Yukawaon Model with Anomaly Free Set of Quarks and Leptons in a U(3) Family Symmetry

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Abstract

In the so-called "yukawaon" model, the (effective) Yukawa coupling constants Y_f^{eff} are given by vacuum expectation values (VEVs) of scalars Y_f (yukawaons) with 3×3 components. So far, yukawaons Y_f have been given as **6** or **6*** of U(3) family symmetry, so that quarks and leptons were not anomaly free in U(3). In this paper, yukawaons are given as $\mathbf{8}+\mathbf{1}$ of U(3), so that quarks and leptons are anomaly free. Since VEV relations among yukawaons are also considerably changed, parameter fitting of the model is renewed. After fixing our free parameters by observed mass ratios, we have still two and one remaining free parameters for quark and lepton mixings, respectively, and we obtain successful predictions including magnitudes of CP violation. Also, the effective Majorana neutrino mass is predicted.

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1 Introduction

It is an attractive idea that observed hierarchical structures of masses and mixings of quarks and leptons are caused by a common origin. In the so-called "yukawaon" model [1, 2, 3, 4, 5, 6], Yukawa coupling constants Y_f ($f = u, d, e$, and so on) in the standard model are effectively given by vacuum expectation values (VEVs) of scalars (yukawaons) Y_f with 3×3 components. Although the model is a kind of "flavon" models [7], the aim of the model is somewhat different: We suppose that the observed hierarchical family structure of masses and mixings of quarks and leptons are caused by one common origin, so that they can successfully be understood by accepting one of the hierarchical structures (e.g. charged lepton mass spectra) as input values of the model. We don't use any other family-number dependent input parameters, except for the observed values of the charged lepton masses (m_e, m_μ, m_τ) as input parameters with hierarchical values. Here, the terminology "family-number independent parameters" means, for example, coefficients of a unit matrix $\mathbf{1}$, a democratic matrix X_3, and so on, where

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad X_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{1.1}$$

(For an explicit example of the previous yukawaon model, for example, see Eq.(1.2) later.) Regrettably, at present, we are obliged to accept a few other family-number dependent parameters. For example, we are obliged to use a phase matrix $P = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$ in the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix $V_{CKM} = U_u^\dagger P\, U_d$ [8] and the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) lepton mixing matrix U_{PMNS} [9]. Therefore, our original intention in the yukawaon model is not yet completed at present.

In the earlier stage of yukawaon models [1, 2], the VEV matrices of yukawaons have been described by the following mass matrix relations:

$$
\begin{array}{l}
Y_e = k_e \Phi_e \Phi_e, \qquad \Phi_e = k'_e \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}) \\
Y_u = k_u \Phi_u \Phi_u, \quad \Phi_u = k'_u \Phi_e (\mathbf{1} + a_u X_3) \Phi_e, \\
Y_d = k_d P_d \Phi_e (\mathbf{1} + a_d X_3) \Phi_e P_d, \\
Y_\nu = Y_D Y_R^{-1} Y_D^T, \\
Y_D = Y_e, \\
Y_R = k_R (\Phi_u Y_e + Y_e \Phi_u) + \cdots ,
\end{array}
\tag{1.2}
$$

where Y_e, Y_u, Y_d, Y_ν, Y_D, Y_R, and Y_ν correspond to charged lepton, up-quark, down-quark, neutrinos, Dirac neutrino, Majorana right-handed neutrino mass matrices, respectively, and $P_d = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$. (Here, we have denoted a VEV matrix $\langle Y_f \rangle$ as Y_f simply.) The coefficients a_u and a_d are family-number independent parameters. On the other hand, since we discuss only mass ratios and mixings, the parameters k_e, k_u and so on are not essential in the model. (Hereafter, we omit such common coefficients.)

The model (1.2) could give tribimaximal mixing for U_{PMNS}, but it gave poor fitting for V_{CKM}. Besides, the model could not give the observed large mixing [10] $\sin^2 2\theta_{13} \sim 0.09$, although the value was reported after the proposal of the model (1.2).

In the second stage of the yukawaon model [3], motivated with this new observation, we proposed to change the structure $Y_e = k_e \Phi_e \Phi_e$ into

$$
Y_e = k_e \Phi_0 (\mathbf{1} + a_e X_3) \Phi_0. \tag{1.3}
$$

Here VEV matrix Φ_0 is given by a diagonal matrix $\Phi_0 = \mathrm{diag}(x_1, x_2, x_3)$ on a family basis after breaking of a U(3) family symmetry, so that the charged lepton mass matrix Y_e is not diagonal any longer. Also, the assumption $Y_D = Y_e$ was changed into

$$
Y_D = \Phi_0 (\mathbf{1} + a_D X_2) \Phi_0, \tag{1.4}
$$

where

$$
X_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{1.5}
$$

Furthermore, recently, we have proposed [6] a new neutrino mass matrix with a bilinear form $Y_\nu = (Y_D Y_R^{-1} Y_D)^2$. The model can give reasonable predictions of the quark and lepton mixings (V_{CKM} and U_{PMNS}) together with their masses.

The purpose of the present paper is not to improve such parameter fitting. (Of course, we will give a reasonable parameter fitting under a model with parameters less than those in the previous works.) The major purpose in the present paper is to improve a basic part of the yukawaon model. So far, the yukawaons Y_f have been described as Y_f^{ij}, i.e. $\mathbf{6}^*$ of a family symmetry U(3). The reason is as follows: If we consider a field C of $\mathbf{8}+\mathbf{1}$ of U(3), i.e. $C_i^{\ j}$, and we require a combination ACB by $A^{ik}C_k^{\ l}B_{lj}$, then, we are obliged to have unwelcome combinations $A^{ik}B_{kl}(C^T)^l_{\ j}$ and $(C^T)^i_{\ k}A^{kl}B_{lj}$. In the yukawaon model, the order of multiplication of matrices is essential. Therefore, so far, we have not adopted a yukawaon model with $(Y_f)_i^{\ j}$. However, in the model with $(Y_f)^{ij}$, quarks and leptons f are assigned to $(f_L, f_R) \sim (\mathbf{3}, \mathbf{3}^*)$ of the U(3) family symmetry, so that the fundamental fermions are not anomaly free in the U(3) symmetry. In the present model, the yukawaons are given by $(Y_f)_i^{\ j}$, so that quarks and leptons, themselves, are anomaly free for the U(3) family symmetry. Of course, there is no reason that quarks and leptons must compose an anomaly free set. In the previous model, we have assumed a supersymmetric theory (SUSY), so that the model could become anomaly free by taking whole flavons in the model into consideration, although quarks and leptons, themselves, were not anomaly free. We empirically know that quarks and leptons, which are fundamental entities in the low energy limit (in the standard model limit), compose an anomaly free set of gauge symmetries concerned. It is unnatural that quarks and leptons do not compose an anomaly free set in family gauge symmetry U(3), too.

Another purpose is to make basic modifications of the model as follows: We have already proposed a yukawaon model with bilinear forms $Y_u = \Phi_u \Phi_u$ and $Y_\nu = \Phi_\nu \Phi_\nu = (Y_D Y_R^{-1} Y_D)^2$. On this occasion of the present model-change, we also change the previous model into a new model in which whole yukawaon VEV matrices except for that for the charged leptons are given by bilinear forms, $Y_u = \Phi_u \Phi_u$, $Y_d = \Phi_d \Phi_d$ and $Y_\nu = \Phi_\nu \Phi_\nu = (Y_D Y_R^{-1} Y_D)^2$. Therefore, parameter fitting for masses and mixings should be renewed completely as shown later.

In the next section, we discuss why it is hard to build a model with yukawaons of $\mathbf{8}+\mathbf{1}$ of U(3), and we give a basic idea how to inherit the successful results in the previous yukawaon model with $(\bar{Y}_f)^{ij}$ into a new model with $(Y_f)_i^{\ j}$.

In Sec.3, we give a new yukawaon model with $(Y_f)_i^{\ j}$. We give VEV matrices relations among flavons (yukawaons). Those flavons are distinguished by R charges. Assignments of R charges are discussed in Sec.4. Renewed parameter fitting is given in Sec.5. Finally, Sec.6 is devoted to concluding remarks.

2 How to get $(\hat{Y}_f)_i^{\ j}$

We assume that a would-be Yukawa interaction is given as follows:

$$W_Y = \frac{y_D}{\Lambda}(\nu^c)^i(\hat{Y}_D^T)_i{}^j \ell_j H_u + \frac{y_e}{\Lambda}(e^c)^i(\hat{Y}_e)_i{}^j \ell_j H_d + \frac{y_u}{\Lambda}(u^c)^i(\hat{Y}_u)_i{}^j q_j H_u + \frac{y_d}{\Lambda}(d^c)^i(\hat{Y}_d)_i{}^j q_j H_d$$

$$+ y_R(\nu^c)^i (Y_R)_{ij}(N^c)^j + y'_D(\nu^c)^i(\hat{Y}_D)_i{}^j N_j + y_N N_i (E_N)^{ij} N_j, \tag{2.1}$$

where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are $\mathrm{SU}(2)_L$ doublets, and N and N^c are new $\mathrm{SU}(2)_L$ singlet leptons. The last three terms are added in order to give the neutrino mass matrix with a form $M_\nu = (Y_D Y_R^{-1} Y_D)^2$ as we show in Sec.3. In order to distinguish each yukawaon from others, we assume that Y_f have different R charges from each other together with considering R charge conservation (a global U(1) symmetry in $N = 1$ supersymmetry). (Of course, the R charge conservation is broken at an energy scale Λ, at which the U(3) family symmetry is broken.) Possible assignments of R charges of the flavons are given in Sec.4. Hereafter, for convenience, we denote a flavon A of $\mathbf{6}^*$ as $\bar{A}$, and a flavon A of $\mathbf{8}+\mathbf{1}$ as $\hat{A}$.

First, we would like to give a brief review how it is hard to build a yukawaon model with $(\hat{Y}_f)_i{}^j$. In the previous yukawaon model, the VEVs of yukawaons Y_f are related to VEV matrix of flavons $\bar{\Phi}_f$ which belong to $\mathbf{6}^*$ of U(3) family symmetry and are given by

$$\langle \bar{\Phi}_f \rangle^{ij} = \langle \bar{\Phi}_0 \rangle^{ik} \left[\langle E_0 \rangle_{kl} + a_f \langle X_3 \rangle_{kl} \right] \langle \bar{\Phi}_0 \rangle^{lj}, \tag{2.2}$$

where $\langle E_0 \rangle$ and $\langle X_3 \rangle$ are given by the matrix forms $\mathbf{1}$ and X_3 defined in Eq.(1.1), and $\langle \bar{\Phi}_0 \rangle$ takes a VEV form

$$\langle \bar{\Phi}_0 \rangle = \mathrm{diag}(x_1, x_2, x_3), \tag{2.3}$$

where $x_1^2 + x_2^2 + x_3^3 = 1$. The parameter a_f in Eq.(2.2) is, in general, complex. (However, we will regard a_e and a_D as real, later in Eqs.(3.1) and (3.2).) In Eq.(2.2), we have dropped common coefficients in families, because we pay attention only to the relative ratios among families. In the previous yukawaon model, yukawaon VEV $\langle \bar{Y}_e \rangle$ in the charged leptons was given by $\langle \bar{Y}_e \rangle = \langle \bar{\Phi}_e \rangle$, and that of up-quarks $\langle \bar{Y}_u \rangle$ was given by a bilinear form

$$\langle \bar{Y}_u \rangle^{ij} = \langle \bar{\Phi}_u \rangle^{ik} \langle E_u \rangle_{kl} \langle \bar{\Phi}_u \rangle^{lj}, \tag{2.4}$$

where $\langle E_u \rangle = \mathbf{1}$. If we want to build a model with $(\hat{Y}_e)_i{}^j$, for example, we may take a VEV relation

$$\langle \bar{E}_e \rangle^{ik} \langle \hat{Y}_e \rangle_k{}^j = \langle \bar{\Phi}_e \rangle^{ij}. \tag{2.5}$$

However, note that the VEV relation (2.5) holds only for a case in which the parameter a_e defined in Eq.(2.2) is real, because, if a_e is complex, the $i = j$ component of $\langle \bar{\Phi}_e \rangle^{ij}$ is complex, while the $i = j$ component of $\langle \hat{Y}_e \rangle_i{}^j$ must be real (we consider $\langle \bar{E}_e \rangle = \mathbf{1}$). Fortunately, for the charged lepton sector, the parameter a_e is real in the actual parameter fitting, so that we can choose the relation (2.5).

On the other hand, for the up-quark sector, we must choose a complex parameter a_u in the VEV relation (2.2), and, besides, we want to build a model with $\langle\hat{Y}_u\rangle_i{}^j$ instead of Eq.(2.4) with $\langle\bar{Y}_u\rangle^{ij}$. Therefore, we cannot adopt a prescription analogous to Eq.(2.5). As far as the transformation property is concerned, for example, we may consider a relation

$$\langle\hat{Y}_u\rangle_i{}^j = \langle\Phi_u\rangle_{ik}\langle\bar{\Phi}_u\rangle^{kj}, \tag{2.6}$$

where $\langle\bar{\Phi}_u\rangle$ and $\langle\Phi_u\rangle$ are given by

$$\begin{aligned}\langle\bar{\Phi}_u\rangle^{ij} &= \langle\bar{\Phi}_0\rangle^{ik}\left[\langle E_0\rangle_{kl} + a_u\langle X_3\rangle_{kl}\right]\langle\bar{\Phi}_0\rangle^{lj},\\ \langle\Phi_u\rangle_{ij} &= \langle\Phi_0\rangle_{ik}\left[\langle\bar{E}_0\rangle^{kl} + a_u^*\langle\bar{X}_3\rangle^{kl}\right]\langle\Phi_0\rangle_{lj},\end{aligned} \tag{2.7}$$

correspondingly to Eq.(2.2). However, phenomenological success for simultaneous fitting of the mass ratios and CKM and PMNS mixings has been obtained only from the form (2.4), and if we adopt the form (2.6), we cannot obtain reasonable fitting. The problem is how to relate $\langle\bar{\Phi}_u E_u\bar{\Phi}_u\rangle$ with $\mathbf{6}^*$ to $\langle\hat{Y}_u\rangle$ with $\mathbf{8}+\mathbf{1}$.

Let us consider a yukawaon VEV $\langle\hat{Y}_u\rangle$ which gives the same mass spectrum and up-quark mixing U as those $\langle\bar{Y}_u\rangle$ has. The mass spectrum and up-quark mixing of $\langle\hat{Y}_u\rangle$ can be obtained by the diagonalization

$$(U^\dagger)_i{}^k\langle\hat{Y}_u\rangle_k{}^l U_l{}^j = \langle\hat{D}\rangle_i{}^j, \tag{2.8}$$

where $\langle D\rangle$ is a diagonal matrix. On the other hand, the mass spectrum and mixing are obtain by the diagonalization

$$(U^\dagger)_i{}^k\langle\bar{Y}_u^\dagger\rangle_{kl}\langle\bar{Y}_u\rangle^{lm}U_m^{\ j} = [\langle\bar{D}^\dagger\rangle\langle\bar{D}\rangle]_i{}^j, \tag{2.9}$$

i.e.

$$(U^T)^i{}_k\langle\bar{Y}_u\rangle^{kl}U_k{}^j = \langle\bar{D}\rangle^{ij}, \tag{2.10}$$

because VEV of the yukawaon with $\mathbf{6}^*$ is symmetric. Here, note that the mixing matrix U denotes a mixing among u_{Li} which are members of SU$(2)_L$ doublet q_i, as seen from the definition of $(\hat{Y}_q)_i{}^j$ (2.1). Since the matrices $\langle\hat{D}\rangle$ and $\langle\bar{D}\rangle$ are diagonal, those can be related by a E matrix as follows

$$\langle\hat{D}\rangle_i{}^j = \langle E\rangle_{ik}\langle\bar{D}\rangle^{kj}. \tag{2.11}$$

This means that $\langle\hat{Y}_u\rangle$ can related to $\langle\bar{Y}_u\rangle$ as follows:

$$\langle\hat{Y}_u\rangle_i{}^j = U_i{}^k\langle\hat{D}\rangle_k{}^l(U^\dagger)_l{}^j = U_i{}^k\left[E\,U^T\bar{Y}_uU\right]_k^{\ l}(U^\dagger)_l{}^j = (U\,E\,U^T)_{ik}\langle\bar{Y}_u\rangle^{kj}. \tag{2.12}$$

Therefore, when we put

$$(C_u)_{ij} \equiv U_i{}^k E_{kl}(U^T)^l{}_j, \tag{2.13}$$

we can relate $\langle\hat{Y}_u\rangle$ to $\langle\bar{Y}_u\rangle$ as

$$\langle\hat{Y}_u\rangle = C_u\langle\bar{Y}_u\rangle. \tag{2.14}$$

From the definition (2.13), the quantity C_u must satisfy the following relations:

$$C_u^\dagger C_u = C_u C_u^\dagger = \mathbf{1}, \qquad C_u^T = C_u, \tag{2.15}$$

so that

$$\langle \bar{Y}_u^\dagger \rangle_{ij} = (C_u)_{ik} \langle \bar{Y}_u \rangle^{kl} (C_u)_{lj}. \tag{2.16}$$

Note that since the matrix C_u gives a mixing among the quarks u^c as seen in (2.14), the explicit value of C_u does not affect the observed values of up-quark mass ratios and CKM matrix parameters.

Similarly, we introduce C_d and we can consider

$$\langle \hat{Y}_d \rangle = C_d \langle \bar{Y}_d \rangle. \tag{2.17}$$

In the present paper, we assume flavons C_f ($f = u$ and d) whose VEV matrices $\langle C_f \rangle$ satisfy the same relations as (2.15), and thereby we will build a model with $(\hat{Y}_f)_i^{\ j}$.

3 Model

First, let us summarize our VEV relations among flavons prior to their derivation from a superpotential.

For lepton sector, since we consider the parameters a_e and a_D are real, we can adopt VEV relations with a type (2.5):

$$\langle \bar{E} \rangle^{ik} \langle \hat{Y}_e \rangle_k^{\ j} + \langle \hat{Y}_e^T \rangle^i_{\ k} \langle \bar{E} \rangle^{kj} = \langle \bar{\Phi}_0 \rangle^{ik} \left[\langle E_0 \rangle_{kl} + a_e \langle X_3 \rangle_{kl} \right] \langle \bar{\Phi}_0 \rangle^{lj}, \tag{3.1}$$

$$\langle \hat{Y}_D \rangle_i^{\ k} \langle E_D \rangle_{kj} + \langle \hat{Y}_D^T \rangle_i^{\ k} \langle E_D \rangle_{kj} = \langle \Phi_0 \rangle_{ik} \left[\langle \bar{E}_0 \rangle^{kl} + a_D \langle \bar{X}_2 \rangle^{kl} \right] \langle \Phi_0 \rangle_{lj}. \tag{3.2}$$

The neutrino Dirac yukawaon $\hat{Y}_D$ takes somewhat complicated structure in order to make flavon sector anomaly free (see later). Although we speculated a mechanism [5] for the form $\bar{X}_2$ by introducing additional family symmetry U(3)$'$, in the present work, we do not refer to such a mechanism. The origin of the structure $\bar{X}_2$ is left as our future task.

For quark sector, according to the previous section, we introduce two flavons C_u and C_d:

$$\langle \hat{Y}_u \rangle_i^{\ j} = \langle C_u \rangle_{ik} \langle \bar{P}_u \rangle^{kl} \langle \Phi_u \rangle_{lm} \langle \bar{E} \rangle^{mn} \langle \Phi_u \rangle_{no} \langle \bar{P}_u \rangle^{oj}, \tag{3.3}$$

$$\langle \Phi_u \rangle_{ij} = \langle E \rangle_{ik} \langle \bar{\Phi}_0 \rangle^{kl} \left[\langle E_0 \rangle_{lm} + a_u \langle X_3 \rangle_{lm} \right] \langle \bar{\Phi}_0 \rangle^{mn} \langle E \rangle_{nj}, \tag{3.4}$$

$$\langle \hat{Y}_d \rangle_i^{\ j} = \langle C_d \rangle_{ik} \langle \bar{\Phi}_d \rangle^{kl} \langle E \rangle_{lm} \langle \bar{\Phi}_d \rangle^{mj}, \tag{3.5}$$

$$\langle \bar{\Phi}_d \rangle^{ij} = \langle \bar{\Phi}_0 \rangle^{ik} \left[\langle E_0 \rangle_{kl} + a_q \langle X_3 \rangle_{kl} \right] \langle \bar{\Phi}_0 \rangle^{lj} + \xi_0^d \langle \bar{E} \rangle. \tag{3.6}$$

The last term (ξ_0^d term) in Eq.(3.6) has been introduced from a phenomenological requirement as we discuss in Sec.5. Derivation of the condition $R(\bar{\Phi}_d) = R(\bar{E})$ is given in the next section. As to the origin of the form $\langle E \rangle = \mathbf{1}$, we will also discuss in the next section. Note that the combination $\langle\bar{\Phi}_0\rangle[\langle E_0\rangle + a_f\langle X_3\rangle]\langle\bar{\Phi}_0\rangle$ is commonly connected to the yukawaon VEVs $\langle\hat{Y}_e\rangle$, $\langle\hat{Y}_u\rangle$ and $\langle\hat{Y}_d\rangle$ except for $\langle\hat{Y}_D\rangle$. R charges of $\hat{Y}_e$, $\hat{Y}_u$ and $\hat{Y}_d$ are distinguished by number of the flavons $\bar{E}$ and E concerned (see Eq.(4.8) in the next section), so that the parameters a_f can take different values according to individual sectors.

From the Yukawa interactions (2.1), we consider that the neutrino mass matrix M_ν is given by a triplicate seesaw mechanism as

$$M_\nu^{ij} \simeq \langle\hat{Y}_D^T\rangle^i{}_k\langle Y_R^{-1}\rangle^{kl}\langle\hat{Y}_D\rangle_l{}^m\langle E_N^{-1}\rangle_{mm'}\langle\hat{Y}_D^T\rangle^{m'}_{l'}\langle Y_R^{-1}\rangle^{l'k'}\langle\hat{Y}_D\rangle_{k'}^{\ j}\,, \tag{3.7}$$

where we must assume

$$y_N^2|\langle E_N\rangle|^2 \gg y_R^2|\langle Y_R\rangle|^2 \gg y_D^2|\langle\hat{Y}_D\rangle|^2, \tag{3.8}$$

In order to obtain good seesaw approximation (3.7). We consider that there are hierarchical structures not only among the VEV values of $|\langle E_N\rangle|^2$, $|\langle Y_R\rangle|^2$ and $|\langle\hat{Y}_D\rangle|^2$, but also among the coupling constants y_N^2, y_R^2 and y_D^2. Here, the Majorana mass matrix $\langle Y_R\rangle$ has the following VEV relation

$$\langle Y_R\rangle_{ij} = \langle\hat{Y}_e\rangle_i{}^k\langle\Phi_u\rangle_{kj} + \langle\Phi_u\rangle_{ik}\langle\hat{Y}_e^T\rangle^k{}_j. \tag{3.9}$$

These VEV matrix relations are obtain SUSY vacuum conditions. For example, the VEV relations (3.1) can be obtained by requiring a SUSY vacuum condition $\partial W/\partial\Theta_e = 0$ for the following superpotential:

$$W_e = \left\{\lambda_e\left[(\hat{Y}_e^T)^i{}_k(\bar{E})^{kj} + (\bar{E})^{ik}(\hat{Y}_e)_k^{\ j}\right] + \frac{\lambda'_e}{\Lambda}(\bar{\Phi}_0)^{ik}\left[(E_0)_{kl} + a_e(X_3)_{kl}\right](\bar{\Phi}_0^T)^{lj}\right\}(\Theta_e)_{ji}. \tag{3.10}$$

Since we assume that the Θ field always takes $\langle\Theta\rangle = 0$ and since SUSY vacuum conditions in other fields always contain the VEV matrix $\langle\Theta\rangle$, such conditions do not play any effective role in obtaining VEV relations.

The VEV relations (3.2), (3.3), (3.5) and (3.7) are obtained from the following superpotential similar to (3.10):

$$W_D = \left\{\lambda_D\left[((\hat{Y}_D)_i{}^k(E_D)_{kj} + (E_D)_{ik}(\hat{Y}_D^T)^k{}_j\right] + \frac{\lambda'_D}{\Lambda}(\Phi_0)_{ik}\left((\bar{E}_0)^{kl} + a_D(\bar{X}_2)^{kl}\right)(\Phi_0)_{lj}\right\}(\bar{\Theta}_D)^{ji}, \tag{3.11}$$

$$W_u = \left\{\mu_u(\hat{Y}_u)_i{}^j + \frac{\lambda_u}{\Lambda^4}(C_u)_{ik}(\bar{P}_u)^{kl}(\Phi_u)_{lm}(\bar{E})^{mn}(\Phi_u)_{no}(\bar{P}_u)^{oj}\right\}(\hat{\Theta}_u)_j{}^i, \tag{3.12}$$

$$W_d = \left\{\mu_d(\hat{Y}_d)_i{}^j + \frac{\lambda_d}{\Lambda^2}(C_d)_{ik}(\bar{\Phi}_d)^{kl}(E)_{lm}(\bar{\Phi}_d)^{mj}\right\}(\hat{\Theta}_u)_j{}^i, \tag{3.13}$$

Table 1: Assignments of SU(2)$_L\times$SU(3)$_c\times$U(3). R charges are discussed in Sec.4.

	ℓ	e^c	ν^c	N	N^c	q	u^c	u^c	H_u	H_d
SU(2)$_L$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
SU(3)$_c$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{1}$	$\mathbf{1}$
U(3)	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{1}$	$\mathbf{1}$

$\hat{Y}_e$	$\hat{Y}_D$	Y_R	$\hat{Y}_u$	$\hat{Y}_d$	Φ_u	$\bar{\Phi}_d$	$\bar{P}_u$	P_u	C_u	C_d	$\bar{C}_u$	$\bar{C}_d$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{6}$	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{6}^*$

Φ_0	$\bar{\Phi}_0$	E_0	X_3	$\bar{E}_0$	$\bar{X}_2$	$\bar{E}$	E	E_D	$\bar{E}_N$	Θ_e	$\bar{\Theta}_D$	$\hat{\Theta}_u$	$\hat{\Theta}_d$	$\bar{\Theta}_R$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{8+1}$	$\mathbf{8+1}$	$\mathbf{6}^*$

$$W_R = \left\{\mu_R (Y_R)_{ij} + \lambda_R \left[(\hat{Y}_e)_i^{\ k}(\Phi_u)_{kj} + (\Phi_u)_{ik}(\hat{Y}_e^T)^k_{\ j}\right]\right\}(\bar{\Theta}_R)^{ji}, \tag{3.14}$$

respectively. Note that, only for the Dirac neutrino mass term, an additional flavon Φ_0 is introduced. We assume that the VEV matrix $\langle\Phi_0\rangle$ takes the same value of $\langle\bar{\Phi}_0\rangle$ because of the D-term condition.

In spite of the idea (3.11), for the VEV relations for Φ_u and $\bar{\Phi}_d$, we use a new type of the superpotential

$$W_q = \mu_q \mathrm{Tr}\left\{\left[\Phi_u + \frac{\lambda_u}{\Lambda^4} E\,\bar{\Phi}_0(E_0 + a_d X_3)\bar{\Phi}_0 E\right]\left[\bar{\Phi}_d + \frac{\lambda_d}{\Lambda^2}\bar{\Phi}_0(E_0 + a_u X_3)\bar{\Phi}_0\right]\right\}. \tag{3.15}$$

SUSY vacuum conditions $\partial W/\partial\bar{\Phi}_d = 0$ and $\partial W/\partial\Phi_u = 0$ lead to the VEV relations (3.4) and (3.6), respectively (except for ξ_0^d term in Eq.(3.6)). Other conditions $\partial W/\partial\bar{\Phi}_0 = 0$, $\partial W/\partial E_0 = 0$, and so on are satisfied identically under the VEV relations (3.4) and (3.6) (and also (3.1)).

Note that in the present model, the yukawaons belong to $\mathbf{8+1}$ of U(3), so that sum of the anomaly coefficients obviously take zero for lepton and quark sectors:

$$\begin{array}{l} \sum_{\mathrm{leptons}} A = 3A(\mathbf{3}) + 3A(\mathbf{3}^*) = 0, \\ \sum_{\mathrm{quarks}} A = 6A(\mathbf{3}) + 6A(\mathbf{3}^*) = 0. \end{array} \tag{3.16}$$

Besides, in the flavon sector, too, sum of the anomaly coefficients becomes zero:

$$\sum_{\mathrm{flavons}} A = 11A(\mathbf{6}) + 11A(\mathbf{6}^*) + 6A(\mathbf{8+1}) = 0, \tag{3.17}$$

as seen in Table 1. (In Table 1, flavons P_u, $\bar{C}_u$ and $\bar{C}_d$ which did not appear in Eqs.(3.1) - (3.8) have been added. Roles of those flavons are discussed in the next section.)

4 R charge assignments

As seen in Eqs.(3.1) - (3.8) and also in Table 1, we have many flavons in the present model. Those are distinguished by their R charges. At preset, we cannot fix the R charges of flavons uniquely, because we have a considerable number of flavons compared with the number of the VEV relations required.

First, we discuss R charges for flavons E whose VEV matrices have the same matrix forms $\mathbf{1}$:

$$(\bar{E})^{ij}, \quad (E)_{ij}, \quad (E_0)_{ij}, \quad (\bar{E}_0)^{ij}, \quad (\bar{E}_N)_{ij}, \tag{4.1}$$

In order to obtain the VEV form $\langle E \rangle \propto \mathbf{1}$, we consider a superpotential with the following type:

$$W_E = \frac{\lambda_1}{\Lambda}\mathrm{Tr}[\bar{E}_A E_B \bar{E}_C E_D] + \frac{\lambda_2}{\Lambda}\mathrm{Tr}[\bar{E}_A E_B]\mathrm{Tr}[\bar{E}_C E_D], \tag{4.2}$$

which leads to VEV relations $\bar{E}_A E_B \bar{E}_C \propto \bar{E}_C$, $E_D \bar{E}_A E_B \propto E_D$, and so on. We assume superpotential with the type (4.2) for possible combinations of flavons given in (4.1). As a result of taking such all combinations among the flavons E's, we obtain $\langle E \rangle \propto \mathbf{1}$. However, we must take care that the value $R(E_0)$ should be different from the value $R(E)$ and the value $R(\bar{E}_0)$ should be different from other $\bar{E}$, because E_0 and $\bar{E}_0$ have the same R charges as those of X_3 and $\bar{X}_2$, respectively, so that they mix with X_3 and $\bar{X}_2$, respectively, while, for other E's, such mixings with X_3 and $\bar{X}_2$ should be forbidden:

$$R(E_0) = R(X_3) \neq R(E), \quad R(\bar{E}_0) = R(\bar{X}_2) \neq R(\bar{E}), R(\bar{E}_N). \tag{4.3}$$

On the other hand, for a VEV matrix of $\bar{P}_u$, we assume the VEV matrix

$$\langle \bar{P}_u \rangle = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1). \tag{4.4}$$

By introducing a new flavon P_u, and by taking

$$R(P_u) + R(\bar{P}_u) = 1. \tag{4.5}$$

we suppose that the pair $P_u \bar{P}_u$ satisfy a superpotential which is similar type to Eq.(4.2):

$$W_P = \frac{\lambda_1}{\Lambda}\mathrm{Tr}[\bar{E} E\, \bar{P}_u P_u] + \frac{\lambda_2}{\Lambda}\mathrm{Tr}[\bar{E}\, E\,]\mathrm{Tr}[\bar{P}_u P_u]. \tag{4.6}$$

Then, we can obtain $\langle P_u \rangle \langle \bar{P}_u \rangle = \mathbf{1}$. We consider (4.4) is a specific solution of $\langle P_u \rangle \langle \bar{P}_u \rangle = \mathbf{1}$.

Also, in order to have the VEV relation (2.14),

$$\langle C_q\rangle\langle C_q^\dagger\rangle = \mathbf{1} \qquad (q=u,d), \tag{4.7}$$

we introduce additional flavons $\bar{C}_u$ and $\bar{C}_d$, and by taking R charges as

$$R(C_u)+R(\bar{C}_u)=1, \qquad R(C_d)+R(\bar{C}_d)=1, \tag{4.8}$$

we consider a superpotential

$$W_C = \frac{\lambda_1}{\Lambda}\mathrm{Tr}[\bar{C}_u C_u \bar{C}_d C_d] + \frac{\lambda_2}{\Lambda}\mathrm{Tr}[\bar{C}_u\, C_u]\mathrm{Tr}[\bar{C}_d C_d]. \tag{4.9}$$

Thereby, we obtain the VEV relations

$$\langle \bar{C}_q\rangle\langle C_q\rangle = \mathbf{1}. \tag{4.10}$$

(However, The relations (4.8) do not mean $R(C_u)=R(C_d)$.) When we consider $\langle\bar{C}_q\rangle = \langle C_q^\dagger\rangle$ from D term condition in SUSY formulation, we can obtain the relation (4.7).

The VEV relations (3.1), (3.4) and (3.6) demand the following R charge relations

$$R(\hat{Y}_e)+R(\bar{E}) = R(\Phi_u)-2R(E) = R(\bar{\Phi}_d) = 2R(\bar{\Phi}_0)+R(E_0) \equiv R_0. \tag{4.11}$$

On the other hand, from the superpotential (3.15), we obtain

$$R(\Phi_u)+R(\bar{\Phi}_d)=2. \tag{4.12}$$

From Eqs.(4.11) and (4.12), we obtain

$$R(\bar{\Phi}_d) = 1-R(E) = R(\bar{E}), \tag{4.13}$$

where we have used $R(\bar{E})+R(E)=1$. Thus, we can see that the flavon $\bar{\Phi}_d$ can mix with the flavon $\bar{E}$. However, note that we can also get

$$R(\Phi_u) = 2R(E)+R(\bar{E}), \tag{4.14}$$

from the relation $R(\Phi_u)+R(\bar{\Phi}_d) = 2(R(E)+R(\bar{E}))=2$ and Eq.(4.13). This means that $(\Phi_u)_{ij}$ can mix with $(E\bar{E}\,E)_{ij}$.

5 Parameter fitting

We summarize our mass matrices in the present model as follows:

$$M_e = \Phi_0(\mathbf{1}+a_e X_3)\Phi_0, \tag{5.1}$$

$$M_D = \Phi_0(\mathbf{1}+a_D X_2)\Phi_0, \tag{5.2}$$

$$M_u = P_u\Phi_u\Phi_u P_u, \qquad \Phi_u = \Phi_0\left(\mathbf{1}+a_u e^{i\alpha_u}X_3\right)\Phi_0 + \xi_0^u\mathbf{1}, \tag{5.3}$$

$$M_d = \Phi_d \Phi_d, \qquad \Phi_d = \Phi_0 \left(\mathbf{1} + a_d e^{i\alpha_d} X_3 \right) \Phi_0 + \xi_0^d \mathbf{1}, \tag{5.4}$$

$$M_\nu = Y_D Y_R^{-1} Y_D \cdot Y_D Y_R^{-1} Y_D, \qquad Y_R = Y_e \Phi_u + \Phi_u Y_e, \tag{5.5}$$

where the term $\xi_0^u \mathbf{1}$ [$\xi_0^d \mathbf{1}$] denotes mixing effect between Φ_u [$\bar{\Phi}_d$] and $(E\bar{E}E)$ [$\bar{E}$], as they were discussed in Eqs.(4.13) and (4.14). Here, for convenience, we have dropped the notations "$\langle$" and "$\rangle$". And also, we have denoted $\bar{\Phi}_d$, $\hat{Y}_e$, $\cdots$, as Φ_d, Y_e, $\cdots$, simply. Since we are interested only in the mass ratios and mixings, we use dimensionless expressions $\Phi_0 = \text{diag}(x_1, x_2, x_3)$, $P_u = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$, and $E = \text{diag}(1,1,1)$. [Therefore, the parameters a_e, a_D, $\cdots$, are re-refined by Eqs.(5.1)-(5.5)]. Although the parameters a_f are, in general, complex, the parameters a_e and a_D in the lepton sectors must be real as seen in the relations (3.1) and (3.2). We have denoted the parameters a_u and a_d in the quark sectors as $a_u e^{i\alpha_u}$ and $a_d e^{i\alpha_d}$.

Besides, we require "economy of the number of parameters". We neglect parameters which play no essential roles in numerical fitting to the mixings and mass ratios as far as possible. As we discussed in Eqs.(4.13) and (4.14), the term $\xi_0^u \mathbf{1}$ [$\xi_0^d \mathbf{1}$] denotes mixing effect between Φ_u [$\bar{\Phi}_d$] and $(E\bar{E}E)$ [$\bar{E}$]. Therefore, we speculate that the value of ξ_0^u is an order of $(\xi_0^d)^3$. Since we guess that value of ξ_0^d is small (in fact, $\xi_0^d \sim 10^{-2}$ as we see later), we neglect the parameter ξ_u with 10^{-6}, so that we put

$$\xi_0^u = 0, \tag{5.6}$$

in the present parameter fitting. On the other hand, although there are no reasons that values of the parameters ϕ_1 and ϕ_2 are small, we put

$$\phi_1 = 0, \tag{5.7}$$

by way of trial.

Therefore, in the present model, we have 8 adjustable parameters, a_e, a_D, (a_u, α_u), (a_d, α_d), ξ_0^d, and ϕ_2 except for (x_1, x_2, x_3), for the 16 observable quantities (6 mass ratios in the up-quark-, down-quark-, and neutrino-sectors, 4 CKM mixing parameters, and 4+2 PMNS mixing parameters). In order to fix these parameters, we use, as input values, the observed values for m_c/m_t, m_u/m_c, $\sin^2 2\theta_{12}$, $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32}$, m_d/m_s, m_s/m_b, $|V_{us}|$, and $|V_{cb}|$ as shown later. The relative ratios of parameters (x_1, x_2, x_3) in Φ_0 are fixed by the ratios of the charged lepton masses m_e/m_μ and m_μ/m_τ under a_e given. The process of fixing parameters are summarized in Table. 2.

Now let us present the details of parameter fitting. Since the mass ratios of the up quarks and the lepton mixing parameter $\sin^2 2\theta_{12}$ depends only on a_e and (a_u, α_u), we first fix the following parameter values of a_e and (a_u, α_u)

$$(a_e, a_u, \alpha_u) \sim (8.0, -1.273, -1.4^\circ), \tag{5.8}$$

which are fixed from the observed values of m_c/m_t, m_u/m_c, and $\sin^2 2\theta_{12}$:

$$r_{12}^u \equiv \sqrt{\frac{m_u}{m_c}} = 0.045^{+0.013}_{-0.010}, \qquad r_{23}^u \equiv \sqrt{\frac{m_c}{m_t}} = 0.060 \pm 0.005, \tag{5.9}$$

Table 2: Process for fitting parameters.

Step	Inputs	N_{inp}	Parameters	N_{par}	Predictions
1st	$\frac{m_u}{m_c}$, $\frac{m_c}{m_t}$ $\sin^2 2\theta_{12}$	3	a_e a_u, α_u	3	
2nd	R_ν	1	a_D	1	$\sin^2 2\theta_{13}$, $\sin^2 2\theta_{23}$, δ_{CP}^ℓ 2 Majorana phases, $\frac{m_{\nu 1}}{m_{\nu 2}}$, $\frac{m_{\nu 2}}{m_{\nu 3}}$
3rd	$\frac{m_s}{m_b}$, $\frac{m_d}{m_s}$, $\lvert V_{us}\rvert$,	3	a_d, α_d, ξ_0^d	3	
4th	$\lvert V_{cb}\rvert$	1	ϕ_2	1	$\lvert V_{ub}\rvert$, $\lvert V_{td}\rvert$, δ_{CP}^q
option	Δm_{32}^2		$m_{\nu 3}$		$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$, $\langle m\rangle$
$\sum N_{...}$		8		8	

at $\mu = m_Z$ [11], and $\sin^2 2\theta_{12} = 0.857 \pm 0.024$ [12]. (These values will be fine-tuned in whole parameter fitting of U_{PMNS} and V_{CKM} later.) Note that we do not change the mass matrix structures for M_e, M_u, and M_ν from the previous paper [4]. However M_d is different from the previous model, so that we refit these parameters in order to reproduce the observed CKM mixing parameters too as seen later.

Secondly, let us investigate lepton sector. In the present model, lepton mixing parameters depend only the parameter a_D after we fix a_e and (a_u, α_u) as (5.8). We illustrate the behaviors of lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared difference ratio R_ν versus the parameter a_D. We draw curves of the lepton mixing parameters and R_ν as functions of a_D in Fig. 1. As seen in Fig.1, the predictions of $\sin^2 2\theta_{12}$ and R_ν are sensitive to the parameter a_D, while the prediction of $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ are insensitive to a_D. Using Fig. 1, we do fine tuning of the parameter a_D as

$$a_D = 9.32, \tag{5.10}$$

in order to fit the observed values [12] given by

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024, \tag{5.11}$$

$$R_\nu \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{m_{\nu 2}^2 - m_{\nu 1}^2}{m_{\nu 3}^2 - m_{\nu 2}^2} = \frac{(7.50 \pm 0.20) \times 10^{-5}\ \mathrm{eV}^2}{(2.32^{+0.12}_{-0.08}) \times 10^{-3}\ \mathrm{eV}^2} = (3.23^{+0.14}_{-0.19}) \times 10^{-2}, \tag{5.12}$$

$$\sin^2 2\theta_{23} > 0.95, \tag{5.13}$$

and

$$\sin^2 2\theta_{13} = 0.095 \pm 0.010. \tag{5.14}$$

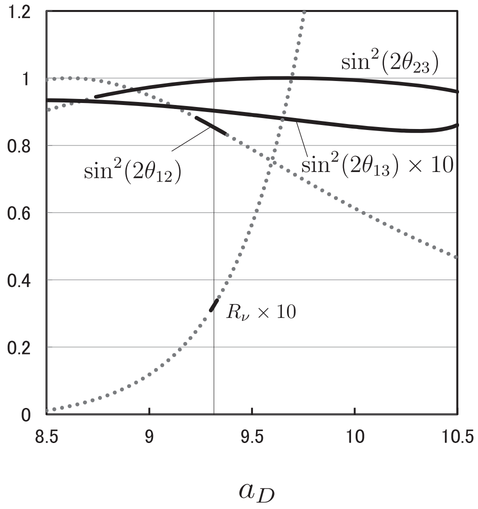

Figure 1: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio R_ν versus the parameter a_D. We draw curves of the lepton mixing parameters as functions of a_D, with taking $a_e = 8.0$, $a_u = -1.273$, and $\alpha_u = -1.4°$. The solid and dotted parts of the curves are, respectively, within and out of the observed ranges given by (3.10)-(3.12). We find that the parameter a_D around $a_D = 9.32$ is consistent with all the observed values [12].

Next, we discuss quark sector. Since we have fixed the four parameters a_e, a_u, α_u, and a_D, we have remaining four parameters a_d, α_d, ξ_0^d, and ϕ_2 for eight observables (2 down-quark mass ratios and 4+2 CKM mixing parameters). The following parameters a_d, α_d, and ξ_0^d

$$a_d = -1.338, \quad \alpha_d = -14.3°, \quad \xi_0^d = 0.0147 \tag{5.15}$$

are fixed to fit the observed down-quark mass ratios at $\mu = m_Z$ [11]

$$r_{23}^d \equiv \frac{m_s}{m_b} = 0.019^{+0.006}_{-0.006}, \quad r_{12}^d \equiv \frac{m_d}{m_s} = 0.053^{+0.005}_{-0.003}, \tag{5.16}$$

and the observed CKM mixing matrix element [12]

$$|V_{us}| = 0.2252 \pm 0.0009. \tag{5.17}$$

Therefore, all the CKM mixing parameters are described only by one parameter ϕ_2. We draw curves of the CKM mixing matrix elements as functions of ϕ_2 in Fig. 2.

As shown in Fig. 2, all the experimental constraints on CKMs are satisfied by fine tuning the parameter ϕ_2 around

$$\phi_2 = 26.5°. \tag{5.18}$$

Here we use the values for the other observed CKM mixing matrix elements [12] given by

$$|V_{cb}| = 0.0409 \pm 0.0011, \quad |V_{ub}| = 0.00415 \pm 0.00049, \quad |V_{td}| = 0.0084 \pm 0.0006. \tag{5.19}$$

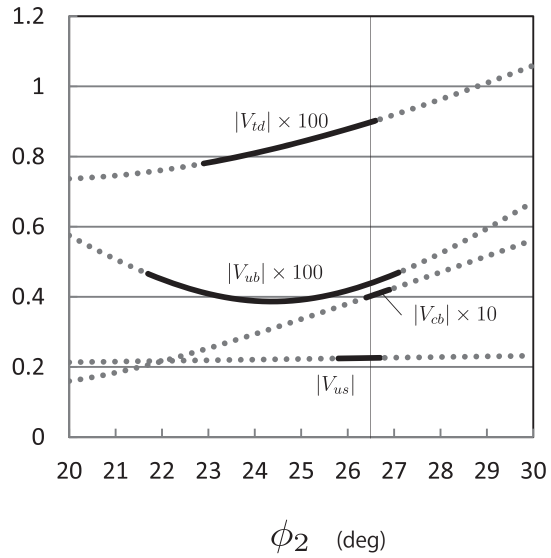

Figure 2: CKM mixing matrix elements $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$, and $|V_{td}|$ versus the parameter ϕ_2. We draw curves of the CKM mixing matrix elements as functions of ϕ_2, with taking $a_e = 8.0$, $a_u = -1.273$, $\alpha_u = -1.4°$, $a_d = -1.338$, $\alpha_d = -14.3°$, and $\xi_d = 0.0147$. The solid and dotted parts of the curves are, respectively, within and out of the observed ranges given by (5.17) and (5.19). We find that the parameter ϕ_2 around $\phi_2 = 26.5°$. is consistent with all the observed values [12].

Finally, we do fine-tuning of whole parameter values in order to give more improved fitting with the whole data. Our final result is as follows: under the parameter values

$$a_e = 8.0,\ (a_u, \alpha_u) = (-1.273, -1.4°),\ (a_d, \alpha_d) = (-1.338, -14.3°),\ \xi_0^d = 0.0147,$$

$$a_D = 9.32,\ \phi_2 = 26.3°, \tag{5.20}$$

we obtain

$$r_{12}^u = 0.0358, \quad r_{23}^u = 0.0599, \quad r_{12}^d = 0.0547, \quad r_{23}^d = 0.0129, \tag{5.21}$$

$$\sin^2 2\theta_{23} = 0.993, \quad \sin^2 2\theta_{12} = 0.852, \quad \sin^2 2\theta_{13} = 0.0903, \quad R_\nu = 0.0329, \tag{5.22}$$

$$\delta_{CP}^\ell = 179° \quad (J^\ell = 6.3 \times 10^{-4}), \tag{5.23}$$

$$|V_{us}| = 0.2256, \quad |V_{cb}| = 0.0402, \quad |V_{ub}| = 0.00439, \quad |V_{td}| = 0.00898, \tag{5.24}$$

$$\delta_{CP}^q = 75.1° \quad (J^q = 3.7 \times 10^{-5}). \tag{5.25}$$

Here, δ_{CP}^ℓ and δ_{CP}^q are Dirac CP violating phases in the standard conventions of U_{PMNS} and V_{CKM}, respectively.

It should be noted that our prediction $\sin^2 2\theta_{13} = 0.0903$ is well consistent with the observed value in (5.14). Also note that our prediction $\sin^2 2\theta_{23} = 0.993$ is roughly consistent with recently observed values $\sin^2 2\theta_{23} = 0.950^{+0.035}_{-0.036}$ and $\sin^2 2\bar{\theta}_{23} = 0.97^{+0.03}_{-0.08}$ by MINOS [13]. Our model predicts $\delta^\ell_{CP} = 179^\circ$ which indicates small CP violating effect in the lepton sector. Note that a recent global analysis [14] has suggested the best fit value for δ^ℓ_{CP} as $\delta^\ell_{CP} = 1.1\pi$.

We can also predict neutrino masses, for the parameters given (5.20) ,

$$m_{\nu 1} \simeq 0.0011 \text{ eV}, \quad m_{\nu 2} \simeq 0.0090 \text{ eV}, \quad m_{\nu 3} \simeq 0.0499 \text{ eV}, \tag{5.26}$$

by using the input value [13] $\Delta m^2_{32} \simeq 0.00241$ eV^2. We also predict the effective Majorana neutrino mass [15] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = \left| m_{\nu 1} (U_{e1})^2 + m_{\nu 2} (U_{e2})^2 + m_{\nu 3} (U_{e3})^2 \right| \simeq 4.6 \times 10^{-3} \text{ eV}. \tag{5.27}$$

6 Concluding remarks

In conclusion, we have made a revision of formulation in the yukawaon model as follows: (i) We have assigned the yukawaons Y_f to $\mathbf{8}+\mathbf{1}$ of U(3) family symmetry (not previous $\mathbf{6}^*$), so that the model can become anomaly free in U(3), i.e. $\sum A(\text{lepton}) = 0$, $\sum A(\text{quark}) = 0$ and $\sum A(\text{flavon}) = 0$. (ii) Mass matrices, not only M_u but also M_d, are given with a bilinear form $[\Phi_0(\mathbf{1} + a_f X_3)\Phi_0]^2$, so that the parameter fitting has renewed thoroughly. (iii) The neutrino mass matrix M_ν has been given by a triplicate seesaw. (iv) In the present paper, we have used only U(3) family, to which quarks and leptons belong, although we have discussed the yukawaon model based on U(3)×U(3)$'$ in the previous model [5, 6]. Therefore, in this paper, we did not refer the origin of the VEV matrix form X_2, (1.5). This problem is our future task.

As a result of this new parameter fitting, we have obtained the following phenomenological results: (i) We can still obtain reasonable mass rations and quark and lepton mixings, in spite of reduced number of the free parameters compared with the previous yukawaon model. (ii) For the CP violation parameter in the lepton sector, δ^ℓ_{CP}, we have found $\delta^\ell_{CP} \simeq \pi$, so that the CP violation in the lepton sector is very small. (iii) We have obtained an almost maximal mixing $\sin^2 2\theta_{23} = 0.99$ in spite of obtaining a sizable value of $\sin^2 2\theta_{13} = 0.09$. We pay attention to the recent observed value by MINOS [13], $\sin^2 2\theta_{23} = 0.950^{+0.035}_{-0.036}$. Our predicted value exists on the upper value with one σ. We interest in that the error is reduced in the near future.

It seems that phenomenological success in the present work encourages our ambitious idea that the observed hierarchical structure in the family mixings and mass ratios of quarks and leptons are caused only by one common origin, i.e. they can be understood by accepting the observed charged lepton mass matrix. However, in the present model, too, we have still the family-dependent parameter ϕ_2 and the unwelcome VEV matrix form X_2. We will need further investigation in order to realize our goal.

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