Structure of Right-Handed Neutrino Mass Matrix

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Abstract

Recently, Nishiura and the author have proposed a unified quark-lepton mass matrix model under a family symmetry U(3)×U(3)′. The model can give excellent parameter-fitting to the observed quark and neutrino data. The model has a reasonable basis as far as the quark sector, but the form of the right-handed neutrino mass matrix M_R does not have a theoretical grand, that is, it was nothing but a phenomenological assumption. In this paper, it is pointed out that the form of M_R is originated in structure of neutrino mass matrix for (ν_i, N_α) where ν_i $(i = 1, 2, 3)$ and N_α $(\alpha = 1, 2, 3)$ are U(3)-family and U(3)′-family triplets, respectively.

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1 Introduction

Recently, Nishiura and the author [**?**, **?**] have proposed a unified mass matrix model under a family symmetry U(3)×U(3)′:

$$(\bar{f}_L^i \quad \bar{F}_L^\alpha) \left(\begin{array}{cc} (0)_i^{\ j} & \langle\Phi_f\rangle_i^{\ \beta} \\ \langle\bar{\Phi}_f\rangle_\alpha^{\ j} & -\langle\hat{S}_f\rangle_\alpha^{\ \beta} \end{array} \right) \left(\begin{array}{c} f_{Rj} \\ F_{R\beta} \end{array} \right), \quad (f = u, d, e) \tag{1.1}$$

where f_i $(i = 1, 2, 3)$ and F_α $(\alpha = 1, 2, 3)$ are U(3)-family and U(3)′-family triplets, respectively, so that we obtain a Dirac mass matrix of f-sector as follows,

$$(\hat{M}_f)_i^{\ j} = \langle\Phi_f\rangle_i^{\ \alpha} \langle\hat{S}_f^{-1}\rangle_\alpha^{\ \beta} \langle\bar{\Phi}_f\rangle_\beta^{\ j}, \tag{1.2}$$

under a seesaw approximation. (Hereafter, we denote U(3)-family nonet scalars as a notation $(\hat{A})_i^{\ j}$ and anti-6-plet scalars as a notation $(\bar{A})^{ij}$.) Here, the VEV matrices $\langle\Phi_f\rangle$ are given by

$$\begin{array}{l} \langle\Phi_e\rangle = m_{0e}\mathrm{diag}(z_1, z_2, z_3), \\ \langle\Phi_d\rangle = m_{0d}\mathrm{diag}(z_1, z_2, z_3), \\ \langle\Phi_u\rangle = m_{0u}\mathrm{diag}(z_1 e^{i\phi_1}, z_2 e^{i\phi_2}, z_3 e^{i\phi_3}). \end{array} \tag{1.3}$$

Since we assume that the U(3)′ symmetry is broken into a discrete symmetry S_3, the vacuum expectation value (VEV) of $\hat{S}_f$ has, in general, to take a VEV form

$$\langle\hat{S}_f\rangle = m_{0f}\left(\mathbf{1} + b_f X_3\right), \tag{1.4}$$

where $\mathbf{1}$ and X_3 are defined by

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{1.5}$$

and b_f are complex parameters. We take b_e as $b_e = 0$, so that the parameters z_i are fixed as

$$z_i = \frac{\sqrt{m_{ei}}}{\sqrt{m_{e1} + m_{e2} + m_{e3}}}, \tag{1.6}$$

where $(m_{e1}, m_{e2}, m_{e3}) = (m_e, m_\mu, m_\tau)$. We may approximately regard m_{ei} in Eq.(1.6) as the observed charged lepton masses m_{ei}^{obs}. However, note that the vales m_{ei} in Eq.(1.6) are not always the eigenvalues $(\hat{M}_e)_i{}^i$ $(i = 1, 2, 3)$ of $\hat{M}_e$ given in Eq.(1.2), and m_{ei} are, in general, given by $\hat{M}_i{}^i = k_0 m_{ei}$ $(i = 1, 2, 3)$ with an arbitrary family-number-independent constant k_0.

The model [**?**] can successfully describe the observed quark masses and Cabibbo-Maskawa-Kobayashi (CKM) [**?**] mixings, especially, not only ratios among $m_{ui} = (m_u, m_c, m_t)$ and among $m_{id} = (m_d, m_s, m_b)$, but also ratios m_{ui}/m_{dj} when we take $m_{0u} = m_{0d}$. (The quark mass matrix structure has first been proposed by Fusaoka and the author [**?**] from the phenomenological point of view.)

In the neutrino sector, according to the conventional neutrino seesaw model [**?**], we consider that the Majorana mass matrix of the left-handed neutrino is given by

$$(M_\nu)_{ij} = (\hat{M}_\nu)_i{}^k (M_R^{-1})_{kl} (\hat{M}_\nu^T)^l{}_j, \tag{1.7}$$

under the Majirana mass matrix M_R of the right-handed neutrino ν_R with a large mass scale, where $\hat{M}_\nu$ is a Dirac mass matrix of neutrinos defined as $(\bar{\nu}_L)^i (\hat{M}_\nu)_i{}^j (\nu_R)_j$. However, in the U(3)×U(3)′ model, the structure of M_R has been given by a somewhat strange form

$$M_R \propto \Phi_\nu \hat{M}_u + \hat{M}_u^T \Phi_\nu + \xi_R \hat{M}_\nu (\hat{M}_\nu)^T, \tag{1.8}$$

where

$$\hat{M}_\nu \equiv \Phi_\nu \bar{\Phi}_\nu, \tag{1.9}$$

$$\Phi_\nu = m_{0\nu} \mathrm{diag}(z_1, z_2, z_3), \tag{1.10}$$

similar to Eq.(1.3). Note that M_R in Eq.(1.8) includes the up-quark mass matrix $\hat{M}_u$. When we use the VEV values of $\hat{M}_u$ fitted in the quark sector, the neutrino mass matrix M_R is described by only one parameter ξ_R, and we can obtain excellent fitting [**?**] for the observed neutrino masses and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [**?**] mixings. (A form M_R which is related up-quark mass matrix M_u has first been proposed by the author [**?**] in somewhat different context $M_R \propto (M_u^D)^{1/2} M_e^D + \cdots$.)

The model has a reasonable basis as far as the quark sector, while the form M_R (1.8) does not have a theoretical grand, and it was nothing but a phenomenological assumption. In this paper, it is pointed out the structure of M_R is originated in the structure of neutrino mass matrix for (ν_i, N_α) where ν_i and N_α are U(3)-family and U(3)′-family triplets, respectively.

2 Basic idea

Correspondingly to the seesaw mass matrix (1.6), we consider a seesaw mass matrix

$$(M_\nu)_{ij} = (\Phi_\nu)_i{}^\alpha (M_R^{-1})_{\alpha\beta} (\Phi_\nu^T)^\beta{}_j. \tag{2.1}$$

(Hereafter, in order to make the transformation property in U(3) and U(3)′ symmetry visual, we use symbols ∘ and • instead of indexes $i, j, \cdots$ and $\alpha, \beta, \cdots$.) From the definition (1.1) of the Majorana neutrino mass matrix $(\bar{M}_R)^{\circ\circ}$, we introduce a Majorana mass matrix, $(M_R)^{\bullet\bullet}$, for $(N_R)_\bullet$, as follows:

$$\bar{M}_R^{\circ\circ} = (\bar{\Phi}_\nu^T)^\circ{}_\bullet (\bar{M}_R)^{\bullet\bullet} (\bar{\Phi}_\nu)_\bullet{}^\circ. \tag{2.2}$$

When we neglect U(3)×U(3)′ indexes, from the relation (1.8), i.e.

$$(M_R)^{\circ\circ} = \xi_R(\Phi_\nu)^4 + \left\{ \Phi_\nu \hat{M}_u + (\hat{M}_u \Phi_\nu)^T \right\}, \tag{2.3}$$

we can write $(\bar{M}_R)^{\bullet\bullet}$ as follows:

$$(M_R)^{\bullet\bullet} = \xi_R(\Phi_\nu)^2 + \left\{ \Phi_\nu^{-1} \hat{M}_u + (\Phi_\nu^{-1} \hat{M}_u)^T \right\}$$

$$= \xi_R(\Phi_\nu)^2 + \left\{ P_u \hat{S}_u^{-1} \bar{P}_u \Phi_\nu + (transposed) \right\}, \tag{2.4}$$

where $\Phi_u \propto \Phi_\nu P_u$ and P_u is a scalar with VEV values

$$P_u = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}). \tag{2.5}$$

For example, by considering U(3)×U(3)′ transformation, we may consider $(M_R)^{\bullet\bullet}$ as

$$(M_R)^{\bullet\bullet} = (M_R^{1st})^{\bullet\bullet} + (M_R^{2nd})^{\bullet\bullet}, \tag{2.6}$$

$$(M_R^{1st})^{\bullet\bullet} = (\Phi_\nu^T)^\bullet{}_\circ (E^{-1})^{\circ\circ} (\Phi_\nu)_\circ{}^\bullet, \tag{2.7}$$

$$(M_R^{2nd})^{\bullet\bullet} = \left\{ (\bar{E}^T)^{\bullet\circ} (P_u)_{\circ\bullet} (\hat{S}_u^T)^\bullet{}_\bullet (P_u^{-1})^{\bullet\circ} (\Phi_\nu)_\circ{}^\bullet + (transposed) \right\}, \tag{2.8}$$

where we have neglected family-independent parameters.

In the next section, we will discuss a model which leads to the VEV relation (2.6).

3 Mass matrix of (ν_L, ν_R, N_L, N_R)

For convenience, in this section, we neglect family-independent parameters.

The first term (2.7) suggests a seesaw-like scenario. Therefore, we would like to consider that the second term is also derived from a seesaw-like scenario in the neutrino mass matrix for (ν_L, ν_R, N_L, N_R):

$$(M_R^{2nd})^{\bullet\bullet} = \left\{ \langle \bar{E}^T \rangle^{\bullet\circ} \langle \hat{S}'^{-1}_u \rangle_{\circ}{}^{\circ} \langle \Phi_\nu \rangle_{\circ}{}^{\bullet} + (transposed) \right\}, \tag{3.1}$$

where $\langle \hat{S}'_u \rangle_{\circ}{}^{\circ}$ should be given by

$$\langle \hat{S}'_u \rangle_{\circ}{}^{\circ} = \langle P_u \rangle_{\circ\bullet} \langle \hat{S}_u^T \rangle^{\bullet}{}_{\bullet} \langle P_u^{-1} \rangle^{\bullet\circ}. \tag{3.2}$$

In this model, $(\bar{M}_R^{1st})$ and $(\bar{M}_R^{2nd})$ can be understood by seesaw scenarios (2.7) and (3.1), while the relation (3.2) cannot be understood by seesaw scenario. Therefore, we consider that the form (3.2) is obtained from a SUSY vacuum condition $\partial W/\partial \Theta = 0$ for the following superpotential:

$$W = \mathrm{Tr}\left[\left\{ (\hat{S}'_u)_{\circ}{}^{\circ} (P_u)_{\circ\bullet} + (P_u)_{\circ\bullet} (\hat{S}_u^T)^{\bullet}{}_{\bullet} \right\} (\Theta)^{\bullet\circ} \right] + (transposed), \tag{3.3}$$

where Θ is a flavon with $\langle \Theta \rangle = 0$.

The structures $(M_R^{1st})^{\bullet\bullet}$ and $(M_R^{2nd})^{\bullet\bullet}$ suggest the following mass matrix for $((\nu_L)_\circ, (\nu_R^c)^\circ, (N_L)_\bullet, (N_R^c)^\bullet)$:

$$\begin{array}{c} ((\bar{\nu}_L)^\circ \quad (\bar{\nu}_R^c)_\circ \quad (\bar{N}_L)^\bullet \quad (\bar{N}_R^c)_\bullet) \\ \times \begin{pmatrix} \langle E \rangle_{\circ\circ} & \langle \hat{S}'_u \rangle_{\circ}{}^{\circ} & \langle P_u \rangle_{\circ\bullet} & \langle \Phi_\nu \rangle_{\circ}{}^{\bullet} \\ \langle \hat{S}'^T_u \rangle^{\circ}{}_{\circ} & (\quad)^{\circ\circ} & (\quad)^{\circ}{}_{\bullet} & \langle \bar{E}^T \rangle^{\circ\bullet} \\ \langle P_u^T \rangle_{\bullet\circ} & (\quad)_{\bullet}{}^{\circ} & (\quad)_{\bullet\bullet} & \langle \hat{S}_u \rangle_{\bullet}{}^{\bullet} \\ \langle \Phi_\nu^T \rangle^{\bullet}{}_{\circ} & \langle \bar{E} \rangle^{\bullet\circ} & \langle \hat{S}_u^T \rangle^{\bullet}{}_{\bullet} & (\quad)^{\bullet\bullet} \end{pmatrix} \begin{pmatrix} (\nu_L^c)^\circ \\ (\nu_R)_\circ \\ (N_L^c)^\bullet \\ (N_R)_\bullet \end{pmatrix} . \end{array} \tag{3.4}$$

Here, $(\quad)^{\bullet\bullet}$ is a room for would-be $(\bar{M}_R)^{\bullet\bullet}$. Thus, we can assign all scalars (flavons) in this mass matrix (3.4) without duplication.

Finally, we would like to comment on R charge assignment. We adopt R charge assignment for flavons (scalars) A and fermions ψ as follows

$$R(\bar{A}) = R(A), \quad R(\bar{\psi}_{L/R}) \neq R(\psi_{L/R}). \tag{3.5}$$

For example, in Eq.(3.4), we have defined the flavon $\hat{S}_u$ as $(\bar{N}_L)^\bullet (\hat{S}_u)_{\bullet}{}^{\bullet} (N_R)_\bullet$. This does not always mean $R(U_L) = R(N_L)$ and $R(U_R) = R(N_R)$ where (U_L, U_R) are components of (F_L, F_R) in the sector $f = u$. It means only

$$R(N_L) + R(N_R) = R(U_L) + R(U_R). \tag{3.6}$$

Thus, we can put the flavon $\hat{S}_u$ on the desirable position in the neutrino mass matrix (3.4). As we already stressed, it has an important meaning that we could assign all scalars (flavons) in this mass matrix (3.4) without duplication. It means that we can uniquely assign those flavons without mixing under suitable R-charge assignment for $(\nu_{L/R}, N_{L/R})$.

Also, note that, in the mass matrix (3.4), there is no $(E)_{\circ\bullet}$ and $(\bar{P}_u)^{\circ\bullet}$ in spite of the existence $(\bar{E})^{\circ\bullet}$ and $(P_u)_{\circ\bullet}$. This is possible only under the selection rule (3.5). For example, note that a conjugate term of the term $(\bar{\nu}_L)^{\circ}(P_u)_{\circ\bullet}(N_L^c)^{\bullet}$ is not $(\bar{N}_R^c)_{\bullet}(\bar{P})^{\bullet\circ}(\nu_R)_{\circ}$, but $(\bar{N}_L^c)_{\bullet}(\bar{P}_u)^{\bullet\circ}(\nu_L)_{\circ}$. Since

$$\begin{aligned} R((\bar{E})^{\bullet\circ}) &= 2 - R((\nu_R)_{\circ}) - R((\bar{N}_R^c)_{\bullet}), \\ R((\bar{P}_u)^{\bullet\circ}) &= 2 - R((\nu_L)_{\circ}) - R((\bar{N}_L^c)_{\bullet}), \end{aligned} \tag{3.7}$$

if we take

$$R((\nu_R)_{\circ}) + R((\bar{N}_R^c)_{\bullet}) \neq R((\nu_L)_{\circ}) + R((\bar{N}_L^c)_{\bullet}), \tag{3.8}$$

we can regard $(P_u)_{\circ\bullet}$ and $\bar{E}^{\circ\bullet}$ as separate flavons.

4 Scales of VEV matrices

In the recent study [?] in the U(3)×U(3)′ model, it has been concluded that flavon VEVs with U(3)×U(3)′ indexes $A_{\bullet\bullet}$, $B_{\bullet\circ}$ and $C_{\circ\circ}$ take the following scales

$$\langle A_{\bullet\bullet}\rangle \sim \Lambda_1 \sim 3\times 10^7\,\mathrm{TeV}, \quad \langle B_{\bullet\circ}\rangle \sim \Lambda_2 \sim 3\times 10^4\,\mathrm{TeV}, \quad \langle C_{\circ\circ}\rangle \sim \Lambda_3 \sim 9\,\mathrm{TeV}. \tag{4.1}$$

In order that the seesaw scenario M_R^{1st}, Eq.(2.7), holds, the flavon scales have to satisfy the relation

$$\langle E_{\circ\circ}\rangle \gg \langle (\Phi_\nu)_{\circ}{}^{\bullet}\rangle. \tag{4.2}$$

As we have proposed in Ref.[?], we adopt such the mechanism $\langle \Phi_\nu\rangle = \xi_\nu \langle \Phi_e\rangle$ with $\xi_\nu \ll 1$.

For the seesaw scenario M_R^{2nd}, Eq.(2.8), a VEV relation

$$\langle \bar{E}^{\bullet\circ}\rangle \sim \langle (\Phi_\nu)_{\circ}{}^{\bullet}\rangle \ll \langle (\hat{S}'_u)_{\circ}{}^{\circ}\rangle, \tag{4.3}$$

is required. We consider that a scale of the flavon $(\hat{S}'_u)$ is an exceptional case against the general rule (4.1) in spite of its indexes $(\ )_{\circ}{}^{\circ}$, because the VEV relation has to be

$$\langle (\hat{S}'_u)_{\circ}{}^{\circ}\rangle \sim \langle (\hat{S}_u)_{\bullet}{}^{\bullet}\rangle \sim \Lambda_1. \tag{4.4}$$

from the consistency among the scales in Eq.(3.3).

Here, we would like to comment on a scale of SU(2)$_L$. Flavons Φ_f and $\hat{S}_F$ given in (1.1) are singlets in the vertical symmetry SU(3)$_c$×SU(2)$_L$×U(1)$_Y$, so that they have only indexes of horizontal symmetry (family symmetry). The mass matrix (1.1) does not correspond to the real masses of quarks and leptons, but it represents the Yukawa coupling constant. Therefore, note that f_L does not mean $f_L = (u, d, e)_L$, and that f_L has to be $\mathrm{SU}(2)_L$ singlet. In Ref.[?], the fermions f_L have been defined as follows:

$$f_L \equiv (f_u, f_d, f_\nu, f_e)_L \equiv \left(\frac{1}{\Lambda_H} H_u^\dagger q_L, \frac{1}{\Lambda_H} H_d^\dagger q_L, \frac{1}{\Lambda_H} H_u^\dagger \ell_L, \frac{1}{\Lambda_H} H_d^\dagger \ell_L \right) \tag{4.5}$$

where

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \ell_L = \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}, \quad H_u = \begin{pmatrix} H_u^0 \\ H_u^- \end{pmatrix}, \quad H_d = \begin{pmatrix} H_d^+ \\ H_d^0 \end{pmatrix}. \tag{4.6}$$

Note that f_L is singlet in $\mathrm{SU}(2)_L$, but it has $\mathrm{U}(1)_Y$ charge. Therefore, correspondingly, f_R, F_L and F_R are $\mathrm{SU}(2)_L$ singlets, while they have $\mathrm{U}(1)_Y$ charge. (We consider that $(F_u, F_d)_{L/R}$ have $\mathrm{SU}(3)_c$ indexes.) For example, the selection of $\hat{S}_f$ in $\Phi_f \hat{S}_f^{-1} \bar{\Phi}_f$ (for example, $\hat{S}_u$ in $\Phi_u \hat{S}_f^{-1} \bar{\Phi}_u$) is done by R charge, not by flavor symmetry and/or $\mathrm{U}(1)_Y$ charge.

Our purpose in this paper was to discuss the neutrino mass matrix. The neutrino Dirac mass matrix $(\hat{M}_\nu)_\circ{}^\circ$ given in Eq.(1.7) comes from the term $(1/\Lambda_H)\bar{\ell} H_u^\dagger = (1/\Lambda_H)(\bar{\nu}_L H_u^0 + \bar{e}_L H_u^-)$ with $\langle H_u^- \rangle = 0$, not from Eq.(3.4).

5 Concluding remarks

Since the previous U(3)×U(3)′ model could give successful predictions for the observed quark masses and CKM mixings under a reasonable theoretical scenario, while the success in the neutrino sector was still phenomenological level. We have investigated a possible neutrino mass matrix structure in context of the U(3)×U(3)′ model. As we stressed in Sec.3, it is essential that flavons are uniquely assigned in the neutrino mass matrix (3.4) without duplication. Under suitable R charge assignment, especially under assumption $R(\bar{\psi}) \neq R(\psi)$, we can put $\hat{S}_u$, which was defined as $\bar{U}_L \hat{S}_u U_R$ in the up-quark sector, into the neutrino sector without confusion.

In conclusion, we have succeeded in giving a theoretical basis to the semi-empirical part (the structure of the right-handed neutrino mass matrix M_R) in the previous U(3)×U(3)′ model. As a result, the U(3)×U(3)′ model has been able to become more realistic as a unified mass matrix model of quarks and neutrinos.

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