Quark and Lepton Mass Matrices
Described by Charged Lepton Masses

Yoshio Koide[a] **and Hiroyuki Nishiura**[b]

[a] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
E-mail address: koide@kuno-g.phys.sci.osaka-u.ac.jp
[b] *Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata,*
Osaka 573-0196, Japan
E-mail address: hiroyuki.nishiura@oit.ac.jp

Abstract

Recently, we proposed a unified mass matrix model for quarks and leptons, in which, mass ratios and mixings of the quarks and neutrinos are described by using only the observed charged lepton mass values as family-number-dependent parameters and only six family-number-independent free parameters. In spite of quite few parameters, the model gives remarkable agreement with observed data (i.e. CKM mixing, PMNS mixing and mass ratios). Taking this phenomenological success seriously, we give a formulation of the so-called Yukawaon model in details from a theoretical aspect, especially for the construction of superpotentials and R charge assignments of fields. The model is considerably modified from the previous one, while the phenomenological success is kept unchanged.

1 Introduction

It is a big concern in the flavor physics to investigate the origin of the observed hierarchical structures of masses and mixings of quarks and leptons. Recently, a unified mass matrix model for quarks and leptons was proposed [1]: In the model, mass ratios and mixings of the quarks and neutrinos are described by using only the observed charged lepton mass values as "family-number-dependent" parameters and only six "family-number-independent" free parameters. In spite of quite few parameters, the model gives remarkable coincidence with observed all data, i.e. Cabibbo-Kobayashi-Maskawa (CKM) mixing [2, 3] in quark sector and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [4, 5] mixing in lepton sector, and quark and lepton mass ratios. Besides, the model gives very interesting predictions for leptonic CP violation parameter $\delta^{\ell}_{CP} \simeq -70° \simeq -\delta^{q}_{CP}$ and effective Majorana neutrino mass $\langle m \rangle \simeq 21$ meV. We list those numerical results in Table 1, which was quoted from Ref.[1].

The previous paper has focused on only phenomenological aspect of the model and shown the phenomenological success which should be taken seriously. However, discussions on the theoretical aspect of the model was somewhat not sufficient. So, in this paper, we shall give a formulation of a new Yukawaon model from the theoretical aspect.

Table 1: Predicted values vs. observed values. [Quoted from [1]]. The predicted values are obtained only from inputs of the six family-number-independent parameters $b_u = -1.011$, $b_d = -3.3522$, $\beta_d = 17.7°$, $(\tilde{\phi}_1, \tilde{\phi}_2) = (-176.05°, -167.91°)$, and $\xi_R = 2039.6$. The observed values were quoted from Ref.[6].

	$|V_{us}|$	$|V_{cb}|$	$|V_{ub}|$	$|V_{td}|$	$\delta^q_{CP}(°)$	r^u_{12}	r^u_{23}	r^d_{12}	r^d_{23}
Predicted	0.2257	0.03996	0.00370	0.00917	81.0	0.061	0.060	0.049	0.027
Observed	0.22536	0.0414	0.00355	0.00886	69.4	0.045	0.060	0.053	0.019
	± 0.00061	± 0.0012	± 0.00015	$^{+0.00033}_{-0.00032}$	± 3.4	$^{+0.013}_{-0.010}$	± 0.005	$^{+0.005}_{-0.003}$	$^{+0.006}_{-0.006}$
	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	$R_\nu\,(10^{-2})$	$\delta^\ell_{CP}(°)$	$m_{\nu 1}$ (eV)	$m_{\nu 2}$ (eV)	$m_{\nu 3}$ (eV)	$\langle m \rangle$ (eV)
Predicted	0.8254	0.9967	0.1007	3.118	-68.1	0.038	0.039	0.063	0.021
Observed	0.846	0.999	0.093	3.09	no data	no data	no data	no data	$< \mathrm{O}(10^{-1})$
	± 0.021	$^{+0.001}_{-0.018}$	± 0.008	± 0.15					

In the so-called Yukawaon model [7, 8, 9, 10, 11, 12, 13, 14, 15, 16], we regard Yukawa coupling constants $(Y_f)_{ij}$ ($f = u, d, \nu, e$) as effective quantities $(Y^{eff}_f)_{ij}$ which are given by vacuum expectation values (VEVs) of scalares Y_f as $(Y^{eff}_f)_{ij} = y_f \langle (Y_f) \rangle_{ij} / \Lambda$. The model is a sort of flavon model [17, 18]. (For recent works, for instance, see [19, 20].) The model is based on family symmetries U(3)×U(3)′. The U(3)×U(3)′ symmetries are broken at $\mu = \Lambda$ and $\mu = \Lambda'$. (We assume $\Lambda \ll \Lambda'$.) The symmetry U(3)′ is broken into S_3 at an energy scale Λ' through the vacuum expectation values (VEVs) of flavons $(S_f)^\beta_\alpha$ which take a form "unit matrix **1** plus democratic matrix X_3", $\langle S_f \rangle = v_{Sf}(\mathbf{1} + b_f X_3)$. Here, f is sector names $f = u, d, \nu, e$, and indices i, j and α, β are those in U(3) and U(3)′, respectively. (For the details, see Eq.(2.7) later.) The flavons S_f play an essential role as we discuss in Eq.(2.3). The parameters b_f are typical "family-number independent parameters" in the Yukawaon model, and they determine not only mass spectrum of the fermion f but also mixing among f_i. On the other hand, the U(3) family symmetry is completely broken by VEV of a flavon Φ_0 at $\mu = \Lambda$ as we discuss in Eq.(2.10) later. We do not consider any subgroups of U(3). Instead, from the practical point of view, we use the observed charged lepton masses for inputs of the VEV $\langle \Phi_0 \rangle \equiv v_0 \mathrm{diag}(z_1, z_2, z_3)$, which are only "family-number dependent parameters" in the Yukawaon model. We do not ask the origin of the values of z_i in this paper. This is future task in our investigation. Furthermore, the last basic hypothesis in the Yukawaon model is that the VEV forms of flavons are diagonal except for $\langle S_f \rangle$ which take the S_3 invariant forms. The details are discussed in the next section.

The VEV relations in the model are derived from superpotentials which are invariant under the U(3)×U(3)′ and constructed by using suitable R charge assignments. Once we give superpotential form invariant under U(3)×U(3)′ with R charge conservation, we can uniquely obtain our desirable VEV relation as seen in Sec.3. Therefore, the R charge assignment is crucial for the phenomenological success. However, the explicit forms of superpotentials and R charge as-

signments were not presented in the previous paper [1]. The purpose of the present paper is to give explicit superpotential forms and R charge assignments in details. In this paper, we obtain more natural R charge assignment than that in the previous model [1]. The new R charges assignment is given in Table 2. This assignment causes a change of the formulation given in the previous paper, including a new relation among the phase parameters in VEV of a flavon P and the observed charged lepton masses.

A flavon P_u presented later in (2.13) with VEV of phase matrix type $\langle P \rangle = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$ plays an essential role in the Yukawaon model and the phases (ϕ_1, ϕ_2, ϕ_3) are described in term of charged lepton mass values [21]. In the the previous model, we did not discuss the explicit mechanism but referred to a mechanism given in Ref. [21], which cannot be straightforwardly applied to the model [1] because the R charge assignment in the model [1] is different from that in the model [21]. The new R charge assignment in the Table 2 is also different from [21] and [1], so that we are led to a new relation among the phase parameters and the observed charged lepton masses. As a result, we obtain different parameter values $(\phi_1, \phi_2, \phi_3) \equiv (\tilde{\phi}_1 + \phi_0, \tilde{\phi}_2 + \phi_0, \phi_0)$, while, as far as the values $(\tilde{\phi}_1, \tilde{\phi}_2)$ are concerned, we can obtain the same values as those in the previous paper [1]. (However, this does not mean that present model is identical with the previous one [1]. Note that, in the CKM fitting, only the values $(\tilde{\phi}_1, \tilde{\phi}_2)$ are observable and ϕ_0 is not observable, while the parameter ϕ_0 is observable in the U(3) family model.) The details are given in Sec.4.

The present paper is arranged as follows: In Sec.2, the basic postulation in the Yukawaon model [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]and the VEV relations in the previous paper [1] are reviewed without showing the explicit superpotentials. In Sec.3, we will discuss superpotentials which give special VEV forms playing an essential role in the phenomenological investigation. In Sec.4, we will discuss the relation of phase parameters defined by Eq.(2.13) to the charged lepton masses. Finally, Sec.5 is devoted to summary and concluding remarks.

2 Basic assumptions in the Yukawaon model and its VEV relations

We investigate flavor physics from the point of view of family symmetry. It is unnatural that the Yukawa coupling constants Y_f explicitly break the family symmetry. Therefore, in order that the Hamiltonian is invariant under the symmetry, we must consider that Y_f are effective coupling constants Y_f^{eff} which are given by vacuum expectation values (VEVs) of scalars ("Yukawaons" [7, 8, 9, 10, 11, 12, 13, 14, 15, 16]) Y_f with 3×3 components for each sector f:

$$(Y_f^{eff})_i{}^j = \frac{y_f}{\Lambda} \langle Y_f \rangle_i{}^j \quad (f = u, d, \nu, e), \tag{2.1}$$

where Λ is an energy scale of the effective theory. All the flavons in the Yukawaon model are expressed by 3×3 components. Would-be Yukawa interactions are given by

$$H_Y = \frac{y_\nu}{\Lambda} (\bar{\ell}_L)^i (\hat{Y}_\nu)_i{}^j (\nu_R)_j H_u + \frac{y_e}{\Lambda} (\bar{\ell}_L)^i (\hat{Y}_e)_i{}^j (e_R)_j H_d + y_R (\bar{\nu}_R)^i (Y_R)_{ij} (\nu_R^c)^j$$

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Table 2: Transformation properties and R charges of flavons, quarks, leptons, and Higgs scalars in the present model. In the table, we omit row on U(3)$'$ for quarks, leptons and Higgs scalars, since it is obvious that those are singlets of U(3)$'$. Also, we omit row on SU(2) for other flavons, because it is obvious that those are SU(2) singlets. We always consider a flavon $\bar{A}$ correspondingly to a flavon A.

quark, lepton, Higgs	ℓ	ν^c	e^c	q	u^c	d^c	H_u	H_d
SU(2)	**2**	**1**	**1**	**2**	**1**	**1**	**2**	**2**
U(3)	**3**	**3***	**3***	**3**	**3***	**3***	**1**	**1**
R charge	$\frac{5}{4}$	$-\frac{1}{4}$	$-\frac{1}{4}$	0	$\frac{1}{2}$	$+\frac{3}{2}$	0	0

flavon	$\hat{Y}_e$	$\hat{Y}_\nu$	$\hat{Y}_d$	$\hat{Y}_u$	Φ_{0e}	$\Phi_{0\nu}$	Φ_{0d}	Φ_{0u}	P_e	P_ν	P_d	P_u
U(3)	**8+1**	**8+1**	**8+1**	**8+1**	**3**	**3**	**3**	**3**	**3**	**3**	**3**	**3**
U(3)$'$	**1**	**1**	**1**	**1**	**3***	**3***	**3***	**3***	**3**	**3**	**3**	**3**
R charge	1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1

$\hat{S}_e$	$\hat{S}_\nu$	$\hat{S}_d$	$\hat{S}_u$	Θ_{0e}	$\Theta_{0\nu}$	Θ_{0d}	Θ_{0u}	Φ_0	E	$\bar{Y}_R$	Θ_R	$\hat{\Theta}_\phi$
1	**1**	**1**	**1**	**3**	**3**	**3**	**3**	**6**	**6**	**6***	**6**	**8+1**
8+1	**8+1**	**8+1**	**8+1**	**3***	**3***	**3***	**3***	**1**	**1**	**1**	**1**	**1**
1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{5}{2}$	$-\frac{1}{2}$	1

$$+\frac{y_u}{\Lambda}(\bar{q}_L)^i(\hat{Y}_u)_i{}^j(u_R)_j H_u + \frac{y_d}{\Lambda}(\bar{q}_L)^i(\hat{Y}_d)_i{}^j(d_R)_j H_d, \tag{2.2}$$

where we have assumed a U(3) family symmetry, and $\ell_L = (\nu_L, e_L)$ and $q_L = (u_L, d_L)$ are SU(2)$_L$ doublets. H_u and H_d are two Higgs doublets. Those Yukawaons $\hat{Y}_f$ are distinguished from each other by R charges. Hereafter, for convenience, we use notations $\hat{A}$, A, and $\bar{A}$ for fields with $\mathbf{8+1}$, $\mathbf{6}$, and $\mathbf{6}^*$ of U(3), respectively. In addition, another types of flavons $A_i{}^\alpha$ and $A_{i\alpha}$ appear in the present U(3)$\times$U(3)$'$ model. Since we pay attention only to the index of U(3), we denote anti-flavons of those as $\bar{A}^i{}_\alpha$ and $\bar{A}^{i\alpha}$, respectively.

In the present model, we have flavons $(\hat{Y}_f)_i{}^j$, $(\Phi_{0f})_i{}^\alpha$, $(P_f)_i{}^\alpha$, $(S_f)_\alpha{}^\beta$, and $(\Theta_{0f})_i{}^\alpha$. We assume that VEV matrices of those flavons take diagonal forms (except for $(S_f)_\alpha{}^\beta$ which take S_3 invariant forms) at our basic flavor basis as we discuss later. In addition to those flavons, we consider other flavons $(\Phi_0)_{ij}$, $(\bar{Y}_R)^{ij}$, $(\Theta_R)_{ij}$, E_{ij}, $\hat{E}_i{}^j$, and $\hat{\Theta}_\phi$. The transformation properties and the R charges of those flavons are listed in Table 2. A role of each flavon will be discussed step by step below.

Let us list VEV relations which are our goal: The following (i)-(v) are used in the previous model [1], while (vi) is revised from the previous paper. We will derive these relations from superpotentials presented in Sec.3 in this paper.

(i) The VEV of the Yukawaons $\langle \hat{Y}_f \rangle$ are given by the following relations:

$$\langle \hat{Y}_f \rangle_i{}^j = k_f \, \langle \Phi_{0f} \rangle_i{}^\alpha \langle (S_f)^{-1} \rangle_\alpha{}^\beta \langle \bar{\Phi}_{0f}^T \rangle_\beta{}^j \quad (f = u, d, \nu, e). \tag{2.3}$$

The factor S_f^{-1} in Eq.(2.3) comes from a seesaw-like scenario by assuming new heavy fermions F_α and considering the following 6×6 mass matrix model:

$$(\bar{f}_L^i \quad \bar{F}_L^\alpha) \begin{pmatrix} (\hat{Y}_f)_i{}^j & (\Phi_{0f})_i{}^\beta \\ (\bar{\Phi}_{0f}^T)_\alpha{}^j & -(S_f)_\alpha{}^\beta \end{pmatrix} \begin{pmatrix} f_{Rj} \\ F_{R\beta} \end{pmatrix}. \tag{2.4}$$

Here $f_{L(R)}$ and $F_{L(R)}$ are, respectively, left (right) handed light and heavy fermions fields. Exactly speaking, we have to read $\bar{f}_L$ in Eq.(2.4) as $\bar{f}_L H_{u/d}/\Lambda$. However, for convenience, we have denoted those as $\bar{f}_L$ simply. Such a seesaw-like scenario with the democratic form of S_f has been proposed by Fusaoka and one of the authors (YK) [22] in order to understand the observed fact

$$m_t \sim \Lambda_{weak}, \qquad m_u \sim m_d \sim m_e. \tag{2.5}$$

In the Yukawaon model, when we consider $|\hat{Y}_f| \ll |\Phi_{0f}| \ll |S_f|$ (i.e. $\Lambda \ll \Lambda'$), we obtain a mass matrix for $\bar{f}_L'$ and f_R',

$$M_f \simeq \hat{Y}_f + \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}, \tag{2.6}$$

after the block diagonalization of Eq.(2.4). Regrettably, this relation (2.6) is not one we want, because the first term $\hat{Y}_f$ in Eq.(2.6) is independent of the second term $\Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}$. Therefore, in the previous paper [1], we have put some phenomenological assumptions in order to obtain the relation (2.3). We are still not satisfied with the scenario given in the previous paper [1], and we think that the scenario should be improved. However, for simplicity, in the present paper, we still denote Dirac mass matrices M_f as Eq.(2.3).

(ii) The VEV form of $\langle S_f \rangle$, which is due to the symmetry breaking $U(3)' \to S_3$, is given by

$$\langle \hat{S}_f \rangle = v_{Sf}(\mathbf{1} + b_f X_3), \tag{2.7}$$

where $\mathbf{1}$ and X_3 are defined as

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{2.8}$$

The parameters b_f in Eq. (2.7) are typical examples of "family-number-independent" parameters. However, we will be obliged to take $b_e = 0$ and $b_\nu = 0$ in the lepton sector $f = e$ and $f = \nu$ as seen in Eq.(3.1) later. Therefore, the Dirac mass matrices M_e and M_ν take diagonal forms

without b_f parameters (except for parameters in $\langle\Phi_0\rangle$ (see Eq.(2.10))), and quark mass matrices M_u and M_d are described only by parameters b_u and b_d, respectively.

Although we have used the discrete symmetry S_3, our aim in the Yukawaon model is to understand all mass spectra and mixing on the basis of U(3)×U(3)′ symmetries without introducing any additional subgroups, e.g. S_2, A_4, SU(2), and so on. If we adopt such symmetries, we will be obliged to accept unwelcome family-number dependent parameters. This is against with the aim of Yukawaon model. Instead, from the practical point of view, we use only the observed charged lepton masses m_{ei} as a result of U(3) symmetry breaking as seen in Eq.(2.10).

(iii) Motivated by the relation given below in (2.11), we assume that the VEV forms $\langle\Phi_{0f}\rangle$ are diagonal in the flavor basis in which $\langle S_f\rangle$ take the forms (2.7), and are given by

$$\langle\Phi_{0f}\rangle_i{}^\alpha = k_{0f}\langle\Phi_0\rangle_{ik}\langle\bar{P}_f\rangle^{k\alpha}. \tag{2.9}$$

The VEV of a flavon Φ_0, $\langle\Phi_0\rangle$ was defined by

$$\langle\Phi_0\rangle = v_0\,\mathrm{diag}\,(z_1, z_2, z_3) \propto \mathrm{diag}\,(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \tag{2.10}$$

where z_i is normalized as $z_1^2 + z_2^2 + z_3^2 = 1$. The $\langle\Phi_0\rangle$ plays a crucial role in the phenomenological investigation of the Yukawaon model. The existence of such the VEV matrix $\langle\Phi_0\rangle$ was suggested by a phenomenological success of the charged lepton mass relation [23, 24, 25]. (For a recent work, for see [26].)

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \tag{2.11}$$

which is excellently satisfied by the observed charged lepton masses (pole masses) as $K = (2/3) \times (0.999989 \pm 0.000014)$. If we accept the flavon Φ_0 with its VEV (2.10), the charged lepton relation (2.11) is simply expressed as

$$K = \frac{\mathrm{Tr}[\langle\Phi_0\rangle\langle\Phi_0]}{(\mathrm{Tr}[\langle\Phi_0\rangle])^2}. \tag{2.12}$$

However, the purpose of the Yukawaon model is not to understand the relation (2.11). Therefore, in the Yukawaon model, we do not ask the origin of the charged lepton mass spectrum. It is a future task to understand the origin of the mass values (m_e, m_μ, m_τ). In this paper we accept the observed charged lepton mass values as fundamental family-dependent parameters, and we give a unified description of quark and lepton mass matrices.

On the other hand, we know that there exist the CKM mixing in quark sector and the PMNS mixing in lepton sector. Therefore, true regularity in the mass spectra ought to be disturbed by such mixings. The relation (2.11) is a specific case only for the charged leptons. We consider that a fundamental flavor basis in flavor physics is a basis in which the charged lepton mass

matrix is diagonal. Moreover, we speculate that all masses and mixings of quarks and leptons might be described by inputting the observed charged lepton mass values. Under this idea, the Yukawaon model has been introduced and investigated [7, 8, 9, 10, 11, 12, 13, 14, 15, 16].

(iv) The VEV form $\langle P_f \rangle$ are defined as

$$\langle P_u \rangle = v_P \, \text{diag}(e^{i\phi_1}, \ e^{i\phi_2}, \ e^{i\phi_3}), \quad \langle P_d \rangle = v_P \mathbf{1}, \quad \langle P_\nu \rangle = v_P \mathbf{1}, \qquad \langle P_e \rangle = v_P \mathbf{1}. \qquad (2.13)$$

As we show $\langle P_f \rangle \langle \bar{P}_f \rangle = \mathbf{1}$ in Sec.3, the special choices (2.13) are obviously ansatzes. (Also see a comment below Eq.(3.10) later.) The parameters (ϕ_1, ϕ_2, ϕ_3) in Eq. (2.13) are typical examples of "family-number-dependent" parameters. However, in Sec.4, we will show that the parameters (ϕ_1, ϕ_2, ϕ_3) are described by the charged lepton masses $(m_e, m_\mu. m_\tau)$ with help of two family-number-independent parameters. The origin of these VEV forms will be discussed in Sec.3.

(v) A neutrino mass matrix is given as

$$(M_\nu^{Majorana})_{ij} = \langle \hat{Y}_\nu \rangle_i{}^k \langle \bar{Y}_R^{-1} \rangle_{kl} \langle \hat{Y}_\nu^T \rangle^l{}_j, \qquad (2.14)$$

by adopting the conventional seesaw mechanism [27, 28, 29, 30]. Here in this paper, differently from the previous paper [1], we assume the following VEV structure of the Y_R (the Majorana mass matrix of the right-handed neutrinos ν_R):

$$\langle \bar{Y}_R \rangle^{ij} = k_R \left[\left(\langle \bar{\Phi}_0 \rangle^{ik} \langle \hat{E} \rangle_k{}^l \langle \hat{Y}_u \rangle_l{}^j + \langle \hat{Y}_u^T \rangle^i{}_k \langle \hat{E}^T \rangle^k{}_l \langle \bar{\Phi}_0 \rangle^{lj} \right) + \xi_R \langle \hat{Y}_\nu^T \rangle^i{}_k \langle \bar{E} \rangle^{kl} \langle \hat{Y}_\nu \rangle_l{}^j \right], \qquad (2.15)$$

where $\langle \hat{E} \rangle = \langle \bar{E} \rangle = v_E \mathbf{1}$. The form of the first term in Eq.(2.15), $\bar{\Phi}_0 \hat{Y}_u$, was first introduced in Ref.[31]. The new form (2.15) for $\langle \bar{Y}_R \rangle$ has been adopted in this paper in order for the R charge assignment to be more natural.

Since we deal with mass ratios and mixings only, the common coefficients k_f, v_{Sf}, and so on does not affect the numerical results, so that hereafter we omit such coefficients even if those have dimensions.

3 R charges and superpotentials

In this section, we demonstrate how to derive the VEV relations presented in Sec.2 from superpotentials given bellow. Hereafter, for convenience, we have sometimes dropped the notations "$\langle$" and "$\rangle$".

Superpotentials for flavons are determied under U(3)×U(3)′ symmetries and R charge conservation. Once we fix R charge assignment, our superpotentials are uniquely determined without ambiguity.

First, let us show guidelines on the assignment of R charges:

(i) In the Yukawaon model, it is required that all flavons take R charges with $R \geq 0$ except for special flavons Θ which always take $\langle \Theta \rangle = 0$. In the conventional models, a U(1) charge Q

is usually assumed, and thereby, only terms with $Q = 0$ are allowed in the Hamiltonian. When there are terms A and B whose R charges are $Q_A = 0$ and $Q_B = 0$, the combined term $A \cdot B$ is also allowed in the Hamiltonian. We do not want such the situation. Therefore, we require that all flavons take $R \geq 0$, and thereby, it is forbidden that a term which already has $R = 2$ makes an unwelcome higher-dimensional term combined with another flavon (or term).

(ii) We require that R charges are conserved even under the mixing between f_L and F_L (and also between f_R and F_R) in Eq.(2.4). Therefore, flavons given in Eq.(2.4) have to satisfy the following relations:

$$R(f_L) = R(F_L) \equiv r_{fL}, \quad R(f_R) = R(F_R) \equiv r_{fR}, \tag{3.1}$$

$$R(S_f) = R(\Phi_f) = R(\bar{\Phi}_f) = R(\hat{Y}_f) \equiv r_f. \tag{3.2}$$

(Eq.(3.2) does not mean $r_{fL} = r_{fR}$.) Correspondingly to Eq.(3.1), we require

$$R(\bar{A}) = R(A), \tag{3.3}$$

for any flavons A and anit-flavons $\bar{A}$. Here $R(A)$ denotes R charge of flavon A, and so on.

(iii) Values of R charges should be as possible as simple, by taking the basic VEV relations into consideration. According to the conventional SUSY models, we assign zero for the Higgs scalar doublets H_u and H_d, e.g. $R(H_u) = R(H_d) = 0$. Our R charge assignment is shown in Table 2.

Next, let us discuss a simple case which gives $b_e = b_\nu = 0$ by considering the following superpotential:

$$W_S = \sum_{f=e,\nu} \left\{ \lambda_{1f}[(S_f)_\alpha^\beta (P_f)_{\beta k}(\bar{P}_f)^{k\alpha}] + \lambda_{2f}[(S_f)_\alpha^\alpha][(P_f)_{\beta k}(\bar{P}_f)^{k\beta}] \right\}, \tag{3.4}$$

where we have taken R charges of S_f and P_f as

$$R(S_f) + 2R(P_f) = 2 \quad (f = e, \nu). \tag{3.5}$$

The form (3.4) is newly adopted in this paper. (Eq.(3.4) is somewhat improved from the previous paper [1].) The vacuum condition for the superpotential (3.4) leads to

$$S_f = \mathbf{1}, \quad P_f \bar{P}_f = \mathbf{1}. \quad (f = e, \nu) \tag{3.6}$$

The result $S_f = \mathbf{1}$ means $b_f = 0$, so that

$$\hat{Y}_e \propto \hat{Y}_\nu \propto \Phi_0 \bar{\Phi}_0. \tag{3.7}$$

On the other hand, from the relation (2.9), we obtain

$$R(P_f) = r_f - r_0, \tag{3.8}$$

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where $r_0 \equiv R(\Phi_0)$, so that, from the R charge relation (3.5), we obtain

$$r_e = r_\nu = \frac{2}{3}(r_0 + 1). \tag{3.9}$$

However, (3.6) and (3.9) do not mean that $\hat{Y}_e$ and $\hat{Y}_\nu$ are an identical flavon.

We take a specific VEV form

$$P_f = \mathrm{diag}(e^{i\phi_1^f}, e^{i\phi_2^f}, e^{i\phi_3^f}), \tag{3.10}$$

from the general form $P_f \bar{P}_f = \mathbf{1}$ in (3.6) by assuming $\langle \bar{P}_f \rangle = \langle P_f \rangle^\dagger$ and by assuming that the VEV matrix is diagonal. Since the VEV matrix form of S_e (and also S_ν) is diagonal, the VEV matrix P_e is commutable with S_e, so that the phase parameters ϕ_i^e in P_e cannot play any physical role in $\bar{P}_e(S_e^{-1})P_e$. Therefore, we have simply put $P_f = \mathbf{1}$ for $f = e, \nu$ in Eq.(2.14) with Eqs.(2.9) and (2.3).

For $f = u, d$, we assume the following superpotential:

$$W_{Pq} = \frac{1}{\Lambda} \left\{ \lambda_{1P} \mathrm{Tr}[P_u \bar{P}_u P_d \bar{P}_d] + \lambda_{2P} \mathrm{Tr}[P_u \bar{P}_u] \mathrm{Tr}[P_d \bar{P}_d] \right\}, \tag{3.11}$$

where we take

$$R(P_u) + R(P_d) = 1. \tag{3.12}$$

The SUSY vacuum condition leads to $P_u \bar{P}_u = \mathbf{1}$ and $P_d \bar{P}_d = \mathbf{1}$.

From Eq.(3.12), we obtain an R charge relation

$$r_u + r_d = 1 + 2r_0. \tag{3.13}$$

On the other hand, we obtain

$$r_u + r_d = r_\nu + r_e = \frac{4}{3}(r_0 + 1), \tag{3.14}$$

from the superpotential W_ϕ given in (4.1). Eqs.(3.13) and (3.14) fix the value of r_0 as

$$r_0 = \frac{1}{2}, \quad \Rightarrow \quad r_e = r_\nu = 1. \tag{3.15}$$

For the VEV relations (2.9) and (2.15), we consider somewhat tricky prescription: We assume existence of Θ fields which always take VEV values $\langle \Theta \rangle = 0$. For example, in order to obtain the VEV relation (2.9), we assume the following superptential

$$W_0 = \mu_{0f}(\Phi_{0f})_i{}^\alpha (\bar{\Theta}_{0f})_\alpha{}^i + \lambda_{0f}(\Phi_0)_{ik}(\bar{P}_f)^{k\alpha}(\bar{\Theta}_{0f})_\alpha{}^i. \tag{3.16}$$

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From $\partial W_0/\partial \bar{\Theta}_{0f} = 0$, we obtain VEV relation (2.9). On the other hand, a derivative of W_0 with respective to other flavon, for example, Φ_0 leads to $\partial W_0/\partial \Phi_0 = \lambda_{0f}\bar{P}_f\Theta_{0f} = 0$. However, the result always includes Θ_{0f}, so that the condition does not lead to any new VEV relations. This prescription is very useful when one flavon appears in the different (two or more) superpotentials. (For example, instead of $W = (\mu A + \lambda BC)\Theta$, we may consider $W = (\mu A + \lambda BC)(\mu A + \lambda BC)^\dagger$. However, then we have an R charge constraint $R(A) = R(B) + R(C) = 1$ addition to $R(A) = R(B) + R(C)$. Beside, the SUSY vacuum condition $\partial W/\partial A = 0$ will lead to unwelcome relation if there is another potential term which includes the flavon A.) Of course, such Θ flavon prescription is a big ansatz in the Yukawaon model. We have to search for more reasonable prescription in future.

Similarly, for the VEV relation (2.15), we assume the following new superpotential presented in this paper,

$$W_R = \left\{ \mu_R(\bar{Y}_R)^{ij} + \frac{\lambda_R}{\Lambda} \left((\bar{\Phi}_{0\nu})^i{}_\alpha (\bar{P}_d)^{\alpha k}(\hat{Y}_u)_k{}^j + (\hat{Y}_u^T)^i{}_k(\bar{P}_d^T)^{k\alpha}(\bar{\Phi}_{0\nu})_\alpha{}^j + \xi_R(\hat{Y}_\nu^T)^i{}_k \bar{E}^{kl}(\hat{Y}_\nu)_l{}^j \right) \right\} (\Theta_R)_{ji},$$

(3.17)

together with

$$W_E = \frac{\lambda_{E1}}{\Lambda}[(E)_{ik}(\bar{E})^{kl}(E)_{lm}(\bar{E})^{mi}] + \lambda_{E2}[(E)_{ik}(\bar{E})^{ki}][(E)_{jl}(\bar{E})^{lj}], \qquad (3.18)$$

where we have taken

$$R(E) = R(\bar{E}) = \frac{1}{2}. \qquad (3.19)$$

Moreover, the form of W_R, Eq.(3.17) requires a relation

$$r_u + r_0 + r_E = 2r_\nu + r_E, \qquad (3.20)$$

so that we obtain r_u and r_d as follows:

$$r_u = \frac{3}{2}, \quad r_d = \frac{1}{2}. \qquad (3.21)$$

Thus, in this section, we have derive the R charges of flavons from the desirable superpotential forms. In other words, this means that if we start from the R charge assignment given in Table 2, we can uniquely reach to the desirable superpotential forms.

4 Relation between (ϕ_1, ϕ_2, ϕ_3) and (m_e, m_μ, m_τ)

In this section, we give a relation which connects the phase parameters (ϕ_1, ϕ_2, ϕ_3) ($\phi_i = \phi_i^u - \phi_i^d$) with the family-number-dependent input parameters (z_1, z_2, z_3). Although the basic idea has already given in Ref.[21], the explicit relation is renewed in the present paper as follows: We consider the following superpotential in this paper.

$$W_\phi = \left\{ \lambda_1 \left[(P_u)_{i\alpha}(\bar{P}_d)^{\alpha j} + (P_d)_{i\alpha}(\bar{P}_u)^{\alpha j} \right] + \lambda_2 \left[(P_\nu)_{i\alpha}(\bar{P}_e)^{\alpha j} + (P_e)_{i\alpha}(\bar{P}_\nu)^{\alpha j} \right] + \lambda_3(\Phi_0)_{ik}(\bar{\Phi}_0)^{kj} \right\} (\hat{\Theta}_\phi)_j{}^i.$$

(4.1)

In order to get Eq.(4.1), it is essential that the VEVs of the flavons satisfy the following R charge relation

$$R(P_u) + R(P_d) = R(P_\nu) + R(P_e) = 2R(\Phi_0). \tag{4.2}$$

The first term in Eq.(4.1) gives a VEV relation

$$\left[(P_u)_{i\alpha}(\bar{P}_d)^{\alpha j} + (P_d)_{i\alpha}(\bar{P}_u)^{\alpha j} \right] \propto \cos\phi_i, \tag{4.3}$$

where $\phi_i = \phi_i^u - \phi_i^d$. On the other hand, VEVs of the second and third terms are proportional to 1 and z_i^2, respectively. Therefore, we obtain a relation

$$\cos\phi_i = a + b z_i^2, \tag{4.4}$$

where the parameters a and b are family-number-independent parameters.

Note that observable parameters in the three phase parameters (ϕ_1, ϕ_2, ϕ_3) are only two. When we denote

$$\phi_1 = \phi_0 + \tilde{\phi}_1, \quad \phi_2 = \phi_0 + \tilde{\phi}_2, \quad \phi_3 = \phi_0, \tag{4.5}$$

the parameter ϕ_0 is not observable. Therefore, we can always choose arbitrary value of ϕ_0, so that the relation (4.4) is satisfied by choosing two family-number-independent parameters a and b suitably. (Note that although the parameter ϕ_0 is not observable in the framework of the standard model (SM), the parameter Φ_0 in the Yukawaon is observable because we consider U(3)×U(3)$'$ which are gauged. The value ϕ_0 will be confirmed by future experiments.)

Explicitly, we can obtain numerical results as follows: By eliminating the parameter a, we obtain a relation

$$\cos\phi_1 - b\, z_1^2 = \cos\phi_2 - b\, z_2^2 = \cos\phi_3 - b\, z_3^2. \tag{4.6}$$

Then, we can obtain a relation for the parameter ϕ_0:

$$b = \frac{\cos\phi_3 - \cos\phi_1}{z_3^2 - z_1^2} = \frac{\cos\phi_3 - \cos\phi_2}{z_3^2 - z_2^2}. \tag{4.7}$$

Since we have obtained the parameter values [1]

$$(\tilde{\phi}_1, \tilde{\phi}_2) = (-176.05°, -169.91°), \tag{4.8}$$

from fitting of the observed CKM mixing data, we obtain family-number-independent parameters (a, b)

$$(a, b) = (1.71573, -0.790018), \tag{4.9}$$

together with a value $\phi_0 = 33.905°$. Note that the value $\phi_0 = 33.905°$ is observable if we consider U(3) family gauge bosons, although it is not observable in the CKM parameter fitting. We predict the phase parameters (ϕ_1, ϕ_2, ϕ_3) as follows:

$$(\phi_1, \phi_2, \phi_3) = (-142.14°, -136.00°, 33.91°). \tag{4.10}$$

In future, those values will be confirmed by family gauge boson experiments.

5 Concluding remarks

In conclusion, we have given formulation of a new Yukawaon model on the basis of seesaw type mass matrix model by presenting U(3)×U(3)$'$ assignments, R charges of flavons, superpotential forms, and so on. In spite of a model with quite few parameters, the model can give a remarkable agreement with the observed quark and lepton mixings and mass ratios. Those phenomenological (numerical) results of the model have already been reported in the previous paper [1]. We emphasize that the phenomenological success highly depends on whether we can assign the R charges reasonably and consistently or not. It is in the present paper that the explicit R charge assignment is completed.

The phenomenological success of the present model seems to suggest the following points:
(a) The observed quark and lepton masses and mixings are caused by a common origin.
(b) Flavor physics should be investigated on a flavor basis in which charged lepton mass matrix is diagonal. (Mass matrix of family gauge bosons is also diagonal in this basis, so that, family gauge bosons will not cause flavor violation in the charged lepton sector [32] .)
(c) Masses and mixings in the quark sector are given by the parameters b_u and b_d in the form of S_f as shown in Eq.(2.7). This mechanism is very interesting.

On the other hand, for the theoretical aspect, the model has still many problems which should be improved in future. For example, in the present paper, we did not discuss explicit scales of Λ and Λ', although we have tacitly assumed that $\langle A_{ij} \rangle \sim \Lambda$, $\langle A_{\alpha\beta} \rangle \sim \Lambda'$ and $\langle A_{i\alpha} \rangle \sim \sqrt{\Lambda\Lambda'}$. The choice is highly correlated in the tininess of neutrino masses. Since we have discussed masses and mixings only, we have neglected the common coefficients and VEV values in the sectors, for example, k_f in Eq.(2.3) v_{Sf} in Eq.(2.7), v_0 in Eq.(2.10), k_R in Eq.(2.15), and so on. Those are our future task.

We believe that the present model can give fruitful suggestions for the study of flavor physics.

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