Phenomenology of Harmless Family Gauge Bosons to K^0-$\bar{K}^0$ Mixing

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Abstract

When we try to consider family gauge bosons with a lower energy scale, a major obstacle is constraints from the observed P^0-$\bar{P}^0$ mixings ($P^0 = K^0, D^0, B^0, B^0_s$). Against such a conventional view, we point out that, in a U(3) family gauge boson model, the bosons are harmless to any P^0-$\bar{P}^0$ mixings independently of explicit values of the family mixings, if masses M_{ij} of the gauge bosons $A_i^{\ j}$ (i, j are family indexes) satisfy a relation $2/M^2_{ij} = 1/M^2_{ii} + 1/M^2_{jj}$. If such the case can be realized together with an inverted mass hierarchy $M^2_{33} \ll M^2_{22} \ll M^2_{11}$, we can consider family gauge bosons with a considerably lower scale, so that we can expect rich signs for family gauge bosons in a TeV scale.

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1 Introduction

It seems to be very attractive to understand "families" ("generations") in quarks and leptons from concept of a symmetry [1]. It is also attractive that such family gauge bosons are visible at our terrestrial energy scale. However, when we try to consider such a visible family gauge boson model, we always meet with constraints from the observed pseudo-scalar-anti-pseudo-scalar (P^0-$\bar{P}^0$) meson mixings (K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, and so on). The constraints are too tight to allow family gauge bosons with lower masses, so that it is usually taken that a scale of the symmetry braking is considerably high (e.g. an order of at least 10^4 TeV). It is usually taken that it is hard to observe gauge boson effects even in the LHC era. However, if the family gauge symmetry really exists, it is rather likely that the effects are certainly visible. If we can built a family gauge boson model in which the gauge bosons do not contribute to the P^0-$\bar{P}^0$ mixings, family gauge boson effects can become visible at a TeV scale.

Recently, a family gauge boson model [2] which considerably loosen such the severe constraints from the P^0-$\bar{P}^0$ mixings have been proposed. The model has the following characteristics:

(i) A family gauge symmetry is U(3), so that a number of the family gauge bosons are nine (not eight).

(ii) The symmetry breaking is not caused by scalars **3** and/or **6** of U(3), but it is caused by a scalar $(\mathbf{3}, \mathbf{3}^*)$ of U(3)$\times$U(3)$'$, which are broken at Λ and Λ' ($\Lambda \ll \Lambda'$), respectively. Therefore, a direct gauge boson mixing $A_i^{\ j} \leftrightarrow A_j^{\ i}$ ($i = 1, 2, 3$) does not appear in this model.

(iii) The family gauge boson mass matrix is diagonal in a diagonal basis of the charged lepton mass matrix M_e. Therefore, in the charged lepton sector, the family number is exactly conserved. (Of course, neutrino states which we can observe through weak interactions are not (ν_1, ν_2, ν_3), but $(\nu_e, \nu_\mu . \nu_\tau)$ which are partners of (e_L, μ_L, τ_L).)
(iv) In the quark sector, since quark mass matrices M_u and M_d are, in general, not always diagonal on the diagonal basis of M_e, so that family number violations at tree level are caused only through the mixing matrices among up- and down-quarks, $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$, where eigenstates of the family symmetry (u_i^0, d_i^0) are given by $(u_i^0, d_i^0) = (U_{ij}^u u_j, U_{ij}^d d_j)$:

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}} \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U_{ik}^{*u} U_{jl}^u (\bar{u}_k \gamma_\mu u_l) + U_{ik}^{*d} U_{jl}^d (\bar{d}_k \gamma_\mu d_l) \right] (A_i^{\ j})^\mu. \tag{1}$$

The form is essential to our discussion, so that we give a brief review of the form (1) in Appendix.
(v) The gauge boson masses M_{ij} are dominantly generated by vacuum expectation values (VEVs) of scalars Ψ_i^α which are $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′, and whose VEVs are given by $\langle \Psi_i^\alpha \rangle = \delta_i^\alpha v_i$ as follows:

$$M^2(A_i^{\ j}) = \frac{1}{2} g_A^2 (|v_i|^2 + |v_j|^2) + \cdots , \tag{2}$$

where "$+ \cdots$ denotes contributions from other scalars which are negligibly small, so that the family gauge boson masses $M_{ij} \equiv M(A_i^{\ j})$ satisfy relations

$$2M_{ij}^2 = M_{ii}^2 + M_{jj}^2. \tag{3}$$

In order to realize the Sumino's cancellation mechanism [3], as we discuss later, we take an inverted mass hierarchy, $v_i \propto 1/\sqrt{m_{ei}}$, i.e.

$$M_{ij}^2 \equiv M^2(A_i^j) = k \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right). \tag{4}$$

Therefore, Eq.(4) gives family gauge boson masses with an inverted mass hierarchy $M_{33} \ll M_{22} \ll M_{11}$. Here, note that the scalar Ψ is different from a scalar Φ which generates charged lepton masses m_{ei}. Since the model gives a VEV relation $\langle \Psi \rangle \langle \Phi^\dagger \rangle \propto \mathbf{1}$, the gauge boson mass matrix is diagonal when the charged lepton mass matrix is diagonal. Also note that Eq.(4) is an approximate relation under $|\langle \Psi \rangle|^2 \gg |\langle \Phi \rangle|^2$. (For more details, see Appendix, Eq.(A.5).)

The model with the inverted mass hierarchy (K-Y model [2]) is an extended version of the Sumino model [3]. In the Sumino model, the gauge coupling constant g_F is not free parameter. Sumino has paid why the charged lepton mass relation [4]

$$m_e + m_\mu + m_\tau = \frac{2}{3} \left(\sqrt{m_e} + \sqrt{m_\tau} + \sqrt{m_\tau} \right)^2 , \tag{5}$$

is well satisfied by the pole masses (not by the running masses). The running masses $m_{ei}(\mu)$ are given by [5]

$$m_{ei}(\mu) = m_{ei}\left[1 - \frac{\alpha_{em}(\mu)}{\pi}\left(1 + \frac{3}{4}\log\frac{\mu^2}{m_{ei}^2(\mu)}\right)\right]. \tag{6}$$

If the factor $\log(m_{ei}^2/\mu^2)$ in Eq.(6) is absent, then the running masses $m_{ei}(\mu)$ are also satisfy the formula (5). Sumino has required that contribution of family gauge bosons to the charged lepton mass $m_{ei}(\mu)$ cancels the factor $\log(m_{ei}^2/\mu^2)$ due to photon. (However, in the present paper, we do not require Sumino's cancellation mechanism, so that we do not refer the details.) In the K-Y model, too, the coupling constant g_F is not a free parameter in the model. In order to cancel a factor $\log(m_{ei}^2/\mu^2)$ in the QED correction by a factor $\log(M_{ij}^2/\mu^2)$ due to family gauge boson exchanges, the gauge boson masses must have inverted masses $M_{ii}^2 \propto m_{ei}^{-1}$. Therefore, the characteristic (v) in the K-Y model, "family gauge bosons with an inverted mass hierarchy", is not an assumption, but inevitable consequence of the model. However, in this paper, we do not require the cancellation mechanism, so that the characteristic (v) is a phenomenological assumption in the present scenario.

However, even in the K-Y model, it is still difficult to reduced the lightest gauge boson mass to a few TeV energy scale [7]. In Sec.2, we point out that if masses M_{ij} of the family gauge bosons $A_i^{\ j}$ satisfy a relation

$$\frac{2}{M_{ij}^2} = \frac{1}{M_{ii}^2} + \frac{1}{M_{jj}^2}, \tag{7}$$

the family gauge bosons do not contribute to the P-$\bar{P}$ mixings at all. Of course, such the mechanism based on the relation (7) is effective only in a model in which there is no direct transition $A_i^{\ j} \leftrightarrow A_j^{\ i}$, i.e. in which gauge bosons interact with quarks and leptons according to Eq.(1).

The purpose of the present paper is to discuss visible effects of the family gauge bosons at TeV scale under the assumption (7) from the phenomenological point of view, but not to build a model with the mass relation (7) from the theoretical point of view. In Sec.2, we demonstrate that the family gauge boson cannot contribute to the P^0-$\bar{P}^0$ mixing at all when we assume the mass relation (7). In Sec.3, phenomenological investigation is given under the assumption (7). We speculate that $M_{33}/(g_F/\sqrt{2}) \sim 5.1$ TeV and $M_{23}/(g_F/\sqrt{2}) \simeq M_{31}/(g_F/\sqrt{2}) \sim 7.3$ TeV, while $M_{12}/(g_F/\sqrt{2}) \sim 500$ TeV. (In the present model, differently from the Sumino model and the K-Y model, we cannot fix the exact values of M_{ij}, since g_F is free parameter.) If g_F is $g_F/\sqrt{2} \sim 0.2$, we can guess that M_{33}, M_{23} and M_{31} are of an order of 1 - 2 TeV, so that we are able to observe those at the LHC with $\sqrt{s} = 14$ TeV via $A_3^{\ 3} \to \tau^+\tau^-$, $A_3^{\ 2} \to \mu^+\tau^-$ and $A_3^{\ 1} \to e^+\tau^-$. The value $M_{12}/(g_F/\sqrt{2}) \sim 500$ TeV is within our reach of our observation of μ-e conversion in the near future experiments. Especially, an observation of $\mu^- N \to e^- N$ (but non-observation of $\mu \to e + \gamma$) will be a promising as a test of the present scenario. Finally, Sec.4 is devoted to concluding remarks.

2 Harmless condition to P-$\bar{P}$ mixings

We start from the family gauge boson interactions given in Eq.(1). The interactions (1) can be derived, for example, from a model U(3)×U(3)′ mode (see Appendix). Then, we can express effective quark current-current interactions with a family number change $\Delta N_{fam} = 2$ as follows:

$$H^{eff} = \frac{1}{2} g_F^2 \left[\sum_i \frac{(\lambda_i)^2}{M_{ii}^2} + 2 \sum_{i<j} \frac{\lambda_i \lambda_j}{M_{ij}^2} \right] (\bar{q}_k \gamma_\mu q_l)(\bar{q}_k \gamma^\mu q_l) \tag{8}$$

where

$$\lambda_1 = U_{1k}^* U_{1l}, \quad \lambda_2 = U_{2k}^* U_{2l}, \quad \lambda_3 = U_{3k}^* U_{3l}. \tag{9}$$

(For example, for a case of K^0-$\bar{K}^0$ mixing are given by $\lambda_1 = U_{11}^{d*} U_{12}^d$, $\lambda_2 = U_{21}^{d*} U_{22}^d$ and $\lambda_3 = U_{31}^{d*} U_{32}^d$.) These λ_i with $k \neq l$ satisfy a unitary triangle condition

$$\lambda_1 + \lambda_2 + \lambda_3 = 0. \tag{10}$$

We define the effective coupling constant G^{eff} in the current-current interaction as

$$G^{eff} = \frac{1}{2} g_F^2 \left[\frac{\lambda_1^2}{M_{11}^2} + \frac{\lambda_2^2}{M_{22}^2} + \frac{\lambda_3^2}{M_{33}^2} + 2 \left(\frac{\lambda_1 \lambda_2}{M_{12}^2} + \frac{\lambda_2 \lambda_3}{M_{23}^2} + \frac{\lambda_3 \lambda_1}{M_{31}^2} \right) \right]. \tag{11}$$

Obviously, a case of $M_{11} = M_{22} = M_{33} = M_{12} = M_{23} = M_{31}$ gives $G^{eff} = 0$, because of $G^{eff} \propto (\lambda_1 + \lambda_2 + \lambda_3)^2$. However, the case is not attractive phenomenologically.

Another case which can give $G^{eff} = 0$ is a case with the relation (7). In fact, the effective coupling constant G^{eff} under the relation (7) is expressed as

$$G^{eff} = \frac{1}{2} g_F^2 \left[\sum_i \frac{(\lambda_i)^2}{M_{ii}^2} + \sum_{i<j} \lambda_i \lambda_j \left(\frac{1}{M_{ii}^2} + \frac{1}{M_{jj}^2} \right) \right] = \frac{1}{2} g_F^2 (\lambda_1 + \lambda_2 + \lambda_3) \left(\frac{\lambda_1}{M_{11}^2} + \frac{\lambda_2}{M_{22}^2} + \frac{\lambda_3}{M_{33}^2} \right), \tag{12}$$

so that, because of the unitary triangle condition (10), we can obtain $G^{eff} = 0$ for any values of the quark mixing.

However, note that if we consider that the U(3) family symmetry is broken by a scalar **6** (and/or **6***), we cannot prevent the P-$\bar{P}$ mixing even with the mass relation (7), because, in such a case, $A_i{}^j$-$A_j{}^i$ mixing directly appears via vacuum expectation value (VEV) of the scalar **6** (and/or **6***). In the K-Y model, the U(3) symmetry is broken only by the scalar $(\mathbf{3}, \mathbf{3}^*)$

of U(3)×U(3)′, so that the effective interactions with $\Delta N_{fam} = 2$ are caused only by Eq.(3). (This suppression mechanism is a kind of the GIM mechanism [6]. This is peculiar to the quark current-current interactions with $\Delta N_{fam} = 2$, and it does not work in quark-lepton interaction with $\Delta N_{fam} = 1$.)

In general, since we have six gauge boson masses and three constraints (7), we can describe five gauge boson mass ratios by two parameters. We define parameters a and b as

$$a \equiv \frac{M_{22}}{M_{33}}, \qquad b \equiv \frac{M_{11}}{M_{33}}. \tag{13}$$

If we assume an inverted mass hierarchy with $b^2 > a^2 > 1$, we obtain the gauge boson mass ratios as follows:

$$M_{33} : M_{23} : M_{31} : M_{22} : M_{12} : M_{11} = 1 : \sqrt{\frac{2}{1+1/a^2}} : \sqrt{\frac{2}{1+1/b^2}} : a : \sqrt{\frac{2}{1+a^2/b^2}}\, a : b, \tag{14}$$

which leads to

$$M_{33} : M_{23} : M_{31} : M_{22} : M_{12} : M_{11} \simeq 1 : \sqrt{2} : \sqrt{2} : a : \sqrt{2}a : b, \tag{15}$$

under the assumption $b^2 \gg a^2 \gg 1$. If we can give two parameters a and b, then we can fix all the gauge bosons mass ratios. (Note that the parameters a and b are fixed by charged lepton mass ratios in the Sumino model and the K-Y model in which the gauge boson masses satisfy the relation (3), while the parameters a and b in the present model are free parameters.)

3 Phenomenology of the family gauge bosons

In this section, we discuss phenomenology of the family gauge bosons whose masses satisfy the harmless condition (7). Let us forget about the theoretical origin of mass spectrum (7) for the time being. We optimistically consider that the relation will be derived by considering a scalar $(\mathbf{6}, \mathbf{6}^*)$ of U(3)×U(3)′ and/or a mixing with another gauge bosons (for example, $[\mathrm{U}(1)]^3$ gauge bosons). In order to investigate the origin of the relation (7) in the near future, it is important to investigate phenomenological aspect.

3.1 Constraints from rare K and B decay searches

First, let us see experimental lower limit of the family gauge bosons. We do not need an explicit value of g_F as far as we discuss phenomenon due to the current-current interactions. It is convenient to define

$$\tilde{M}_{ij}^2 \equiv \frac{M_{ij}^2}{g_F^2/2}. \tag{16}$$

As far as we treat four-Fermi current-current interactions, the value $\tilde{M}_{ij}$ are practically useful rather than M_{ij}. Real mass values M_{ij} are needed only when we discuss a direct observation of A_i^j (for example, $pp \to A_3{}^3 + X \to \tau^+\tau^- X$).

For example, we can estimate a rare decay $K^+ \to \pi^+ e^- \mu^+$ as follows:

$$\frac{Br(K^+ \to \pi^+ e^- \mu^+)}{Br(K^+ \to \pi^0 \mu^+ \nu_\mu)} = (r_{12})^4 \frac{|U_{11}^{*d} U_{22}^d|^2}{|V_{us}|^2} \frac{f(m_{\pi^+}/m_K)}{\frac{1}{2} f(m_{\pi^0}/m_K)} \simeq \frac{2}{|V_{us}|^2} (r_{12})^4, \tag{17}$$

where

$$(r_{ij})^2 = \frac{(g_F^2/2)/M_{ij}^2}{(g_w^2/8)/M_W^2} = \frac{2v_H^2}{\tilde{M}_{12}^2}, \tag{18}$$

$v_H = 246$ GeV, and $f(x)$ is a phase space function $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$. (We have neglected the lepton masses.) Note that the weak interactions are $V - A$, while our family gauge boson interactions are pure V. The present data [8] show $Br(K^+ \to \pi^0 \mu^+ \nu_\mu) = (3.353 \pm 0.034)$ % and $Br(K^+ \to \pi^+ e^- \mu^+) < 1.3 \times 10^{-11}$, so that we obtain a lower limit of $\tilde{M}_{12} \sim 196$ TeV. We show results of lower limits of $\tilde{M}_{ij}$ from the observed rare pseudo-scalar meson decays in Table 1.

Table 1: Experimental lower limit of $\tilde{M}_{ij} \equiv M_{ij}/(g_F/\sqrt{2})$. (For $K^+ \to \pi^+ \nu\bar{\nu}$, see the text.)

	Input		Output [TeV]
$Br(K^+ \to \pi^+ e^- \mu^+)$	$< 1.3 \times 10^{-11}$	$\tilde{M}_{12}$	> 196
$Br(K_L \to \pi^0 e^\mp \mu^\pm)$	$< 7.6 \times 10^{-11}$	$\tilde{M}_{12}$	> 151
$Br(K_L \to \pi^0 \nu\bar{\nu})$	$< 2.6 \times 10^{-8}$	$\tilde{M}_{12}$	> 17.5
$Br(K^+ \to \pi^+ \nu\bar{\nu})$	$(1.7 \pm 1.1) \times 10^{-10}$	$\tilde{M}_{12}$	~ 243
$Br(B^+ \to K^+ \mu^\pm \tau^\mp)$	$< 7.7 \times 10^{-5}$	$\tilde{M}_{23}$	> 4.11
$Br(B^+ \to K^+ \nu\bar{\nu}))$	$< 1.3 \times 10^{-5}$	$\tilde{M}_{23}$	> 5.4
$Br(B^0 \to K^0 \nu\bar{\nu})$	$< 5.6 \times 10^{-6}$	$\tilde{M}_{23}$	> 6.7
$Br(B^0 \to \pi^0 \nu\bar{\nu})$	$< 2.2 \times 10^{-4}$	$\tilde{M}_{31}$	> 4.8

In Table 1, only $Br(K^+ \to \pi^+ \nu\bar{\nu})$ has been reported with a finite value of the branching ratio, $(1.7 \pm 1.1) \times 10^{-10}$ [8]. It is usually taken that this value is consistent with the standard model prediction [9]

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = (0.80 \pm 0.11) \times 10^{-10}. \tag{19}$$

Since our purpose is to find a room for new physics as much as possible, we take the center value of the observed value. Then, we can obtain a value $\tilde{M}_{12} \sim 243$ TeV shown in Table 1. Therefore, exactly speaking, the value $\tilde{M}_{12} \sim 243$ TeV should be regarded as a lower limit of the mass of the family gauge boson $A_2{}^1$. (Here, we have regard $Br(K^+ \to \pi^+ \nu\bar{\nu}) \simeq Br(K^+ \to \pi^+ \nu_e \bar{\nu}_\mu)$.)

Also note that our gauge bosons interact with fermions as a pure vector, while they behave as V-A for decay into neutrinos, because ν_R are extremely heavy.

As seen in Table 1, the data roughly show $\tilde{M}_{12} \geq 250$ TeV and $\tilde{M}_{23} \geq 7$ TeV. If we want a model in which contains a family gauge boson with a TeV scale mass, it seems to be better to consider a family gauge boson model with an inverted mass hierarchy.

Since we consider that the family gauge boson mass matrix is diagonal in the diagonal bases of the charged lepton mass matrix, it is likely that the gauge boson masse ratios are described by the charged lepton mass ratios as in the Sumino model and the K-Y model. Suggested by the K-Y model, by a way of trial, let us assume that the parameters a and b defined by Eq.(13) are given by

$$a = \left(\frac{1/m_\mu}{1/m_\tau} \right)^{n/2}, \quad b = \left(\frac{1/m_e}{1/m_\tau} \right)^{n/2}. \tag{20}$$

In the K-Y model [2], a case of $n = 1$ in Eq.(20) was adopted. However, it has been demonstrated tat the K-Y model with $n = 1$ cannot give a family gauge boson with a TeV scale from a phenomenological study in Ref.[7]. In the K-Y model, the case $n = 1$ has been derived by considering $\langle \Phi \rangle \langle \bar{\Psi} \rangle \propto \mathbf{1}$, where Φ and Ψ are scalars $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×(3)′, as given a review in Appendix. Note that we cannot make a singlet state $(\mathbf{1}, \mathbf{1})$ from three $(\mathbf{3}, \mathbf{3}^*)$, i.e. $\Phi\bar{\Phi}\Psi$. A possible case of $(\mathbf{1}, \mathbf{1})$ with a next smaller n is $\Phi\bar{\Phi}\Phi\bar{\Psi}$, i.e. a case of $n = 3$. Therefore, in the present model, we consider the case of $n = 3$: $a = 68.96$ and $b = 2.050 \times 10^5$. Although we have speculated $\tilde{M}_{12} \sim 250$ TeV in Table 1 from the observed value of $Br(K^+ \to \pi^+ \nu\bar{\nu})$, we conservatively take a value of $\tilde{M}_{12} \sim 500$ TeV in the present estimate. Then, we can speculate

$$\tilde{M}_{33} \sim 5.1 \text{ TeV}, \quad \tilde{M}_{23} \simeq \tilde{M}_{31} \sim 7.3 \text{ TeV}, \quad \tilde{M}_{22} \sim 350 \text{ TeV}, \quad \tilde{M}_{12} \sim 500 \text{ TeV}, \quad \tilde{M}_{11} \sim 1.1 \times 10^6 \text{ TeV}, \tag{21}$$

The predicted mass values $\tilde{M}_{ij}$ do not conflict with the lower limits of the observed values given in Table 1. Obviously, the gauge boson ${A_1}^1$ is invisible. However, ${A_3}^3$ and ${A_2}^1$ have possibility to be observed at the LHC and at the COMET experiment [10], respectively.

3.2 Other visible family gauge boson effects

Let us discuss possible visible effects of the family gauge bosons with the mass spectrum (21).

(i) *Deviation from the e-μ-τ universality in tau decays*

Previously, we have estimated [7] a mass value of $\tilde{M}_{23}$ as $\tilde{M}_{23} = 5.2^{+6.4}_{-1.4}$ TeV, from the deviation $\delta = 0.0020 \pm 0.0016$ in $Br(\tau^- \to \mu^- \nu\bar{\nu}/e^- \nu\bar{\nu})$. (In Ref. [7], the result has been represented in terms of M_{ij}, in which $g_F/\sqrt{2} = 0.4999$ has been taken.) Regrettably, we cannot extract such the value in the present model, because the previous value was extracted under an assumption $\tilde{M}_{23}^2 \ll \tilde{M}_{31}^2$, while since the mass spectrum in the present model gives $\tilde{M}_{23}^2 \simeq \tilde{M}_{31}^2$, the previous value $\tilde{M}_{23} \sim 5.2$ TeV cannot be derived from the present model.

On the other hand, we can see sizable deviations from the e-μ-τ universality in the Υ decays, $\Upsilon \to \tau^+\tau^-/\mu^+\mu^-/e^+e^-$. We have estimated [7] a mass value of M_{33} as $\tilde{M}_{33} = 0.22^{+0.26}_{-0.05}$ TeV. However, the previous value of $\tilde{M}_{33}$ is too small compared with the value given in (21). However, note that upper value in the previous estimate contain infinity if we take 1.3 σ of the observed deviation. Therefore, the previous result is not conflict with the present estimate in (21). The value of $\tilde{M}_{33}$ given in (21) will be confirmed in the Υ decay in the near future.

(ii) *Lepton number violating rare decays of B and K*

For lepton-flavor violating rare decays of B and K, B decays, $B^+ \to K^+\mu^-\tau^+$ and, $B^0 \to K^+\mu^-\tau^+$, and K decays, $K^+ \to \pi^+\mu^-\tau^+$, $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\mu^\mp\tau^\pm$ become soon within our reach as seen in Table 1.

(iii) *μ-e conversion*

Most sensitive test for our scenario is to observe the so-called μ-e conversion. (For a review of the μ-e conversion and more detailed calculations, for example, see Ref.[11] and Ref.[12], respectively.) At present, we do not know values of $|U^{q*}_{11}U^q_{21}|$ $(q = u, d)$. Therefore, it is not practical, at this stage, to estimate a μ-e conversion rate strictly. Instead, we roughly estimate a μ-e conversion rate in the quark level as follows:

$$R_q \equiv \frac{\sigma(\mu^- + q \to e^- + q)}{\sigma(\mu^- + u \to \nu_\mu + d)} \simeq \left(\frac{|U^{q*}_{11}U^q_{21}|}{|V_{ud}|} \frac{g_F^2/2}{\tilde{M}_{12}^2} \frac{M_W^2}{g_w^2/8} \right)^2 = \left(\frac{|U^{q*}_{11}U^q_{21}|}{|V_{ud}|} (r_{12})^2 \right)^2 , \tag{22}$$

where $q = u, d$, and (r_{12}^2) is defined by Eq.(18). It is likely that $|U^u_{21}|^2 \ll |U^d_{21}|^2$. Then, we may regard the ratios R_q as $R_u \ll R_d$, so that we can neglect contribution to nucleon from R_u compared with that from R_d. When we suppose $|U^{d*}_{11}U^d_{21}|/|V_{ud}| \sim |V_{cd}| \sim 10^{-1}$, we can roughly estimate values of R_d for the input values $\tilde{M}_{12} \sim 500$ TeV as $R_d \sim 0.95 \times 10^{-14}$. Present experimental limit is, for instance for Au, $R(Au) \equiv \sigma(\mu^- + Au \to e^- + Au)/\sigma(\mu \text{ capture}) < 7 \times 10^{-13}$ [13]. The estimated values $R_d \sim 10^{-14}$ become within reach of our observation. (Although the estimated value R_d has different physical meaning from the value $R(Au)$, we consider that the order of the value R_d can provide one with useful information.) Since the decay $\mu^- \to e^- + \gamma$ is highly suppressed in the present scenario, if we observe $\mu^- N \to e^- N$ without observation of $\mu^- \to e^- + \gamma$, then it will strongly support our family gauge boson scenario. (The decay $\mu^- \to e^- + \gamma$ can occur through a quark-loop diagram. However, such a diagram is highly suppressed.)

(iv) *Direct production of the light gauge bosons ${A_3}^3$, ${A_3}^2$ and ${A_3}^1$*

It should be noted that the values $\tilde{M}_{33} \simeq 5.1$ TeV and $\tilde{M}_{32} \simeq \tilde{M}_{31} \simeq 7.2$ TeV are not real mass values of these family gauge bosons. The observed masses M_{ij} are given by $M_{ij} = (g_F/\sqrt{2})\tilde{M}_{ij}$. If the gauge coupling constant g_F is $g_F/\sqrt{2} \sim 0.2$, direct productions at the LHC will become hopeful. Then, we can observe typical decay modes ${A_3}^3 \to \tau^-\tau^+$, ${A_3}^2 \to \tau^-\mu^+$ and ${A_3}^1 \to \tau^- e^+$ with the branching fraction 2/15=13.3%. (For example, in the decays of ${A_3}^2$, we have decay modes, $t+\bar{c}$, $b+\bar{s}$, $\tau^- + \mu^+$ and $\nu_\tau + \bar{\nu}_\mu$ with branching fractions 6/15, 6/15, 2/15 and 1/15, respectively.)

Meanwhile, note that our family gauge interactions are only interactions which can interact not only with ν_L but also with ν_R. The branching ratio $Br(A_i^{\ j} \to \nu_i \bar{\nu}_j) = 1/15 = 6.7\%$ is one in the case of Majorana neutrinos. If neutrinos are Dirac neutrinos, the branching ratios is given $Br(A_i^{\ j} \to \nu_i \bar{\nu}_j) = 2/16 = 12.5\%$. Therefore, in future, when the data of the direct production of $A_i^{\ j}$ are accumulated, we will be able to conclude whether neutrinos are Dirac or Majorana by observing whether $Br(A_i^{\ j} \to \nu_i \bar{\nu}_j)$ is 6.7% or 12.5%.

4 Concluding remarks

We have pointed out that if family gauge boson masses satisfy the relation (7), the gauge bosons are harmless to P^0-$\bar{P}^0$ mixing. This is valid only in a model in which there is no direct $A_i^{\ j} \leftrightarrow A_i^{\ j}$ transition and the family number violation is caused only by quark mixing ($U^d \neq \mathbf{1}$ and/or $U^u \neq \mathbf{1}$).

We would like to emphasize the mass relation (7) is promising from a phenomenological point of view. If we had adopted the relation (3) instead of the relation (7), we would obtain a mass relation

$$M_{33} : M_{23} : M_{22} : M_{31} : M_{12} : M_{11} =$$

$$1 : \sqrt{\frac{1+a^2}{2}} : a : \sqrt{\frac{1+b^2}{2}} : \sqrt{\frac{a^2+b^2}{2}} : b \simeq 1 : \frac{a}{\sqrt{2}} : a : \frac{b}{\sqrt{2}} : \frac{b}{\sqrt{2}} : b, \tag{23}$$

under the assumption $b^2 \gg a^2 \gg 1$, instead of the relation (14). As seen by comparing the relation (15) with (23), we can have three light bosons $A_3^{\ 3}$, $A_2^{\ 3}$ and $A_1^{\ 3}$ and two bosons $A_2^{\ 2}$ and $A_1^{\ 2}$ with masses of the order of $a\, M_{33}$ in the model with the relation (7), while, in the model with the relation (3), we have only one lightest boson $A_3^{\ 3}$ and two bosons $A_2^{\ 3}$ and $A_2^{\ 2}$ with masses of the order of $a\, M_{33}$. Therefore, the model with relation (3) cannot give any interesting phenomenology.

However, note that the mechanism of the family symmetry breaking which is caused by a scalar $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′ can give the gauge boson interactions (1), but, at the same time, the mechanism leads to the mass relation (3). Regrettably, we do not know mechanism which gives the interaction (1) and also gives the mass relation (7). Our next task is to derive the relation (7).

Meanwhile, note that we do not need to require that the relation (7) should exactly be satisfied. The relation (7) may approximately be satisfied in practice. For example, when we suppose $M_{22} \simeq M_{12} \simeq M_{11}$, the family gauge boson contribution to Δm_K (also Δm_D) can become negligibly small. The relation (7) is a very useful measure to model-builders who consider a family gauge boson model with a lighter scale.

Anyhow, if the relation (7) is satisfied, at least, approximately, we can speculate many fruitful family gauge boson effects under the mass relation (7). However, the relation (7) is purely phenomenological one. Especially, the numerical values in Eq.(21) should not be taken rigidly. The values are highly dependent on the tentative input $\tilde{M}_{12} \simeq 500$ TeV. The purpose of the present paper is to point out a possibility that masses of the family gauge bosons are considerably small, and it is not to give numerical predictions definitely.

We again would like to emphasize that an observation of $\mu^- N \to e^- N$ without observation of $\mu \to e + \gamma$ will be promising as a test of the present scenario.

We hope that many physicists turn their attention to a possibility of the family gauge bosons with an inverted mass hierarchy.

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Appendix

In the K-Y model [2], charged lepton mass term is generated by a VEV of scalar Φ as follows:

$$\mathcal{H}_{\rm Yukawa} = \frac{y_e}{\Lambda^2} \bar{\ell}_{Li} {\Phi_i}^\alpha {\bar{\Phi}_\alpha}^{\ j} e_{Rj} H_d, \tag{A.1}$$

where $\ell_{Li} = (\nu_i, e_i^-)_L$. Here, for simplicity, we have shown only the charged lepton term. For quark mass terms, we will need further scalars.

On the other hand, family gauge boson masses are generated by another scalar Ψ:

$$\mathcal{H}_{mass} = \frac{1}{2} \left(g_A {A_i}^k \langle {\Psi_k}^\alpha \rangle - g_B \langle ({\Psi_i}^\beta) \rangle {B_\beta}^\alpha \right) \left(g_A \langle {(\Psi^\dagger)_\alpha}^k \rangle A_k^i - g_B {B_\alpha}^\gamma (\langle {(\Psi^\dagger)_\gamma}^i \rangle \right) + \cdots , \tag{A.2}$$

where A and B are gauge bosons of U(3) and U(3)$'$, respectively. The term "$+\cdots$" denotes other contributions except for that from $\Psi = (\mathbf{3}, \mathbf{3}^*)$ of U(3)$\times$U(3)$'$, i.e. contribution from $\Phi = (\mathbf{3}, \mathbf{3}^*)$ which gives quark and lepton masses, contribution from $\Psi' = (\mathbf{1}, \mathbf{6})$ which plays a role in giving $|M_{ij}(B)|^2 \gg |M_{ij}(A)|^2$, and so on. When we assume a VEV form

$$\langle {\Psi_i}^\alpha \rangle = {\delta_i}^\alpha v_i, \tag{A.3}$$

we obtain

$$\mathcal{H}_{mass} = \frac{1}{4} \sum_{i,j} (|v_i|^2 + |v_j|^2) \left(g_A {A_i}^j - g_B {B_i}^j \right)^2 + \cdots . \tag{A.4}$$

Since we suppose $|\langle\Phi\rangle|^2 \ll |\langle\Psi\rangle|^2 \ll |\langle\Psi'\rangle|^2$, i.e. in the limit of large masses of the gauge bosons ${B_\alpha}^\beta$, we obtain

$$M_{ij}^2 \simeq \frac{g_A^2}{2}(|v_i|^2 + |v_j|^2). \tag{A.5}$$

(Hereafter, we denote g_A as g_F.)

In Eq.(A.5), since contributions of scalars which generate quark and lepton masses are very small, the contributions have been neglected. Hereafter, we neglect small contributions denoted by "$+\cdots$". Under this approximation, family gauge bosons interact with quarks and leptons as follows:

$$\mathcal{H}_{fam} \simeq \frac{g_F}{\sqrt{2}}\left[(\bar{e}_i\gamma_\mu e_j) + (\bar{\nu}_i\gamma_\mu\nu_j) + U_{ik}^{*u}U_{jl}^u(\bar{u}_k\gamma_\mu u_l) + U_{ik}^{*d}U_{jl}^d(\bar{d}_k\gamma_\mu d_l)\right]({A_i}^j)^\mu. \tag{A.6}$$

On the flavor basis in which the charged lepton mass matrix M_e is diagonal, the family gauge bosons ${A_i}^j$ are in the eigenstates of mass. On the other hand, quark mass matrices M_u and M_d are, in general, not diagonal. U^u and U^d in Eq.(A.6) are mixing matrices which are described in the diagonal basis of M_e. Therefore, there is no family number violation in the charged lepton sector, while family number violations can appear in the quark sectors through the mixings U^u and U^d.

The interaction form (A.6) is always correct in a model in which mass matrices of the charged leptons and family gauge bosons can be diagonalized simultaneously.

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