

Family Gauge Boson Mass Estimated from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

Yoshio Koide

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

E-mail address: koide@kuno-g.phys.sci.osaka-u.ac.jp

Abstract

It is emphasized that a rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ becomes promising in a future search for a new particle, because the theoretical treatment is well established and the value of the branching ratio $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is sensitive to a search for a new particle with a TeV scale mass. As an example, according to a U(3) family gauge boson model which predicts the lowest family gauge boson with a few TeV mass M_{11} , the branching ratio $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is discussed. If we can obtain, in future, a slightly lower value $Br^{obs} \sim 0.9 \times 10^{-10}$ compared with the present observed value $Br^{obs} = (1.7 \pm 1.1) \times 10^{-10}$, we can conclude $M_{11} \sim$ a few TeV.

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1 Introduction

In the current physics, search for a new particle is one of our big concern. Except for direct searches (e.g. Z' search at the LHC), it is usually tried to investigate a deviation between an observed value and its standard model (SM) prediction. However, in the harmonic events, it is not so easy to evaluate the QCD effects exactly. Leptonic processes will rather be important for such new particle searches. In this paper, we notice a rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ as a promising way for such a new particle search.

The present observed value [1] and a SM prediction [2] are as follows:

$$Br^{obs} \equiv Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{obs} = (1.7 \pm 1.1) \times 10^{-10}, \quad (1.1)$$

$$Br^{SM} \equiv Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{SM} = (0.80 \pm 0.11) \times 10^{-10}. \quad (1.2)$$

Of course, the observed value (1.1) is consistent with the SM value (1.2), so that there is no room of a new particle. However, the observed value (1.1) has a large error at present. As far as the center values are concerned, we see a sizable deviation from each other. The predicted value (1.2) is reliable because the decay is a leptonic one, so that the theoretical treatment is well established. We would like especially to notice that, different from most rare decay searches in which we know only upper limits of the branching ratios, we know that the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has really been observed although Br^{obs} has a large error at present. We may expect that the error in the observed value would be reduced in the near future. The study would be hopeful as the nearest way of a new particle search.

In this paper, we discuss a contribution of a family gauge boson (FGB) to the branching ratio $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ on the basis of a specific FGB model [3] which gives a possibility of a TeV scale mass of the lowest FGB (a brief review will be given later). We will conclude that if the observed value becomes near the SM prediction (1.2), i.e. if we could have $Br^{obs} \sim 0.9 \times 10^{-10}$ in future, we will be able to conclude that a mass M_{11} of the lowest FGB A_1^1 is an order of a few TeV. Of course, on the other hand, if the observed value becomes far from the SM value (1.2), i.e. $Br^{obs} > 1.0 \times 10^{-10}$, the result will rule out the specific scenario of FGBs given in Ref.[3], and it will confirm the conventional FGB model. In either case, the experimental search for the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ will play an important role in investigating the FGB models.

Meanwhile, we would like to talk about why we notice FGBs. In the standard model (SM), the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [5] $V_{CKM} = U_u^\dagger U_d$ is, of course, observable quantity, while the up- and down-quark mixing matrices U_u and U_d are not separately observable quantities. When a family gauge symmetry exists, U_u and U_d can be individually observable. We usually consider that masses M_{ij} of the family gauge bosons (FGBs) A_i^j are larger than an order of 10^4 TeV, because of the observed values of P^0 - \bar{P}^0 mixings ($P = K, D, B, B_s$). Therefore, even if FGBs exist, it will be hard to observe their effects by means of terrestrial experiments.

Even though FGBs are different from a conventional model, we know a FGB model which has been proposed by Sumino [4] and which can give a considerably low mass of the lowest FGB. Sumino has introduced FGBs in order to solve a problem in a charged lepton mass relation [6] $K \equiv (m_e + m_\mu + m_\tau) / (\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$: the mass relation is well satisfied by the pole masses [i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$], while it is not so satisfactory for the running masses [i.e. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$]. The deviation is due to a factor $\log(m_{ei}^2/\mu^2)$ in the QED radiative correction. Sumino has considered that FGBs really exist, and the factor $\log(m_{ei}^2/\mu^2)$ is canceled by a factor $\log(M_{ii}^2/\mu^2)$ in the FGB contribution. In order that the cancellation works correctly as we hope, the FGB masses M_{ii} have to be proportional to the charged lepton masses m_{ei} :

$$M_{ij} = k (m_{ei}^n + m_{ej}^n) \quad (n = \pm 1, \pm 2, \dots), \quad (1.3)$$

and the family gauge coupling constant g_F has to satisfy a relation

$$\left(\frac{g_F}{\sqrt{2}}\right)^2 = \frac{2}{n} e^2 = \frac{4}{n} \left(\frac{g_w}{\sqrt{2}}\right)^2 \sin^2 \theta_w. \quad (1.4)$$

(Although in the original Sumino model [4], n has been taken as $n = 1$, we can, in general, take $n = \pm 1, \pm 2, \dots$.) In the Sumino model, FGB mass matrix is diagonal in the family basis in which the charged lepton mass matrix is diagonal. Therefore, family number violation does not occur in the lepton sector (at the tree level). In contrast to the lepton sector, in the quark sector,

the up- and down-quark mass matrices M_u and M_d are, in general, not diagonal, so that family number violation appears at tree level via quark mixing matrices U_u and U_d . (See Eq.(1.6) later.) Therefore, FGB contribution to P^0 - \bar{P}^0 mixing ($P = K, D, B, B_s$) are allowed only through the quark mixings U_u and U_d . This is a reason that the FGB contribution in the Sumino model is considerably suppressed compared with the conventional FGB model. However, even in the Sumino FGB model, we cannot obtain a visible low mass with a TeV scale.

Recently, a new FGB model [3] with a TeV scale mass of the lowest FGB has been proposed, without conflicting with the observed P^0 - \bar{P}^0 mixings, on the basis of the Sumino U(3) FGB model. The point of the success in the new model [3] compared with the original Sumino model [4] is as follow: when we define the family number in the lepton sector as $(e_1, e_2, e_3) = (e^-, \mu^-, \tau^-)$, the family number in the quark sector is defined as $(d_1, d_2, d_3) = (d, s, b)$ in the Sumino model [4] as the same as the conventional one, while it is defined as

$$(d_1, d_2, d_3) = (b, d, s) \quad [\text{or } (d_1, d_2, d_3) = (b, s, d)], \quad (1.5)$$

in the new model [3]. In addition to the original Sumino model, when the family number assignment is revised from the conventional one to the new one (1.5), we can obtain a lower mass M_{11} without conflicting with the observed K^0 - \bar{K}^0 mixing. However, although a value $M_{11} \sim 1$ TeV has been reported in Ref.[3], the value should be not taken rigidly, because the estimates of P^0 - \bar{P}^0 mixing are still dependent on QCD corrections and on many unfixed parameter values. We need a confirmation from another phenomenon.

In conclusion, in this paper, according to the revised model [3] of the original Sumino model, we take the following interactions of quarks and leptons with FGBs:

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}} \left[\sum_{\ell=\nu, e} (\bar{\ell}_L^i \gamma_\mu \ell_{Lj} - \bar{\ell}_{Rj} \gamma_\mu \ell_R^i) + \sum_{q=u, d} (U_q^*)_{ik} (U_q)_{jl} (\bar{q}_k \gamma_\mu q_l) \right] (A_i^j)^\mu, \quad (1.6)$$

together with the new family number assignment (1.5). The first term (leptonic part) in Eq.(1.6) takes somewhat an unfamiliar form. This is due to the following reason: in the Sumino model, the minus sign for the cancellation has been provided by the U(3) assignment $(e_L, e_R) \sim (\mathbf{3}, \mathbf{3}^*)$ for the left-handed and right-handed charged leptons e_L and e_R . As a result, we have unwelcome situation: (i) The model cannot be anomaly free. (ii) Effective current-current interactions with $\Delta N_{fam} = 2$ (N_{fam} is a family number) appear inevitably. In order to avoid problem (i), we tacitly assume an existence of heavy leptons in the lepton sector. On the other hand, the problem (ii) causes a fatal damage to the P^0 - \bar{P}^0 mixing problem. Therefore, only for quark sector, we restore the Sumino's assignment to the normal assignment $(q_L, q_R) \sim (\mathbf{3}, \mathbf{3})$ of U(3), because we do not need such the Sumino cancellation [4] in the quark sector. Furthermore, in the present paper, according to the modified model [3], we discuss the case $n = 2$ which leads to a FGB mass $M_{11} \sim 1$ TeV in Ref.[3].

2 Estimate of $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$

The value $M_{11} \sim 1$ TeV in Ref.[3] has been estimated from a difference between the observed value of K^0 - \bar{K}^0 mixing and its SM value. Of course, since the present observed value of K^0 - \bar{K}^0 mixing is consistent with the predicted value (with a large error), so that the value $M_{11} \sim 1$ TeV should be regarded as a lower value in an optimistic estimate. On the other hand, the SM value is based on somewhat ambiguous input values (for example, QCD effects and so on). In order to confirm this estimate, in this section, we estimate $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ which includes FGB contributions in addition to the SM contribution. In this estimate, we will use the value (1.2) as the SM contribution. The estimated result will be compared with a value Br^{obs} in a future observation, and thereby a possible value M_{11} will be speculated.

We assume that the difference between the values (1.1) and (1.2) is due to contributions from FGBs, although the value (1.1) have large errors and it seems that the value is consistent with the SM prediction (1.2) at present. However, we consider that the error value in (1.1) will be improved in the near future. Then, the value M_{11} from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ will become rather reliable than that from the observed K^0 - \bar{K}^0 mixing.

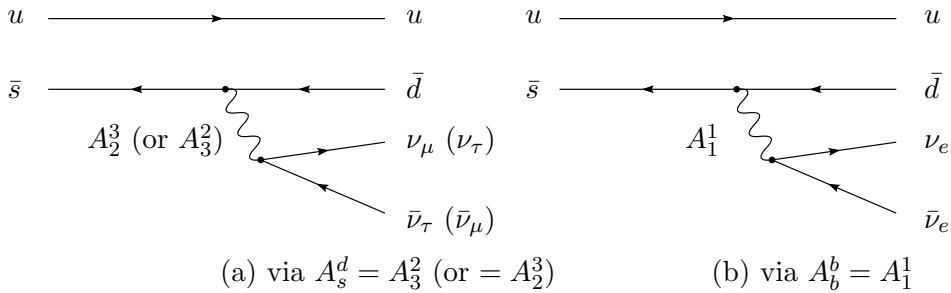


Figure 1: Contributions of FGBs in the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$

The neutrinos $\nu \bar{\nu}$ in the SM prediction mean $\nu_e \bar{\nu}_e + \nu_\mu \bar{\nu}_\mu + \nu_\tau \bar{\nu}_\tau$ because it is caused by exchange of Z . On the other hand, in the FGB contributions, there are two diagrams as seen in Fig. 1. A decay given in Fig.1 (a) is induced via A_s^d (i.e. A_2^3 or A_3^2), so that the final state $\nu \bar{\nu}$ means $\nu_\mu \bar{\nu}_e$. Another one [Fig.1 (b)] is induced via A_1^1 , so that the final state $\nu \bar{\nu}$ means $\nu_e \bar{\nu}_e$. Those decay amplitudes \mathcal{M}_a and \mathcal{M}_b are proportional to factors $1/M_{23}^2$ and $|(U_d)_{bd}|| (U_d)_{bs} | / M_{11}^2$, respectively. Under the approximation $U_u \sim \mathbf{1}$ and $U_d \simeq V_{CKM}$, we obtain $|(U_d)_{bd}|| (U_d)_{bs} | = 3.59 \times 10^{-4}$. On the other hand, we have FGB mass ratios

$$\frac{M_{23}^2}{M_{11}^2} = \frac{1}{2} \left(\frac{m_\tau^2}{m_e^2} + \frac{m_\mu^2}{m_e^2} \right) = 6.05 \times 10^6, \quad (2.1)$$

from Eq.(1.4) with $n = 2$, so that we obtain the ratio of the matrix element \mathcal{M}_b (Fig.1 (b)) to \mathcal{M}_a (Fig.1 (a)) as $|\mathcal{M}_b/\mathcal{M}_a| = 2.17 \times 10^3$. Therefore, we find that the contribution from Fig.1 (a) is negligible compared with that from Fig.1 (b). Hereafter we neglect the contribution from the diagram (a).

Let us denote the observed branching ratio $Br^{obs} \equiv Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{obs}$ as follows:

$$Br^{obs} = k (|\mathcal{M}(\nu_e \bar{\nu}_e)|^2 + |\mathcal{M}(\nu_\mu \bar{\nu}_\mu)|^2 + |\mathcal{M}(\nu_\tau \bar{\nu}_\tau)|^2), \quad (2.2)$$

where $\mathcal{M}(\nu_i \bar{\nu}_i)$ denotes matrix elements of $K^+ \rightarrow \pi^0 \nu_i \bar{\nu}_i$, and k is a constant which includes phase volume for three body decay in the limit of massless neutrinos. In the SM, since

$$\mathcal{M}(\nu_e \bar{\nu}_e) = \mathcal{M}(\nu_\mu \bar{\nu}_\mu) = \mathcal{M}(\nu_\tau \bar{\nu}_\tau) \equiv \mathcal{M}_{SM}, \quad (2.3)$$

the predicted branching ratio in SM Br^{SM} can be expressed as

$$Br^{SM} = 3k|\mathcal{M}_{SM}|^2. \quad (2.4)$$

On the other hand, when we denote a matrix element corresponding to Fig.1 (b) as $\mathcal{M}_{FGB} \equiv \mathcal{M}(\nu_e \bar{\nu}_e)_{FGB}$, the observed branching ratio Br^{obs} is given by

$$Br^{obs} = k (|\mathcal{M}_{SM} + \mathcal{M}_{FGB}|^2 + 2|\mathcal{M}_{SM}|^2) = \frac{1}{3} Br^{SM} \left(\left| 1 + \frac{\mathcal{M}_{FGB}}{\mathcal{M}_{SM}} \right|^2 + 2 \right). \quad (2.5)$$

In order to estimate

$$(R_{11})^2 \equiv \left(\frac{\mathcal{M}_{FGB}}{\mathcal{M}_{SM}} \right)^2 = 3 \frac{Br^{FGB}}{Br^{SM}}, \quad (2.6)$$

we define a parameter

$$\varepsilon \equiv \frac{Br^{SM}(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{Br(K^+ \rightarrow \pi^0 \nu e^+)}. \quad (2.7)$$

We use the following approximate relation

$$\frac{Br^{FGB}(K^+ \rightarrow \pi^+ \nu_e \bar{\nu}_e)}{Br(K^+ \rightarrow \pi^0 \nu e^+)} = \frac{|V_{td}|^2 |V_{ts}|^2 (g_F^2 / 8M_{11}^2)^2 f_+}{\frac{1}{2} |V_{us}|^2 (g_W^2 / 8M_W^2)^2 f_0}, \quad (2.8)$$

where $f_+ \equiv f(m_{\pi^+}/m_{K^+})$, $f_0 \equiv f(m_{\pi^0}/m_{K^+})$, and $f(x)$ is a phase space function $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$. Here, we have neglected the lepton masses. We have also neglected form factor effects in $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K^+ \rightarrow \pi^0 \nu_e e^+$. We have assumed that both

form factors are approximately similar, so that the effect is canceled in our estimate of the ratio. Therefore, we obtain

$$R_{11} = \sqrt{\frac{6}{\varepsilon} \frac{f_+}{f_-}} \xi \frac{g_F^2/8M_{11}^2}{g_W^2/8M_W^2}. \quad (2.9)$$

where $\xi \equiv |V_{td}||V_{ts}|/|V_{us}| = 1.59 \times 10^{-3}$ [1]. $f_+/f_0 = 0.964$ and $g_W^2/8M_W^2 = 1/2v_H^2 = 8.25 \text{ TeV}^{-2}$ (we have used $v_H = 246.22 \text{ GeV}$). In the present paper, we will use the value of ε

$$\varepsilon = \frac{(0.80 \pm 0.11) \times 10^{-10}}{(5.07 \pm 0.04) \times 10^{-2}} = (1.58 \pm 0.22) \times 10^{-9}. \quad (2.10)$$

Thus, by using Eqs.(2.5) - (2.10), we can estimate a value of M_{11} for given value of Br^{obs} . For example, for $Br^{SM} = 8.0 \times 10^{-10}$ and $Br^{obs} = (1.7 \pm 1.1) \times 10^{-10}$ gives

$$M_{11} = 0.54_{-0.13}^{+\infty} \text{ TeV}. \quad (2.11)$$

(The expression (2.11) is somewhat misleading. The value $M_{11} = 0.54 \text{ TeV}$ means a value of M_{11} estimated by using the center value $Br^{obs} = 1.7 \times 10^{-10}$. On the other hand, the value $M_{11} = (0.54 - 0.13) = 0.41 \text{ TeV}$ means a value estimated by using the upper value $Br^{obs} = (1.7 + 1.1) \times 10^{-10} = 2.8 \times 10^{-10}$. The value $M_{11} = \infty$ means nothing but that there is no room for a new physics at present. In the expression $\frac{+\sigma}{-\sigma}$, the values mean neither the theoretical error nor the experimental error. Nevertheless, for the convenience, we will use this expression hereafter.) If the observed value of Br^{obs} becomes slightly improved in future, for example, $Br^{obs} = (1.7 \pm 0.8) \times 10^{-10}$, we will obtain a finite value of $M_{11} = (0.54_{-0.11}^{+0.82}) \times 10^{-10}$.

However, an estimate of M_{11} for larger than a TeV mass should be careful, because it is highly dependent on the value of Br^{SM} . In order to understand this situation, we show the behavior of M_{11} versus $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in Fig.2. As seen in Fig.2, it is impossible to estimate an exact value of M_{11} under the SM prediction (1.2) with the present error.

Nevertheless, we can conclude some important results from Fig.2: (i) A lower limit of Br^{obs} (not a upper limit) gives an upper limit of M_{11} . A mass value M_{11} smaller than 1 TeV has already ruled out by dilepton searches at the LHC [7]. This suggests that if A_1^1 really exists, an observed value of Br^{obs} in the near future must be smaller than 1×10^{-10} . In other words, if $Br^{obs} > 1.0 \times 10^{-10}$ is established, the specific FGB model discussed in this section will be ruled out. (ii) In order to estimate the value of M_{11} , more improvements of the errors are required not only for the observed value Br^{obs} but also the SM estimation Br^{SM} .

So far we are taken $n = 2$ in the FGB mass relation (1.3), because the case can give the mass M_{11} with a few TeV without conflicting with the observed P^0 - \bar{P}^0 mixings = K, D, B, B_s [3]. For reference, here, we would like to comment on the simplest case $n = 1$. Since the case gives a mass ratio $(M_{23}/M_{11})^2 = 1.84 \times 10^3$ instead of the value (2.1), we cannot neglect the

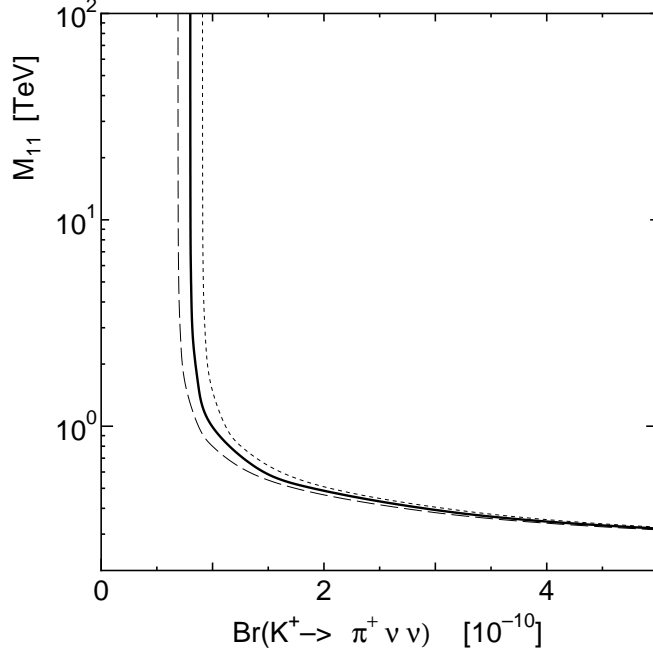


Figure 2: Mass M_{11} versus branching ratio $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. Solid, dashed and dotted curves denote cases of $Br^{SM}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 0.80 \times 10^{-10}$, $(0.80+0.11) \times 10^{-10}$ and $(0.80-0.11) \times 10^{-10}$, respectively. If we take the present observed value $Br^{obs} = (1.7 \pm 1.1) \times 10^{-10}$ we obtain $M_{11} = 0.54_{-0.13}^{+\infty}$ TeV for $Br^{SM} = 0.80 \times 10^{-10}$.

contribution given in Fig.1 (a) compared with Fig.1 (b). Then, we have to modify Br^{obs} given in Eq.(2.2) as $Br(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{obs}$ as follows:

$$Br^{obs} = k (|\mathcal{M}(\nu_e \bar{\nu}_e)|^2 + |\mathcal{M}(\nu_\mu \bar{\nu}_\mu)|^2 + |\mathcal{M}(\nu_\tau \bar{\nu}_\tau)|^2 + |\mathcal{M}(\nu_\mu \bar{\nu}_\tau)|^2), \quad (2.12)$$

(Case of $A_2^3 (= A_d^s)$). Therefore, Eq.(2.5) is modified into

$$Br^{obs} = \frac{1}{3} Br^{SM} (|1 + R_{11}|^2 + 2 + |R_{23}|^2), \quad (2.13)$$

where R_{11} is defined by Eq.(2.6) and R_{23} is given by

$$|R_{23}|^2 = 3 \frac{Br^{FGB}(\nu_\mu \bar{\nu}_\tau)}{Br^{SM}} = \frac{3}{\varepsilon} \frac{(g_F^2/8M_{23}^2)^2}{\frac{1}{2}|V_{us}|^2(g_W^2/8M_W^2)^2}. \quad (2.14)$$

The relation between Br^{obs} and M_{11} is also similar to Fig.2, but the numerical result is slightly changed. For example, for the branching ratio $Br^{obs} = (1.7 \pm 1.1) \times 10^{-10}$ gives

$$M_{23} = 0.65_{-0.14}^{+\infty} \text{ TeV.} \quad (2.15)$$

(For a hypothetical value $Br^{obs} = (1.7 \pm 0.8) \times 10^{-10}$, we obtain $M_{11} = 0.65_{-0.12}^{+0.80}$ TeV.) Although the lowest FGB mass is almost the same as the case of $n = 2$ as far as we see the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, we should note that the strong constraint in the case $n = 1$ still comes from the observed $K^0-\bar{K}^0$ mixing. The result (2.15) suggests only that the constraint from $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is not essential for the case $n = 1$.

3 Concluding remarks

According to a specific FGB model in which the FGB masses are related to the charged lepton masses as shown in Eq.(1.3), the family gauge coupling constant g_F is not free as shown in Eq.(1.4), and the family number assignment in quark sector is twisted as shown in Eq.(1.5). We have estimated the lowest FGB mass M_{11} from the deviation between the observed value (1.1) and the SM estimate value (1.2). We cannot regrettably obtain meaningful value of M_{11} because of the present large errors in Br^{obs} and Br^{SM} . However, we expect improvement of those values in future, because the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ has really been detected differently from other lepton flavor changing rare decays. On the other hand, the estimate of Br^{SM} is reliable because the decay is leptonic one, so that the theoretical treatment is well established.

However, the present estimate was done. basing on non-conventional model. As a reference, let us estimate a FGB mass on the basis of conventional FGB model. In the conventional model in which the family number is assigned as $(d_1, d_2, d_3) = (d, s, b)$, a FGB which can dominantly contribute to the decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is A_1^2 as $\mathcal{M}(K^+ \rightarrow \pi^+ \nu_e \bar{\nu}_\mu)$. Therefore, there is no interference with the final states in the SM estimate:

$$Br^{obs}(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = Br^{SM}(\nu_i \bar{\nu}_i) + Br^{FGB}(\nu_e \bar{\nu}_\mu). \quad (3.1)$$

Therefore, we can estimate in Sec.2, similarly,

$$\frac{Br^{FGB}}{Br^{SB}} = \frac{1}{\varepsilon} \frac{(g_F^2/8M_{12}^2)^2 f_+}{\frac{1}{2}|V_{us}|^2 (g_W^2/8M_W^2)^2 f_0}, \quad (3.2)$$

where ε is defined by Eq.(2.7). Present values (1.1) and (1.2) lead to

$$\tilde{M}_{12} \equiv \frac{M_{12}}{g_F/\sqrt{2}} = 66.6_{-1.1}^{+\infty} \text{ TeV.} \quad (3.3)$$

(Do not confuse A_1^2 in the conventional model with A_1^2 in the model [3] discussed in Sec.2. The FGB A_1^2 in the conventional model means A_d^s in terms of names of down-quarks, while A_1^2

in Sec.2 means A_b^d [or A_b^s].) Note that the gauge coupling constant g_F is unknown parameter in the conventional FGB model, so that we can determine only the value of \tilde{M}_{12} directly (not M_{12}).

This value (3.3) is comparatively low, so that the value is within reach of μ - e conversion experiments in preparation [8]. However, such a low value (3.3) in the conventional model will conflict with the observed K^0 - \bar{K}^0 mixing even we adopt the Sumino RGB model [4]. Only the way in which we can obtain FGBs with lower masses is still to adopt the twisted assignment of quark family numbers [3] in addition to the Sumino FGB model.

In conclusion, the improvement of the values in Br^{obs} and Br^{SM} seems to be very important to the estimate of FGB masses. If the observed value becomes close to the SM prediction (1.2), e.g. if we can have $Br^{obs} \sim 0.9 \times 10^{-10}$ in future, we will be able to conclude that a mass of the lowest FGB, M_{11} , is an order of a few TeV. If the observed value becomes inversely far from the SM value (1.2), e.g. $Br^{obs} > 1.0 \times 10^{-10}$, the result will rule out the specific scenario given in this paper, because such a value of Br^{obs} leads to too low value of M_{11} , so that the case is ruled out by the data $pp \rightarrow e^+e^- + X$ at the LHC [7]. Such a value Br^{obs} will, rather, confirm the conventional FGB model. In either case, the improvement of the observation of $Br(K^+ \rightarrow \pi^+\nu\bar{\nu})$ will play an important role in the FGB searches. We are eager for an improved observation of $K^+ \rightarrow \pi^+\nu\bar{\nu}$.

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