Phenomenology of Harmless Family Gauge Bosons to K^0-$\bar{K}^0$ Mixing

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Abstract

When we try to consider family gauge bosons with a visible lower energy scale, a major obstacle is constraints from the observed P^0-$\bar{P}^0$ mixings ($P^0 = K^0, D^0, B^0, B_s^0$). Against such a conventional view, we point out that in a U(3) family gauge boson model, when masses M_{ij} of the gauge bosons A_i^j (i, j are family indexes) satisfy a relation $2/M_{ij}^2 = 1/M_{ii}^2 + 1/M_{jj}^2$, the bosons are harmless to P^0-$\bar{P}^0$ mixings. In such the case, we can consider family gauge bosons with a considerably lower scale. Then, we can speculate many fruitful new physics in Tera scale physics.

PCAC numbers: 11.30.Hv, 12.60.-i, 14.70.Pw,

1 Introduction

It seems to be very attractive to consider families (generations) in quarks and leptons on the basis of a symmetry [1]. It is also attractive that such family gauge bosons are visible at our terrestrial energy scale. However, when we try to consider such visible family gauge symmetry, we always meet with constraints from the observed pseudo-scalar-anti-pseudo-scalar (P^0-$\bar{P}^0$) meson mixings (K^0-$\bar{K}^0$, D^0-$\bar{D}^0$, and so on). The constraints are too tight to allow family gauge bosons with lower masses, so that it is usually taken that a scale of the symmetry braking is considerably high (e.g. an order of 10^3 TeV). It is taken that it is hard to observe the gauge boson effects even in the LHC era. However, if the family gauge symmetry really exists, it is rather likely that the effects are certainly visible. If we can built a family gauge boson model in which the gauge bosons do not contribute to the P^0-$\bar{P}^0$ mixings, family gauge boson effects can become visible at TeV scale and/or lower scale than TeV.

Recently, a gauge boson model [2] which considerably reduces such the severe constraints from the P^0-$\bar{P}^0$ mixings has been proposed. The model has the following characteristics: (a) Since we consider U(3) family symmetry, a number of the gauge bosons is not eight, but nine. (b) The symmetry breaking is not caused by $\mathbf{3}$ and/or $\mathbf{6}$, but it is caused by $(\mathbf{3}, \mathbf{3}^*)$ of U(2)×U(3)′, which are broken at Λ and Λ' ($\Lambda \ll \Lambda'$). Then, in this model, there is no direct gauge boson mixing ${A_i}^j \leftrightarrow {A_j}^i$ ($i = 1, 2, 3$ denotes family indices). (c) The family number is defined in a family basis in which the charged lepton mass matrix M_e is diagonal, so that the family gauge boson mass matrix is also diagonal. Therefore, in the charged lepton sector, the family number is exactly conserved. (d) In the quark sector, since quark mass matrices M_u and M_d are not always diagonal, so that family number violations are caused only through the mixing matrices among up- and down-quarks, $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$. (e) The gauge bosons interact with quarks and leptons as a pure vector. (f) The gauge boson masses M_{ij} take an inverted mass hierarchy, i.e. $M_{33} \ll M_{22} \ll M_{11}$.

The model (Model II) is an extended version of the Sumino model (Model I) [5]. In Model I, the gauge coupling constant g_F is not free parameter because Sumino has required that contribution of family gauge bosons to the radiative correction of the charged lepton mass m_{ei} cancels that of photon in order to understand why the charged lepton mass relation [4], $m_e + m_\mu + m_\tau = (2/3)(\sqrt{m_e} + \sqrt{m_\tau} + \sqrt{m_\tau})^2$, is well satisfied by the pole masses (not by the running masses). (However, in the present paper, we do not require Sumino's cancellation mechanism, we do not refer the details.) In Model II, too, the coupling constant g_F is not a free parameter in the model. In order to cancel a factor $\log(m_{ei}/\mu)^2$ in the QED correction by a family gauge boson induced factor $\log(M_{ij}/\mu)^2$, the gauge boson masses must have inverted masses $M_{ii}^2 \propto m_{ei}^{-1}$. Therefore, the characteristic (f) in Model II is not an assumption, but inevitable consequence of the model. However, in this paper, we do not require the cancellation mechanism, so that the characteristic (f) is an assumption in the present scenario, as we discuss later.

However, even in Model II, it is still difficult to reduced the lightest gauge boson mass to a few TeV energy scale [3]. In Sec.2, we point out that masses M_{ij} of the family gauge bosons $A_i^{\ j}$ (i, j are family indexes) satisfy a relation

$$\frac{2}{M_{ij}^2} = \frac{1}{M_{ii}^2} + \frac{1}{M_{jj}^2}, \tag{1}$$

the family gauge bosons do not contribute to the P-$\bar{P}$ mixings at all. Of course, a model in which the mass relation (1) can take effect is restricted to a model in which there is no direct transition $A_i^{\ j} \leftrightarrow A_j^{\ i}$ such as in Model II.

The purpose of the present paper is to discuss visible effects of the family gauge bosons at TeV scale under the assumption (1) from the phenomenological point of view, but not to build a model with the mass relation (1) from the theoretical point of view. In Sec.2, we demonstrate that the family gauge boson cannot contribute to the P^0-$\bar{P}^0$ mixing at all when we assume the mass relation (1). In Sec.3, phenomenological investigation is given under the assumption (1). We speculate that $M_{23}/(g_F/\sqrt{2}) \simeq M_{31}/(g_F/\sqrt{2}) \sim 10$ TeV and $M_{33}/(g_F/\sqrt{2}) \sim 8$ TeV, while $M_{12}/(g_F/\sqrt{2}) \sim 180$ TeV. (In the present model, differently from Models I and II, since g_F is free parameter, we cannot fix the exact values of M_{ij}.) If g_F is $g_F/\sqrt{2} < 0.4$, we can guess that M_{33}, M_{23} and M_{31} are of an order of a few TeV, so that we are able to observe those at LHC with $\sqrt{S} = 14$ TeV via $A_3^3 \to \tau^+\tau^-$, $A_3^2 \to \mu^+\tau^-$ and $A_3^1 \to e^+\tau^-$. The value $M_{12}/(g_F/\sqrt{2}) \sim 180$ TeV is within our reach of the observation in near future μ-e conversion experiments. Especially, an observation of $\mu^- N \to e^- N$ versus no observation of $\mu \to e + \gamma$ will be a promising as a test of the present scenario. On the other hand, as we discuss in Sec.4, if we regard a background of such a mass spectrum (1), we are led a family gauge boson model with nearly degenerated masses. Then, we speculate that all family gauge bosons have masses of nearly 200 TeV. In this case, too, we can observe $\mu^- N \to e^- N$ and $K^+ \to \pi^+ e^- \mu^+$ and so on. In Sec.5 is devoted to concluding remarks.

2 Harmless condition to P-$\bar{P}$ mixings

We start the following family gauge boson interactions to quarks and leptons:

$$\mathcal{H}_{fam} = \frac{g_F}{\sqrt{2}} \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U^{*d}_{ik} U^d_{jl} (\bar{d}_k \gamma_\mu d_l) + U^{*u}_{ik} U^u_{jl} (\bar{u}_k \gamma_\mu u_l) \right] (A_i{}^j)^\mu, \tag{2}$$

where U^u and U^d are quark mixing matrices. The interactions (1) can be derived, for example, from a model U(3)×U(3)′ model [2]. Then, we can express effective current-current interactions with a family number change $\Delta N_{fam} = 2$ as follows:

$$H^{eff} = \frac{1}{2} g_F^2 \left[\sum_i \frac{(\lambda_i)^2}{M_{ii}^2} + 2 \sum_{i<j} \frac{\lambda_i \lambda_j}{M_{ij}^2} \right] (\bar{q}_k \gamma_\mu q_l)(\bar{q}_k \gamma^\mu q_l) \tag{3}$$

where

$$\lambda_1 = U^*_{1k} U_{1l}, \quad \lambda_2 = U^*_{2k} U_{2l}, \quad \lambda_3 = U^*_{3k} U_{3l}. \tag{4}$$

(For example, for a case of K^0-$\bar{K}^0$ mixing are given by $\lambda_1 = U^{d*}_{11} U^d_{12}$, $\lambda_2 = U^{d*}_{21} U^d_{22}$ and $\lambda_3 = U^{d*}_{31} U^d_{32}$.) These λ_i with $k \neq l$ satisfy a unitary triangle condition

$$\lambda_1 + \lambda_2 + \lambda_3 = 0. \tag{5}$$

We define the effective coupling constant G^{eff} in the current-current interaction as

$$G^{eff} = \frac{1}{2} g_F^2 \left[\frac{\lambda_1^2}{M_{11}^2} + \frac{\lambda_2^2}{M_{22}^2} + \frac{\lambda_3^2}{M_{33}^2} + 2 \left(\frac{\lambda_1 \lambda_2}{M_{12}^2} + \frac{\lambda_2 \lambda_3}{M_{23}^2} + \frac{\lambda_3 \lambda_1}{M_{31}^2} \right) \right]. \tag{6}$$

Obviously, a case of $M_{11} = M_{22} = M_{33} = M_{12} = M_{23} = M_{31} = 0$ gives $G^{eff} = 0$, because of $G^{eff} \propto (\lambda_1 + \lambda_2 + \lambda_3)^2$. However, the case is not attractive phenomenologically.

Another case which can give $G^{eff} = 0$ is a case with the relation (1). In fact, the effective coupling constant G^{eff} under the relation (1) is expressed as

$$G^{eff} = \frac{1}{2} g_F^2 \left[\sum_i \frac{(\lambda_i)^2}{M_{ij}^2} + \sum_{i<j} \lambda_i \lambda_j \left(\frac{1}{M_{ii}^2} + \frac{1}{M_{jj}^2} \right) \right] = \frac{1}{2} g_F^2 (\lambda_1 + \lambda_2 + \lambda_3) \left(\frac{\lambda_1}{M_{11}^2} + \frac{\lambda_2}{M_{22}^2} + \frac{\lambda_3}{M_{33}^2} \right), \tag{7}$$

so that, because of the unitary triangle condition (5), we can obtain $G^{eff} = 0$ for any values of the quark mixing.

However, note that if we consider that the U(3) family symmetry is broken by a scalar **6** (and/or **6***), we cannot prevent the P-$\bar{P}$ mixing even with the mass relation (1), because, in such a case, ${A_i}^j$-${A_j}^i$ mixing directly appears via vacuum expectation value (VEV) of the scalar **6** (and/or **6***). In Model II, the U(3) symmetry is broken only by the scalar $(\mathbf{3}, \mathbf{3}^*)$ of U(3)×U(3)$'$, so that the effective interactions with $\Delta N_{fam} = 2$ are caused only by Eq.(2).

In order to give the relation (1) in a scenario without a scalar **6**, we have to consider some complicated mechanism. For the present, we do not ask the origin of the mass relation (1). Let us go on to the next section in order to discuss our phenomenological study under the mass relation (1).

3 Phenomenology of the family gauge bosons

In this section, we discuss phenomenology of the family gauge bosons whose masses satisfy the harmless condition (1). Let us forget about the theoretical origin of mass spectrum (1) for the time being. We optimistically consider that the relation will be derived by considering a scalar $(\mathbf{6}, \mathbf{6}^*)$ of U(3)×U(3)$'$ and/or a mixing with another gauge boson (for example, a U(1) gauge boson). In order to investigate the origin of the relation (1) in the near future, it is important to investigate phenomenological aspect.

3.1 Constraints from rare K and B decay searches

First, let us see experimental lower limit of the family gauge bosons. We do not need an explicit value of g_F as far as we discuss phenomenon due to the current-current interactions. So, it is convenient to define

$$\tilde{M}_{ij}^2 \equiv \frac{M_{ij}^2}{g_F^2/2}. \tag{8}$$

As far as we treat current-current interaction, the value $\tilde{M}_{ij}$ are practically observables rather than M_{ij}. M_{ij} are needed only when we discuss a direct observation of A_i^j (for example, $pp \to A_3^3 + X \to \tau^+\tau^- X$).

For example, we can estimate a rare decay $K^+ \to \pi^+ e^- \mu^+$ as follows:

$$\frac{Br(K^+ \to \pi^+ e^- \mu^+)}{Br(K^+ \to \pi^0 \mu^+ \nu_\mu)} = (r_{12})^4 \frac{|U_{11}^{*d} U_{22}^d|^2}{|V_{us}|^2} \frac{f(m_{\pi^+}/m_K)}{\frac{1}{2} f(m_{\pi^0}/m_K)} \simeq \frac{2}{|V_{us}|^2} (r_{12})^4, \tag{9}$$

where

$$(r_{ij})^2 = \frac{(g_F^2/2)/M_{ij}^2}{(g_w^2/8)/M_W^2} = \frac{2 v_H^2}{\tilde{M}_{12}^2}, \tag{10}$$

$v_H = 246$ GeV, and $f(x)$ is a phase space function $f(x) = 1-8x^2+8x^6-x^8-12^4 \log x^2$. (We have neglected the lepton masses.) Note that the weak interactions are $V-A$, while our family gauge boson interactions are pure V. The present data [6] show $Br(K^+ \to \pi^0 \mu^+ \nu_\mu) = (3.353 \pm 0.034)$ % and $Br(K^+ \to \pi^+ e^- \mu^+) < 1.3 \times 10^{-11}$, so that we obtain a lower limit of $\tilde{M}_{12} = M_{12}/(g_F/\sqrt{2})$, $\tilde{M}_{12} > 196$ TeV. We show results of lower limits of $\tilde{M}_{ij} = M_{ij}/(g_F/\sqrt{2})$ from the observed rare pseudo-scalar meson decays in Table 1.

Table 1: Experimental lower limit of $\tilde{M}_{ij} \equiv M_{ij}/(g_F/\sqrt{2})$.

	Input	Output [TeV]	
$Br(K^+ \to \pi^+ e^- \mu^+)$	$< 1.3 \times 10^{-11}$	$\tilde{M}_{12}$	> 196
$Br(K_L \to \pi^0 e^\mp \mu^\pm)$	$< 7.6 \times 10^{-11}$	$\tilde{M}_{12}$	> 151
$Br(K_L \to \pi^0 \nu \bar{\nu})$	$< 2.6 \times 10^{-8}$	$\tilde{M}_{12}$	> 17.5
$Br(K^+ \to \pi^+ \nu \bar{\nu})$	$= (1.7 \pm 1.1) \times 10^{-10}$	$\tilde{M}_{12}$	$= 87^{+26}_{-10}$
$Br(B^+ \to K^+ \mu^\pm \tau^\mp)$	$< 7.7 \times 10^{-5}$	$\tilde{M}_{23}$	> 4.11
$Br(B^+ \to K^+ \nu \bar{\nu}))$	$< 1.3 \times 10^{-5}$	$\tilde{M}_{23}$	> 5.4
$Br(B^0 \to K^0 \nu \bar{\nu})$	$< 5.6 \times 10^{-6}$	$\tilde{M}_{23}$	> 6.7
$Br(B^0 \to \pi^0 \nu \bar{\nu})$	$< 2.2 \times 10^{-4}$	$\tilde{M}_{31}$	> 4.8

In Table 1, only $Br(K^+ \to \pi^+ \nu \bar{\nu})$ has been reported with a finite value of the branching ratio, $(1.7 \pm 1.1) \times 10^{-10}$ [6]. The value $\tilde{M}_{12} = 87^{+26}_{-10}$ TeV in Table 1 has been estimated by $Br(K^+ \to \pi^+ \nu \bar{\nu})$ as $Br(K^+ \to \pi^+ \nu_e \bar{\nu}_\mu)$. Exactly speaking, the $\pi^+ \nu \bar{\nu}$ state consists of all combinations $\sum_{i,j} \pi^+ \nu_i \bar{\nu}_j$. Since the correction depends on a model of the gauge boson mass spectrum, the correction will be discussed later after we discuss the mass spectrum. Also note that our gauge bosons interacts with fermions as a pure vector, while they behaves as V-A for decay into neutrinos, because ν_R are extremely heavy.

Also we can estimate of the gauge boson masses from the observed deviations in the e-μ-τ universality [3] when we assume an inverted mass hierarchy $M_{11} \gg M_{22} \gg M_{33}$:

$$\tilde{M}_{23} = 4.9^{+6.0}_{-1.3} \text{ TeV}, \tag{11}$$

from the deviation $\delta = 0.0020 \pm 0.0016$ in $Br(\tau^- \to \mu^- \nu \bar{\nu}/e^- \nu \bar{\nu})$, and

$$\tilde{M}_{33} = 0.21^{+0.24}_{-0.05} \text{ TeV}, \tag{12}$$

from the deviation from the τ-μ universality in the upsilon decays $\Upsilon \to \tau^+ \tau^- / e^+ e^-$. Note that upper values in these estimates contain infinity if we take 1.3 σ of the observed deviations. These values should be also not taken rigidly.

As seen in Table 1, the data roughly show $\tilde{M}_{12} > 2 \times 10^2$ TeV and $\tilde{M}_{23} > 5$ TeV. If we want a model in which contains a family gauge boson with a TeV scale mass, it seems to be better to consider a family gauge boson model with an inverted mass hierarchy.

In general, since we have six gauge boson masses and three constraints (1), we can describe five gauge boson mass ratios by two parameters. We define parameters a and b as

$$a \equiv \frac{M_{22}}{M_{33}}, \qquad b \equiv \frac{M_{11}}{M_{22}}. \tag{13}$$

If we assume an inverted mass hierarchy with $a \gg 1$ and $b \gg 1$, we obtain the gauge boson mass ratios as follows:

$$M_{33} : M_{23} : M_{31} : M_{22} : M_{12} : M_{11} = 1 : \sqrt{\frac{2}{1+1/a^2}} : \sqrt{\frac{2}{1+1/a^2b^2}} : a : \sqrt{\frac{2}{1+1/b^2}}\, a : ab, \tag{14}$$

which leads to

$$M_{33} : M_{23} : M_{31} : M_{22} : M_{12} : M_{11} \simeq 1 : \sqrt{2} : \sqrt{2} : a : \sqrt{2}a : ab, \tag{15}$$

under the assumption $a \gg 1$ and $b \gg 1$. If we can give two parameters a and b, then we can fix masses of all the gauge bosons.

Since we consider that the family gauge boson mass matrix is diagonal in the diagonal bases of the charged lepton mass matrix, it is likely that the gauge boson masses will be related to the charged lepton masses as in Models I and II. In Model II [2], in order to realize the Sumino mechanism [5], the family gauge boson masses M_{ii} are given by $M_{ii}^2 \propto 1/m_{ei}$ (m_{ei} are charged lepton masses). Suggested by this model, by a way of trial, let us assume that the parameters a and b are given by

$$a = \frac{1/m_\tau}{1/m_\mu} = 16.817, \qquad b = \frac{1/m_\tau}{1/m_\mu} = 206.8. \tag{16}$$

Furthermore, we assume $\tilde{M}_{12} \sim 180$ TeV tentatively. (Although the value conflict with the lower limit from $Br(K^+ \to \pi^+ e^- \mu^+)$, we have optimistically adopted a value which is weighted in favor of the result from $Br(K^+ \to \pi^+ \nu\bar{\nu})$.) Then, we can speculate

$$\tilde{M}_{33} \sim 7.6 \text{ TeV}, \quad \tilde{M}_{23} \simeq \tilde{M}_{31} \sim 10.7 \text{ TeV}, \quad \tilde{M}_{22} \sim 127 \text{ TeV}, \quad \tilde{M}_{12} \sim 180 \text{ TeV}, \quad \tilde{M}_{11} \sim 3.7{\times}10^4 \text{ TeV}, \tag{17}$$

Obviously, the gauge boson A_1^1 is invisible. The masses of A_3^2 and A_3^1 are not conflict with the lower values in Table 1. The value of $\tilde{M}_{23}$ is a value which is within our reach in the near future.

Now, we would like to give a comment on the mass value $\tilde{M}_{12} = 87^{+26}_{-10}$ TeV from $Br(K^+ \to \pi^+ \nu\bar{\nu})$. The value have been estimated by taking only $Br(K^+ \to \pi^+ \nu_e \bar{\nu}_\mu)$ into consideration. If we consider the mass spectrum (17), we must take other contributions of $\nu_i \bar{\nu}_j$ into consideration. The down-quark mixing factors accompanied with the decays into $\nu_i \bar{\nu}_j$ are listed in Table 2. Since

$(\tilde{M}_{23}/\tilde{M}_{12})^2 = 3.54 \times 10^{-3}$ from the assumption (16), the gauge bosons A_1^3, A_3^1, A_2^3 in addition to A_1^2 and A_3^2 can sizably contribute to $Br(K^+ \to \pi^+ \nu\bar{\nu})$. The recalculated value of (decay width $\times(\tilde{M}_{12})^4$)) increases 1.53 times, so that the recalculated value of $\tilde{M}_{12}$ is increases 1.11 times. As a result, the value in Table 1 is slightly corrected into

$$\tilde{M}_{12} = 96^{+29}_{-11} \text{ TeV}. \tag{18}$$

The corrected value (18) is still incompatible with the lower limits from other rare kaon decays in Table 1. We consider that our choice $\tilde{M}_{12} \sim 180$ TeV is reasonable.

Table 2: Down-quark mixing factors in the decay $A_i^j \to \nu_i \bar{\nu}_j$. For reference, the numerical values of $U_{i2}^* U_{j1}$ for the case $U \equiv U^d \simeq V_{CKM}$ are given in the fourth row.

A_1^1	A_2^2	A_3^3	A_1^2	A_2^1	A_1^3	A_3^1	A_2^3	A_3^2
$\nu_e\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\mu$	$\nu_\tau\bar{\nu}_\tau$	$\nu_e\bar{\nu}_\mu$	$\nu_\mu\bar{\nu}_e$	$\nu_e\bar{\nu}_\tau$	$\nu_\tau\bar{\nu}_e$	$\nu_\mu\bar{\nu}_\tau$	$\nu_\tau\bar{\nu}_\mu$
$U_{12}^*U_{11}$	$U_{22}^*U_{21}$	$U_{32}^*U_{31}$	$U_{22}^*U_{11}$	$U_{12}^*U_{21}$	$U_{12}^*U_{31}$	$U_{32}^*U_{11}$	$U_{22}^*U_{31}$	$U_{32}^*U_{21}$
0.2195	0.2192	0.000350	0.9484	0.0507	0.00195	0.0394	0.00844	0.00910

3.2 Other visible family gauge boson effects

Let us discuss other visible effects of the family gauge bosons with the mass spectrum (17).

(i) *Deviation from the e-μ-τ universality in tau decays*:

Regrettably, we cannot extract the value (11) from the tau decays, because the value was extracted only under an assumption $\tilde{M}_{23}^2 \ll \tilde{M}_{31}^2$ against the mass spectrum (17). On the other hand, we can see sizable deviations from the e-μ-τ universality in the Υ decays, $\Upsilon \to \tau^+\tau^-/\mu^+\mu^-/e^+e^-$. However, our expected value 7.6 TeV of $\tilde{M}_{33}$ is too larger than the value (12) obtained from the Υ decays [3]. The deviations will be confirmed in the Υ decay data in future.

(ii) *Lepton number violating rare decays of B and K*:

For lepton-flavor violating rare decays of B and K, B decays, $B^+ \to K^+\mu^-\tau^+$ and, $B^0 \to K^+\mu^-\tau^+$, and K decays, $K^+ \to \pi^+\mu^-\tau^+$, $K_L \to \pi^0\nu\bar{\nu}$ and $K^+ \to \pi^+\mu^\mp\tau^\pm$ become soon within our reach as seen in Table 1.

(iii) *μ-e conversion*:

Most sensitive test for our scenario is to observe the so-called μ-e conversion. (For a review of the μ-e conversion and more detailed calculations, for example, see Ref.[7] and Ref.[8], respectively.) At present, we do not know values of $|U_{11}^{q*}U_{21}^q|$ $(q = u, d)$. Therefore, it is not practical, at this stage, to estimate a μ-e conversion rate strictly. Instead, we roughly estimate a μ-e conversion rate in the quark level as follows:

$$R_q \equiv \frac{\sigma(\mu^- + q \to e^- + q)}{\sigma(\mu^- + u \to \nu_\mu + d)} \simeq \left(\frac{|U_{11}^{q*}U_{21}^q|}{|V_{ud}|} \frac{g_F^2/2}{\tilde{M}_{12}^2} \frac{M_W^2}{g_w^2/8} \right)^2 = \left(\frac{|U_{11}^{q*}U_{21}^q|}{|V_{ud}|} (r_{12})^2 \right)^2, \tag{19}$$

where $q = u, d$. It is likely that $|U^u_{21}|^2 \ll |U^d_{21}|^2$. Then, we may regard the ratios R_q as $R_u \ll R_d$, so that we can neglect contribution to nucleon from R_u compared with that from R_d. When we suppose $|U^{d*}_{11} U^d_{21}|/|V_{ud}| \sim 10^{-1}$, we can roughly estimate values of R_d for the input values $\tilde{M}_{12} \sim 180$ TeV as $R_d \sim 1.4 \times 10^{-13}$. Present experimental limit is, for instance for Au, $R(A_u) \equiv \sigma(\mu^- + Au \to e^- + Au)/\sigma(\mu \text{ capture}) < 7 \times 10^{-13}$ [9]. The estimated values $R_d \sim 1.4 \times 10^{-13}$ become within reach of our observation. (Although the estimated value R_d is different physical quantities from the value $R(A_u)$, we consider that the order of the value R_d can provide one with useful information.) Since the decay $\mu^- \to e^- + \gamma$ is highly suppressed in the present scenario, if we observe $\mu^- N \to e^- N$ without observation of $\mu^- \to e^- + \gamma$, then it will strongly support our family gauge boson scenario.

(iv) *Direct production of the light gauge bosons* ${A_3}^3$, ${A_3}^2$ *and* ${A_3}^1$:

It should be noted that the values $\tilde{M}_{33} \simeq 7.6$ TeV and $\tilde{M}_{32} \simeq \tilde{M}_{31} \simeq 10.7$ TeV are not true mass values of these family gauge bosons. The observed masses M_{ij} are given by $M_{ij} = (g_F/\sqrt{2})\tilde{M}_{ij}$. If the gauge coupling constant g_F is $g_F/\sqrt{2} < 0.4$, direct productions at LHC will become hopeful. Then, we can observe typical decay modes ${A_3}^3 \to \tau^- \tau^+$, ${A_3}^2 \to \tau^- \mu^+$ and ${A_3}^1 \to \tau^- e^+$ with the branching fraction 2/15=13.3%. (For example, in the decays of ${A_3}^2$, we have decay modes, $t + \bar{c}$, $b + \bar{s}$, $\tau^- + \mu^+$ and $\nu_\tau + \bar{\nu}_\mu$ with branching fractions 6/15, 6/15, 2/15 and 1/15, respectively.)

Meanwhile, note that our family gauge interactions are only interactions which can interact not only with ν_L but also with ν_R. The branching ratio $Br(A_i^j \to \nu_i \bar{\nu}_j) = 1/15 = 6.7\%$ is one in the case of Majorana neutrinos. If neutrinos are Dirac neutrinos, the branching ratios is given $Br(A_i^j \to \nu_i \bar{\nu}) = 2/16 = 12.5\%$. Therefore, in future, when the data of the direct production of A_i^j are accumulated, and it can be distinguished whether $Br = 6.7\%$ or $Br = 12.5\%$, we will be able to conclude whether neutrinos are Dirac or Majorana.

4 Family gauge bosons with nearly degenerated masses

In the previous section, we have speculated how we can obtain fruitful phenomenology, if we can build a model with the mass relation (1). However, we do not know the origin of the mass relation (1) at present.

In the present scenario, the absence of direct $A_i^j \leftrightarrow A_j^i$ transitions is essential. As we gave a brief review of such a model in Sec.1, we know that such the model is realized by introducing a scalar $(\mathbf{3}, \mathbf{3})$ of U(3)×U(3)′ into the model. In the model, the gauge boson masses M_{ij} are given by

$$M_{ij}^2 = \frac{1}{2} g_F^2 (v_i^2 + v_j^2), \tag{20}$$

where v_i are defined $\langle (\mathbf{3}, \mathbf{3}^*)_i^\alpha \rangle = \delta_i^\alpha v_i$, so that the masses of the family gauge bosons satisfy

$$2M_{ij}^2 = M_{ii}^2 + M_{jj}^2. \tag{21}$$

Obviously, the mass relations (1) and (21) is not compatible each other.

If we stubbornly adhere to the current theoretical aspect, we have to take the relation (21) prior to the relation (1). We have to consider that the relation (1) holds only approximately.

First, let us show that the relation (1) approximately holds even we take the relation (21) strictly. We define parameters ε_a and ε_b as follows:

$$\frac{M_{22}}{M_{33}} = 1 + \varepsilon_a, \qquad \frac{M_{11}}{M_{22}} = 1 + \varepsilon_b. \tag{22}$$

Then, we can express the effective current-current coupling constant G^{eff} defined by Eq.(6) is expressed as

$$G^{eff} = \frac{1}{\tilde{M}_{33}} \left(\frac{\lambda_1^2}{(1+\varepsilon_a)(1+\varepsilon_b)} + \frac{\lambda_2^2}{1+\varepsilon_a} + \lambda_3^2 \right.$$

$$\left. + \frac{2\lambda_1\lambda_2}{(1+\varepsilon_a)(1+\frac{1}{2}\varepsilon_a)} + \frac{2\lambda_2\lambda_3}{1+\frac{1}{2}\varepsilon_a} + \frac{2\lambda_3\lambda_1}{1+\frac{1}{2}(\varepsilon_a+\varepsilon_b+\varepsilon_a\varepsilon_b)} \right)$$

$$= \frac{1}{\tilde{M}_{33}} \left[\frac{1}{2}(\lambda_3\varepsilon_a - \lambda_1\varepsilon_b)^2 - \frac{1}{4}\varepsilon_a(\lambda_3\varepsilon_a - \lambda_1\varepsilon_b)(3\lambda_3\varepsilon_a^2 - 2\lambda_1\varepsilon_a\varepsilon_b - 3\lambda_1\varepsilon_b^2) + O(\varepsilon_a^4, \cdots) \right]. \tag{23}$$

Therefore, we can regard G^{eff} as $G^{eff} \simeq 0$ when we neglect $O(\varepsilon_a^2, \varepsilon_a\varepsilon_b, \varepsilon_b^2)$. (Besides, if we assume a specific relation $\lambda_3/\lambda_1 = \varepsilon_b/\varepsilon_a$, G^{eff} can regard G^{eff} as $G^{eff} \simeq 0$ until $O(\varepsilon_a^3, \cdots)$.)

In any cases, we practically obtain

$$\tilde{M}_{33} \simeq \tilde{M}_{32} \simeq \tilde{M}_{31} \simeq \tilde{M}_{22} \simeq \tilde{M}_{12} \simeq \tilde{M}_{11} \sim 180 \text{ TeV}. \tag{24}$$

Then, most family gauge boson effects discussed in the previous section will become invisible. Only the effects in $\mu^- N \to e^- N$ and $K \to \pi e^- \mu^+$ are still visible when we assume $\tilde{M}_{12} \sim 180$ TeV.

5 Concluding remarks

We have pointed out that if family gauge boson masses satisfy the relation (1), the gauge bosons are harmless to P^0-$\bar{P}^0$ mixing. This is valid only in a model in which there is no direct $A_i^j \leftrightarrow A_i^j$ transition and the family number violation is caused only by quark mixing ($U^d \neq \mathbf{1}$ and/or $U^u \neq \mathbf{1}$).

We can speculate many fruitful family gauge boson effects under the mass relation (1). However, the relation (1) is purely phenomenological one. The numerical values in Eq.(17) should be taken rigidly. The values are dependent on the tentative input (16). If we take $a = (m_\mu/m_\tau)^{3/2}$, the mass rations become more favorable to the phenomenology, but such the choice of a is not likely to be justified more than the choice $a = m_\mu/m_\tau$.

If we want to a model which is steady in the theoretical aspect, we have to adopt a family gauge boson model with nearly degenerated masses (24) instead of the phenomenological one (1).

In both cases (14) and (24), an observation of $\mu^- N \to e^- N$ versus no observation of $\mu \to e + \gamma$ will be a promising as a test of the present scenario.

However, we believe that Nature is rich. We would like to expect that the case (14) is likely. However, at present, how to derive to relation (1) instead of (21) is an open question. This is a next task to us.

Acknowledgments

The author thanks T. Yamashita for valuable and helpful conversations, especially, for pointing out a mistake in the earlier version of this work. He also thanks H. Yokoya for helpful comments on the direct production of ${A_3}^2$ at LHC, Y. Kuno and H. Sakamoto for useful comments on experimental status of μ-e conversion, and M. Koike for helpful comments on an estimate of the μ-e conversion.

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