

Can Mass of the Lightest Family Gauge Boson be of the Order of TeV?

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Abstract

The observed sign of a deviation from the e - μ universality in tau decays suggests family gauge bosons with an inverted mass hierarchy. Under the constraints from the observed K^0 - \bar{K}^0 and D^0 - \bar{D}^0 mixing, we investigate a possibility that a mass M_{33} of the lightest gauge boson A_3^3 which couples with only the third generation quarks and leptons is of the order of TeV. It is concluded that $M_{33} \sim 1$ TeV is possible if we adopt a specific model phenomenologically.

1. Introduction

We know three generations of quarks and leptons. It seems to be natural to regard those as triplets of a family symmetry SU(3) [1] or U(3). However, so far, one has considered that, even if the family gauge symmetry exists, it is impossible to observe such gauge boson effects, because we know a severe constraint from the observed K^0 - \bar{K}^0 mixing [2] and results from Z' search [3] at the Tevatron. Nevertheless, it is interesting to consider a possibility that a family gauge symmetry really exists and the family gauge bosons are visible at a lower energy scale. If there are family gauge bosons, we will inevitably observe the deviations from the e - μ - τ universality, although whether they are visible or not depends on the breaking scale of the family gauge symmetry. At present, we know only sizable deviations from the e - μ - τ universality in tau decays and upilon decays although they are accompanied with large errors, so that they do not mean violations of the e - μ - τ universality statistically. Nevertheless, they give sufficient curiosity to investigate a possibility of family gauge bosons with a lower mass scale.

In this paper, we pay attention to deviations from the e - μ - τ universality in the tau decays and in the upilon decays. On the other hand, we will give a reconsideration of the constraints from the K^0 - \bar{K}^0 and D^0 - \bar{D}^0 mixings. Although, we will estimate a mass of the lightest family gauge boson from the observed deviations from the e - μ - τ universality in this paper, the value is nothing but a value for reference, because the experimental values have large errors at present. (One of the purposes is to call experimental physicist's attention to the observation of the deviations from the e - μ - τ universality, because they can give an important clue to a family gauge boson model, and the observations are just within our reach because the data have already shown visible deviations.) We will conclude that a mass of the lightest gauge boson A_3^3 can be $M_{33} \sim 1$ TeV, if we consider a family gauge boson model with a highly hierarchical mass spectrum.

The present work has been stimulated by the following observed data in the tau decays: From the present observed branching ratios [2] $Br(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) = (17.41 \pm 0.04)\%$ and

$Br(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = (17.83 \pm 0.04)\%$, we obtain the ratio $R_{Br} \equiv Br(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)/Br(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) = 0.97644 \pm 0.00314$. For convenience, we define parameters δ_μ and δ_e which are measures of a deviation from the e - μ universality as follows:

$$R_{amp} \equiv \frac{1 + \delta_\mu}{1 + \delta_e} = \sqrt{R_{Br} \frac{f(m_e/m_\tau)}{f(m_\mu/m_\tau)}} = 1.0020 \pm 0.0016, \quad (1)$$

where $f(x)$ is known as the phase space function and it is given by $f(x) = 1 - 8x^2 + 8x^6 - x^8 - 12x^4 \log x^2$. Then, the result (1) gives

$$\delta \equiv \delta_\mu - \delta_e = 0.0020 \pm 0.0016. \quad (2)$$

[The values of the deviation parameters δ_μ and δ_e depend on types of the gauge boson interactions, i.e. $(V - A)$, pure V , and so on. In Sec.3, we will discuss corrections for the parameters δ_μ and δ_e which have been defined by Eq.(1).] Of course, from the value (2), we cannot conclude that we found a significant difference of the deviation from the e - μ universality. However, we may speculate a possibility of family gauge bosons. We can consider that the deviation in the tau decays originates in exchange of gauge bosons A_3^2 and A_3^1 which interact as $\tau \rightarrow A_3^2 + \mu$ and $\tau \rightarrow A_3^1 + e$, respectively, as shown in Fig.1.

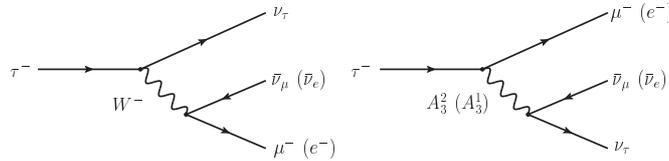


Figure 1: Deviation from e - μ universality in tau decays

Here, let us notice that the observed ratio defined by Eq.(1) shows $R_{amp} > 1$, i.e. $\delta_\mu > \delta_e$. Since the deviations are considered as $\delta_i \sim g_F^2/M_{3i}^2$ ($i = 1, 2$), this suggests that the mass of A_3^1 is larger than that of A_3^2 , i.e. $M_{31}^2 > M_{32}^2$, where $M_{ij} \equiv m(A_i^j)$. This suggests that the deviation (1) is caused by family gauge bosons with an inverted mass hierarchy. (If the gauge boson masses take a normal mass hierarchy, we will obtain $\delta_\tau \simeq 0$ because the gauge boson A_3^3 will take the highest mass.) If it is true, the phenomenological aspect for family gauge bosons will be changed drastically: (i) A family gauge boson with the highest mass is A_3^1 , so that it is in favor of a relaxation of the severe constraint from the observed K^0 - \bar{K}^0 mixing. (ii) The lightest family gauge boson A_3^3 interacts with only quarks and leptons of the third generation, so that the lightest gauge boson search has to be done by $X \rightarrow \tau^+\tau^-$, not by $X \rightarrow e^+e^-$. (The constraint from $Z' \rightarrow \tau^+\tau^-$ search at the Tevatron [4] cannot be apply to this A_3^3 search, because the production rate of A_3^3 is much smaller than that of the conventional Z' boson.) (iii) A large deviation from the μ - τ universality may be seen in the upsilon decays. We consider that it is important to investigate such a possibility phenomenologically.

Such a family gauge symmetry model with an inverted mass hierarchy has recently been proposed by Yamashita and the author [5]. In the present paper, we investigate the possibility

on the basis of this model, because mass ratios M_{ij}/M_{33} and gauge coupling constant g_F are fixed in the model (we refer it as Model I) as we give a brief review in the next section.

We would like to emphasize that, in this model, when the family gauge bosons exist the mass eigenstates, the charged leptons also exist in the mass eigenstates, while quarks, in general, do not exist in the mass eigenstates, as discussed in Sec.4. Therefore, the family gauge boson masses are given related only to the charged lepton masses, and the family number violated processes appear only in the quark sectors, e.g. $\mu \rightarrow e + \gamma$ is forbidden in the tree-level, while $b \rightarrow s + \gamma$ is allowed in the tree-level. Investigation in the present paper is highly dependent on the idea in the model [5].

In Sec.3, we estimate the lightest gauge boson mass M_{33} from the tau decay data (1) and also from the upilon decay data. Regrettably, at present, we cannot obtain a conclusive value of M_{33} because of the large errors.

Usually, a severe constraint is obtained from the observed $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ mixing data. In Sec.4, we discuss the $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ mixings which are caused though quark-family mixings $U_d \neq \mathbf{1}$ and $U_u \neq \mathbf{1}$. Although the constraint become mild for such a model with inverse mass hierarchy, it is still severe if $(U_d)_{21}$ and $(U_u)_{21}$ are sizable. Especially, Model I will be ruled out from candidates which can interpret both data, i.e. the data in tau and upilon decays and the observed $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ mixing data, because Model I has a mass relation $M_{ij} \propto \sqrt{1/m_{ei} + 1/m_{ej}}$ and it gives a small ratio $M_{22}/M_{33} = 4.10$.

In Sec.5, we discuss another models: one has a mass relation $M_{ij} \propto (1/m_{ei} + 1/m_{ej})$, and the other one has a mass relation $M_{ij} \propto 1/m_{ei}m_{ej}$. The former is a minimum reversion of Model I, but it cannot still overcome the constraint from $K^0-\bar{K}^0$ mixing because it gives $M_{22}/M_{33} = 16.8$. The latter can satisfy both constraints because it gives $M_{22}/M_{33} = 283$, but it is difficult to build a model such mass spectrum theoretically.

Sec.6 is devoted to concluding remarks.

2. Family gauge boson model with an inverted mass hierarchy

The family gauge boson model with an inverted mass hierarchy has been proposed stimulated by the Sumino model [6]. Therefore, first, let us give a brief review of the Sumino mechanism. Sumino has seriously taken why the charged lepton mass formula [7] $K \equiv (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$ is so remarkably satisfied with the pole masses: $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$, while if we take the running masses, the ratio becomes $K(\mu) = (2/3) \times (1.00189 \pm 0.000002)$, for example, at $\mu = m_Z$. The deviation comes from the $\log m_{ei}^2$ term in the QED radiative correction [8]. Therefore, Sumino has proposed an idea that the factor $\log m_{ei}^2$ is canceled by a contribution from family gauge bosons. In order to work the Sumino mechanism correctly, the following conditions are essential: (i) The left- and right-handed charged leptons (e_L, e_R) have to be assigned to $(\mathbf{3}, \mathbf{3}^*)$ of the U(3) family symmetry, respectively. (ii) Masses of the gauge bosons are given by $M_{ij}^2 = k(m_{ei} + m_{ej})$. Thus, the factor $\alpha_{em} \log m_{ei}^2$ due to the photon is canceled by a part of $-\alpha_F \log M_{ii}^2 = -\alpha_F(\frac{1}{2} \log m_{ei}^2 + \log 2k)$ due to the family gauge bosons, where $\alpha_F = g_F^2/4\pi$. However, the Sumino model has the following problems: (i) The model is not anomaly free because the charged leptons are assigned as $(e_L, e_R) = (\mathbf{3}, \mathbf{3}^*)$ of a U(3)_{fam} gauge symmetry (this assignment is inevitable in order to the

so-called Sumino's cancellation mechanism [6]); (ii) Effective current-current interactions with $\Delta N_{fam} = 2$ appear because of the $(e_L, e_R) = (\mathbf{3}, \mathbf{3}^*)$ assignment; (iii) The Sumino's cancellation mechanism cannot be applied to a SUSY model, because the vertex type diagram does not work in a SUSY model.

Therefore, in order to evade the above problems, in the revised model [5], we assign the $U(3)_{fam}$ quantum numbers as $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$, so that the model is anomaly free, and the $\Delta N_{fam} = 2$ interactions do not appear at tree level. On the other hand, in order to realize the cancellation mechanism, we must consider that masses M_{ij} of the gauge bosons A_i^j are given as follows:

$$m^2(A_i^j) \equiv M_{ij}^2 = k \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right), \quad (3)$$

differently from those in the Sumino model, $M_{ij}^2 = k(m_{ei} + m_{ej})$, where m_{ei} are charged lepton masses. (Note that $\log M_{ii}^2 = +\frac{1}{2} \log m_{ei}^2 + \log 2k$ in the Sumino model, while $\log M_{ii}^2 = -\frac{1}{2} \log m_{ei}^2 + \log 2k$ in our model).

As well as the Sumino model, the family gauge coupling constant g_F in our model is not a free parameter because the cancellation mechanism:

$$g_F^2 = \frac{3}{2} \zeta e^2 = \frac{3}{2} \zeta g_W^2 \sin^2 \theta_W, \quad (4)$$

where g_W is the weak gauge coupling constant given by $G_F/\sqrt{2} = g_W^2/8M_W^2$, and ζ is a fine tuning parameter. In our model, the parameter ζ is numerically given by $\zeta = 1.752$ ($\zeta \simeq 7/4$) [5]. (Hereafter, in numerical estimates of g_F , we will use input values $\zeta = 7/4$ and $\sin^2 \theta_W = 0.223$.) Only a free parameter in the model is the magnitude of M_{33} because the ratios M_{ij}/M_{33} are fixed by the relation (3): $M_{33} : M_{23} : M_{22} : M_{13} : M_{12} : M_{11} = 1 : 2.98 : 4.10 : 41.70 : 41.80 : 58.97$.

The family gauge boson interactions are given by

$$H_{fam} = g_F (\bar{e}^i \gamma_\mu e_j) (A^\mu)_j^i, \quad (5)$$

because the $U(3)$ triplet assignment for charged leptons is given by $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$ which gives anomaly free configuration. Note that the interaction type is pure vector differently from that in the Sumino model, in which the currents have been given by $(V \pm A)$. (For example, a decay $B_s^0 \rightarrow \tau^- + \mu^+$ via an exchange of family gauge boson A_3^2 is forbidden.)

Note that the family gauge bosons are in the mass-eigenstates on the flavor basis in which the charged lepton mass matrix is diagonal. In this model, a lepton number violating process never occurs at the tree level of the current-current interaction in the charged leptons. As we discuss in Sec.4, since quarks are not in the mass-eigenstates on the diagonal basis of the charged lepton mass matrix, family number changing interactions appear in the quark-quark and quark-lepton interactions. For example, the $K^0 - \bar{K}^0$ mixing is caused only through the quark mixings. The μ - e conversion $\mu^- + N \rightarrow e^- + N$ is also caused through the quark mixings.

3. Mass of the lightest gauge boson

First, on the basis of the model with the gauge boson masses (3), we investigate a possible deviation from the e - μ universality in the tau decays, because the processes are pure leptonic, so

that they are not affected by quark family mixing. (Although the estimate was already discussed in Ref.[5], the purpose was only to estimate an order of the energy scale roughly, and the relation (4) was not used.) In the present model, the deviation from the e - μ universality is characterized by the parameters

$$\delta_i^0 = \frac{g_F^2/M_{3i}^2}{g_W^2/8M_W^2}, \quad (6)$$

where $i = 1, 2$ (i.e. $i = e, \mu$) in the tau decays. Since $(M_{32}/M_{31})^2 = 0.00508$ from the relation (3), we neglect the contribution δ_e^0 compared with the contribution δ_μ^0 hereafter. Since the interactions (5) with the family gauge bosons are pure vector, our parameter δ_μ^0 does not directly mean the observed δ_μ . The effective four Fermi interaction for $\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau$ is given by

$$H^{eff} = \frac{G_F}{\sqrt{2}} \{ [\bar{\mu}\gamma_\rho(1-\gamma_5)\nu_\mu][\bar{\nu}_\tau\gamma^\rho(1-\gamma_5)\tau] + \delta_\mu^0(\bar{\nu}_{L\tau}\gamma_\rho\nu_{L\mu})(\bar{\mu}\gamma^\rho\tau) \}, \quad (7)$$

where we have dropped the term $(\bar{\nu}_{R\tau}\gamma_\rho\nu_{R\mu})$ because ν_R have large Majorana masses. By using Fierz transformation, we can express Eq.(7) as

$$H^{eff} = 4\frac{G_F}{\sqrt{2}} \left\{ \left(1 + \frac{1}{4}\delta_\mu^0\right) (\bar{\mu}_L\gamma_\rho\nu_{L\mu})(\bar{\nu}_{L\tau}\gamma^\rho\tau_L) - \frac{1}{2}\delta_\mu^0(\bar{\mu}_R\nu_{L\mu})(\bar{\nu}_{L\tau}\tau_R) \right\}. \quad (8)$$

Therefore, the observed δ_μ is related to our parameter δ_μ^0 as follows:

$$\delta_\mu = \frac{1}{2} \left(1 - 2x_\mu \frac{g(x_\mu)}{f(x_\mu)}\right) \delta_\mu^0, \quad (9)$$

where $g(x) = 1 + 9x^2 - 9x^4 - x^6 + 6x^2(1+x^2)\log x^2$ and $x_\mu = m_\mu/m_\tau$. Here, we have neglected higher terms of δ_μ^0 . (For more details, for example, see Ref.[9].) The present deviation $\delta \equiv \delta_\mu - \delta_e = (2.0 \pm 1.6) \times 10^{-2}$ gives a family gauge boson mass of A_3^2

$$M_{23} = 2.6_{-0.7}^{+3.2} \text{ TeV}. \quad (10)$$

so that it means the lightest family gauge boson mass

$$M_{33} = 0.87_{-0.22}^{+1.07} \text{ TeV}, \quad (11)$$

from the mass relation (3). However, at present, the numerical result (10) [also (11)] should not be taken rigidly, because, for example, if we change the input value δ from the input value $\delta = 0.0020 \pm 0.0016$ to $\delta = 0.0020 \pm 0.0016 \times 1.25$, the predicted upper value of M_{33} will become $M_{33} \rightarrow \infty$.

At present we have another data of deviations from the e - μ - τ universality, i.e. data of upilon decays $\Upsilon(1S) \rightarrow \ell^+\ell^-$ ($\ell = e, \mu, \tau$). For the time being, we neglect family mixing among quark families. Then, the $b\bar{b}$ sector couples only to the family gauge boson A_3^3 in addition to the

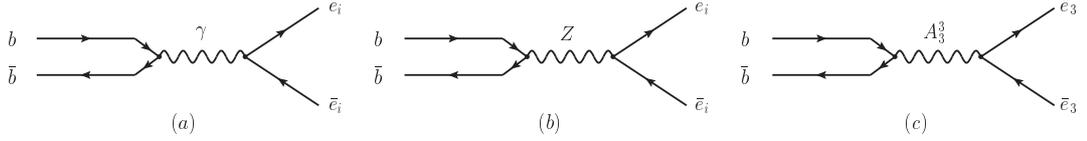


Figure 2: Deviation from e - μ - τ universality in upsilon decay

standard gauge bosons (photon and Z boson) as seen in Fig.2. Present experimental data [2] $Br(\Upsilon(1S) \rightarrow \tau^+\tau^-) = (2.60 \pm 0.10)\%$, $Br(\Upsilon(1S) \rightarrow \mu^+\mu^-) = (2.48 \pm 0.05)\%$, and $Br(\Upsilon(1S) \rightarrow e^+e^-) = (2.38 \pm 0.11)\%$ gives $R_{Br} \equiv Br(\Upsilon \rightarrow \tau^+\tau^-)/Br(\Upsilon \rightarrow \mu^+\mu^-) = 1.048 \pm 0.046$, which leads to

$$R_{amp} = 1 + \delta_{\tau/\mu} = 1.028 \pm 0.022, \quad (12)$$

where R_{amp} has been defined by $R_{amp} \equiv \sqrt{R_{Br}/R_{kine}}$,

$$R_{kine}^{\tau/\mu} = \frac{1 + 2\frac{m_\tau^2}{M^2}}{1 + 2\frac{m_\mu^2}{M^2}} \sqrt{\frac{1 - 4\frac{m_\tau^2}{M^2}}{1 - 4\frac{m_\mu^2}{M^2}}}. \quad (13)$$

Also, we obtain $R_{amp}^{\mu/e} = 1 + \delta_{\mu/e} = 1.021 \pm 0.051$. However, hereafter, we will not utilize the data on $Br(\Upsilon(1S) \rightarrow e^+e^-)$ because of its large error. Since the contributions from photon, Z boson, and A_3^3 boson, are characterized by $1/q^2$, $1/(q^2 - M_Z^2)$ and $1/(q^2 - M_{33}^2)$ with $q^2 = M_\Upsilon^2$, respectively, the sign of the deviation δ_τ has to be negative considering naively, while the observed result (12) has denoted that it is positive. Therefore, we assume that quark fields are assigned as $(q_L, q_R) \sim (\mathbf{3}^*, \mathbf{3}^*)$ of the $U(3)$ family symmetry, differently from that in the charged lepton sector, $(e_L, e_R) \sim (\mathbf{3}, \mathbf{3})$. [The model is still anomaly free in spite of this modification, differently from the Sumino model with $(\mathbf{3}, \mathbf{3}^*)$.] Since we can neglect the Z boson contribution compared with the photon contribution, the deviation parameter δ_τ is given

$$\delta_\tau = \frac{g_F^2}{e^2/3} \frac{M_\Upsilon^2}{M_{33}^2}, \quad (14)$$

where the factor $1/3$ has originated in the electric charge of b quark. The observed deviation $\delta_\tau = 0.028 \pm 0.022$ gives

$$M_{33} = (112_{-26}^{+130}) \text{ GeV}. \quad (15)$$

This value is considerably small compared with the result (11) from the tau decay data. However, the upper bound of M_{33} is sensitive to the input value of δ_τ .

Although we cannot obtain a conclusive value of M_{33} after all, it should be noted that those results show that the determination of M_{33} is within our reach. We hope to obtain more accurate data on the deviation from e - μ - τ universality in near future.

4. Family-number violating processes due to quark mixing

So far, we have not discussed family mixing in the quark sectors. In the present model, the family number is defined by a flavor basis in which the charged lepton mass matrix M_e

is diagonal, while, in general, quark mass matrices M_u and M_d are not diagonal in this basis. When we denote quarks in the mass eigenstates as $u = (u, c, t)$ and $d = (d, s, b)$, and those in the family eigenstates as $u^0 = (u_1^0, u_2^0, u_3^0)$ and $d^0 = (d_1^0, d_2^0, d_3^0)$, the family mixing matrices are defined as $q_L^0 = U_L^q q_L$ (and also $L \rightarrow R$) ($q = u, d$). Quark mass matrices M_q are diagonalized as $(U_L^q)^\dagger M_q U_R^q = D_q$, and the quark mixing matrix V_{CKM} [10] is given by $V_{CKM} = (U_L^u)^\dagger U_L^d$. [Since $(\nu_i, e_i^-)_L$ are doublets in $SU(2)_L$, we can regard the eigenstates of the family symmetry as the eigenstates of weak interactions.] Since we know $V_{CKM} \neq \mathbf{1}$, we cannot take the mixing matrices U_L^u and U_L^d as $U_L^u = \mathbf{1}$ and $U_L^d = \mathbf{1}$ simultaneously. Under this definition of the mixing matrices, the family gauge bosons interact with quarks as follows:

$$H_{fam} = g_F \sum_{q=u,d} (\bar{q}^{0i} \gamma_\mu q_j^0) (A^\mu)_i^j = g_F \sum_{q=u,d} (A_\mu)_i^j [(U_L^{q*})_{ik} (U_L^q)_{jl} (\bar{q}_{Lk} \gamma^\mu q_{Ll}) + (L \rightarrow R)]. \quad (16)$$

In the investigation of the upilon decays in Sec.3, we may consider that b - s mixing (i.e. U_{31}^d and U_{32}^d) is highly suppressed, considering the observed CKM mixing $|V_{ub}| \sim 10^{-3}$ and $|V_{cb}| \sim 10^{-2}$. If the mixing is sizable, we would observe a decay $\Upsilon \rightarrow \mu^\pm \tau^\mp$ (the data [2] show $Br(\Upsilon \rightarrow \mu^\pm \tau^\mp) < 6.0 \times 10^{-6}$). We can consider that the estimate in Eq.(12) with neglecting the b - s - d mixing is reasonable.

The greatest interest to us is whether we can take a lower value of M_{33} without contradicting the constraint from the observed K^0 - \bar{K}^0 mixing. The K^0 - \bar{K}^0 mixing is caused by A_1^1 , A_2^2 and A_3^3 exchanges only when the down-quark mixing $U_{L/R}^d \neq \mathbf{1}$ exists:

$$H^{eff} = g_F^2 \left[\frac{1}{M_{33}^2} (U_{31}^{d*} U_{32}^d)^2 + \frac{1}{M_{22}^2} (U_{21}^{d*} U_{22}^d)^2 + \frac{1}{M_{11}^2} (U_{11}^{d*} U_{12}^d)^2 \right] (\bar{s} \gamma_\mu d) (\bar{s} \gamma^\mu d) + h.c., \quad (17)$$

where, for simplicity, we have taken $U_L^d = U_R^d$. If we assume the vacuum-insertion approximation, we obtain

$$\Delta m_K^{fam} = \left[\frac{1}{M_{33}^2} (U_{31}^{d*} U_{32}^d)^2 + \frac{1}{M_{22}^2} (U_{21}^{d*} U_{22}^d)^2 + \frac{1}{M_{11}^2} (U_{11}^{d*} U_{12}^d)^2 \right] \times 0.7738 \times 10^{-12} \text{ [TeV]}, \quad (18)$$

where the value of M_{33} is taken in a unit of TeV, and we have used values $f_K = 0.1561$ GeV, $m_s(0.5\text{GeV}) = 0.513$ GeV and $m_d(0.5\text{GeV}) = 0.0259$ GeV. On the other hand, the observed value [2] is $\Delta m_K = (4.484 \pm 0.006) \times 10^{-18}$ TeV, and the standard model has a share of $\Delta m_K \sim 2 \times 10^{-18}$ TeV (for example, see Ref.[11], and for recent work, for instance, see the second one in Refs.[1]).

We know the observed values of V_{CKM} parameters [2], but we do not know the mixing values U^d and U^u separately. By way of trial, let us take $U^d = U(\frac{1}{2}\theta_{12}, \frac{1}{2}\theta_{23}, \frac{1}{2}\theta_{13}, \delta_d)$ and $U^u = U(-\frac{1}{2}\theta_{12}, -\frac{1}{2}\theta_{23}, -\frac{1}{2}\theta_{13}, \delta_u)$ corresponding to the standard expression of the CKM matrix $V_{CKM} = U(\theta_{12}, \theta_{23}, \theta_{13}, \delta)$ with $\theta_{12} = 13.02^\circ$, $\theta_{23} = 2.36^\circ$, $\theta_{13} = 0.201^\circ$ and $\delta = 69.0^\circ$. In order to reproduce the observed V_{CKM} , we must take $\delta_u \sim 80^\circ$ and $\delta_d \simeq 25^\circ$. This tentative choice

gives

$$|U^d| = \begin{pmatrix} 0.9935 & 0.1134 & 0.00176 \\ 0.1133 & 0.9933 & 0.02060 \\ 0.003985 & 0.02029 & 0.9998 \end{pmatrix}, \quad |U^u| = \begin{pmatrix} 0.9935 & 0.1134 & 0.00176 \\ 0.1134 & 0.9933 & 0.02060 \\ 0.00266 & 0.02051 & 0.9998 \end{pmatrix}. \quad (19)$$

Of course, there is a possibility that U^u and U^d take large mixings with opposite signs each other, e.g. $|\theta_{12}^d| \gg |\theta_C|$ and $|\theta_{12}^u| \gg |\theta_C|$, but $\theta_{12}^d - \theta_{12}^u = \theta_C$ (θ_C is the Cabibbo angle). Such a case will give not only large mixings in $K^0-\bar{K}^0$, $D^0-\bar{D}^0$, \dots systems, but also large rates of μ - e conversion ($\mu N \rightarrow eN$) and $s \rightarrow d + \gamma$. In this paper, we do not consider such a case, and we consider a case $U^d \sim U^u \sim V_{CKM}$ with small mixings.

If we take the mixing U^d given in Eq.(19) on trial, from $|U_{31}^{d*}U_{32}^d|^2 = 6.539 \times 10^{-9}$, $|U_{21}^{d*}U_{22}^d|^2 = 0.1268$ and $|U_{21}^{d*}U_{22}^d|^2 = 0.1269$, we find that the second term gives a severe constraint

$$M_{22} > 99 \text{ TeV}, \quad (20)$$

where we have set $|\Delta m_K^{fam}|_{max} = 10^{-18}$ TeV optimistically.

The most easy way to evade the constraint from $K^0-\bar{K}^0$ is to assume $U^d \simeq \mathbf{1}$. Then, the constraint from the $K^0-\bar{K}^0$ mixing disappears. However, then, we must consider $U^u = V_{CKM}^\dagger$ instead of $U^d = V_{CKM}$. Then, we will meet a similar problem on the observed $D^0-\bar{D}^0$ mixing: The $D^0-\bar{D}^0$ mixing gives

$$\Delta m_D^{fam} = \left[\frac{1}{M_{33}^2} (U_{31}^{u*}U_{32}^u)^2 + \frac{1}{M_{22}^2} (U_{21}^{u*}U_{22}^u)^2 + \frac{1}{M_{11}^2} (U_{11}^{u*}U_{12}^u)^2 \right] \times 0.98974 \times 10^{-11} \text{ [TeV]}, \quad (21)$$

where we have used (center values) $f_D = 0.2067$ GeV, $m_c(m_c) = 1.275$ GeV and $m_u(2\text{GeV}) = 0.0023$ GeV. On the other hand, the present observed value [2] is $\Delta m_D^{obs} = (8.38_{-2.9}^{+2.8}) \times 10^{-18}$ TeV. If we again take the mixing U^u given in Eq.(19), from values $|U_{31}^{u*}U_{32}^u|^2 = 2.979 \times 10^{-9}$, $|U_{21}^{u*}U_{22}^u|^2 = 0.1269$ and $|U_{21}^{u*}U_{22}^u|^2 = 0.1269$, we again find that a constraint

$$M_{22} > 251 \text{ TeV}, \quad (22)$$

where we have set $|\Delta m_D^{fam}|_{max} = 2 \times 10^{-18}$ TeV.

Of course, these constraints (20) and (22) are dependent of the mixing values U^d and U^u and the setting of $|\Delta_{K,D}^{fam}|_{max}$, so that those values should be rigidly taken. Optimistically, we consider

$$M_{22} \gtrsim 10^2 \text{ TeV}. \quad (23)$$

5. Search for another models

As seen in the previous section, we have concluded that a mass of the gauge boson A_2^2 must be larger than a few hundred TeV. However, if we take, for example, $M_{22} = 300$ TeV, then Model I [5] predicts masses $M_{23} = 218$ TeV and $M_{33} = 73$ TeV, which are too large compared with the values (10) and (15), respectively, so that we cannot see deviations from the e - μ - τ

universality in the tau and upilon decays. If we adhere to the idea that the family gauge boson effects are visible, we are obliged to abandon Model I.

The motivation in Model I is in the idea that the family gauge boson contribution cancels the logarithmic term $\log m_{ei}^2$ in the QED radiative correction, so that the characteristic of the model is that the gauge coupling constant is related to the electroweak gauge coupling constants. Therefore, we could discuss the gauge boson mass values explicitly in this paper.

Let us consider a minor change of Model I keeping the idea of a model with an inverted mass hierarchy. In the model I, the mass relation (3) has been obtained by the following mechanism: We assume a scalar Φ_i^α which is $(\mathbf{3}, \mathbf{3}^*)$ of $U(3) \times U(3)'$ families and whose VEVs $\langle \Phi \rangle$ give the charged lepton mass matrix M_e as $(M_e)_{ij} = k_e \langle \bar{\Phi}_\alpha^i \rangle \langle \Phi_j^\alpha \rangle$ and $\langle \Phi_i^\alpha \rangle \propto \delta_{i\alpha} \sqrt{m_{ei}}$. We also consider another scalar Ψ_i^α whose VEV dominantly gives family gauge boson masses (we have been assume $|\langle \Psi \rangle| \gg |\langle \Phi \rangle|$) and satisfies a relation $\langle \Psi \rangle \langle \Phi \rangle \propto \mathbf{1}$. Then we can obtain the gauge boson mass relation (3). Similarly, if we introduce a scalar Y_e^{ij} with $\langle Y_e^{ij} \rangle \propto \langle \Phi_\alpha^i \rangle \langle \Phi_\alpha^j \rangle$ [Φ_α^i is $(\mathbf{3}, \mathbf{3})$ of $U(3) \times O(3)$] and we assume a relation $\langle Y_e \rangle \langle \Psi \rangle \propto \mathbf{1}$, we can obtain a gauge boson mass relation

$$m^2(A_i^j) \equiv M_{ij}^2 = k \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right)^2. \quad (24)$$

which gives the mass ratios $M_{33} : M_{22} : M_{11} = 1 : 16.82 : 3.477 \times 10^2$.

Note that, in this revised model (Model II), the cancellation condition of the factor $\log m_{ei}^2$ is given by

$$\varepsilon_i + \varepsilon_0 = \log \frac{m_{ei}^2}{m_{e3}^2} + \zeta_I \log \left(\frac{M_{i1}^2 M_{i2}^2 M_{i3}^2}{M_{31}^2 M_{32}^2 M_{33}^2} \right)_I = \log \frac{m_{ei}^2}{m_{e3}^2} + \zeta_{II} \log \left(\frac{M_{i1}^2 M_{i2}^2 M_{i3}^2}{M_{31}^2 M_{32}^2 M_{33}^2} \right)_{II}, \quad (25)$$

with $\varepsilon_i = 0$. Here, in the second term, only m_{ei} -dependent part is extracted. Since the gauge boson masses satisfy the relation

$$\left(\frac{M_{ij}}{M_{33}} \right)_{II}^2 = \left(\frac{M_{ij}}{M_{33}} \right)_I^4, \quad (26)$$

the ζ parameter defined by Eq.(4) satisfies

$$\zeta_{II} = \frac{1}{2} \zeta_I, \quad (27)$$

By these modifications for g_F^2 and M_{ij}/M_{33} , we obtain a revised value of M_{33} ,

$$(M_{33})_{II}^\tau = 206_{-52}^{+253} \text{ GeV}, \quad (M_{33})_{II}^\Upsilon = 79_{-18}^{+92} \text{ GeV}, \quad (28)$$

from the observed deviation (2) in the tau decays and from the observed deviation (12) in the $\Upsilon(1S)$ decays, respectively. Such small values of $M_{33} \sim 10^2$ GeV cannot be ruled out from the

Table 1: Numerical results in typical models. Values of M_{23} and M_{33} have been extracted from the observed deviations from tau and upsilon decays, respectively. Example values for M_{23} and M_{33} have been deduced from the lower value of M_{22} which has been obtained from the observed $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ mixings (note that those values are dependent on the value of g_F). The values of M_{ij} are presented in a unit of TeV.

	Model I	Model II	Model B
ζ	$1.752 \equiv \zeta_I$	$\frac{1}{2}\zeta_I$	$\frac{1}{3} = 0.1903\zeta_I$
α_F	0.022254	0.01127	0.004293
$M_{33} : M_{23} : M_{22}$	1 : 2.98 : 4.10	1 : 8.91 : 16.8	1 : 16.8 : 283
M_{23}^τ	$2.6^{+3.2}_{-0.7}$	$1.84^{+2.25}_{-0.46}$	$1.13^{+2.25}_{-0.20}$
M_{33}^Υ	$0.112^{+0.130}_{-0.026}$	$0.079^{+0.092}_{-0.018}$	$0.049^{+0.056}_{-0.011}$
$M_{22}^{K,D}$	$\gtrsim 300$	$\gtrsim 200$	$\gtrsim 130$
Example	$M_{22} \equiv 300$ $M_{23} = 218$ $M_{33} = 73$	$M_{22} \equiv 200$ $M_{23} = 106$ $M_{33} = 12$	$M_{22} \equiv 130$ $M_{23} = 7.7$ $M_{33} = 0.46$

current lower bound [4] by the $X \rightarrow \tau^+\tau^-$ search at the Tevatron, because the production rate of A_3^3 is much smaller than that of the conventional Z' boson. However, since Model II predicts, at most, $M_{22} \sim 20$ TeV even $M_{33} \sim 1$ TeV, the model cannot clear the constraint (23) from the observed $K^0-\bar{K}^0$ and $D^0-\bar{D}^0$ mixings.

If we adhere to the idea of the Sumino's cancellation mechanism and the idea of an inverted mass hierarchy, we can consider a model with the following mass spectrum

$$M_{ij} = k_B \frac{1}{m_{ei}m_{ej}}. \quad (29)$$

(Since this model is not a minor change of Model I, we call it as Model B hereafter.) Note that as seen in Eq.(25), exactly speaking, Models I and II cannot give $\varepsilon_i = 0$ exactly (although they approximately satisfy the cancellation condition such as the charged lepton mass relation practically holds), while Model B with the mass spectrum (29) can exactly cancel the QED $\log m_{ei}^2$ term because of $(M_{i1}^2 M_{i2}^2 M_{i3}^2)/(M_{31}^2 M_{32}^2 M_{33}^2) = (m_{e3}/m_{ei})^6$. Since Model B can give the mass ratio $(M_{22}/M_{33})^2 = 8.00 \times 10^4$, we can clear the constraint (23) for $M_{33} \sim 1$ TeV. For the predictions based on Model B, we list those in Table 1 together with the results in Models I and II. As seen in Table 1, Model B seems to be in favor of the observed values. However, the big problem of Model B is that we cannot build a model with such the family gauge boson mass spectrum (29) at present.

6. Concluding remarks

In conclusion, it has been pointed out that the sign of the deviation from the e - μ universality in the tau decays suggests an existence of family gauge bosons with an inverted mass hierarchy.

Stimulated this fact, we have investigated possible phenomenology of a specific model (Model I) [5] with an inverted mass hierarchy proposed by Yamashita and the author. Since the gauge coupling constant g_F is not free parameter in this model, the observed values of the deviations from the e - μ - τ universality in the tau decays and upsilin decays can, in principle, determine the family gauge boson masses: $M_{23} = 2.6_{-0.7}^{+3.2}$ TeV and $M_{33} = 0.112_{-0.026}^{+0.120}$ TeV, respectively. Regrettably, at present, the data have large errors, so that we could not obtain a conclusive value of M_{33} .

On the other hand, in the present model, the gauge bosons A_i^j are in the mass-eigenstates on the family basis in which the charged lepton mass matrix is diagonal, while the quarks are, in general, not in the mass-eigenstates in the family basis, so that family-mixings $U^u \neq \mathbf{1}$ and $U^d \neq \mathbf{1}$ appear. We have also investigated K^0 - \bar{K}^0 and D^0 - \bar{D}^0 mixings, because it is the biggest obstacle to a family gauge boson model with a lower scale. The observed values of Δm_K and Δm_D put a severe constraint for the mass M_{22} : $M_{22} \gtrsim 3 \times 10^2$ TeV. This constraint gives a conclusion that the deviations from the e - μ - τ universality are invisible in the tau decays and upsilin decays. (Since the upper values of the predicted gauge boson masses become infinity if we take 1.3σ of the errors in the data, the conclusion $M_{22} \gtrsim 3 \times 10^2$ TeV does not contradict the results (10) and (15) considering their large errors, but such a case does not give visible effects of the family gauge bosons.)

Since we want family gauge bosons whose effects are visible in a lower scale physics, we have discussed alternative model in Sec.5. Model II is a minor change of Model I, and it is possible to build such a model in fact, but the model cannot cope with both results, that from the tau and upsilin decays, and that from K^0 - \bar{K}^0 and D^0 - \bar{D}^0 mixings.

Of course, there is an option that we abandon the Sumino's cancellation mechanism. Then, the gauge coupling constant g_F can become free parameter independently of the gauge boson mass spectrum, so that we can fit all data freely. However, in Sec.5, we have not taken such the option. We have inherited the ideas in the Sumino mechanism and in [5]. If we abandon the Sumino mechanism, we will lose the motivation to consider a family gauge boson model with the inverted mass hierarchy. In Model B, the Sumino mechanism exactly holds (in Models I and II, the mechanism holds only approximately). Model B can give interesting phenomenology, but we have not been able to build such a model explicitly at present. This is a future task to us.

We again would like to emphasis that if we improve the error values in the deviations from the e - μ - τ universality in the tau decays and upsilin decays, we can determine the values of family gauge boson masses, i.e. the determination is within our reach.

If we leave the constraint from the K^0 - \bar{K}^0 and D^0 - \bar{D}^0 mixings, we can expect fruitful physics not only in TeV but also sub-TeV regions. For example, we expect a direct search for A_3^3 at the LHC. (For the details of the direct search for the lightest family gauge boson A_3^3 at the LHC, we shall report elsewhere.) Very recently, an interesting decay model via a family changing neutral gauge boson has been pointed out [12]. Although we have discussed Model I, II and B in Sec.5, those are only examples. The essential idea is that the family gauge bosons have an inverted mass hierarchy. If we adopt a view of such the family gauge boson model with the inverted mass hierarchy, it will offer to us fruitful new physics experimentally and theoretically. Further studies are our future tasks.

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