ$, $\|(\hat{M}_\nu)_\circ{}^\circ\|$, is given by

$$||\hat{M}_\nu^{Maj}|| \equiv \xi_M ||\hat{M}_e|| = \xi_M \frac{\langle H_u \rangle}{\Lambda_H} \frac{(\Lambda_2)^2}{\Lambda_1}. \tag{5.2}$$

Since $\|\hat{M}_e\|$ is given by the order of $(\langle H_d \rangle / \Lambda_H)(\Lambda_2)^2/\Lambda_1$ and we consider an additional seesaw (1.8), a ratio of the scales $\|M^{Maj}\|/\|\hat{M}_e\|$ is given by

$$R_{\nu/e} \equiv \frac{\|M_\nu^{Maj}\|}{\|\hat{M}_e\|} = (\xi_M)^2 \frac{v_H}{\Lambda_H} \frac{(\Lambda_2)^2}{\Lambda_1} \frac{1}{\|\bar{Y}_R\|}, \tag{5.3}$$

where $v_{Hu} = v_{Hd} \equiv v_H$ is defined in (2.6). In order to estimate the ratio (5.3), we have to build a model for $\bar{Y}_R$.

5.1 VEV structure of Majorana mass matrix $\bar{Y}_R$

For convenience, we denote the form $\langle \bar{Y}_R \rangle$ by the following terms

$$\mu_R \langle \bar{Y}_R \rangle^{\circ\circ} = \mu_R \left[(\bar{Y}_R^{1st})^{\circ\circ} + (\bar{Y}_R^{2nd})^{\circ\circ} \right], \tag{5.4}$$

with $\|(\bar{Y}_R^{1st})\| \gg \|(\bar{Y}_R^{2nd})\|$. In Eq.(5.4) , we denote the first and second terms in $\bar{Y}_R$ as $(\bar{Y}_R^{1st})$ and $(\bar{Y}_R^{2nd})$, but it does not mean that they are two new flavons. Considering that the mass hierarchy in the neutrino sector is mild compared with other sectors $f = u, d, e$, we assume that the first term of M_ν^{Maj}, $\hat{Y}_\nu (\bar{Y}_R^{1st})^{-1} \bar{Y}_\nu$ takes a form of diagonal matrix, and the observed mass differences and the PMNS mixing come form in the second term $\bar{Y}_R^{2nd}$. Since $\langle \hat{Y}_\nu \rangle \propto Z^2$ (Z is defined in (4.4)), we take a form $\bar{Y}_R^{1st} \propto Z^2$.

Since we want that the R charge of $\bar{Y}_R$ is as smaller as possible in order to avoid appearance of many combinations with the same R charge, we take a form of $(\bar{Y}_R^{1st})^{\circ\circ}$ with $R = 13/3$, $(\bar{\Phi}_e^T)^\circ{}_\bullet \bar{E}^{\bullet\bullet} (\bar{\Phi}_e)_\bullet{}^\circ (\Phi_u)_\circ{}^\bullet (\bar{\Phi}_u)_\bullet{}^\circ$, i.e.

$$\mu_R (\bar{Y}_R^{1st})^{\circ\circ} = \frac{1}{2} \frac{1}{(\Lambda_1)^2} \left\{ (\bar{\Phi}_e^T)^\circ{}_\bullet \bar{E}^{\bullet\bullet} (\hat{Y}_{eu})_\bullet{}^\bullet (\bar{\Phi}_u)_\bullet{}^\circ + (transposed) \right\} \tag{5.5}$$

Here, in order to adjust the scale of $\bar{Y}_R^{1st}$ compared with $\bar{Y}_R^{2nd}$, we introduce a new flavon $\hat{Y}_{eu}$ defined by

$$\mu_{eu}(\hat{Y}_{eu})_\bullet{}^\bullet = (\bar{\Phi}_e)_\bullet{}^\circ(\Phi_u)_\circ{}^\bullet. \tag{5.6}$$

Then, we obtain

$$\mu_R\|(\bar{Y}_R^{1st})^{\circ\circ}\| = (\eta_\Phi)^2\frac{\Lambda_1}{\mu_{eu}}\frac{(\Lambda_2)^4}{(\Lambda_1)^2}. \tag{5.7}$$

Next, we discuss a term $\bar{Y}_R^{2nd}$. We suppose that the deviation term $\bar{Y}_R^{2nd}$ gives Pontcorvo-Maki-Nakagawa-Sakata (PMNS) [15] mixing and neutrino mass ratios. Since we want to inherit the form of $\langle \bar{Y}_R\rangle$ from the previous model, in which the form of Y_R has included a term proportional to up-quark mass matrix $\hat{M}_u$ [16]. However, in the present model, there is no Yukawaon $\hat{Y}_u$. Therefore, we introduce a new flavon whose VEV is proportional to $\langle \hat{S}_u^{-1}\rangle$ with use of a superpotential given by

$$W_{S'_u} = \left\{\bar{E}^{\circ\bullet}(\hat{E})_\bullet{}^\bullet + (\bar{S}'_u)^{\circ\bullet}(\hat{S}_u)_\bullet{}^\bullet\right\}(\Theta'_{Su})_{\bullet\circ}, \tag{5.8}$$

where, for simplicity, we have drop the coefficients λ_1 and λ_2 although we suppose $\lambda_1 \simeq \lambda_2 \simeq 1$. Here, since $R(\hat{E}_\bullet{}^\bullet) = 10/5$ and $R(E_{\circ\bullet}) = 1/3$, the flavon $\hat{\Theta}'_{Su}$ has zero VEV value and R charge $R = 0$. The flavon $(\bar{S}'_u)^{\circ\bullet}$ with $R = 5/3$ can obtain the VEV form proportional to $\langle \hat{S}_u^{-1}\rangle$. Thus, we take a small deviation term $\bar{Y}_R^{2nd}$ as follows:

$$\mu_R(\bar{Y}_R^{2nd})^{\circ\circ} = \frac{1}{2}\frac{1}{(\Lambda_1)^2}\left\{(\bar{\Phi}_d)^\circ{}_\bullet(\bar{\Phi}_u^T)^\bullet{}_\circ(\bar{S}'_u)^{\circ\bullet}(\bar{\Phi}_u)_\bullet{}^\circ + (transposed)\right\}. \tag{5.9}$$

Then, we estimate of a scale of (5.9) as

$$\mu_R\|(\bar{Y}_R^{2nd})\| = \frac{(\Lambda_2)^4}{(\Lambda_1)^2}. \tag{5.10}$$

Therefore, we obtain the ratio

$$R_{2/1} \equiv \frac{\|\bar{Y}_R^{2nd}\|}{\|\bar{Y}_R^{1st}\|} = \frac{1}{(\eta_\Phi)^2}\frac{\mu_{eu}}{\Lambda_1}. \tag{5.11}$$

5.3 Parameter ξ_R

Parameter fitting for neutrino data such as neutrino masses and PMNS lepton mixing matrix is done under the following dimensionless re-expression:

$$\tilde{M}_\nu^{Maj} = (\xi_M)^4 Z^2 \tilde{Y}_R^{-1} Z^2, \tag{5.12}$$

where

$$\tilde{Y}_R = Z^4 + \xi_R Z^2 P(\mathbf{1} + b_u X_3)^{-1} P^\dagger Z, \tag{5.13}$$

$$\begin{aligned} Z &= \mathrm{diag}(z_1, z_2, z_3), \\ P &= \mathrm{diag}(e^{i\tilde{\phi}_1}, e^{i\tilde{\phi}_2}, 1). \end{aligned} \tag{5.14}$$

(Z and P do not mean new flavons. Those are noting but dimensionless 3×3 matrices.) The parameter ξ_R corresponds to the ratio $R_{2/1}$ defined in (5.11). Since the parameter values b_u and $(\tilde{\phi}_1, \tilde{\phi}_2)$ have already determined from the CKM fitting as shown in (2.25) and (2.26), only a free parameter in the neutrino sector is ξ_R in (5.13). (The parameter fitting is practically the same as one in the previous model [7]. However, note that the present parameter ξ_R corresponds to $2/\xi_R$ in the previous model.) The best fitting value of ξ_R is

$$\xi_R = 0.9806 \times 10^{-3}. \tag{5.15}$$

The detail fitting is reviewed in Appendix B.

5.4 Scales of μ parameters

We have four flavon mass parameters μ_ν, $\mu_{S\nu}$, μ_{eu} and μ_R defined by (2.18), (2.22), (5.6), and (5.5), respectively. So far, we have considered three VEV scales Λ_1, Λ_2, and Λ_3 as shown in (1.7) and also in Table 1. Therefore, let us put the following selection rules for μ parameters defined by

$$\mu_A \|A\| = \|G\|, \tag{5.16}$$

where G is a combination of some flavons including factor $1/(\Lambda_1)^n$: (i) We assume that our parameters μ_A such as μ_ν, $\mu_{S\nu}$, μ_{eu}, and μ_R are given by some of those three scales Λ_1, Λ_2 and Λ_3. (ii) If our choice $\mu_A = \Lambda_a$ $(a = 1, 2, 3)$, as a result, gives a VEV value of $\|A\|$ which is larger than the maximally allowed scale of $\|A\|$, e.g. Λ_1 for $\|(\hat{Y}_{eu})_\bullet{}^\bullet\|$, Λ_2 for $\|(\Phi_\nu)_\circ{}^\bullet\|$, and Λ_3 for $\|(\bar{Y}_R)^{\circ\circ}\|$, then, the choice is ruled out. (iii) If the choice $\mu_A = \Lambda_a$ gives a VEV value $\|A\|$ which is two rank lower compared with $\|A\|_{max}$, the choice is also ruled out.

For example, let us see the case (5.6):

$$\mu_{eu} \|(\hat{Y}_{eu})_\bullet{}^\bullet\| = \eta_\Phi (\Lambda_2)^2 = \eta_\Phi \rho \Lambda_1 \Lambda_3. \tag{5.17}$$

If we tale $\mu_{eu} = \Lambda_3$, we obtain $\|\hat{Y}_{eu}\| = \eta_\Phi \rho \Lambda_1$ from Eq.(5.17). Since $\|(\hat{Y}_{eu})_\bullet{}^\bullet\| \leq \Lambda_1$, the case $\mu_{eu} = \Lambda_3$ is not ruled out by the rule (ii). However, the case give $R_{2/1} = (\eta_\Phi)^{-2} \sim O(1)$ from (5.11). This contradicts with the parameter fitting result (5.15). Therefore, we rule out this case $\mu_{eu} = \Lambda_3$. On the other hand, if we take $\mu_{eu} = \Lambda_1$, we obtain $\|(\hat{Y}_{eu})_\bullet{}^\bullet\| = \rho \Lambda_3$. However, the value is two rank small compared with the maximal value Λ_1, so that we rule out the case $\mu_{eu} = \Lambda_1$ by the rule (iii). As a result, we choose $\mu_{eu} = \Lambda_2$, and then we obtain

$$\|(\hat{Y}_{eu})_\bullet{}^\bullet\| = \eta_\Phi \Lambda_2. \tag{5.18}$$

Then, from the relation (5.11), we obtain

$$\xi_R \equiv R_{2/1} = \frac{1}{(\eta_\Phi)^2} \frac{\Lambda_2}{\Lambda_1}. \tag{5.19}$$

Similarly, from the relation (2.18), we have

$$\mu_\nu \|(\Phi_\nu)_\circ{}^\bullet\| = \eta_\Phi \frac{(\Lambda_2)^2}{\Lambda_1}\Lambda_2 = \eta_\Phi \rho \Lambda_3 \Lambda_2 = \rho \Lambda_3 \|(\Phi_e)_\circ{}^\bullet\|. \tag{5.20}$$

If we take take $\mu_\nu = \Lambda_3$, we obtain $\|(\Phi_\nu)_\circ{}^\bullet\| = \rho\|(\Phi_e)_\circ{}^\bullet\|$, i.e. $\xi_\nu = \rho > 1$. The result is not our desired one, because our aim is to understand tiny neutrino masses by $\xi_\nu \ll 1$. Therefore, we choose $\mu_\nu = \Lambda_2$, and we get

$$\|(\Phi_\nu)_\circ{}^\bullet\| = \eta_\Phi \rho \Lambda_3 = \rho \frac{\Lambda_3}{\Lambda_2} \eta_\Phi \Lambda_2 = \frac{\Lambda_2}{\Lambda_1} \|(\hat{S}_e)_\bullet{}^\bullet\|, \tag{5.21}$$

so that we have

$$\xi_\nu = \frac{\Lambda_2}{\Lambda_1} = (\eta_\Phi)^2 \xi_R. \tag{5.22}$$

from (5.11).

Similarly, when we choose $\mu_{S\nu} = \Lambda_2$ in Eq.(2.23), from

$$\mu_{S\nu} (\hat{S}_\nu)_\bullet{}^\bullet = \eta_S \Lambda_1 \frac{(\Lambda_2)^2}{\Lambda_1} = \eta_S \rho \Lambda_1 \Lambda_3, \tag{5.23}$$

we obtain

$$\|(\hat{S}_\nu)_\bullet{}^\bullet\| = \eta_S \frac{\Lambda_2}{\Lambda_1} \Lambda_1 = \frac{\Lambda_2}{\Lambda_1} \|\hat{S}_e\|, \tag{5.24}$$

so that we have

$$\xi_{S\nu} = \frac{\Lambda_2}{\Lambda_1} = (\eta_\Phi)^2 \xi_R = \xi_\nu, \tag{5.25}$$

from Eq.(5.22). Therefore, from the relation (2.24), we obtain

$$\xi_M = (\xi_\nu)^2 (\xi_{S\nu})^{-1} = \xi_\nu = (\eta_\phi)^2 \xi_R. \tag{5.26}$$

Finally, we discuss a scale of μ_R. From (5.7), by regarding Y_R as $Y_R \simeq Y_R^{1st}$, we can write

$$\mu_R \|(\bar{Y}_R)^{\circ\circ}\| = \frac{1}{(\Lambda_1)^2} (\eta_\Phi)^2 \Lambda_2 \Lambda_1 \Lambda_2 \Lambda_2 = \rho (\eta_\Phi)^2 \Lambda_2 \Lambda_3. \tag{5.27}$$

If we take $\mu_R = \Lambda_2$, we obtain

$$\|(\bar{Y}_R)^{\circ\circ}\| = \rho(\eta_\Phi)^2\Lambda_3. \tag{5.28}$$

Since $\|(\bar{Y}_R)^{\circ\circ}\|$ cannot have a larger scale than Λ_3, we have a constraint

$$\rho(\eta_\Phi)^2 \leq 1, \quad i.e. \;\; \rho \leq (\eta_\Phi)^{-2} = 3^{2/3}. \tag{5.29}$$

Comparing the fitting value $\rho = 1.37$ with the value $(\eta_\Phi)^{-1} = 3^{1/3} = 1.44$ in (4.14), it is likely that the value of ρ is given in unit of $3^{1/3}$, so that we regard the value of ρ as

$$\rho = \frac{1}{\eta_\Phi} = 3^{1/3}. \tag{5.30}$$

Then, this choice satisfies the condition $\|Y_R\| \leq \Lambda_3$ in (5.28).

6 Estimate of flavon VEV scales

Similarly to Eq.(4.1), from the diagonalization of (5.12) with the parameter values (2.25), (2.26), and (5.15), we obtain

$$(m_{\nu1}, m_{\nu2}, m_{\nu3}) = (0.30892, 0.31689, 0.51026)\, m_{0\nu}. \tag{6.1}$$

Here, we have defined m_{0R} by

$$m_{0\nu} = (\xi_M)^2\frac{(m_{0e})^2}{m_{0R}}. \tag{6.2}$$

From the observed values [17] $\Delta m^2_{32} = 0.000244$ eV^2 and $\Delta m^2_{21}/\Delta m^2_{32} = 0.0309$, we estimate $m_{\nu3} = 0.063$ eV. Thus we obtain

$$m_{0\nu} = 1.235 \times 10^{-13} \text{ TeV}. \tag{6.3}$$

Then, from Eq.(6.2) with (6.3), and (4.7), we obtain ,

$$m_{0R} = (\xi_M)^2\frac{(m_{0e})^2}{m_{0\nu}} = (\xi_M)^2 \times 2.7710 \times 10^7 \text{ TeV}. \tag{6.4}$$

From Eqs.(5.26) and (4.14), we estimate

$$m_{0R} = (\eta_\Phi)^4(\xi_R)^2 \times 2.7710 \times 10^7 \text{ TeV} = 6.158 \text{ TeV}. \tag{6.5}$$

Therefore, from (5.28) with (5.30), i.e. $\|Y_R\| = \eta_\Phi\Lambda_3$, we obtain

$$\Lambda_3 = (\eta_\Phi)^{-1}m_{0R} = 8.883 \text{ TeV}. \tag{6.6}$$

In conclusion, we obtain

$$\begin{aligned}
\Lambda_3 &= 8.883 \text{ TeV}, \\
\Lambda_2 &= \rho\Lambda_3/\xi_R = 1.306 \times 10^4 \text{ TeV}, \\
\Lambda_1 &= \Lambda_2/\xi_R = 1.332 \times 10^7 \text{ TeV}.
\end{aligned} \tag{6.7}$$

Finally, we estimate the value of Λ_H defined in (2.5). From (2.5), we use a relation

$$m_{0e} = \|\Psi_e(\hat{S}_e)^{-1}\bar{\Phi}_e\| = \frac{v_{Hd}}{\Lambda_H}(\eta_\Phi)^2(\eta_S)^{-1}\frac{(\Lambda_2)^2}{\Lambda_1} = \frac{v_H}{\Lambda_H}(\eta_\Phi)^2(\eta_S)^{-1}\rho\Lambda_3, \tag{6.8}$$

where $m_{0e} = 1.8499 \times 10^{-3}$ TeV, $v_H = 173.9 \times 10^{-3}$ TeV, and $(\eta_\Phi)^2(\eta_S)^{-1}\rho = 3^{-2/3}$, so that we obtain

$$\Lambda_H = 401.4 \text{ TeV}. \tag{6.9}$$

The result (6.9) gives $(v_H)/\Lambda_H = 0.4332 \times 10^{-3}$.

7 Concluding remarks

In conclusion, we have investigated a unified quark and lepton mass matrix model on the basis of U(3)×U(3)′ family symmetry. We have inherited a basic aim of a series of the so-called Yukawaon models [3]. However, in the present model, we have not assumed existence of Yukawaons. Instead, we have introduced triplet fermions F_α of U(3)′ in addition to quarks and leptons f_i which are triplets of U(3), and we assumed a seesaw-like mechanism in (2.1).

The U(3)′ symmetry is broken into S_3 at a scale $\mu = \Lambda_1$, and the U(3) symmetry is broken at $\mu = \Lambda_2$ by VEVs of flavons $\langle (\Phi_f)_i^{\ \alpha} \rangle$, which are $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′. However, in this paper, we do not ask the origin of the VEV form $\langle \Phi_f \rangle \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, and do not discuss what mechanism leads to such a VEV form. We consider that it is too early to investigate the origin of the values m_{ei}. It is our future task.

The purpose of the present paper is not to give parameter fitting for quark and lepton masses and mixing because the VEV structures are the same as previous model, although the present model is completely different from the previous one. In the present model, we have only six free family-number-independent parameters for quark and lepton mass ratios and mixing as well as in the previous model. We have used the values $b_u = -1.011$, $b_d = -3.522\, e^{i\, 17.7^\circ}$, $(\tilde{\phi}_1, \tilde{\phi}_2)$, and $\xi_R = 0.4903 \times 10^{-3}$, whose values have been quoted from the previous paper [1].

In this paper, we have investigated the symmetry-breaking scales Λ_1 and Λ_2 from phenomenological study in the neutrino data. Our essential hypothesis is that all flavon VEVs (and also μ parameters) take one of three scales Λ_1, Λ_2, and Λ_3 except for $H_{u/d}$ and Λ_H. First, we have fixed the ratio $\Lambda_2/\Lambda_1 \sim \Lambda_3/\Lambda_2 \sim 10^{-3}$ from the ϕ_i-m_{ei} relation with observed data as discussed in Sec.3. (We would like to emphasize that the relations (3.3) with the numerical fitting values (3.5) are essential for our result $\Lambda_2/\Lambda_1 \sim \Lambda_3/\Lambda_2$, and the relations (3.3) are completely different from the previous one [1, 2].) Then, we have obtain the value of $\Lambda_1 \sim 4 \times 10^7$ TeV, $\Lambda_2 \sim 2 \times 10^4$ TeV, and $\Lambda_3 \sim 10^1$ TeV. Here, the value $\xi_R \sim 10^{-3}$ has been obtained for the parameter ξ_R which is only the free parameter in the neutrino sector after remaining free parameters have been fixed by fitting the observed quark mass ratios and CKM mixing . The scale Λ_3 has been fixed from the scale of the Majorana mass matrix $\langle (\bar{Y}_R)^{\circ\circ} \rangle$.

Note that our estimate of Λ_1, Λ_2, and Λ_3 is highly dependent on the VEV structure of the flavon $\bar{Y}_R$, which corresponds to the Majonara mass matrix of right-handed neutrinos ν_R. We have inherited the VEV structure from the previous model, which has given excellent description of neutrino masses and mixing with only one parameter ξ_R phenomenologically. The structure is dependent on the R charge assignment. The present R charge assignment $(r_u, r_e, r_d, r_\nu) = (1/3, 1, 6/3, 10/3)$ is one of a few possible cases which do not cause theoretical trouble, although it has no theoretical ground. Investigation of the logical necessity of this assignment is a future task to us.

In Sec.4, we have discussed somewhat trifling parameterization with an additional ansatz. The orders of three VEV scales can roughly be obtained without these parameters in Sec.4. However, owing to this additional parametrization, we can understand $m_{0u}/m_{0e} = 3$ and $\rho \equiv c_2/c_1 \simeq 3^{1/3}$ consistently.

Although the present model still leaves some of tasks in future, we consider that the outline of the model is worthy to notice.

Acknowledgments

The authors thank T. Yamashita and Y. Sumino for helpful discussions on the effective superpotential and helpful comments. The work was supported by JSPS KAKENHI Grant number JP16K05325 (Y.K.).

Appendix A: Brief review of Sumino's family gauge boson model

A charged lepton mass relation is well-known:

$$K(m_{ei}) \equiv \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\tau} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}. \tag{A.1}$$

The formula (A.1) was speculated on the basis of a U(3) family symmetry [12]. Therefore, the U(3) formula is satisfied only at an energy scale $\mu = \Lambda_X$ and only with charged lepton masses m_{ei}, where whole the electroweak and strong interactions are switched off. On the other hand, 10 years after the proposal of the U(3) formula, the precise value of the tau lepton mass was reported by ARGUS, BES and CLEO collaborations [18], so that an empirical formula

$$K(m_{ei}^{pole}) = (2/3) \times (0.999989 \pm 0.000014), \tag{A.2}$$

attracted considerable notice. However, we should distinguish the empirical formula (A.2) from the U(3) model formula (A.1). The coincidence of (A.2) may be an accidental coincidence. The formula (A.1) is never satisfied by pole masses.

Nevertheless, in 2009, Sumino [8] has directed his attention to the empirical coincidence (A.2). The deviation of $K(m_{ei}^{run})$ from $K = 2/3$ by using running masses m_{ei}^{run} comes mainly from a logarithmic term $\log m_{ei}$ [19] in the QED loop diagram. Therefore, Sumino has considered family gauge boson masses M_{ii} which are proportional to the charged lepton masses m_{ei}, and thereby, he has assumed the factor $\log m_{ei}$ in the QED term is canceled by the factor $\log M_{ii}$ in the corresponding family gauge boson diagram.[3] The Sumino model is based on a U(3)×O(3) gauge model and the charged lepton mass matrix M_e is given by

$$M_{ij} = (\Phi)_{i\alpha}(\Phi)_{\alpha j}, \tag{A.3}$$

where Φ is a scalar of $(\mathbf{3}, \mathbf{3})$ of U(3)×O(3) with VEV $\langle\Phi\rangle = k\,\mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, so that the U(3) familly gauge boson masses M_{ij} have

$$M_{ij}^2 \propto (m_{ei} + m_{ej}), \tag{A.4}$$

in the limit of $\Lambda_{U(3)} \ll \Lambda_{O(3)}$. Sumino has speculated that the scale Λ_2, at which the formula (A.1) is created, is of the order of 10^3 TeV [8].

The present model based on U(3)×U(3)′ in this paper is an extended version of the Sumino model. Therefore, one of our interest in this paper is to know what is the scale of U(3). However, note that our present model has nothing to do with the charged lepton mass relation (A.1) and/or Sumino's cancellation mechanism. In our model, the parameters (z_1, z_2, z_3) are given by the observed charged lepton masses, and the purpose of the model is not to give the values for (z_1, z_2, z_3).

Appendix B: Parameter fitting for masses and mixings

As far as quark sector is concerned, the VEV matrices are exactly the same as those in the previous paper, although the model is completely different from the previous one. Therefore, we quote only the numerical results of the previous paper [1, 2]. For the details, see Ref.[1, 2]. Note that the parameter fitting in Ref.[1, 2] has been done at $\mu = m_Z$, so that, for the charged lepton mass values, too, we have used the values at $\mu = m_Z$, not pole mass values.

In our model, the parameters (b_u, b_d) and $(\tilde{\phi}_1, \tilde{\phi}_2)$ are fixed by quark masses and CKM quark mixing fitting. We have chosen the parameter values as

$$b_u = -1.011, \qquad b_d = -3.522\, e^{i\ 17.7^\circ}, \tag{B.1}$$

[3] It seems that some of readers somewhat misunderstand this cancellation mechanism. Sumino's cancellation does not mean a complete cancellation. Obviously the cancellation mechanism is effective only for one-loop diagram. Moreover, there are another diagrams, for example, Higgs diagram, stau diagram, and so on. Sumino mechanism is applicable only within the range of present experimental errors, and within the not so wide energy range. Sumino said that the deviation will be observed if the present error of tau lepton mass will be improved by one order.

Table 2: Predicted values vs. observed values in quark sector. Input values are given by Eq.(2.25) and (2.26). r^q_{12} and r^q_{23} are defined by $r^u_{12} = \sqrt{m_u/m_c}$, $r^u_{23} = \sqrt{m_c/m_t}$, $r^d_{12} = m_d/m_s$ and $r^d_{23} = m_s/m_b$. The predicted values are same as those in the previous paper[1].

	$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert$	$\lvert V_{td}\rvert$	$\delta^q_{CP}(^\circ)$	r^u_{12}	r^u_{23}	r^d_{12}	r^d_{23}
Predicted	0.2257	0.03996	0.00370	0.00917	81.0	0.061	0.060	0.049	0.027
Observed	0.22536	0.0414	0.00355	0.00886	69.4	0.045	0.060	0.053	0.019
	± 0.00061	± 0.0012	± 0.00015	$^{+0.00033}_{-0.00032}$	± 3.4	$^{+0.013}_{-0.010}$	± 0.005	$^{+0.005}_{-0.003}$	$^{+0.006}_{-0.006}$

and

$$(\tilde{\phi}_1, \tilde{\phi}_2) = (-176.05^\circ,\ -167.91^\circ). \tag{B.2}$$

Then, we can obtain reasonable quark mass ratios and CKM mixing as shown in Table.2.

Parameter fitting for neutrino data is done under the following dimensionless re-expression:

$$\tilde{M}^{Maj}_\nu = (\xi_M)^4 Z^2 \tilde{Y}_R^{-1} Z^2, \tag{B.3}$$

where

$$\tilde{Y}_R = Z^4 + \xi_R Z^2 P(\mathbf{1} + b_u X_3)^{-1} P^\dagger Z, \tag{B.4}$$

$$\begin{array}{l} Z = \mathrm{diag}(z_1, z_2, z_3), \\ P = \mathrm{diag}(e^{i\tilde{\phi}_1}, e^{i\tilde{\phi}_2}, 1). \end{array} \tag{B.5}$$

(Z and P do not mean new flavons. Those are noting but dimensionless 3×3 matries.)

Since the parameter values b_u and $(\tilde{\phi}_1, \tilde{\phi}_2)$ have already determined from the CKM fitting as shown in (B.1) and (B.2), only a free parameter in the neutrino sector is ξ_R in (B.4) as discussed in subsection 5.3. (The parameter fitting is practically the same as one in the previous model [7]. However, note that the present parameter ξ_R corresponds to $2/\xi_R$ in the previous model.)

Therefore, as shown in the previous paper, the PMNS mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, CP violating Dirac phase parameter $\delta^\ell_{\mathrm{CP}}$, and the neutrino mass squared difference ratio $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32}$ are turned out to be functions of the remaining only one parameter ξ_R. In Fig. 1, we draw the curves as functions of our new ξ_R with taking $b_u = -1.011$, and $b_d = -3.3522$, $\beta_d = 17.7^\circ$, and $(\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ)$. From Fig.1, it turns out that the best fitting value of ξ_R in the present paper is

$$\xi_R = 0.9806 \times 10^{-3}, \qquad (\ \frac{2}{\xi_R} = 2039.6\ ), \tag{B.6}$$

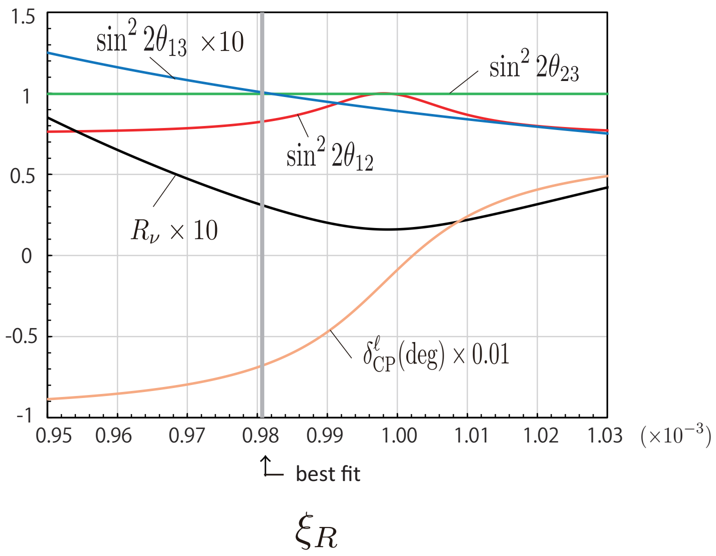

Figure 1: ξ_R dependence of the lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared difference ratio $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32}$. For the input values of b_u and (ϕ_1, ϕ_2), see the text.

Table 3: Predicted values vs. observed values in neutrino sector. Input value is only ξ_R given by Eq.(B.6) except for values (B.1) and (B.2) in the quark sector.

	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	R_ν (10^{-2})	$\delta^\ell_{CP}(^\circ)$	$m_{\nu 1}$ (eV)	$m_{\nu 2}$ (eV)	$m_{\nu 3}$ (eV)	$\langle m \rangle$ (eV)
Predicted	0.8254	0.9967	0.1007	3.118	-68.1	0.038	0.039	0.063	0.021
Observed	0.846	0.999	0.093	3.09	no data	no data	no data	no data	$< \mathrm{O}(10^{-1})$
	± 0.021	$^{+0.001}_{-0.018}$	± 0.008	± 0.15					

which gives the predictions

$$\sin^2 2\theta_{12} = 0.8254, \quad \sin^2 2\theta_{23} = 0.9967, \quad \sin^2 2\theta_{13} = 0.1007, \quad \delta^\ell_{\mathrm{CP}} = -68.1^\circ, \quad R_\nu = 0.03118, \tag{B.7}$$

as shown in Table 3. These predictions are in good agreement with the observed values [17].

The neutrino masses are predicted as $m_{\nu 1} \simeq 0.038$ eV, $m_{\nu 2} \simeq 0.039$ eV, and $m_{\nu 3} \simeq 0.063$ eV, by using the input value [17] $\Delta m^2_{32} \simeq 0.00244$ eV^2. We have also predicted the effective Majorana neutrino mass [20] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = \left| m_{\nu 1}(U_{e1})^2 + m_{\nu 2}(U_{e2})^2 + m_{\nu 3}(U_{e3})^2 \right| \simeq 21 \ \ \mathrm{meV}. \tag{B.8}$$

It is interesting that the model predicts $\delta^\ell_{CP} = -68^\circ$, which shows $\delta^\ell_{CP} \simeq -\delta^q_{CP}$. It is also worthwhile noticing that we obtain a large value 21 meV for the effective Majorana neutrino mass $\langle m \rangle$ in spite of the normal hierarchy for the neutrino mass in our model.

References

[1] Y. Koide and H. Nishiura, Phys. Rev. **D 92**, 111301(R) (2015).

[2] Y. Koide and H. Nishiura, Mod. Phys. Lett. A **31**, 1650125 (2016).

[3] Y. Koide, Phys. Rev. **D 79**, 033009 (2009); Phys. Lett. **B 680**, 76 (2009); H. Nishiura and Y. Koide, Phys. Rev. **D 83**, 035010 (2011) ; Y. Koide and H. Nishiura, Euro. Phys. J. **C 72**, 1933 (2012); Y. Koide, J. Phys. **G 38**, 085004 (2011); Y. Koide and H. Nishiura, Euro. Phys. J. **C 73**, 2277 (2013); JHEP **04**, 166 (2013); Phys. Rev. **D 88**, 116004 (2013); Phys. Rev. **D 90**, 016009 (2014).

[4] C. D. Froggatt and H. B. Nelsen, Nucl. Phys. **B 147**, 277 (1979). For recent works, for instance, see R. N. Mohapatra, AIP Conf. Proc. **1467**, 7 (2012); A. J. Buras *et al.*, JHEP **1203**, 088 (2012).

[5] Y. Koide and H. Fusaoka, Z. Phys. **C 71**, 459 (1996).

[6] Y. Koide and H. Nishiura, Phys. Lett. **B 712**, 396 (2012).

[7] Y. Koide and H. Nishiura, Phys. Rev. **D 91**, 116002 (2015).

[8] Y. Sumino, Phys. Lett. **B 671**, 477 (2009); JHEP, 0905 (2009).

[9] Y. Koide and T. Yamashita, Phys. Lett. **B 711**, 384 (2012).

[10] N. Cabibbo, Phys. Rev. Lett. **10**, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49**, 652 (1973).

[11] Y. Koide, Phys. Lett. **B 736**, 499 (2014).

[12] Y. Koide, Lett. Nuovo Cim. **34**, 201 (1982); Phys. Lett. B **120**, 161 (1983); Phys. Rev. D **28**, 252 (1983).

[13] P. Minkowski, Phys. Lett. **B 67**, 421 (1977); M. Gell-Mann, P. Ramond, and R. Slansky, Proceedings of the Supergravity Stony Brook Workshop, New York, 1979, edited by P. Van Nieuwenhuizen and D. Freedman (North-Holland, Amsterdam, 1979); T. Yanagida, Proceedings of the Workshop on Unified Theories and Baryon Number in the Universe, Tsukuba, Japan 1979, edited by A. Sawada and A. Sugamoto [KEK Report No. 79-18, Tsukuba]; R. Mohapatra and G. Senjanovic, Phys. Rev. Lett. **44**, 912 (1980).

[14] Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Rev. **D 77**, 113016 (2008). Also see H. Fusaoka and Y. Koide, Phys. Rev. **D 57**, 3986 (1998).

[15] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957); **34**, 247 (1957); Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28**, 870 (1962).

[16] Y. Koide, J. Phys. **G 35**, 125004 (2008); Phys. Lett. **B 665**, 227 (2008).

[17] K. A. Olive *et al.* (Particle Data Group), Chinese Phys. C, **38**, 090001 (2014).

[18] H. Albrecht *et al.* ARGUS collab., Phys.=Lett. **B292**, 221 (1992); J. Z. Bai *el al.* BES collab., Phys. Rev. Lett. **69**, 3021 (1982); M. Daoudi *et al.* CLEO collab., a talk given at the XXVI int. Conf. on High Energy Physics, Dallas, 1992.

[19] H. Arason, *et al.*, Phys. Rev. **D 46**, 3945 (1992).

[20] M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Phys. Lett. **B 103**, 219 (1981) and **B 113**, 513 (1982).