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Neutrino Mass Matrix Model with a Bilinear Form

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Abstract

A neutrino mass matrix model with a bilinear form $M_\nu = k_\nu (M_D M_R^{-1} M_D^T)^2$ is proposed within the framework of the so-called yukawaon model, which has been proposed for the purpose of a unified description of the lepton mixing matrix U_{PMNS} and the quark mixing matrix V_{CKM}. The model has only two adjustable parameters for the PMNS mixing and neutrino mass ratios. (Other parameters are fixed from the observed quark and charged lepton mass ratios and the CKM mixing.) The model gives reasonable values $\sin^2 2\theta_{12} \simeq 0.85$ and $\sin^2 2\theta_{23} \sim 1$ and $\sin^2 2\theta_{13} \sim 0.09$ together with $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32} \sim 0.03$. Our prediction of the effective neutrino mass $\langle m \rangle$ in the neutrinoless double beta decay takes a sizable value $\langle m \rangle \simeq 0.0034$ eV.

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1 Introduction

Most particle physicists think that the mass spectra and mixing patterns of quarks and leptons, the Cabibbo-Kobayashi-Maskawa mixing matrix V_{CKM} [1] and the Pontecorvo-Maki-Nakagawa-Sakata mixing matrix U_{PMNS} [2], will be understood by a unified description. As one of such models, "yukawaon" model [3, 4, 5, 6] has been proposed. The model, which is a kind of "flavon" model [7], has the following characteristics: (i) Yukawa coupling constants Y_f ($f = u, d, e, \cdots$) in the standard model are understood as vacuum expectation values (VEVs) of scalars ("yukawaon") with 3×3 components, i.e. by $y_f \langle Y_f \rangle / \Lambda$, where Λ is an energy scale of the effective theory. (ii) The hierarchical structures of the effective Yukawa coupling constants originate only in a fundamental VEV matrix $\langle \Phi_0 \rangle$, whose hierarchical structure is ad hoc assumed and whose VEVs are fixed by the observed charged lepton masses. (Of course, the goal in the yukawaon model is to understand the VEVs of the fundamental scalar Φ_0 itself on the basis of a dynamical consideration.) In the yukawaon model, in principle, there are no family-number-dependent parameters except for $\langle \Phi_0 \rangle$. (Regrettably, in order to obtain reasonable values of quark mixing matrix V_{CKM}, at present, we need a phase matrix P_u (or P_d) with two phase parameters [5, 6]. The final goal of our model is also to remove such family dependent parameters.) (iii) Relations among VEV matrices are obtained from SUSY vacuum conditions for a given superpotential under family symmetries and R charges. (It is a characteristic in the yukawaon model that adjustable parameters are quite few because the observed charged lepton mass values are used as the input values.)

In a series of previous yukawaon models [4, 5, 6], quark and lepton mass matrices were given as follows:

$$\begin{aligned} &M_e = k_e \Phi_0 (\mathbf{1} + a_e X_3) \Phi_0, \\ &M_\nu = M_D M_R^{-1} M_D^T, \\ &M_D = M_e, \\ &M_R = k_R (\hat{M}_u M_e + M_e \hat{M}_u) + \xi_\nu \text{ term}, \\ &P_u M_u P_u = k_u \hat{M}_u \hat{M}_u, \\ &\hat{M}_u = k'_u \Phi_0 (\mathbf{1} + a_u X_3) \Phi_0, \\ &M_d = k_d \left[\Phi_0 (\mathbf{1} + a_d X_3) \Phi_0 + m_d^0 \mathbf{1} \right], \end{aligned} \tag{1.1}$$

where M_u, M_d, M_ν, and M_e are, respectively, mass matrices of up, down quarks, light neutrinos, and charged leptons. M_D and M_R are the Dirac and right handed Majorana neutrino mass matrices, respectively. Here, we introduced a common mass matrix form

$$M_f = k_f \Phi_0 (\mathbf{1} + a_f X_3) \Phi_0. \tag{1.2}$$

(For convenience, we have dropped the notations "$\langle$" and "$\rangle$" on the VEV matrix $\langle \Phi_0 \rangle$). X_3, $\mathbf{1}$ and P_u are also VEV matrices of other scalar fields. The matrices Φ_0, X_3, $\mathbf{1}$ and P_u are defined by

$$\Phi_0 = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.3}$$

and $P_u = \text{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$, respectively. The form $(\mathbf{1} + a_e X_3)$ is due to a family symmetry breaking U(3)$\to$ S$_3$ [6] as we discuss later. The coefficients a_f play an essential role in obtaining the mass ratios and mixings, while the family-number independent coefficients k_f and k'_u do not. The values of (x_1, x_2, x_3) with $x_1^2 + x_2^2 + x_3^2 = 1$ are fixed by the observed charged lepton mass values under the given value of a_e. In order to get reasonable fits for lepton mixing angles $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{12}$ keeping reasonable fits for up-quark mass ratios, an additional term, "ξ_ν-term", was added in the mass matrix M_R. In this model, only the up-quark mass matrix M_u is given by a bilinear form of $P_u M_u P_u = k_u \hat{M}_u \hat{M}_u$. Such a form was suggested by a phenomenological fact $M_u^{diag} \sim (M_d^{diag})^2$.

In this paper, we shall present the new mass matrix model. The different points of the new model from the previous ones are as follows: (i) there is no ξ_ν-term and (ii) Considering the correspondence between quark and lepton mass matrices ($M_e \leftrightarrow M_d$; $M_\nu \leftrightarrow M_u$), we try to consider a form

$$M_\nu = k_\nu \hat{M}_\nu \hat{M}_\nu, \tag{1.4}$$

corresponding to the up-quark mass matrix form $M_u = k_u \hat{M}_u \hat{M}_u$ because we assumed the form $M_d \sim M_e = k_e \Phi_0 (\mathbf{1} + a_e X_3) \Phi_0$. Here $\hat{M}_\nu$ is given by

$$\hat{M}_\nu = M_D M_R^{-1} M_D^T, \tag{1.5}$$

and we take

$$M_D = k_D \Phi_0^T (\mathbf{1} + a_D X_2) \Phi_0, \tag{1.6}$$

$$M_R = k_R (\hat{M}_u M_e + M_e \hat{M}_u). \tag{1.7}$$

Note that the form of M_D is given by (1.6) against other mass matrix forms (1.2). The matrix form X_2 [8] is defined by

$$X_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{1.8}$$

By this revision (1.4) with rather simple structures of yukawaon VEV structures, we can obtain reasonable prediction of $\sin^2 2\theta_{13} \sim 0.09$ together with other PMNS and CKM parameters and quark and lepton mass ratios. (The big drawback in the previous yukawaon model (1.1) was that the model could not give the observed large value [9] $\sin^2 2\theta_{13} \sim 0.09$.)

In the next section, we give a new model for quark and lepton mixings and their mass ratios on the basis of a revised yukawaon model. In Sec.3, we discuss parameter fitting of observed values only for the PMNS mixing and neutrino mass ratios because we revised the model only in the neutrino sector. The parameter values in the down-quark sector are effectively unchanged, so that we can obtain the same predictions for the down-quark mass ratios and CKM matrix parameters without changing the successful results in the previous paper [8].

2 VEV matrix relations

We assume that a would-be Yukawa interaction is given as follows:

$$W_Y = \frac{y_e}{\Lambda} e_i^c \bar{Y}_e^{ij} \ell_j H_d + \frac{y_\nu}{\Lambda^2} (\ell_i H_u) \bar{Y}_\nu^{ij} (\ell_j H_u) + \frac{y_d}{\Lambda} d^{ci} Y_{ij}^d q^j H_d + \frac{y_u}{\Lambda} u^{ci} Y_{ij}^u q^j H_u, \tag{2.1}$$

where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are SU(2)$_L$ doublets. Assignments of these fields to family symmetries U(3)$\times$U(3)$'$ are given in Table 1. Note that in Eq.(2.1) there are no SU(2)$_L$ singlet neutrinos. We have straightforwardly defined the neutrino mass matrix M_ν by the second term in Eq.(2.1). Although we denoted in Eq.(1.5) as if the matrix M_D is a Dirac neutrino mass matrix, the matrix M_D does not have a meaning of the Dirac mass matrix [see Eq.(2.3) later]. Under the definition of $\bar{Y}_\ell$ (Y^q) in Eq.(2.1), the quark mixing matrix V_{CKM} and the lepton mixing mixing matrix U_{PMNS} are given by $V_{CKM} = U_u^\dagger U_d$ and $U_{PMNS} = U_e^\dagger U_\nu$, respectively, where U_f are defined by $U_f^\dagger M_f^\dagger M_f U_f = D_f^2$ (D_f are diagonal). (Here and hereafter, sometimes, we denote $\bar{Y}_\ell$ and Y^q as Y_f for simplify. In order to distinguish each yukawaon from others, we assume that Y_f have different R charges from each other together with R charge conservation. (Of course, the R charge conservation is broken at the energy scale Λ.)

Table 1: Assignments of SU(2)$_L\times$SU(3)$_c\times$U(3)$\times$U(3)$'$ and R charges

	ℓ	e^c	q	u^c	d^c	H_u	H_d	$\bar{Y}_e$	$\bar{Y}_\nu$	Y^d	Y^u
SU(2)$_L$	**2**	**1**	**2**	**1**	**1**	**2**	**2**	**1**	**1**	**1**	**1**
SU(3)$_c$	**1**	**1**	**3**	**3***	**3***	**1**	**1**	**1**	**1**	**1**	**1**
U(3)	**3**	**3**	**3***	**3***	**3***	**1**	**1**	**6***	**6***	**6**	**6**
U(3)$'$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
R	r_ℓ	r_{ec}	r_q	r_{uc}	r_{dc}	r_{Hu}	r_{Hd}	$\bar{r}_{Ye}$	$\bar{r}_{Y\nu}$	r_{Yd}	r_{Yu}

We obtain VEV matrix relations from the superpotential which is invariant under the family symmetries U(3)$\times$U(3)$'$ and R charge conservation. In the yukawaon model, the VEV matrix relations are phenomenological ones, and they are dependent on the R charge assignments. Since derivations of the VEV matrix relations are essentially similar to those in the previous papers [3, 4, 5, 6, 8] although the assignments of U(3)$\times$U(3)$'$and R charges are different. Besides, in order to derive the desirable mass matrix relations, we must to consider a complicated superpotential form. The purpose of the present paper is not to derive those mass matrix relations uniquely, but it is to investigate the neutrino mass matrix $M_\nu = k_\nu \hat{M}_\nu \hat{M}_\nu$ from the phenomenological point of view. Therefore, the derivation of the mass matrix relations are discussed in Appendix.

In the present section, we present only the results:

$$\langle \bar{Y}_e \rangle = k_e \langle \bar{\Phi}_0 \rangle \left(\mathbf{1} + a_e X X^T \right) \langle \bar{\Phi}_0^T \rangle, \tag{2.2}$$

$$\langle \bar{Y}_\nu \rangle = k_\nu \langle \hat{Y}^\nu \rangle \langle \hat{Y}^\nu \rangle, \tag{2.3}$$

$$\langle \hat{Y}^\nu \rangle = k'_\nu \langle \bar{Y}_D \rangle \langle \hat{Y}_R \rangle \langle \bar{Y}_D \rangle = k''_\nu \langle \bar{Y}_D \rangle (\langle \bar{Y}_R \rangle)^{-1} \langle \bar{Y}_D \rangle, \tag{2.4}$$

$$\langle \bar{Y}_D \rangle = k_D \langle \bar{\Phi}_0^T \rangle \left(\mathbf{1} + a_D X^T X \right) \langle \bar{\Phi}_0 \rangle, \tag{2.5}$$

$$\langle \bar{Y}_R \rangle = k_R \left(\langle \bar{Y}_e \rangle \langle \hat{Y}^u \rangle + \langle \hat{Y}^u \rangle \langle \bar{Y}_e \rangle \right), \tag{2.6}$$

$$\langle Y^u \rangle = k_u \langle \hat{Y}^u \rangle \langle \hat{Y}^u \rangle, \tag{2.7}$$

$$\langle \hat{Y}^u \rangle = k'_u \langle \bar{\Phi}_0 \rangle \left(\mathbf{1} + a_u X X^T \right) \langle \bar{\Phi}_0^T \rangle, \tag{2.8}$$

$$\langle \bar{P}_d \rangle \langle Y^d \rangle \langle \bar{P}_d \rangle = k_d \langle \bar{\Phi}_0 \rangle \left(\mathbf{1} + a_d X X^T \right) \langle \bar{\Phi}_0^T \rangle + \xi_0^d \mathbf{1}, \tag{2.9}$$

where X has phenomenologically been introduced in the previous model [8] and it has the form

$$\frac{1}{v_X}\langle X\rangle_{\alpha i} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}_{\alpha i} . \tag{2.10}$$

The form (2.10) leads to

$$\left(\langle X\rangle\langle X^T\rangle\right)_{\alpha\beta} = \frac{3}{2}(X_3)_{\alpha\beta}, \quad \left(\langle X^T\rangle\langle X\rangle\right)_{ij} = \frac{3}{2}(X_2)_{ij}, \tag{2.11}$$

together with $\langle X\rangle\langle X\rangle = \langle X\rangle$, where X_3 and X_2 is defined by Eqs. (1.3) and (1.8), respectively, and, for simplicity, we have put $v_X = 1$ because we are interested only in the relative ratios among the family components. At present, there is no idea for the origin of this form (2.10). We may speculate that this form is related to a breaking pattern of U(3)×U(3)′ (for example, discrete symmetries U(3)×U(3)′ →S_2×S_3). In the present paper, the form (2.11) is only ad hoc assumption. However, as seen later, we can obtain a good fitting for the neutrino mixing angle $\sin^2 2\theta_{13}$ due to this assumption.

3 Parameter fitting

We again summarize our mass matrix model as follows:

$$\bar{Y}_e = k_e \bar{\Phi}_0 (\mathbf{1} + a_e X_3)\bar{\Phi}_0^T, \tag{3.1}$$

$$\bar{Y}_\nu = k_\nu \hat{Y}_\nu \hat{Y}_\nu, \tag{3.2}$$

$$\hat{Y}_\nu = k''_\nu \bar{Y}_D \bar{Y}_R^{-1} \bar{Y}_D, \tag{3.3}$$

$$\bar{Y}_D = k_D \bar{\Phi}_0^T (\mathbf{1} + a_D e^{i\alpha_D} X_2)\bar{\Phi}_0, \tag{3.4}$$

$$\bar{Y}_R = k_R \left(\bar{Y}_e \hat{Y}^u + \hat{Y}^u \bar{Y}_e\right), \tag{3.5}$$

$$Y^u = k_u \hat{Y}^u \hat{Y}^u, \tag{3.6}$$

$$\hat{Y}^u = k'_u \bar{\Phi}_0 \left(\mathbf{1} + a_u e^{i\alpha_u} X_3\right) \bar{\Phi}_0^T, \tag{3.7}$$

$$\bar{P}_d Y^d \bar{P}_d = k_d[\bar{\Phi}_0(\mathbf{1} + a_d X_3)\bar{\Phi}_0^T + \xi_0^d \mathbf{1}], \tag{3.8}$$

where, for convenience, we have dropped the notations "⟨" and "⟩". In numerical calculations, we use dimensionless expressions $\bar{\Phi}_0 = \mathrm{diag}(x_1, x_2, x_3)$ (with $x_1^2 + x_2^2 + x_3^2 = 1$), $\bar{P}_d = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$, and $E = \mathrm{diag}(1, 1, 1)$. The parameters are re-refined by Eqs.(3.1)-(3.8). In Eqs.(3.4) and (3.7), since we assume that the parameters a_e and a_d are real, while a_u and a_D are complex in our $M_D \leftrightarrow M_u$ and $M_e \leftrightarrow M_d$ correspondence scheme, we have denoted a_u and a_D as $a_u e^{i\alpha_u}$ and $a_D e^{i\alpha_D}$, respectively.

Table 2: Process for fitting parameters. Of course, since these parameters listed in each step can slightly affect predicted values listed in the other steps, we need fine tuning after the step 5th. New parameter fitting in the present paper is started from the 5th step.

Step	Inputs	N_{inp}	Parameters	N_{par}	Predictions
1st	$\frac{m_e}{m_\mu}$, $\frac{m_\mu}{m_\tau}$ $\frac{m_u}{m_c}$, $\frac{m_c}{m_t}$	4	$\frac{x_1}{x_2}$, $\frac{x_2}{x_3}$ a_e, a_u	4	
2nd	$\|V_{us}\|$, $\|V_{cb}\|$, $\|V_{ub}\|$	3	α_u, (ϕ_1, ϕ_2)	3	$\|V_{td}\|$, δ^q_{CP}
3rd	$\frac{m_s}{m_b}$	1	a_d	1	
4th	$\frac{m_d}{m_s}$	1	m^0_d	1	not affect to other predictions
5th	$\sin^2 2\theta_{12}$ R_ν	2	a_D α_D	2	$\sin^2 2\theta_{13}$, δ^ℓ_{CP}, 2 Majorana phases $\sin^2 2\theta_{23}$, $\frac{m_{\nu 1}}{m_{\nu 2}}$, $\frac{m_{\nu 2}}{m_{\nu 3}}$
option	Δm^2_{atm}		$m_{\nu 3}$		$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$, $\langle m \rangle$
$\sum N_{\dots}$		11		11	

In this model, we have 2 parameters (a_D, α_D) for neutrino sector, 4 parameters a_D, ξ^d_0 and (ϕ_1, ϕ_2) for down-quark mass ratios and V_{CKM}, and 3 parameters a_e, (a_u, α_u) for charged lepton mass ratios and up-quark mass ratios as shown in Table 2. Especially, it is worthwhile noticing that the neutrino mass ratios and U_{PMNS} are described only two parameters after a_e and (a_u, α_u) have been fixed from the observed CKM mixing and up-quark mass ratios. There is effectively no change in the mass matrix structures except for Y_ν from the previous paper [8], so that we can use, even in the present model too, the same parameter values for a_e and (a_u, α_u) taken in the previous study, which are given by

$$a_e = 7.5, \quad (a_u, \alpha_u) = (-1.35, 7.6^\circ), \tag{3.9}$$

Therefore, as far as PMNS mixing and neutrino mass ratios are concerned, we have only two free parameters (a_D, α_D) in the present neutrino mass matrix model.

At present, the observed values [10] are as follows:

$$\sin^2 2\theta^{obs}_{12} = 0.857 \pm 0.024, \quad \sin^2 2\theta^{obs}_{23} > 0.95, \quad \sin^2 2\theta^{obs}_{13} = 0.098 \pm 0.013, \tag{3.10}$$

$$R^{obs}_\nu \equiv \frac{(\Delta m^2_{21})^{obs}}{(\Delta m^2_{32})^{obs}} = \frac{(7.50 \pm 0.20) \times 10^{-5}\ \text{eV}^2}{(2.32^{+0.12}_{-0.08}) \times 10^{-3}\ \text{eV}^2} = (3.23^{+0.14}_{-0.19}) \times 10^{-2}. \tag{3.11}$$

Since the parameters (a_D, α_D) are sensitive to the observables $\sin^2 2\theta^{obs}_{12}$ and R^{obs}_ν, we use the observed values of $\sin^2 2\theta_{12}$ and R_ν in order to fix our parameter values (a_D, α_D). In Fig.1, we illustrate an allowed parameter region of (a_D, α_D) obtained from the observed values [10]

$\sin^2 2\theta_{12}^{obs} = 0.875 \pm 0.024$ and $R_\nu^{obs} \times 10 = 0.323^{+0.014}_{-0.019}$. As seen in Fig.1, the observed values uniquely fix the parameter values (a_D, α_D) as

$$(a_D, \alpha_D) = (8.7, 12^\circ). \tag{3.12}$$

It is worthwhile noticing that the parameter values (3.12) uniquely give a prediction of $\sin^2 2\theta_{13} \simeq 0.09$.

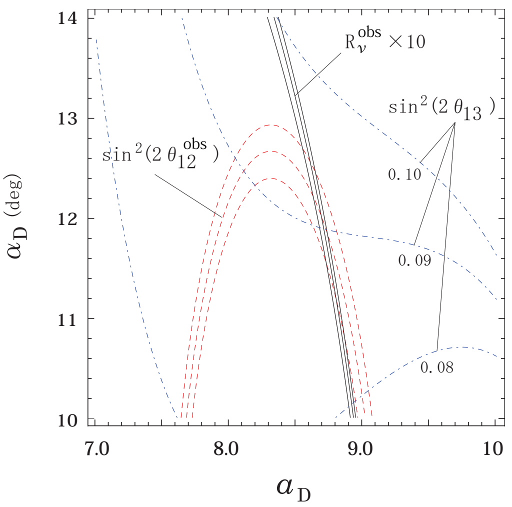

Figure 1: Allowed parameter region in (a_D, α_D) plane. The solid and dashed curves indicate the border and center curves of the allowed region which are obtained from the observe values $\sin^2 2\theta_{12}^{obs} = 0.875 \pm 0.024$ and $R_\nu^{obs} \times 10 = 0.324^{+0.014}_{-0.019}$, respectively. The dot-dashed curves represent contour curves of $\sin^2 2\theta_{13} = 0.08$, 0.09, and 0.10.

For reference, in Fig.2, we illustrate behaviors of $\sin^2 2\theta_{12}$ and R_ν versus α_D for the case of $a_D = 8.7$. The choice $\alpha_D = 12^\circ$ gives excellent fittings to the observed values of $\sin^2 2\theta_{12}$ and R_ν simultaneously:

$$\sin^2 2\theta_{12} = 0.8544, \quad R_\nu = 0.0331. \tag{3.13}$$

Then, we obtain our predictions for $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ using (3.12) as follows:

$$\sin^2 2\theta_{23} = 0.9962, \quad \sin^2 2\theta_{13} = 0.0907, \tag{3.14}$$

which are in excellent agreement with the observed values in given in Eq.(3.10).

The fixing of the parameters (a_D, α_D), Eq.(3.12), leads to the prediction of the CP violating phase parameter in the lepton sector too:

$$\delta_{CP}^\ell = 127^\circ \quad (J^\ell = 2.74 \times 10^{-2}). \tag{3.15}$$

We can also predict neutrino masses:

$$m_{\nu 1} = 0.00061 \text{ eV}, \quad m_{\nu 2} = 0.00899 \text{ eV}, \quad m_{\nu 3} = 0.05011 \text{ eV}, \tag{3.16}$$

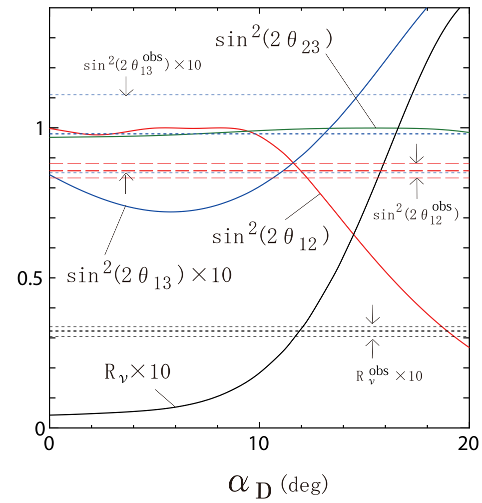

Figure 2: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio R_ν versus the phase parameter α_D for $a_D = 8.7$. The horizontal lines denote observed values $\sin^2 2\theta_{12}^{obs} = 0.875 \pm 0.024$, $\sin^2 2\theta_{13}^{obs} \times 10 = 0.98 \pm 0.13$ and $R_\nu^{obs} \times 10 = 0.324^{+0.014}_{-0.019}$. Our predicted value for $\sin^2 2\theta_{23}$ is well satisfied the obtained experimental bound $\sin^2 2\theta_{23}^{obs} > 0.95$.

by using the input value [11] $\Delta m_{32}^2 = 0.00243$ eV^2. We also predict the effective Majorana neutrino mass [12] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = \left| m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 \right| = 0.0034 \text{ eV}. \tag{3.17}$$

This value is a magnitude which may be observed by a future neutrinoless double beta decay experiment.

Finally, we list the predicted values of the CKM mixing parameters and down-quark mass ratios, although they are essentially the same as those in the previous model [8]:

$$|V_{us}| = 0.2271, \quad |V_{cb}| = 0.0394, \quad |V_{ub}| = 0.00347, \quad |V_{td}| = 0.00780, \tag{3.18}$$

$$\delta_{CP}^q = 59.6^\circ \quad (J^q = 2.6 \times 10^{-5}), \tag{3.19}$$

$$r_{12}^u = \sqrt{\frac{m_d}{m_s}} = 0.00465, \qquad r_{23}^u = \sqrt{\frac{m_d}{m_b}} = 0.0614. \tag{3.20}$$

$$r_{12}^d = \frac{m_d}{m_s} = 0.0569, \qquad r_{23}^d = \frac{m_d}{m_b} = 0.0240. \tag{3.21}$$

Here, we have used $a_d = 25$, $\xi_0^d = 0.0115$, and $(\phi_1, \phi_2) = (177.0^\circ, 197.4^\circ)$. The observed values are as follows: $|V_{us}| = 0.2252 \pm 0.0009$, $|V_{cb}| = 0.0409 \pm 0.0011$, $|V_{ub}| = 0.00415 \pm 0.00049$,

$|V_{td}| = 0.0084 \pm 0.0006$, $J = (2.96^{+0.20}_{-0.16}) \times 10^{-5}$ [10], and $r^u_{12} = 0.045^{+0.013}_{-0.010}$, $r^u_{23} = 0.060 \pm 0.005$, $r^d_{12} = 0.053^{+0.005}_{-0.003}$, $r^d_{23} = 0.019 \pm 0.006$ [13].

4 Concluding remarks

In conclusion, we have proposed a new neutrino mass matrix form within the framework of the yukawaon model, in which we have only two adjustable parameters, (a_D, α_D), for PMNS mixing and neutrino mass ratios. We obtain reasonable results for PMNS mixing and neutrino mass ratios as shown in Eqs.(3.13) - (3.17) for the parameter values $(a_D, \alpha_D) = (8.7, 12^\circ)$. As seen in Fig.2, it is worthwhile noticing that only when we choose a reasonable value of $R_\nu \simeq 0.033$, we obtain a reasonable value of $\sin^2 2\theta_{13} \simeq 0.09$. Also, we would like to emphasize that our prediction gives a sizable value of $\langle m \rangle \simeq 0.0034$ eV in spite of the normal mass hierarchy model (in spite of $m_{\nu 1} \simeq 0.0006$ eV, $m_{\nu 2} \simeq 0.00899$ eV and $m_{\nu 3} \simeq 0.05011$ eV).

Of course, we have also obtained reasonable results for CKM mixing and quark mass ratios as same as those in the previous paper [8].

The present model gives successful results from the phenomenological point of view. However, we still have some of theoretical problems. One of the major problems is why only Y_D takes the mass matrix form with X_2 (not X_3). In Ref.[6], the form X_3 has been understood by a symmetry breakdown U(3)×U(3)′ → U(3)×S$_3$. However, for the form X_2, the model is still in a phenomenological level. We have been able to remove the unnatural term [ξ_ν term in Eq.(1.2)], but we still have ξ^d_0 term in the Y_d sector. These problems are our future tasks.

Appendix

In this Appendix, we discuss a derivation of the mass matrix relations (2.2)-(2.9) from superpotential. We assume the following superpotential $W = W_e + W_\nu + W_R + W_D + W_u + W_d$:

$$W_e = \left\{ \mu_e \bar{Y}_e^{ij} + \frac{\lambda_e}{\Lambda} (\bar{\Phi}_0)^{i\alpha} \left(E''_{\alpha\beta} + \frac{a_e}{\Lambda^2} X_{\alpha k} \bar{E}^{kl} X^T_{l\beta} \right) (\bar{\Phi}_0^T)^{\beta j} \right\} \Theta^e_{ji}, \tag{A.1}$$

$$W_\nu = \frac{1}{\Lambda} \left[\lambda_\nu (E')^\alpha_k \bar{Y}_\nu^{kl} (E'^T)^\beta_l + \lambda'_\nu (\hat{Y}_\nu^T)^{\alpha\gamma} E''_{\gamma\delta} \hat{Y}_\nu^{\delta\beta} \right] \Theta^\nu_{\beta\alpha} + \left[\mu_\nu \hat{Y}_\nu^{\alpha\beta} + \frac{\hat{\lambda}_\nu}{\Lambda} \bar{Y}_D^{\alpha\gamma} \hat{Y}^R_{\gamma\delta} \bar{Y}_D^{\delta\beta} \right] \hat{\Theta}^\nu_{\beta\alpha}, \tag{A.2}$$

$$W_D = \left[\mu_D \bar{Y}_D^{\alpha\beta} + \frac{\lambda_D}{\Lambda} (\bar{\Phi}_0^T)^{\alpha k} \left(E_{kl} + \frac{\lambda'_D}{\Lambda^2} X^T_{k\beta} (\bar{E}'')^{\beta\gamma} X_{\gamma l} \right) \bar{\Phi}_0^{l\beta} \right] \Theta^D_{\beta\alpha}, \tag{A.3}$$

$$W_R = \left[\frac{\lambda_R}{\Lambda} \bar{Y}_R^{ik} (E')^\gamma_k \hat{Y}^R_{\gamma\alpha} + \mu_R (\bar{E}')^i_\alpha \right] (\hat{\Theta}_R)^\alpha_i + \left[\mu_R \bar{Y}_R^{ij} + \frac{\lambda'_R}{\Lambda} \left(\bar{Y}_e^{ik} \hat{Y}^u_{kl} \bar{E}^{lj} + \bar{E}^{ik} \hat{Y}^u_{kl} \bar{Y}_e^{lj} \right) \right] \Theta^R_{ji}, \tag{A.4}$$

$$W_u = \left(\mu_u Y^u_{ij} + \frac{\lambda_u}{\Lambda} \hat{Y}^u_{ik} \bar{E}^{kl} \hat{Y}^u_{lj} \right) \bar{\Theta}_u^{ji}$$

$$+\frac{1}{\Lambda}\left[\lambda'_u \bar{E}_u^{ik}\hat{Y}^u_{kl}\bar{E}_u^{lj} + \lambda''_u(\bar{\Phi}_0)^{i\alpha}\left((E''_u)_{\alpha\beta} + \frac{a_u}{\Lambda^2}X_{\alpha k}\bar{E}_u^{kl}X^T_{l\beta}\right)(\bar{\Phi}_0^T)^{\beta j}\right]\hat{\Theta}^u_{ji}, \qquad (A.5)$$

$$W_d = \left[\frac{\lambda_d}{\Lambda}\lambda_d \bar{P}_d^{ik}Y^d_{kl}\bar{P}_d^{lj} + \frac{\lambda'_d}{\Lambda}(\bar{\Phi}_0)^{i\alpha}\left((E''_d)_{\alpha\beta} + \frac{a_d}{\Lambda^2}X_{\alpha k}\bar{E}_d^{kl}X^T_{l\beta}\right)(\bar{\Phi}_0^T)^{\beta j} + \mu_d \bar{E}_d^{ij}\right]\Theta^d_{ji}. \qquad (A.6)$$

The VEV matrix relations (2.2) - (2.9) are obtained from SUSY vacuum conditions, $\partial W/\partial\Theta_A = 0$ ($A = e, \nu, \cdots$). Since we assume that all Θ fields take $\langle\Theta\rangle = 0$, SUSY vacuum conditions with respect to another fields do not lead to meaningful relations, because such conditions always contain, at least, one $\langle\Theta\rangle$.

In Eqs.(A.5) and (A.6), we have introduced fields E''_u, E''_d, $\bar{E}_u$ and $\bar{E}_d$ in addition to E'' and $\bar{E}$ in order to distinguish the R charges of $\hat{\Theta}^u$ and Θ^d from Θ^e. All VEV matrices $\langle E\rangle$ are given by the forms $\langle E\rangle \propto \mathbf{1}$ as seen in (A.10). The VEV matrix relations (2.2) - (2.9) have already been presented by replacing $\langle E\rangle \to \mathbf{1}$.

We list the assignments of SU$(2)_L\times$SU$(3)_c\times$U$(3)\times$U$(3)'$ and R charges for additional fields in Table 3. The assignments of R charges are done so that the total R charge of the superpotential term is $R(W) = 2$. We have 17 constraints on the R charges of the fields from Eqs.(2.1) and (A.1) - (A.6), while we have 34 fields even except for Θ fields in Tables 1 and 3. Therefore, we cannot uniquely fix R charge assignments of those fields. Here, let us give only typical constraints:

$$2r_X = r''_E - \bar{r}_E = r_E - \bar{r}''_E = r''_{Eu} - \bar{r}_{Eu} = r''_{Ed} - \bar{r}_{Ed}, \qquad (A.7)$$

$$2r_0 = \bar{r}_{Ye} - r''_E = \bar{r}_{YD} - r_E = \hat{r}_{Yu} + 2\bar{r}_{Eu} - r''_{Eu} = \hat{r}_{Yd} + 2\bar{r}_{Pd} - r''_{Ed}. \qquad (A.8)$$

From Eq.(A.7), we obtain $r''+\bar{r}''_E = r_E+\bar{r}_E$. When we take $R(E'')+R(\bar{E}'') = R(E)+R(\bar{E}) = R(P^d) + R(\bar{P}_d) = 1$, we can introduce the following superpotential:

$$W_{E,P} = \frac{\lambda_1}{\Lambda}\mathrm{Tr}[\bar{E}E\bar{P}_dP_d] + \frac{\lambda_2}{\Lambda}\mathrm{Tr}[\bar{E}E]\mathrm{Tr}[\bar{P}_dP_d], \qquad (A.9)$$

from which, we obtain relations $\langle E\rangle\langle\bar{E}\rangle \propto \mathbf{1}$ and $\langle P_d\rangle\langle\bar{P}_d\rangle \propto \mathbf{1}$. We assume following specific solutions of those relations:

$$\frac{1}{v_E}\langle E\rangle = \frac{1}{\bar{v}_E}\langle\bar{E}\rangle = \mathbf{1}, \qquad (A.10)$$

$$\frac{1}{v_P}\langle P_d\rangle^\dagger = \frac{1}{\bar{v}_P^*}\langle\bar{P}_d\rangle = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1), \qquad (A.11)$$

as the explicit forms of $\langle E\rangle$, $\langle\bar{E}\rangle$ and $\langle\bar{P}_d\rangle$. We assume similar superpotential forms for $(E,\bar{E})$, $(E_u,\bar{E}_u)$, $(E_d,\bar{E}_d)$, $(E'',\bar{E}'')$, $(E''^u,\bar{E}''_u)$, $(E''_d,\bar{E}''_d)$ and $(E',\bar{E}')$.

The term $\mu_d E_d$ in Eq.(A.6) has been introduced in order to adjust the down-quark mass ratio m_d/m_s as seen in Sec.3. Similar bias terms E and E_u cannot appear in the lepton and up-quark sectors, because we take $R(E) \neq R(E_d)$ and $R(E_u) \neq (E_d)$.

Table 3: Assignments of $\mathrm{SU}(2)_L \times \mathrm{SU}(3)_c \times \mathrm{U}(3) \times \mathrm{U}(3)'$ and R charges

	$\hat{Y}_\nu$	$\bar{Y}_D$	$\bar{Y}_R$	$\hat{Y}^R$	$\hat{Y}^u$	Θ^e	Θ^ν	$\hat{\Theta}^\nu$	Θ^D	Θ^R	$\hat{\Theta}_R$	$\bar{\Theta}_u$	$\hat{\Theta}^u$	Θ^d
$\mathrm{SU}(2)_L$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$\mathrm{SU}(3)_c$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
SU(3)	**1**	**1**	$\mathbf{6}^*$	**1**	**6**	**6**	**1**	**1**	**1**	**6**	**3**	$\mathbf{6}^*$	**6**	**6**
$\mathrm{U}(3)'$	$\mathbf{6}^*$	$\mathbf{6}^*$	**1**	**6**	**1**	**1**	**6**	**6**	**6**	**1**	$\mathbf{3}^*$	**1**	**1**	**1**
R	$\hat{r}_{Y\nu}$	$\bar{r}_{YD}$	$\bar{r}_{YR}$	$\hat{r}_{YR}$	$\hat{r}_{Yu}$	$r_{\Theta e}$	$r_{\Theta\nu}$	$\hat{r}_{\Theta\nu}$	$r_{\Theta D}$	$r_{\Theta R}$	$\hat{r}_{\Theta R}$	$\bar{r}_{\Theta u}$	$\hat{r}_{\Theta u}$	$r_{\Theta d}$

	Φ_0	X	E	$\bar{E}$	E'	$\bar{E}'$	E''	$\bar{E}''$
$\mathrm{SU}(2)_L$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$\mathrm{SU}(3)_c$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
SU(3)	$\mathbf{3}^*$	**3**	**6**	$\mathbf{6}^*$	**3**	$\mathbf{3}^*$	**1**	**1**
$\mathrm{U}(3)'$	$\mathbf{3}^*$	**3**	**1**	**1**	$\mathbf{3}^*$	**3**	**6**	$\mathbf{6}^*$
R	r_0	$\frac{1}{2}(r_E + r''_E - 1)$	r_E	$1 - r_E$	r'_E	$1 - r'_E$	r''_E	$1 - r''_E$

	E_u	$\bar{E}_u$	E_d	$\bar{E}_d$	E''_u	$\bar{E}''_u$	E''_d	$\bar{E}''_d$	P^d	$\bar{P}_d$
$\mathrm{SU}(2)_L$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$\mathrm{SU}(3)_c$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
SU(3)	**6**	$\mathbf{6}^*$	**6**	$\mathbf{6}^*$	**1**	**1**	**1**	**1**	**6**	$\mathbf{6}^*$
$\mathrm{U}(3)'$	**1**	**1**	**1**	**1**	**6**	$\mathbf{6}^*$	**6**	$\mathbf{6}^*$	**1**	**1**
R	r_{Eu}	$1 - r_{Eu}$	r_{Ed}	$1 - r_{Ed}$	r''_{Eu}	$1 - r''_{Eu}$	r''_{Ed}	$1 - r''_{Ed}$	r_{Pd}	$1 - r_{Pd}$

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