Spectroscopy of Family Gauge Bosons

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Abstract

Spectroscopy of family gauge bosons is investigated based on a U(3) family gauge boson model proposed by Sumino. In his model, the family gauge bosons are in mass eigenstates in a diagonal basis of the charged lepton mass matrix. Therefore, the family numbers are defined by $(e_1, e_2, e_3) = (e, \mu, \tau)$, while the assignment for quark sector are free. For possible family-number assignments (q_1, q_2, q_3), under a constraint from K^0-$\bar{K}^0$ mixing, we investigate possibilities of new physics, e.g. production of the lightest family gauge boson at the LHC, $\mu^- N \to e^- N$, rare K and B decays, and so on.

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1 Introduction

The most exciting subject in particle physics is to understand the origin of "flavor". It seems to be very attractive to understand "families" ("generations") in quarks and leptons from concept of a symmetry [1]. Since the observed masses of quarks and leptons are in range of $10^{-3} - 10^{2}$ GeV, we may suppose a possibility that the lightest family gauge boson can be observed by terrestrial experiments, e.g. at the LHC.

However, when we try to consider such a visible family gauge boson model, we always meet with constraints from the observed pseudo-scalar-anti-pseudo-scalar meson mixings P^0-$\bar{P}^0$ $(P = K, D, B, B_s)$. The constraints are too tight to allow family gauge bosons with lower masses. It is usually taken that a scale of the symmetry braking is considerably high (e.g. an order of, at least, 10^4 TeV). However, there is a family gauge boson model [2] in which such severe constraints from the P^0-$\bar{P}^0$ mixings can be considerably loosen. In the model, the family gauge symmetry is U(3), so that a number of the family gauge bosons are nine (not eight), and quarks and leptons interacts with the family gauge bosons ${A_i}^j$ is given by

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}} \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U^{*u}_{ik} U^{u}_{jl} (\bar{u}_k \gamma_\mu u_l) + U^{*d}_{ik} U^{d}_{jl} (\bar{d}_k \gamma_\mu d_l) \right] ({A_i}^j)^\mu, \tag{1.1}$$

where (u^0_i, d^0_i) are eigenstates of the family symmetry U(3) and those are define by $(u^0_i, d^0_i) = (U^u_{ij} u_j, U^d_{ij} d_j)$. (The expression (1.1) is based on an extended version [2] of the Sumino model [3]. See in the next section.) Note that in the limit of no quark mixing, the family number is exactly conserved, so that the whole P^0-$\bar{P}^0$ mixings are forbidden. (A brief review is given in the next section.)

Another remarkable point in the Sumino model is that the family gauge coupling constant g_F and ratios among the family gauge boson masses M_{ij} are not free, and when once a model is settled, g_F and M_{ij}/M_{kl} are fixed. Therefore, the model can give a clear answer to observations.

The family number in the Sumino model [3] is defined by the charged lepton sector $e_i = (e, \mu, \tau)$ and the gauge boson masses are given proportionally to the charged lepton masses. On the other hand, family number in the quark sector may be $d_i^0 = (d^0, s^0, b^0)$, but it may be an inverted assignment $d_i^0 = (b^0, s^0, d^0)$, and also a twisted assignment $d_i^0 = (b^0, d^0, s^0)$. (Of course, we consider the same assignments for u_i^0 because of SU$(2)_L$ symmetry.) There are six possible assignments of (u_i^0, d_i^0) correspondingly to $e_i = (e, \mu, \tau)$. (Hereafter, for convenient, we will denote q_i^0 as q_i simply.)

In the present paper, based on the Sumino model [3] (and also an extended Sumino model [2]), we investigate visible effects of the family gauge bosons, i.e. the deviations from the e-μ-τ universality, rare K and B decays, μ-e conversion, direct production of the lightest family gauge boson, and so on. We will conclude that the case with a twisted assignment $d_i^0 = (b^0, d^0, s^0)$ can give rich phenomenology to us.

2 Sumino mechanism

Priori to our investigation, let us give a brief review of the Sumino model and its extended version.

The necessity of the family gauge bosons was first pointed out by Sumino [3]. Sumino has paid why the charged lepton mass relation [4]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\tau} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}, \tag{2.1}$$

is well satisfied by the pole masses (not by the running masses). The running masses $m_{ei}(\mu)$ are given by [5]

$$m_{ei}(\mu) = m_{ei}\left[1 - \frac{\alpha_{em}(\mu)}{\pi}\left(1 + \frac{3}{4}\log\frac{\mu^2}{m_{ei}^2(\mu)}\right)\right]. \tag{2.2}$$

If the factor $\log(m_{ei}^2/\mu^2)$ in Eq.(2.2) is absent, then the running masses $m_{ei}(\mu)$ are also satisfy the formula (2.1). Sumino has required that contribution of family gauge bosons to the charged lepton mass $m_{ei}(\mu)$ cancels the factor $\log(m_{ei}^2/\mu^2)$ due to photon. That is, in the collection factors,

$$\varepsilon_0 + \varepsilon_i \equiv e^2 \log\frac{m_{ei}^2}{\mu^2} - 2\left(\frac{g_F}{\sqrt{2}}\right)^2 \log\frac{M_{ii}^2}{\mu^2}, \tag{2.3}$$

the factor ε_i must be $\varepsilon_i = 0$. (ε_0 denotes a family-number independent part.) In the Sumino model, the family gauge boson masses M_{ii} are given $M_{ii}^2 \propto m_{ei}$, so that we can give $\varepsilon_i = 0$ by adjusting the gauge coupling constant g_F suitably. (The details are given later.)

Note that in the Sumino model, the minus sign for the cancellation comes form a U(3) assignment of the left-handed and right-handed charged leptons e_L and e_R, $(e_L, e_R) = (\mathbf{3}, \mathbf{3}^*)$ of U(3). As a result, we obtain a somewhat unfamiliar gauge current-current interaction form

$$\mathcal{H}_{fam}^{Sumino} = \frac{g_F}{\sqrt{2}} \sum_{f=u,d,\nu,e} \left(\bar{f}_L^i \gamma_\mu f_{Lj} - \bar{f}_{Rj} \gamma_\mu f_R^i\right) (A_i^{\ j})^\mu. \tag{2.4}$$

However, when the assignment $(e_L, e_R) = (\mathbf{3}, \mathbf{3}^*)$ is extended to all quarks and leptons (f_L, f_R), we have unwelcome situation: (i) The model cannot be anomaly free. (ii) Effective current-current interactions with $\Delta N_{fam} = 2$ (N_{fam} is a family number) appear inevitably.

In order to evade these problems, an extended version of the Sumino model (K-Y model) [2] has been proposed by Yamashita and the author: (i) U(3) assignment is $(f_L, f_R) = (\mathbf{3}, \mathbf{3})$, so that the model is anomaly free. (ii) In order to obtain the minus sign of cancellation, the family gauge boson masses are given by an inverted mass hierarchy

$$M^2(A_i^{\ j}) \equiv M_{ij}^2 = k \left(\frac{1}{m_{ei}^n} + \frac{1}{m_{ej}^n} \right) + \cdots , \tag{2.5}$$

where "$+\cdots$" denotes contributions from other scalars which are negligibly small. (Here, although the number n is $n = 1$ in the original K-Y model [2], we have denoted an extended case with $n \neq 1$ for convenience of later discussion.) Note that although only one scalar Φ gives charged lepton masses and family gauge boson masses in the Sumino model [3], while, in the K-Y model [2], there are two scalars Ψ and Φ which are $(\mathbf{3}, \mathbf{3}')$ of U(3)×U(3)′. Only Φ can gives charged lepton masses as $m_{ei}\delta_i^{\ j} \propto \langle \Phi_i^{\ \alpha} \rangle \langle \bar{\Phi}_\alpha^{\ j} \rangle$. On the other hand, only Ψ contributes dominantly gauge boson masses, i.e. $M_{ij}^2 \propto \langle \Psi_i^{\ \alpha} \rangle \langle \bar{\Psi}_\alpha^{\ j} \rangle$ with $\langle \Psi \rangle \langle \bar{\Phi} \rangle = k\mathbf{1}$. (For a case of $n \geq 2$, see later.) Therefore, the Sumino cancellation mechanism is satisfied only approximately.

In the present investigation, it is essential that the family gauge boson interactions are given by Eq.(1.1). The interaction (1.1) has been derived from the following scenario: The family symmetry breaking is not caused by scalars $\mathbf{3}$ and/or $\mathbf{6}$ of U(3), but it is caused by a scalar $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′, which are broken at Λ and Λ' ($\Lambda \ll \Lambda'$), respectively. (In the original Sumino model, the scalar was $(\mathbf{3}, \mathbf{3})$ of U(3)×O(3). In the present investigation, the difference is not essential.) Therefore, a direct gauge boson mixing $A_i^{\ j} \leftrightarrow A_j^{\ i}$ ($i = 1, 2, 3$) does not appear in this model. The U(3)×U(3)′ is dominantly broken by a scalar Ψ_i^α which is $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′, i.e. by a vacuum expectation value (VEV) $\langle \Psi_i^\alpha \rangle = v_i \delta_i^\alpha$ as in the K-Y model [2]. In the limit of $\Lambda' \gg \Lambda$, we obtain the U(3) family current interaction (1.1). In the quark sector, since quark mass matrices M_u and M_d are, in general, not always diagonal on the diagonal basis of M_e, so that family number violations at tree level are caused only through the mixing matrices among up- and down-quarks, $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$.

On the other hand, the gauge boson masses M_{ij} are also dominantly generated by VEV of scalar Ψ_i^α which is $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)′, and whose VEV is given by $\langle \Psi_i^\alpha \rangle = \delta_i^\alpha v_i$. Then, we obtain family gauge boson masses

$$M^2(A_i^{\ j}) \equiv M_{ij}^2 = \frac{1}{2} g_A^2 (|v_i|^2 + |v_j|^2) + \cdots \, , \tag{2.6}$$

where "$+\cdots$" denotes contributions from other scalars which are negligibly small, so that the family gauge boson masses $M_{ij} \equiv M(A_i^{\ j})$ approximately satisfy relations

$$2M_{ij}^2 \simeq M_{ii}^2 + M_{jj}^2 . \tag{2.7}$$

Here, the assumption $|\langle \Psi \rangle|^2 \gg |\langle \Phi \rangle|^2$ is essential. For example, in a case B_1 which is discussed later, we consider that the largest component of $\langle \Phi \rangle$ is of an order of 10^2 GeV, while the largest component of $\langle \Psi \rangle$ is of an order of 10^7 GeV,

In the present paper, we investigate the following Cases A and B which satisfy the Sumino cancellation mechanism: Case A with an inverted mass hierarchy and Case B with a normal gauge boson mass hierarchy. In both cases, the gauge boson masses are given by Eq.(2.6), so that the gauge boson masses satisfy the relation (2.7). Since we still consider $m_{ei} \delta_i^{\ j} \propto \langle \Phi_i^{\ \alpha} \rangle \langle \bar{\Phi}_\alpha^{\ j} \rangle$, the difference between Case A and Case B is only in a relation of the VEV $\langle \Psi \rangle$ to the VEV $\langle \Psi \rangle$.

Case A: The inverted gauge boson mass hierarchy (K-Y model like)

Charged lepton masses are given by Eq.(2.5). Here, we also consider cases with $n \neq 1$ in addition to the case with $n = 1$ in the original K-Y model. For example, for $n = 2$ we suppose $\langle \Phi_i^{\ \alpha} \rangle \langle \bar{\Psi}_\alpha^{\ j} \rangle \langle \Phi_j^{\ \beta} \rangle \propto \langle E_i^{\ \beta} \rangle$, where $\langle \bar{E}_\alpha^j \rangle = v_E \mathrm{diag}(1, 1, 1)$.

The gauge boson masses satisfy the relation (2.7), mass ratios can be expressed as follow:

$$M_{33} : M_{32} : M_{22} : M_{31} : M_{21} : M_{11} = 1 : \sqrt{\frac{a^2+1}{2}} : a : \sqrt{\frac{b^2+1}{2}} : \sqrt{\frac{b^2+a^2}{2}} : b, \tag{2.8}$$

where

$$a \equiv \frac{M_{22}}{M_{33}} = \left(\frac{m_\tau}{m_\mu} \right)^{n/2} , \quad b \equiv \frac{M_{11}}{M_{33}} = \left(\frac{m_\tau}{m_e} \right)^{n/2} . \tag{2.9}$$

Sumino cancellation condition $g_F^2/2 = (3/2)\zeta e^2$ in the K-Y model is rewritten as

$$\left(\frac{g_F}{\sqrt{2}} \right)^2 \simeq \frac{1}{n} \frac{3}{2} \zeta \, e^2 , \tag{2.10}$$

because of $\log M_{ii}^2 = -n \log m_{ei} + \text{const}$. Here, the coupling constant g_F is defined by

$$\mathcal{H}_{fam}^{K-Y} = \frac{g_F}{\sqrt{2}} \sum_{f=u,d,\nu,e} (\bar{f}^i \gamma_\mu f_j)(A_i^{\ j})^\mu. \tag{2.11}$$

(For convenience of comparison with the Sumino model, the coupling constant g_F in the original K-Y model [2] has been changed into $g_F/\sqrt{2}$.) Note that, differently from the original Sumino model, the cancellation in the K-Y model is satisfied only approximately. The factor ζ in Eq.(2.10) is a fine tuning factor which gives $K(\mu) \simeq 2/3$ almost independently of μ, and it is numerically given by $\zeta = 1.752$ in the case of $n = 1$.

Case B: The normal gauge boson mass hierarchy (the original Sumino model type)

Gauge boson masses are given by

$$M_{ij}^2 = k(m_{ei}^n + m_{ej}^n). \tag{2.12}$$

Although in the original Sumino model [3], the scalar Φ gives the gauge boson masses M_{ij} and the charged lepton masses m_{ei}, in the present investigation, we also consider other possibilities in addition to the case with $n = 1$. For example, a case with $n = 2$ is realized by a VEV relation $\langle \Psi_i^\alpha \rangle \langle \bar{E}_\alpha^j \rangle = \langle \Phi_i^\alpha \rangle \langle \bar{\Phi}_\alpha^j \rangle$. Then, the cancellation condition is given by

$$\left(\frac{g_F}{\sqrt{2}} \right)^2 = \frac{2}{n} e^2 = \frac{4}{n} \left(\frac{g_w}{\sqrt{2}} \right)^2 \sin^2 \theta_w, \tag{2.13}$$

because of $\log M_{ii}^2 = n \log m_{ei} + \text{const}$.

From Eq.(2.12), the gauge boson mass ratios are expressed by

$$M_{11} : M_{12} : M_{22} : M_{13} : M_{23} : M_{33} = 1 : \sqrt{\frac{a^2+1}{2}} : a : \sqrt{\frac{b^2+1}{2}} : \sqrt{\frac{b^2+a^2}{2}} : b, \tag{2.14}$$

where

$$a \equiv \frac{M_{22}}{M_{11}} = \left(\frac{m_\mu}{m_e} \right)^{n/2}, \quad b \equiv \frac{M_{33}}{M_{11}} = \left(\frac{m_\tau}{m_e} \right)^{n/2}. \tag{2.15}$$

In the original Sumino model, the currents with an unwelcome form as shown in Eq.(2.4) appear inevitably. We want less contribution of the family gauge bosons to the P^0-$\bar{P}^0$ mixing. Therefore, in the present investigation in Case B, we slightly change the original Sumino model into a modified model where leptons $\ell_i = (\nu_i, e_i^-)$ are still assigned to $(\ell_L, \ell_R) = (\mathbf{3}, \mathbf{3}^*)$, while quarks $q_i = (u_i, d_i)$ are assigned to $(q_L, q_R) = (\mathbf{3}, \mathbf{3})$, so that the quark sector is anomaly free. In Case B, the gauge boson interactions are given by

$$\mathcal{H}_{fam}^{(B)} = \frac{g_F}{\sqrt{2}} \left[\sum_{f=\nu,e} (\bar{f}_L^i \gamma_\mu f_{Lj} - \bar{f}_{Rj} \gamma_\mu f_R^i) + \sum_{f=u,d} (\bar{f}^i \gamma_\mu f_j) \right] (A_i^{\ j})^\mu, \tag{2.16}$$

instead of Eq.(2.4). However, the lepton currents with the unwelcome form still appear. (We will provide additional heavy leptons in order to remove anomaly in the lepton sector.)

Finally we would like to emphasize that we assume that the family symmetry U(3) is assumed for all cases, so that the condition between g_F and e is unchanged in Case A (and also Case B). (For example, Eq.(2.3) is satisfied model-independently in Case B.) However, since the relations between $\langle\Psi\rangle$ and $\langle\Phi\rangle$ (i.e. between M_{ii} and m_{ei}) are model-dependent even the family symmetry U(3) is assumed in common, so that in Eqs.(2.5), (2.8) - (2.10) and (2.12) - (2.15), the factor n has appeared model-dependently.

3 Quark family arrangements and P^0-$\bar{P}^0$ mixing

Effective quark current-current interactions with $\Delta N_{fam}=2$ are given by

$$H^{eff}=\frac{1}{2}g_F^2\left[\sum_i \frac{(\lambda_i)^2}{M_{ii}^2}+2\sum_{i<j}\frac{\lambda_i\lambda_j}{M_{ij}^2}\right](\bar{q}_k\gamma_\mu q_l)(\bar{q}_k\gamma^\mu q_l) \tag{3.1}$$

where

$$\lambda_1=U_{1k}^{q*}U_{1l}^{q},\quad \lambda_2=U_{2k}^{q*}U_{2l}^{q},\quad \lambda_3=U_{3k}^{q*}U_{3l}^{q}. \tag{3.2}$$

For example, in a case of K^0-$\bar{K}^0$ mixing, λ_i are given by

$$\lambda_1=U_{11}^{d*}U_{12}^{d},\quad \lambda_2=U_{21}^{d*}U_{22}^{d}\quad \lambda_3=U_{31}^{d*}U_{32}^{d}. \tag{3.3}$$

These λ_i with $k\neq l$ satisfy a unitary triangle condition

$$\lambda_1+\lambda_2+\lambda_3=0. \tag{3.4}$$

We define the effective coupling constant G^{eff} in the current-current interaction as

$$G^{eff}=\frac{1}{2}g_F^2\left[\frac{\lambda_1^2}{M_{11}^2}+\frac{\lambda_2^2}{M_{22}^2}+\frac{\lambda_3^2}{M_{33}^2}+2\left(\frac{\lambda_1\lambda_2}{M_{12}^2}+\frac{\lambda_2\lambda_3}{M_{23}^2}+\frac{\lambda_3\lambda_1}{M_{31}^2}\right)\right]. \tag{3.5}$$

Note that all family gauge bosons contribute to the P^0-$\bar{P}^0$ mixing as seen in Eq.(3.1).

In order to demonstrate numerical results, we tentatively assume $U^u\simeq\mathbf{1}$ and $U^d\simeq V_{CKM}$ (V_{CKM} is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix [7]). Alternative case with $U^u\simeq V_{CKM}^\dagger$ and $U^d\simeq\mathbf{1}$ can give no contributions to K^0-$\bar{K}^0$, B^0-$\bar{B}^0$ and B_s^0-$\bar{B}_s^0$ mixings, so that it is good news for the present purpose. However, the case brings a more severe constraint on the gauge boson masses from the observed value of D^0-$\bar{D}^0$ mixing.

The assumption $U^d\simeq V_{CKM}$ leads to values of λ_i,

$$\lambda_1\simeq 0.220,\quad \lambda_2\simeq -0.219,\quad \lambda_3\simeq -0.00035. \tag{3.6}$$

Therefore, in the limit of $\lambda_3 \simeq 0$ and $\lambda_1 \simeq -\lambda_2$, we obtain approximate relation

$$G_K^{eff} \simeq \frac{g_F^2}{2}\frac{\lambda_1^2}{M_{11}^2} \quad \left(\text{or } G_K^{eff} \simeq \frac{g_F^2}{2}\frac{\lambda_2^2}{M_{22}^2}\right). \tag{3.7}$$

Thus, the K^0-$\bar{K}^0$ mixing put a severe constraint on the lower bound of the family gauge boson mass M_{11} for $M_{11} < M_{22}$ (or M_{22} for $M_{22} < M_{11}$)]. When we use the observed value [6] $\Delta m_K^{obs} = (3.484 \pm 0.006) \times 10^{-18}$ TeV and a tentative standard model (SM) value [8] $\Delta m_K^{SM} \sim 2 \times 10^{-18}$ TeV, we obtain a lower limit of the value $M_{11}/(g_F/\sqrt{2})$ [or $M_{22}/(g_F/\sqrt{2})$] ~ 340 TeV, where we have used a vacuum-insertion approximation (with no QCD correction)

$$\Delta m_K^{fam} = \frac{1}{6} G_K^{eff} f_K^2 f_K (1 + 2S_K), \tag{3.8}$$

and $S_K = m_K^2/(m_s + m_d)^2$. If we give the parameters a and b in Eq.(2.9) [or (2.15)], we can estimate G^{eff} without approximation (3.7). In the next section, we will calculate constraints for $M_{ij}/(g_F/\sqrt{2})$ directly from Eq.(3.5) and by using V_{CKM} with CP violation phase.

Here, note that the CKM matrix V_{CKM} is defined in the generation basis $u_i = (u, c, t)$ and $d_i = (d, s, b)$. Therefore, the notations M_{ij} in Eqs.(3.1) are different from those defined by the diagonal bases of the charged lepton mass matrix M_e. In this paper, we investigate various assignments of $q_i = (q_1, q_2, q_3)$. As far as quark sector is concerned, the use of generation basis $d_i = (d, s, b)$ is convenient. Therefore, hereafter, for example, for Case B_1 with family number $d_i = (b, s, d)$ (the case is defined in the next section), we denote M_{11}, M_{12}, M_{22}, $\cdots$ as M_{bb}, M_{bs}, M_{ss}, $\cdots$, respectively, in order to distinguish those from M_{ij} defined in the family numbers. (For convenience, we use down-quark names as the quark family numbers.) The physics is highly dependent on the quark family assignments. The details are discussed in the next section.

4 Which quark-family assignment is favorable ?

We find that K^0-$\bar{K}^0$ mixing puts the most severe constraints on the family gauge boson masses M_i compared with other P^0-$\bar{P}^0$ mixings. As seen in Eqs.(3.6) and (3.7), because of $|\lambda_b|^2 \ll |\lambda_s|^2 \simeq |\lambda_d|^2$, the observed K^0-$\bar{K}^0$ mixing put a constraint on M_{dd} or M_{ss}, but it does not put a constraint on M_{bb}. Therefore, for our purpose of visible family gauge bosons, we should regard the third generation quark (t, b) as $(t, b) = (u_3, d_3)$ in Case A with the inverted gauge boson mass hierarchy, and (t, b) as $(t, b) = (u_1, d_1)$ in Case B with the normal gauge boson mass hierarchy. As a result, we have the following four candidates of the quark family assignments: Case A_1: $(d_1, d_2, d_3) = (d, s, b)$, Case A_2: $(d_1, d_2, d_3) = (s, d, b)$, Case B_1: $(d_1, d_2, d_3) = (b, s, d)$ and Case B_2: $(d_1, d_2, d_3) = (b, d, s)$. Cases A_1 with $n = 1$ and B_1 with $n = 1$ correspond to the K-Y model and the Sumino model, respectively.

In Table 1, we list gauge boson masses M_{ij} estimated from $\Delta m_K^{fam} \sim 1.4 \times 10^{-8}$ TeV for these four cases with typical values of n, where n is defined by Eq.(2.9) [or Eq.(2.15)]. [Exactly speaking, since the value $\Delta m_K^{fam} \sim 1.4 \times 10^{-8}$ TeV means those which we can take as large as possible, the values of $\tilde{M}_{ij}$ in Table 1 ($\tilde{M}_{ij}$ are defined by Eq.(4.1) below) denote lower limits of $\tilde{M}_{ij}$.] In the evaluations of λ_i we have taken not only of the magnitudes of V_{CKM} elements, but also the CP phase [6] into consideration. In Table 1, for convenience, numerical values of masses are given by

$$\tilde{M}_{ij}^2 \equiv \frac{M_{ij}^2}{g_F^2/2}. \tag{4.1}$$

As far as we treat four-Fermi current-current interactions, the value $\tilde{M}_{ij}$ are practically useful rather than M_{ij}. Real mass values M_{ij} are needed only when we discuss a direct observation of A_i^j. In the numerical estimates of $\tilde{M}_{ij}$, note that the expression M_{ij} given by Eq.(2.8) [and also Eq.(2.14)] have been described in the family numbers which defined by $(e_1, e_2, e_3) = (e^-, \mu^-, \tau^-)$, while the formula (3.5) with Eq.(3.2) have been described by using the quark generation-number $(d_1, d_2, d_3) = (d, s, b)$.

As seen in Table 1, Case A_1 and Case A_2 lead to large values of $\tilde{M}_{ij}$, so that these cases are not interesting to us. Case A with $n \geq 3$ can have $\tilde{M}_{33}$ smaller than a few TeV, but the case gives $\tilde{M}_{11} \sim 10^6$ TeV. Phenomenology in Case A_1 with $n = 1$ has already been investigated in Refs.[2, 9]. Phenomenology for Case B with $d_i = (d, s, b)$ has investigated in Ref.[10]. The results for visible effects of the family gauge bosons was negative.

We consider that Case B with $n = 2$ is phenomenologically most attractive, because the lightest family gauge boson ${A_1}^1$ has mass of an order of a few TeV which is visible at the LHC (remember $M_{11} = (g_F/\sqrt{2})\tilde{M}_{ii}$). Besides, even the heaviest gauge boson has, at most, a mass of an order of 10^4 TeV.

5 Phenomenology of the family gauge bosons in Cases B_1 and B_2

In this section, let us investigate phenomenology of the family gauge bosons in Cases B_1 and B_2 with $n = 2$. From a point of view of model-building, too, the case $n = 2$ is not so unlikely, because we can consider a VEV relation ${\langle\Psi\rangle_i}^\alpha = {\langle\Phi\rangle_i}^\beta \langle\bar{E}\rangle_\beta^j {\langle\Phi\rangle_j}^\alpha$, where $\langle\bar{E}\rangle = \mathbf{1}$. In this case, from Eq.(2.13), the gauge coupling constant $g_F/\sqrt{2}$ is given by

$$\left.\frac{g_F}{\sqrt{2}}\right|_{n=2} = e = 0.30684, \tag{5.1}$$

where, for convenience, we have used [6] $\alpha(m_\tau) = 1/133.471$.

5.1 Direct production of the lightest gauge boson ${A_1}^1$

From the value given in Table 1 and the value (5.1), the mass of gauge boson ${A_1}^1$ is

$$M_{11} \simeq 0.543 \text{ TeV} \;\; (0.540 \text{ TeV}) \;\; \text{for Case } B_1 \;\; (\text{Case } B_2), \tag{5.2}$$

Table 1: Family gauge boson masses estimated from $\Delta m_K^{fam} \sim 1.4 \times 10^{-8}$ TeV. Here, we have used parameter values $a = (m_\tau/m_\mu)^{n/2} = (16.8167)^{n/2}$ and $b = (m_\tau/m_e)^{n/2} = (3477.15)^{n/2}$ for Case A, and $a = (m_\mu/m_e)^{n/2} = (206.768)^{n/2}$ and $b = (m_\tau/m_e)^{n/2} = (3477.15)^{n/2}$ for Case B. In this table, for convenience, numerical values of masses are given by $\tilde{M}_{ij} \equiv M_{ij}/(g_F/\sqrt{2})$ in a unit of TeV.

Case	Family gauge boson masses										
(A)	M_{11}	$>$	M_{12}	$>$	M_{13}	$>$	M_{22}	$>$	M_{23}	$>$	M_{33}
Ratios	b		$\sqrt{\frac{b^2+a^2}{2}}$		$\sqrt{\frac{b^2+1}{2}}$		a		$\sqrt{\frac{a^2+1}{2}}$		1
(A_1)	$\tilde{M}_{dd}$	$>$	$\tilde{M}_{ds}$	$>$	$\tilde{M}_{db}$	$>$	$\tilde{M}_{ss}$	$>$	$\tilde{M}_{sb}$	$>$	$\tilde{M}_{bb}$
$n = 1/2$	1209		884.5		862.5		319.0		251.5		157.5
$n = 1$	5062		3588		3580		352.0		256.2		85.8
$n = 2$	73342		51861		51860		354.7		251.3		21.1
$n = 3$	1.1×10^6		7.4×10^5		7.4×10^5		356.0		251.8		5.16
(A_2)	$\tilde{M}_{ss}$	$>$	$\tilde{M}_{sd}$	$>$	$\tilde{M}_{sb}$	$>$	$\tilde{M}_{dd}$	$>$	$\tilde{M}_{db}$	$>$	$\tilde{M}_{bb}$
$n = 1/2$	1205		881.4		859.5		317.8		250.7		156.0
$n = 1$	5042		3574		3566		350.7		255.2		85.5
$n = 2$	73035		51644		51644		353.2		250.2		21.0
$n = 3$	1.2×10^6		7.5×10^5		7.5×10^5		354.5		250.7		5.14
(B)	M_{11}	$<$	M_{12}	$<$	M_{22}	$<$	M_{13}	$<$	M_{23}	$<$	M_{33}
Ratios	1		$\sqrt{\frac{a^2+1}{2}}$		a		$\sqrt{\frac{b^2+1}{2}}$		$\sqrt{\frac{b^2+a^2}{2}}$		b
(B_1)	$\tilde{M}_{bb}$	$<$	$\tilde{M}_{bs}$	$<$	$\tilde{M}_{ss}$	$<$	$\tilde{M}_{bd}$	$<$	$\tilde{M}_{sd}$	$<$	$\tilde{M}_{dd}$
$n = 1/2$	63.5		176.0		240.7		347.5		384.4		487.4
$n = 1$	22.5		229.8		324.2		940.2		967.6		1329
$n = 2$	1.77		258.3		365.3		4344		4352		6144
(B_2)	$\tilde{M}_{bb}$	$<$	$\tilde{M}_{bd}$	$<$	$\tilde{M}_{dd}$	$<$	$\tilde{M}_{bs}$	$<$	$\tilde{M}_{ds}$	$<$	$\tilde{M}_{ss}$
$n = 1/2$	63.1		174.9		239.2		345.4		382.0		484.3
$n = 1$	22.4		228.7		322.7		935.8		963.1		1323
$n = 2$	1.76		257.3		363.9		4327		4334		6119

It should be noted that the gauge boson ${A_1}^1$ can interact only with the third generation quarks (t, b), although it does with the first generation leptons (ν_e, e) for leptons. Therefore, the gauge boson ${A_1}^1$ will be produced by gluon fusion (Fig.1) as

$$p + p \to A_1^1 + b + \bar{b} + X \to e^+ e^- + X, \tag{5.3}$$

at the LHC. (In future, we will also observe ${A_1}^1$ production in the ILC as $e^+ + e^- \to A_1^1$.)

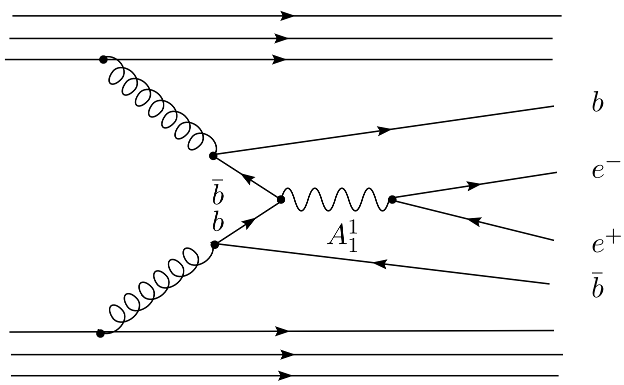

Figure 1: ${A_1}^1$ production at the LHC.

We have decay modes of ${A_1}^1$ into $t + \bar{t}$, $b + \bar{b}$, $e^- + e^+$ and $\nu_e + \bar{\nu}_e$ with branching fractions as follows:

$$\begin{array}{l} Br({A_1}^1 \to t\bar{t}) = Br({A_1}^1 \to b\bar{b}) = \frac{6}{15} = 40\%, \\ Br({A_1}^1 \to e^- e^+) = \frac{2}{15} = 13.3\%, \quad Br({A_1}^1 \to \nu_e \bar{\nu}_e) = \frac{1}{15} = 6.7\%. \end{array} \tag{5.4}$$

Note that the branching ratio $Br({A_1}^1 \to \nu_e\bar{\nu}_e) = 1/15 = 6.7\%$ is one in the case of Majorana neutrinos. If neutrinos are Dirac neutrinos, the branching ratios is given $Br({A_1}^1 \to \nu_e\bar{\nu}_e) = 2/16 = 12.5\%$. The large difference between both is due to the large leptonic branching ratio in the family gauge boson decays. Therefore, in future, when data of the direct production of ${A_1}^1$ are accumulated, we will be able to conclude whether neutrinos are Dirac or Majorana by observing whether $Br({A_1}^1 \to \nu_e\bar{\nu}_e)$ is 6.7% or 12.5%.

The search for ${A_1}^1$ production at the LHC is done by a similar way of the Z' search (for a review, see, for example, [11]). Although there has been an experimental report on Z' search [12], the result cannot be applicable for ${A_1}^1$ search, because ${A_1}^1$ cannot interact with the first generation quarks, so that the cross section is considerably small compared with Z' production. The cross section of ${A_1}^1$ in the original Sumino model has been discussed in Ref.[10], but the case was a different family gauge boson ${A_1}^1$ which can interact with the first generation quarks.

Since the purpose of the present paper is to give an overview of the family gauge bosons with visible energy scale, estimate of the production rate $\sigma(pp \to A_1^1)$ will be given elsewhere.

If the real mass M_{11} is smaller than 500 GeV, we may expect an observation at the ILC in future, too.

5.2 Contribution of family gauge bosons to the rare decay $K^+ \to \pi^+ \nu\bar{\nu}$

Let us estimate contributions of family gauge bosons to the rare decay $K^+ \to \pi^+ \nu\bar{\nu}$, because only a finite value of the branching ratio has been reported [6] at present:

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{obs} = (1.7 \pm 1.1) \times 10^{-10}. \tag{5.5}$$

It is usually taken that this value is consistent with the standard model prediction [13]

$$Br(K^+ \to \pi^+ \nu\bar{\nu})_{SM} = (0.80 \pm 0.11) \times 10^{-10}. \tag{5.6}$$

We are interested in whether Case B is consistent or not with the present experimental result (5.5).

In the present model, all family gauge bosons can, in principle, contribute to each rare decay mode. For example, in Cases B$_1$ and B$_2$, a transition $K \to \pi$ is mediated by the gauge bosons ${A_s}^d \equiv {A_2}^3$ and ${A_s}^d \equiv {A_3}^2$, respectively. However, as seen in Table 1, the mass of M_{23} is of the order of 10^3 TeV, so that the effect is invisible. Remember that family-number violating transitions are possible in the quark sector. Since the effective mass value of $\tilde{M}_{11} \equiv \tilde{M}_{bb}$ is too small, the contribution of ${A_1}^1$ is dominated compared with other gauge boson exchanges even considering the existence of the suppression factor $|U^{d*}_{bd} U^d_{bs}|$ (the value is 0.0155 in the approximation $U^d \simeq V_{CKM}$). Then, the branching ratio due to the family gauge boson exchange ${A_1}^1$ are estimated as follows: $K^+ \to \pi^+ e^- \mu^+$ as follows:

$$\frac{Br(K^+ \to \pi^+ \nu_e \bar{\nu}_e)_{fam}}{Br(K^+ \to \pi^0 \mu^+ \nu_\mu)} = \eta_B \xi^2 \frac{f(m_{\pi^+}/m_K)}{\frac{1}{2} f(m_{\pi^0}/m_K)} (r_{11})^4 \tag{5.7}$$

where

$$(r_{ij})^2 = \frac{(g_F^2/2)/M_{ij}^2}{(g_w^2/8)/M_W^2} = \frac{2v_H^2}{\tilde{M}_{ij}^2}, \tag{5.8}$$

$v_H = 246$ GeV, and $f(x)$ is a phase space function $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$. (We have neglected the lepton masses.) Here, the factor ξ denotes mixing effects in quarks, and in this case, ξ is given by

$$\xi = \frac{|V_{td}^* V_{ts}|}{|V_{us}|}, \tag{5.9}$$

where we have used the approximation $U^d \simeq V_{CKM}$. The factor $\frac{1}{2}$ in the denominator of Eq.(5.7) is due to $\pi^0 = (u\bar{u} - d\bar{d})/\sqrt{2}$. The factor η denotes difference of effective current-current interactions: When we denote the currents for weak interactions $(\bar{\nu}\gamma_\mu(1-\gamma_5)e)$ as $(V-A)$

symbolically, the factor η_B for a final state of $\nu\bar{\nu}$ is given by $\eta_B^{\nu\nu} = [(|V|^2+|A|^2)/4]/(|V|^2+|A|^2) = 1/4$ because only the left-handed neutrino ν_L can contribute as seen in Eq.(2.16). [In contrast to the case $\nu\bar{\nu}$, for a final state of e^+e^-, it is given by $\eta_B^{ee} = 1/2$.]

We obtain numerical results

$$Br(K^+ \to \pi^+ \nu_e \bar{\nu}_e)_{fam} = 1.1 \times 10^{-10} \ (0.91 \times 10^{-10}) \ \text{ for } \tilde{M}_{11} = 1.8 \ (1.9) \text{ TeV}, \tag{5.10}$$

form Eq.(5.7). This value is just favorable to the difference between the observed one (5.5) and the SM one (5.6), i.e. $(1.7-0.8)\times 10^{-10}$. (However, it should be noted that result (5.10) is only an approximate one, because we have neglected interference with the final state mode from the standard model. The numerical result should be taken rigidly.)

For rare B and K decays, we can estimate their branching ratios by a similar way to Eq.(5.7). We investigate only the decay modes via the family gauge boson ${A_2}^1$, because other gauge bosons are considerably heavy, so that such gauge boson effects are obviously invisible. Note that since the family number in the quark sector is assigned unconventionally, for example, the gauge boson ${A_2}^1$ causes the decay $B \to K + e^+ + \mu^-$ with mixing factor $\xi = |U_{bb}U_{ss}|/|V_{us}|$ for Case B$_1$ with ${A_1}^2 \equiv {A_b}^s$, and $B \to \pi + e^+ + \mu^-$ with mixing factor $\xi = |U_{bb}U_{dd}|/|V_{us}|$ for Case B$_2$ with ${A_1}^2 \equiv {A_b}^d$. Differently from the decay $K \to \pi\nu_e\bar{\nu}_e$, the lightest gauge boson ${A_1}^1$ cannot contribute. Since rare B and K decays via the lightest family gauge boson ${A_1}^1$ yields final states e^+e^- and $\nu_e\bar{\nu}_e$, such decay modes are confused with decay modes via photon and Z boson. The lightest gauge boson ${A_i}^j$ with $i \neq j$ is ${A_2}^1$. The branching ratios of decay modes via ${A_2}^1$ are, for example, as follows:

$$\begin{array}{lll} \text{Case B}_1: & Br(B^+ \to K^+\mu^-e^+) \simeq 2.1 \times 10^{-11}, & Br(B^0 \to K^0\mu^-e^+) \simeq 2.1 \times 10^{-11}, \\ \text{Case B}_2: & Br(B^+ \to \pi^+\mu^-e^+) \simeq 2.1 \times 10^{-11}, & Br(B^0 \to \pi^0\mu^-e^+) \simeq 1.0 \times 10^{-11}, \end{array} \tag{5.11}$$

where we have assumed $U^d \simeq V_{CKM}$. These results are invisible for a time, because the present experimental lower limits [6] are $Br(B^+ \to K^+\mu^-e^+) < 9.1 \times 10^{-8}$ and $Br(B^+ \to \pi^+\mu^\mp e^\pm) < 1.7 \times 10^{-7}$. The family gauge boson ${A_2}^1$ can also contribute to rare K decays. However, the predicted branching ratios are of orders of $10^{-15} - 10^{-17}$ because small values of quark mixing factors, so that the effects invisible.

5.3 $\mu^- + N(A,Z) \to e^- + N(A,Z)$

So far, phenomenological merits of Cases B$_1$ and B$_2$ has been almost equal. In this subsection, we would like to emphasize that $\mu^- N \to e^- N$ is visible in Case B$_2$, while it is invisible in Case B$_1$.

Most sensitive test in the near future for Cases B$_1$ and B$_2$ is to observe the so-called μ-e conversion. (For a review of the μ-e conversion and more detailed calculations, for example, see

Ref.[14] and Ref.[15], respectively.) The present experimental limit is, for instance, for Au, [17]

$$R(Au) \equiv \frac{\sigma(\mu^- + Au \to e^- + Au)}{\sigma(\mu \text{ capture})} < 7 \times 10^{-13}. \tag{5.12}$$

The reaction $\mu^- N \to e^- N$ is caused by an exchange of the family gauge boson ${A_2}^1$. It means the exchange of ${A_2}^1 \equiv {A_s}^b$ [${A_2}^1 \equiv {A_d}^b$] in Case B$_1$ [Case B$_2$]. At present, we do not know values of $|U^q_{ij}|$ $(q = u, d)$. Therefore, it is not practical, at this stage, to estimate a μ-e conversion rate strictly. Instead, we roughly estimate a μ-e conversion rate in the quark level as follows:

$$R_q \equiv \frac{\sigma(\mu^- + q \to e^- + q)}{\sigma(\mu^- + u \to \nu_\mu + d)} \simeq \left(\xi \frac{g_F^2/2}{\tilde{M}_{12}^2} \frac{M_W^2}{g_w^2/8} \right)^2 = \left(\xi (r_{12})^2 \right)^2, \tag{5.13}$$

where $q = u, d$, and (r_{12}^2) is defined by Eq.(5.8). (Although the estimated value R_q has different physical meaning from the value $R(Au)$, we consider that the order of the value R_q can provide one with useful information.) In Eq.(5.13), ξ is a quark mixing factor similar to Eq.(5.9), and the value of ξ is given by $\xi = |U^{d*}_{sd} U^d_{bd}|/|V_{ud}| = 2.00 \times 10^{-3}$ [$\xi = |U^{d*}_{dd} U^d_{bd}|/|V_{ud}|) = 0.867 \times 10^{-2}$] in Case B$_1$ [in Case B$_2$] under the approximation $U^d \simeq V_{CKM}$. In this approximation, we may regard the ratios R_q as $R_u \ll R_d$, so that we can neglect contribution to nucleon from R_u compared with that from R_d. Then, we can roughly estimate values of R_q

$$R_q \simeq R_d \sim 1.32 \times 10^{-17} \ (2.52 \times 10^{-16}) \ \text{ for Case B}_1 \text{ (Case B}_2\text{)}, \tag{5.14}$$

where we have used $\tilde{M}_{12} = 260$ TeV from Table 1.

In the near future, the COMET experiment [16] will reach a single-event sensitivity of 2.6×10^{-17}. Therefore, the value $R_q \sim 10^{-16}$ in Case B$_2$ become within reach of our observation, but the value $R_q \sim 1.32 \times 10^{-17}$ in Case B$_1$ is critical for its observation.

Since the decay $\mu^- \to e^- + \gamma$ is highly suppressed in the present scenario, if we observe $\mu^- N \to e^- N$ without observation of $\mu^- \to e^- + \gamma$, then it will strongly support our family gauge boson scenario. (The decay $\mu^- \to e^- + \gamma$ can occur through a quark-loop diagram. However, such a diagram is highly suppressed.)

5.4 Deviations from the e-μ-τ universality

Previously, we pointed out [9] a possibility of a deviation from the e-μ universality in tau decays $\tau \to \mu\nu\bar{\nu}/e\nu\bar{\nu}$ by assuming $\tilde{M}_{23} \ll \tilde{M}_{31}$. However, in the present model, we cannot observe such a deviation because the mass spectrum in the present model gives $\tilde{M}_{23} \simeq \tilde{M}_{31}$, and besides, we have a large value $\tilde{M}_{23} \sim 10^3$ TeV in Case B.

On the other hand, we have a possibility of sizable deviations from the e-μ-τ universality in the Υ decays $\Upsilon \to \tau^+\tau^-/\mu^+\mu^-/e^+e^-$, because the value of $\tilde{M}_{11} \equiv \tilde{M}_{bb}$ is considerably small in Case B. We have matrix elements for the decays $\Upsilon \to \tau^+\tau^-/\mu^+\mu^-/e^+e^-$, as follows: $\mathcal{M}_{\tau\tau} = \mathcal{M}_{\mu\mu} \equiv \mathcal{M}_{SM}$ and $\mathcal{M}_{ee} = \mathcal{M}_{SM} + \mathcal{M}_{fam} = \mathcal{M}_{SM}(1-\varepsilon)$, where

$$\varepsilon \simeq \frac{g_F^2/2}{(e/3)^2}\frac{M_\Upsilon^2}{M_{11}^2 - M_\Upsilon^2} \simeq \frac{9}{e^2}\frac{M_\Upsilon^2}{\tilde{M}_{11}^2} = 2.64\times 10^{-3}. \tag{5.15}$$

Therefore, we can expect a deviation

$$1 - \frac{Br(\Upsilon \to e^+e^-)}{Br(\Upsilon \to \mu^+\mu^-)} \simeq 2\varepsilon = 0.0053. \tag{5.16}$$

At present, we have not observed such a deviation [6]. However, the value (5.16) will become visible in future experiments.

6 Concluding remarks

We have investigated possibility of visible family gauge boson effects for six family assignments in the quark sector $(d_1, d_2, d_3) = (d, s, b)$, $(d_1, d_2, d_3) = (s, d, b)$, and so on, under the Sumino cancellation condition. In the Sumino model, the family number is defined by the diagonal basis of the charged lepton mass matrix $M_e = \mathrm{diag}(m_e, m_\mu, m_\tau)$. The P^0-$\bar{P}^0$ mixings $(P = K, D, B, B_s)$ are caused only through quark mixings $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$. We have found that the most interesting case is Case B$_2$, $(d_1, d_2, d_3) = (b, d, s)$. In Case B$_2$, a direct production of ${A_1}^1$ at the LHC, μ-e conversion $\mu^- N \to e^- N$, and a deviation from e-μ-τ universality in the Υ decay will be observed in future experiments. Also, Case B$_1$, $(d_1, d_2, d_3) = (b, s, d)$, is attractive, although the case is somewhat hard to observe in $\mu^- N \to e^- N$ compared with Case B$_2$.

In Case B, the leptons take a Sumino-like structure (so that Sumino's cancellation mechanism is satisfied), while quarks takes a twisted family-number assignment. At present, there is no theoretical ground for such family-number assignments. In order to make the twisted family-number assignment $(d_1, d_2, d_3) = (b, d, s)$ more reliable, we, at least, have to build a unified mass matrix model of quarks and leptons under such the twisted family-number assignment. It is a task in future.

We hope that many physicists turn their attention to a possibility of visible family gauge bosons and of a twisted family-number assignment versus generation-numbers.

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