Quark and Lepton Mass Matrices Described by Charged Lepton Masses

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Abstract

Recently, we proposed a unified mass matrix model for quarks and leptons, in which, mass ratios and mixings of the quarks and neutrinos are described using only the observed charged lepton mass values as family-number-dependent parameters and only six family-number-independent free parameters. In spite of quite few parameters, the model gives remarkable agreement with observed data (i.e. CKM mixing, PMNS mixing and mass ratios). Moreover, the model gives very interesting predictions for leptonic CP violation parameter $\delta^{\ell}_{CP} \simeq -70^\circ \simeq \delta^{q}_{CP}$ and effective neutrino mass $\langle m \rangle \simeq 21$ meV. The present paper gives a theoretical framework of the model in details.

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1 Introduction

It is a big concern in the flavor physics to investigate the origin of the observed hierarchical structures of masses and mixings of quarks and leptons. Recently, a unified mass matrix model for quarks and leptons was proposed [1]: In the model, mass ratios and mixings of the quarks and neutrinos are described using only the observed charged lepton mass values as "family-number-dependent" parameters and only six "family-number-independent" free parameters. In spite of quite few parameters, the model gives remarkable coincidence with observed all data, i.e. Cabibbo-Kobayashi-Maskawa (CKM) mixing [2] in quark sector and Pontecorvo-Maki-Nakagawa-Sakata (PMNS) [3] mixing in lepton sector, and quark and lepton mass ratios. Besides, the model gives very interesting predictions for leptonic CP violation parameter $\delta^{\ell}_{CP} \simeq -70^\circ \simeq \delta^{q}_{CP}$ and effective Majorana neutrino mass $\langle m \rangle \simeq 21$ meV. The model is a sort of flavon model [4]. The model is based on family symmetries U(3)×U(3)′. The symmetry U(3)′ is broken into S_3, so that vacuum expectation values (VEVs) of flavons $(S_f)_\alpha^{\ \beta}$ (f is sector names $f = u, d, \nu, e$) take a form "unit matrix plus democratic matrix", and U(3) is broken by a VEV form of a flavon $(\Phi_0)_{ij}$, $\langle \Phi_0 \rangle \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$. (Here, indices i, j and α, β are those in U(3) and U(3)′, respectively.) The VEV relations in the model are derived from superpotentials which are invariant under the U(3)×U(3)′ and constructed by using suitable R

charge assignments. However, since the previous paper [1] was given by a form of a short paper from a point of view that the phenomenological results should be rapidly informed, the explicit forms of superpotentials and R charge assignments were omitted. The purpose of the present paper is to give explicit superpotential forms and R charge assignments in datails.

Practically, as we will discuss later, all VEV relations among flavons are derived from supersymmetric vacuum conditions for superpotentials invariant under R charge conservation and family symmetries such as U(3)×U(3)′ which are broken at $\mu = \Lambda$ and $\mu = \Lambda'$. (We assume $\Lambda \ll \Lambda'$.) In order to obtain suitable VEV relations, the R charge assignments are crucial. It should be noticed that those VEV relations are directly given by forms of multiplication (not by forms of the sum) differently from usual mass matrix models.

The present paper is arranged as follows: In Sec.2, the basic postulation in the Yukawaon model [5] and the VEV relations in the previous paper [1] are reviewed without showing the explicit superpotentials. In Sec.3, we will discuss superpotentials which give special VEV forms, which play an essential role in the phenomenological investigation. In Sec.4, we will discuss the relation of phase parameters defined by Eq.(2.13) to the charged lepton masses. In Sec.5, we give a possible R charge assignment in this model. Finally, Sec.6 is devoted to summary and concluding remarks. In Appendix A, we give a brief review of the results of phenomenological investigation, which have already been given in the previous short paper [1]. In Appendix B, we review a relation of Yukawaon model to the seesaw model which has also been discussed in the previous paper [1].

2 Basic assumptions in the Yukawaon model and its VEV relations

We investigate flavor physics from the point of view of a family symmetry. It is unnatural that the Yukawa coupling constants Y_f explicitly break the family symmetry. Therefore, in order that the Hamiltonian is invariant under the symmetry, we must consider that Y_f are effective coupling constants Y_f^{eff} which are given by vacuum expectation values (VEVs) of scalars ("Yukawaons" [5]) Y_f with 3×3 components for each sector f:

$$(Y_f^{eff})_i{}^j = \frac{y_f}{\Lambda} \langle Y_f \rangle_i{}^j \qquad (f = u, d, \nu, e), \tag{2.1}$$

where Λ is an energy scale of the effective theory. All the flavons in the Yukawaon model are expressed by 3×3 components. Would-be Yukawa interactions are given by

$$H_Y = \frac{y_\nu}{\Lambda} (\bar{\ell}_L)^i (\hat{Y}_\nu)_i{}^j (\nu_R)_j H_u + \frac{y_e}{\Lambda} (\bar{\ell}_L)^i (\hat{Y}_e)_i{}^j (e_R)_j H_d + y_R (\bar{\nu}_R)^i (Y_R)_{ij} (\nu_R^c)^j$$

$$+ \frac{y_u}{\Lambda} (\bar{q}_L)^i (\hat{Y}_u)_i{}^j (u_R)_j H_u + \frac{y_d}{\Lambda} (\bar{q}_L)^i (\hat{Y}_d)_i{}^j (d_R)_j H_d, \tag{2.2}$$

where we have assumed a U(3) family symmetry, and $\ell_L = (\nu_L, e_L)$ and $q_L = (u_L, d_L)$ are $SU(2)_L$ doublets. H_u and H_d are two Higgs doublets. Those Yukawaons $\hat{Y}_f$ are distinguished from each other by R charges. Hereafter, for convenience, we use notations $\hat{A}$, A, and $\bar{A}$ for fields with $\mathbf{8}+\mathbf{1}$, $\mathbf{6}$, and $\mathbf{6}^*$ of U(3), respectively.

Table 1: Transformation properties and R charges of flavons in the present model. (Quarks and leptons and Higgs scalars are omitted in this table, because these quantum numbers are obvious.) Except for $\bar{Y}_R$ and Θ_R, we always consider a flavon $\bar{A}$, for a flavon A.

flavon	$\hat{Y}_e$	$\hat{Y}_\nu$	$\hat{Y}_d$	$\hat{Y}_u$	Φ_{0e}	$\Phi_{0\nu}$	Φ_{0d}	Φ_{0u}	P_e	P_ν	P_d	P_u
U(3)	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
U(3)$'$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
R charge	1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{1}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	0	1

flavons	$\hat{S}_e$	$\hat{S}_\nu$	$\hat{S}_d$	$\hat{S}_u$	Θ_{0e}	$\Theta_{0\nu}$	Θ_{0d}	Θ_{0u}	Φ_0	E	$\bar{Y}_R$	Θ_R	$\hat{\Theta}_\phi$
U(3)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{8+1}$
U(3)$'$	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
R charge	1	1	$\frac{1}{2}$	$\frac{3}{2}$	1	1	$\frac{3}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{8}{3}$	$-\frac{2}{3}$	0

In the present model, we have flavons $(\hat{Y}_f)_i{}^j$, $(\Phi_{0f})_i{}^\alpha$, $(P_f)_i^\alpha$, $(\hat{S}_f)_\alpha{}^\beta$, and $(\Theta_f)_i{}^\alpha$. In addition to those flavons, we consider other flavons $(\Phi_0)_{ij}$, $(\bar{Y}_R)^{ij}$, $(\Theta_R)_{ij}$, E_{ij}, and $\hat{\Theta}_\phi$. The transformation properties and the R charges of those flavons are listed in Table 1. A role of each flavon will be discussed step by step below.

First, let us list VEV relations which are our goal:

(i) The VEV of the Yukawaons $\langle\hat{Y}_f\rangle$ are given by the following relations:

$$\langle\hat{Y}_f\rangle_i{}^j = k_f\ \langle\Phi_{0f}\rangle_i{}^\alpha \langle(S_f)^{-1}\rangle_\alpha{}^\beta \langle\bar{\Phi}_{0f}^T\rangle_\beta{}^j \qquad (f = u, d, \nu, e). \tag{2.3}$$

The factor S_f^{-1} in Eq.(2.3) comes from a seesaw-like scenario: Let us suppose the following 6×6 mass matrix model:

$$(\bar{f}_L^i\ \ \bar{F}_L^\alpha) \left(\begin{array}{cc} (\hat{Y}_f)_i{}^j & (\Phi_{0f})_i{}^\beta \\ (\bar{\Phi}_{0f}^T)_\alpha{}^j & -(S_f)_\alpha{}^\beta \end{array} \right) \left(\begin{array}{c} f_{Rj} \\ F_{R\beta} \end{array} \right). \tag{2.4}$$

Here $f_{L(R)}$ and $F_{L(R)}$ are, respectively, left (right) handed light and heavy fermions fields. Exactly speaking, we have to read $\bar{f}_L$ in Eq.(2.4) as $\bar{f}_L H_{u/d}/\Lambda$. However, for convenience, we have denoted those as $\bar{f}_L$ simply. Such a seesaw-like scenario with the democratic form of S_f has been proposed by Fusaoka and one of the authors (YK) [6] in order to understand the observed fact

$$m_t \sim \Lambda_{weak}, \qquad m_u \sim m_d \sim m_e. \tag{2.5}$$

In the Yukawaon model, when we consider $|\hat{Y}_f| \ll |\Phi_{0f}| \ll |S_f|$ (i.e. $\Lambda \ll \Lambda'$), we obtain a mass matrix for $\bar{f}'_L$ and f'_R,

$$M_f \simeq \hat{Y}_f + \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}, \tag{2.6}$$

after the block diagonalization of Eq.(2.4). Regrettably, this relation (2.6) is not one we want, because the first term $\hat{Y}_f$ in Eq.(2.6) is independent of the second term $\Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}$. Therefore, in the previous paper [1], in order to obtain the relation (2.3), we have put somewhat phenomenological assumptions. In Appendix B, we will give a brief review on this mechanism proposed in the previous paper [1]. However, in this paper, we start our discussion from the VEV relation (2.3) without repeating the same thing.

(ii) The VEV form of $\langle \hat{S}_f \rangle$, which is due to the symmetry breaking $\mathrm{U}(3)' \to \mathrm{S}_3$, is given by

$$\langle \hat{S}_f \rangle = v_{Sf} (\mathbf{1} + b_f X_3), \tag{2.7}$$

where $\mathbf{1}$ and X_3 are defined as

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{2.8}$$

The parameters b_f in Eq. (2.7) are typical examples of "family-number-independent" parameters. However, we will take $b_e = 0$ and $b_\nu = 0$ in the charged lepton sector $f = e$ and neutrino sector $f = \nu$ in the phenomenological investigation in this model. In Sec.3, we give a superpotential which gives $b_e = 0$ and $b_\nu = 0$.

(iii) The VEV form $\langle \Phi_{0f} \rangle$ are given by

$$\langle \Phi_{0f} \rangle_i^\alpha = \frac{1}{\Lambda} \langle \Phi_0 \rangle_{ik} \langle \bar{P}_f \rangle^{k\alpha}. \tag{2.9}$$

(iv) The VEV of a flavon Φ_0, $\langle \Phi_0 \rangle$ was defined by

$$\langle \Phi_0 \rangle = v_0 \, \mathrm{diag}\,(z_1, z_2, z_3) \propto \mathrm{diag}\,(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \tag{2.10}$$

where z_i is normalized as $z_1^2 + z_2^2 + z_3^2 = 1$. The $\langle \Phi_0 \rangle$ plays a crucial role in the phenomenological investigation of the Yukawaon model. However, the form of $\langle \Phi_0 \rangle$ is at present, only assumption. The existence of such a flavon which takes a diagonal form in the flavor basis in which the flavons with $\mathbf{8} + \mathbf{1}$ of $\mathrm{U}(3)'$ take a S_3 invariant form (2.7) is the most basic postulation in the Yukawaon model. The form (2.10) was suggested by a charged lepton mass relation [7]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \tag{2.11}$$

which is excellently satisfied by the observed charged lepton masses (pole masses) as $K = (2/3) \times (0.999989 \pm 0.000014)$. If we accept the flavon Φ_0 with its VEV (2.10), the charged lepton relation (2.11) is simply expressed as

$$K = \frac{\mathrm{Tr}[\Phi_0 \Phi_0]}{(\mathrm{Tr}[\Phi_0])^2}. \tag{2.12}$$

Thus, the flavon Φ_0 was introduced into the Yukawaon model motivated by this relation (2.12). On the other hand, we know that there exist the CKM mixing in quark sector and the PMNS mixing in lepton sector. Therefore, it is absolutely impossible that masses of all quarks and leptons satisfy relations similar to Eq.(2.11). The relation (2.11) is a specific relation only for the charged leptons. We consider that a fundamental flavor basis in flavor physics is a basis in which the charged lepton mass matrix is diagonal. Moreover, we speculate that all masses and mixings of quarks and leptons might be described by inputing the observed charged lepton mass values. Under such the idea, the Yukawaon model has been introduced and investigated [5].

(v) The VEV form $\langle P_f \rangle$ are defined as

$$\langle P_u \rangle = v_P \,\mathrm{diag}(e^{i\phi_1},\ e^{i\phi_2},\ e^{i\phi_3}), \quad \langle P_d \rangle = v_P \mathbf{1}, \quad \langle P_\nu \rangle = v_P \mathbf{1}, \qquad \langle P_e \rangle = v_P \mathbf{1}. \tag{2.13}$$

The parameters (ϕ_1, ϕ_2, ϕ_3) in Eq. (2.13) are typical examples of "family-number-dependent" parameters. However, in Sec.4, we will show that the parameters (ϕ_1, ϕ_2, ϕ_3) are discribed by the chaged lepton masses $(m_e, m_\mu . m_\tau)$ with help of two family-number-independent parameters. The origin of these VEV forms will be discussed in Sec.3.

(vi) A neutrino mass matrix is given, by adopting the conventional seesaw mechanism [8], as

$$(M_\nu^{Majorana})_{ij} = \langle \hat{Y}_\nu \rangle_i{}^k \langle \bar{Y}_R^{-1} \rangle_{kl} \langle \hat{Y}_\nu^T \rangle^l{}_j. \tag{2.14}$$

Here, the following VEV structure of the Y_R (the Majorana mass matrix of the right-handed neutrinos ν_R) is assumed:

$$\langle \bar{Y}_R \rangle^{ij} = k_R \frac{1}{\Lambda^2} \left[\left(\langle \bar{\Phi}_0 \rangle^{ik} \langle \hat{E} \rangle_k{}^l \langle \hat{Y}_u \rangle_l{}^j + \langle \hat{Y}_u^T \rangle^i{}_k \langle \hat{E}^T \rangle^k{}_l \langle \bar{\Phi}_0 \rangle^{lj} \right) + \xi_R \langle \hat{Y}_\nu^T \rangle^i{}_k \langle \bar{E} \rangle^{kl} \langle \hat{Y}_\nu \rangle_l{}^j \right], \tag{2.15}$$

where $\langle \hat{E} \rangle = \langle \bar{E} \rangle = v_E \mathbf{1}$. The form of the first term in Eq.(2.15), $\Phi_0 Y_u$, was first introduced in Ref.[9]. Since then, the form of Y_R has played a crucial role in the Yukawaon models. (The form (2.15) has been done a minor change from one which was given in the previous paper [1], but the VEV relation is unchanged effectively.)

Since we deal with mass ratios and mixings only, the common coefficients k_f, v_{Sf}, and so on does not affect the numerical results, so that hereafter we omit such coefficients even it those have dimensions.

3 Examples of superpotentials

In this section, we demonstrate how to derive the VEV relations from superpotentials given. Hereafter, for convenience, we have sometimes dropped the notations "$\langle$" and "$\rangle$".

First, let us discuss a simple case which gives $b_e = b_\nu = 0$ by considering the following superpotential:

$$W_S = \sum_{f=e,\nu} \left\{ \lambda_{1f}[(S_f)_\alpha^{\ \beta}(P_f)_{\beta k}(\bar{P}_f)^{k\alpha}] + \lambda_{2f}[(S_f)_\alpha^{\ \alpha}][(P_f)_{\beta k}(\bar{P}_f)^{k\beta}] \right\}, \tag{3.1}$$

where we have taken R charges of S_f and P_f as

$$R(S_f) + 2R(P_f) = 2. \quad (f = e, \nu) \tag{3.2}$$

Here and hereafter, we take $R(\bar{A}) = R(A)$ for arbitrary flavons A. (Eq.(3.1) is slightly improved from the previous paper [1].) The vacuum condition for the superpotential (3.1) leads to

$$S_f = \mathbf{1}, \quad P_f \bar{P}_f = \mathbf{1}. \quad (f = e, \nu) \tag{3.3}$$

The result $S_f = \mathbf{1}$ means $b_f = 0$, so that

$$\hat{Y}_e \propto \hat{Y}_\nu \propto \Phi_0 \bar{\Phi}_0. \tag{3.4}$$

However, this does not mean that $\hat{Y}_e$ and $\hat{Y}_\nu$ are an identical flavon.

A specific solution of the result $P_f \bar{P}_f = \mathbf{1}$ is

$$P_f = \mathrm{diag}(e^{i\phi_1^f}, e^{i\phi_2^f}, e^{i\phi_3^f}). \tag{3.5}$$

Since the VEV matrix form of S_f $(f = e, \nu)$ is diagonal, the VEV matrix P_f is commutable with S_f, so that the phase parameters ϕ_i^f in P_f cannot play any physical role in $\bar{P}_f(S_f^{-1})P_f$. Therefore, we put $P_f = \mathbf{1}$ simply for $f = e, \nu$.

For $f = u, d$, we consider the following superpotential:

$$W_{Pq} = \frac{1}{\Lambda} \left\{ \lambda_{1P} \mathrm{Tr}[P_u \bar{P}_u P_d \bar{P}_d] + \lambda_{2P} \mathrm{Tr}[P_u \bar{P}_u] \mathrm{Tr}[P_d \bar{P}_d] \right\}, \tag{3.6}$$

where we take

$$R(P_u) + R(P_d) = 1. \tag{3.7}$$

The SUSY vacuum condition leads to $P_u \bar{P}_u = \mathbf{1}$ and $P_d \bar{P}_d = \mathbf{1}$.

As seen in Eq.(2.13), we have assumed $\phi_i^u \neq 0$ in the up-quark sector, while $\phi_i^d = 0$. Here, we would like to give a comment on this assumption. VEV matrices Φ_0, P_f and S_f in Eq.(2.3) take symmetric forms. Therefore, when we denote the VEV matrix relations Eq.(2.3) simply as

$$(\hat{Y}_f)_i^{\ j} = (\Phi_0 P_f^* (S_f^{-1}) P_f \Phi_0)_i^{\ j}, \tag{3.8}$$

we obtain

$$\begin{aligned}(\hat{Y}_f^T)^j{}_i &= (\Phi_0 P_f (S_f^{-1}) P_f^* \Phi_0)^j{}_i, \\ (\hat{Y}_f^\dagger)^j{}_i &= (\Phi_0 P_f^* (S_f^{-1})^* P_f \Phi_0)^j{}_i.\end{aligned} \tag{3.9}$$

We assume that the VEV matrix $\hat{Y}_f$ satisfies either

$$(\hat{Y}_f^T)^j{}_i = (\hat{Y}_f)_i{}^j, \tag{3.10}$$

or

$$(\hat{Y}_f^\dagger)^j{}_i = (\hat{Y}_f)_i{}^j. \tag{3.11}$$

(Note that the requirements (3.10) and (3.11) are not $(\hat{Y}_f^T)_i{}^j = (\hat{Y}_f)_i{}^j$ and $(\hat{Y}_f^\dagger)_i{}^j = (\hat{Y}_f)_i{}^j$, respectively.) When we require the case (3.10), we obtain $P_f^* = P_f$ (i.e $\phi_i^f = 0$). While, when we require the case (3.11), we obtain $S_f^* = S_f$ (i.e. $\beta_f = 0$). We do not consider a case with $\phi_i^f = 0$ and $\beta_f = 0$. We require the constraint (3.10) for up-quark sector, so that we obtain $\phi_i^u \neq 0$ and $\beta_u = 0$. On the other hand, we require the constraint (3.11) for down-quark sector, so that we obtain $\phi_i^d = 0$ and $\beta_d \neq 0$.

For the VEV relations (2.9) and (2.15), we consider somewhat tricky prescription: We assume existence of Θ fields which always take VEV values $\langle\Theta\rangle = 0$. For example, in oder to obtain the VEV relation (2.9), we assume the following superptential

$$W_0 = \mu_{0f} (\Phi_{0f})_i{}^\alpha (\Theta_{0f})_\alpha^{\ i} + \lambda_{0f} (\Phi_0)_{ik} (P_f)^{k\alpha} (\Theta_{0f})_\alpha^{\ i}. \tag{3.12}$$

From $\partial W_0/\partial\Theta_{0f} = 0$, we obtain VEV relation (2.9). On the other hand, a derivative of W_0 with respective to other flavon, for example, Φ_0 leads to $\partial W_0/\partial\Phi_0 = \lambda_{0f} \bar{P}_f \Theta_{0f} = 0$. However, the result always includes Θ_{0f}, so that the condition does not any new VEV relations. This prescription is very useful when one flavon appears in the different (two or more) superpotentials.

Similarly, for the VEV relation (2.15), we assume the following superpotential

$$W_R = \left\{ \mu_R \bar{Y}_R^{ij} + \frac{\lambda_R}{\Lambda} \left((\bar{\Phi}_0)^{ik} (\hat{E})_k{}^l (\hat{Y}_u)_l{}^j + (\hat{Y}_u^T)^i{}_k (\hat{E}^T)^k{}_l (\bar{\Phi}_0)^{lj} + \xi_R (\hat{Y}_\nu^T)^i{}_k \bar{E}^{kl} (\hat{Y}_\nu)_l{}^j \right) \right\} (\Theta_R)_{ji}, \tag{3.13}$$

together with

$$W_E = \lambda_{E1} [(\hat{E})_i{}^j (E)_{jk} (\bar{E})^{ki}] + \lambda_{E2} [(\hat{E})_i{}^i] \, [(E)_{jk} (\bar{E})^{kj}], \tag{3.14}$$

where we have taken

$$R(\hat{E}) = R(E) = R(\bar{E}) = \frac{2}{3}. \tag{3.15}$$

4 Relation between (ϕ_1, ϕ_2, ϕ_3) and (m_e, m_μ, m_τ)

In the VEV form of the flavon P_u given by Eq.(2.13), the parameters (ϕ_1, ϕ_2, ϕ_3) are typically family-number-dependent parameters. Now, we let us express these parameters $\phi_i = (\phi_1, \phi_2, \phi_3)$ by using the charged lepton masses $m_{ei} = (m_e, m_\mu, m_\tau)$. Such a mechanism has already been proposed in Ref.[10]. However, flavons in the preset model are different from those in the model [10], in which the Yukawaons $\hat{Y}_f$ were given by a bi-linear form $\hat{Y}_f = \Phi_f \bar{\Phi}_f$ (there are no Φ_f in the present model). Therefore, in the present paper, we give the following superpotential differently from that in Ref.[10]:

$$W_\phi = \left\{ \lambda_1 \left[(\Phi_{0u})_{i\alpha} (\bar{\Phi}_{0d})^{\alpha j} + (\Phi_{0d})_{i\alpha} (\bar{\Phi}_{0u})^{\alpha j} \right] + \lambda_2 (\Phi_{0\nu})_i{}^\alpha (\bar{\Phi}_{0e})_\alpha{}^j + \lambda_3 (\hat{Y}_\nu)_i{}^k (\hat{Y}_e)_k{}^j \right\} (\hat{\Theta}_\phi)_j{}^i . \tag{4.1}$$

The first term gives a VEV relation

$$\left[(\Phi_{0u})_{i\alpha} (\bar{\Phi}_{0d})^{\alpha j} + (\Phi_{0d})_{i\alpha} (\bar{\Phi}_{0u})^{\alpha j} \right] \propto 2 \delta_i^j z_i^2 \cos\phi_i . \tag{4.2}$$

On the other hand, VEVs of the second and third terms give z_i^2 and z_i^4, respectively. Therefore, we obtain a relation

$$\cos\phi_i = a + b z_i^2 , \tag{4.3}$$

where the parameters a and b are family-number-independent parameters.

Note that observable parameters in the three phase parameters (ϕ_1, ϕ_2, ϕ_3) are only two. When we denote

$$\phi_1 = \phi_0 + \tilde{\phi}_1, \quad \phi_2 = \phi_0 + \tilde{\phi}_2, \quad \phi_3 = \phi_0, \tag{4.4}$$

the parameter ϕ_0 is not observable. Therefore, we can always choose arbitrary value of ϕ_0, so that the relation (4.3) is satisfied by choosing two family-number-independent parameters a and b suitably.

Explicitly, we can obtain numerical results as follows: We obtain a relation

$$\cos\phi_1 - b\, z_1^2 = \cos\phi_2 - b\, z_2^2 = \cos\phi_3 - b\, z_3^2, \tag{4.5}$$

by eliminating the parameter a. Then, can obtain a relation for the parameter ϕ_0:

$$b = \frac{\cos\phi_3 - \cos\phi_1}{z_3^2 - z_1^2} = \frac{\cos\phi_3 - \cos\phi_2}{z_3^2 - z_2^2} . \tag{4.6}$$

Since we have obtained the parameter values [1]

$$(\tilde{\phi}_1, \tilde{\phi}_2) = (-176.05^\circ, -169.91^\circ), \tag{4.7}$$

from fitting of the observed CKM mixing data, we obtain family-number-independent parameters ϕ_0 and (a, b)

$$\phi_0 = 33.905^\circ, \quad (a, b) = (1.71573, -0.790018). \tag{4.8}$$

5 R-charge assignments

In the Yukawaon model, the R-charge assignment is very curcial from the phenomenological point of view. If we assign unsuitable R charges, we will obtain unwelcome VEV matrix combinations among flavons. If different flavons, say A and B, which have the same transformation property in U(3)×U(3)′, have the same R charge, then those flavons cause unwelcome mixing $A \leftrightarrow B$. The R-charge assignments must be carefully done.

In the present Yukawaon model, we still have too many flavons lager than a number of VEV relations, so that we cannot uniquely determine R chages of flavons. Therefore, we put the following rules on the R charge assignments for simplicity: (i) a flavon A and $\bar{A}$ have the same R charge:

$$R(\bar{A}) = R(A). \tag{5.1}$$

(ii) We consider that R charge is conserved even after mixing between fermions f and F in the seesaw mass matrix (2.4):

$$R(\hat{Y}_f) = R(\Phi_{0f}) = R(S_f) \equiv r_f. \tag{5.2}$$

On the other hand, from the relation (2.9), we obtain

$$R(P_f) = r_f - r_0, \tag{5.3}$$

so that, from the R charge relation (3.2), we obtain

$$r_e = r_\nu = \frac{2}{3}(r_0 + 1). \tag{5.4}$$

For the quark sector, we obtain an R charge relation

$$r_u + r_d = 1 + 2r_0, \tag{5.5}$$

from Eq.(3.7). On the other hand, we obtain

$$r_u + r_d = r_\nu + r_e = \frac{4}{3}(r_0 + 1), \tag{5.6}$$

from the superpotental (4.1). Eqs.(5.4) and (5.5) fix the value of r_0 as

$$r_0 = \frac{1}{2}, \quad \Rightarrow \quad r_e = r_\nu = 1. \tag{5.7}$$

Moreover, the form of $\bar{Y}_R$, Eq.(3.11) requires a relation

$$r_u = 2r_\nu - r_0, \tag{5.8}$$

because of R charge relation (3.13), so that we obtain r_u and r_d as follows:

$$r_u = \frac{3}{2}, \quad r_d = \frac{1}{2}. \tag{5.9}$$

In other words, under the R charge assignment given in Eqs.(5.7) and (5.9), we can uniquely obtain the VEV relations given in Secs.2-4. R charges of whole flavons in the present models have been summarized in Table 1.

6 Concluding remarks

In conclusion, we have given formulation of a new Yukawaon model on the basis of seesaw type mass matrix model by presenting U(3)×U(3)$'$ assignments, R charges of flavons, superpotential forms, and so on. In spite of quite few parameter model, the model can give a remarkable agreement with the observed quark and lepton mixings and mass ratios. The phenomenological (numerical) results of the model has already been reported in the previous paper [1], which is referred in Appendix A briefly.

The phenomenological success seems to suggest the following points:
(a) The observed quark and lepton masses and mixings are caused by a common origin.
(b) Flavor physics should be investigated on a flavor basis in which charged lepton mass matrix is diagonal. (Mass matrix of family gauge bosons is also diagonal in this basis, so that, family gauge bosons will not cause flavor violation in the charged lepton sector [11] .)
(c) Masses and mixings in the quark sector are given by parameters b_u and b_d in the form of S_f as shown in Eq.(2.3). This mechanism is very interesting.

On the other hand, for the theoretical aspect, the model has still many problems which should be improved in future. For example, (i) a mechanism discussed in Appendix B will be revised in near future. (ii) In the present paper, we did not discuss explicit scales of Λ and Λ', although we have tacitly assumed that $\langle A_{ij} \rangle \sim \Lambda$, $\langle A_{\alpha\beta} \rangle \sim \Lambda'$ and $\langle A_{i\alpha} \rangle \sim \sqrt{\Lambda\Lambda'}$. The choice is highly correlated in the tininess of neutrino masses. Since we have discussed masses and mixings only, we have neglected the common coefficents and VEV values in the sectors, for example, k_f in Eq.(2.3) v_{Sf} in Eq.(2.7), v_0 in Eq.(2.10), k_R in Eq.(2.15), and so on. Those are our future task.

We believe that the present model can give fruitful suggestions for the study of flavor physics.

Appendix A: Numerical results

The VEV relations in the present model are exactly the same as those in the previous model [1]. The purpose of the present paper is to give formulations which we cannot give in the previous short paper, and not to give numerical refitting on the model. Therefore, in the present paper, we quote only the numerical results of the previous paper [1]. For the details, see Ref.[1]. Note that the parameter fitting in Ref.[1] has been done at $\mu = m_Z$, so that, even for the charged lepton masses inputted, we have used values at $\mu = m_Z$, not pole mass values.

We summarize our mass matrices for the numerical analysis as follows:

$$\hat{Y}_u = \Phi_0 \bar{P}_u (\mathbf{1} + b_u X_3)^{-1} P_u \bar{\Phi}_0, \quad \hat{Y}_d = \Phi_0 (\mathbf{1} + b_d e^{i\beta_d} X_3)^{-1} \bar{\Phi}_0, \quad \hat{Y}_e = \Phi_0 \bar{\Phi}_0, \tag{A.1}$$

$$M_\nu^{Majorana} = \hat{Y}_\nu \bar{Y}_R^{-1} \hat{Y}_\nu^T, \quad \bar{Y}_R = \bar{\Phi}_0 \hat{Y}_u + \hat{Y}_u^T \bar{\Phi}_0 + \xi_R \hat{Y}_e^T \hat{Y}_e, \quad \hat{Y}_\nu = \Phi_0 \bar{\Phi}_0. \tag{A.2}$$

For convenience of numerical fitting, we re-define all VEV matrices of flavons as dimensionless matrices, i.e. $\bar{P}_u = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, $\Phi_0 = \mathrm{diag}(z_1, z_2, z_3)$, and so on.

In the present model, we have six family-number-independent parameters. b_u and (b_d, β_d) are fixed by the observed quark mass ratios [12]. the parameters (ϕ_1, ϕ_2) are fixed by the observed CKM mixing parameters. (For convenience, we count the parameters (ϕ_1, ϕ_2) as family-number-independent parameters.) ξ_R is fixed by the neutrino mass squared difference ratio $R_\nu \equiv (m_{\nu 2}^2 - m_{\nu 1}^2)/(m_{\nu 3}^2 - m_{\nu 2}^2)$. Our input parameter values are

$$b_u = -1.011, \quad b_d = -3.3522, \quad \beta_d = 17.7^\circ, \quad (\phi_1, \phi_2) = (-176.05^\circ, -167.91^\circ), \quad \xi_R = 2039.6, \tag{A.3}$$

Then, our predicted results are in excellent agreement with the observed values as seen in Table 2. Note that PMNS mixing is predicted with no free parameter after mass ratios and CKM mixing are fitted.

It is interesting that the model predicts $\delta_{CP}^\ell = -68^\circ$, which shows $\delta_{CP}^\ell \simeq -\delta_{CP}^q$. It is also worthwhile noticing that we obtain a large value 21 meV for the effective Majorana neutrino mass $\langle m \rangle$ in spite of the normal hierarchy for the neutrino mass in our model.

Appendix B: Yukawaon model based on a seesaw scenario

A notable point in the recent work [1] compared with conventional Yukawaon models [5] is that we have adopted a seesaw-type mass matrix (2.4). However, the relation (2.6), which is derived from (2.4), is not our desired relation $Y_f \simeq \phi_{0f} S_f^{-1} \bar{\Phi}_{0f}$. Therefore, in the previous paper [1], in order to obtain the relation (2.3), we have put somewhat phenomenological assumptions: [Assumption 1] The VEV value $\hat{Y}_f$ and the VEV value $M_F = -S_f$ take the same scale transformation (we denote the scale transformation as a parameter ζ_f):

$$M_f = \zeta_f \hat{Y}_f + \frac{1}{\zeta_f} \Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}. \tag{B.1}$$

Table 2: Predicted values vs. observed values. [Quoted from [1]].

	$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert$	$\lvert V_{td}\rvert$	$\delta^q_{CP}(^\circ)$	r^u_{12}	r^u_{23}	r^d_{12}	r^d_{23}
Predicted	0.2257	0.03996	0.00370	0.00917	81.0	0.061	0.060	0.049	0.027
Observed	0.22536	0.0414	0.00355	0.00886	69.4	0.045	0.060	0.053	0.019
	± 0.00061	± 0.0012	± 0.00015	${}^{+0.00033}_{-0.00032}$	± 3.4	${}^{+0.013}_{-0.010}$	± 0.005	${}^{+0.005}_{-0.003}$	${}^{+0.006}_{-0.006}$
	$\sin^2 2\theta_{12}$	$\sin^2 2\theta_{23}$	$\sin^2 2\theta_{13}$	R_ν (10^{-2})	$\delta^\ell_{CP}(^\circ)$	$m_{\nu 1}$ (eV)	$m_{\nu 2}$ (eV)	$m_{\nu 3}$ (eV)	$\langle m\rangle$ (eV)
Predicted	0.8254	0.9967	0.1007	3.118	-68.1	0.038	0.039	0.063	0.021
Observed	0.846	0.999	0.093	3.09	no data	no data	no data	no data	$< \mathrm{O}(10^{-1})$
	± 0.021	${}^{+0.001}_{-0.018}$	± 0.008	± 0.15					

[Assumption 2] The VEV value $\hat{Y}_f$ is taken so that M_f takes a locally minimum value under the ζ_f transformation:

$$\frac{\partial M_f}{\partial \zeta_f} = \hat{Y}_f - \frac{1}{\zeta_f^2}\Phi_{0f} S_f^{-1} \bar{\Phi}_{0f} = 0. \qquad (B.2)$$

Then, we obtain

$$\hat{Y}_f = \frac{1}{\zeta_f^2}\Phi_{0f} S_f^{-1} \bar{\Phi}_{0f}, \quad \text{i.e.} \quad M_f = \frac{2}{\zeta_f}\Phi_{0f} S_f^{-1} \bar{\Phi}_{0f} = 2\zeta \hat{Y}_f. \qquad (B.3)$$

The above derivation is somewhat an easygoing way, thus, it may be revised into more natural scenario in future.

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