

Effective Valence Quark Model and a Possible Dip in $dBr(B \rightarrow K\ell\bar{\ell})/dq^2$

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Abstract

In rare B meson decays $B \rightarrow K\ell^+\ell^-$, a possible contribution of $\ell^+\ell^-$ emission via photon from the “spectator” quark q ($q = u, d$) in the B meson ($q\bar{b}$) is investigated in addition to the conventional one $\bar{b} \rightarrow \bar{s} + \gamma \rightarrow \bar{s} + \ell^+ + \ell^-$. If such a contribution is sizable compared with the standard estimate of $B \rightarrow K\ell^+\ell^-$, we will observe visible difference between $d\Gamma(B^0 \rightarrow K^0\ell^+\ell^-)/dq^2$ and $d\Gamma(B^+ \rightarrow K^+\ell^+\ell^-)/dq^2$ in q^2 dependence ($q^2 \equiv m_{\ell\ell}^2$). Besides, as a result of the interference between the conventional one and a new one, a dip appears in $d\Gamma(B \rightarrow K\ell^+\ell^-)/dq^2$ at a small region of q^2 . The interference effect in the B^0 decay will also be observed differently from that in the B^+ decay. The calculation is done based on a semi-classical approach, a valence quark model. In the present model, the photon emission from the spectator quark q , $d \rightarrow d + \gamma$ ($u \rightarrow u + \gamma$) is independent of the b - s transition mechanism, and the characteristic results are due to a straightforward estimate of the quark propagator which cannot be incorporated into the factorization method. The model is not a valence quark “dominant” model, so that, for example, the valence quarks in the final state carry only 24% of the energy-momentum of the kaon.

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1. INTRODUCTION

Recent observations of the bottom meson decays $B \rightarrow K\ell^+\ell^-$ by Belle [1] and BABAR [2] seem to reveal an interesting feature: the observed q^2 dependence of the differential branching fraction, $dBr(B \rightarrow K\ell^+\ell^-)/dq^2$, seems to have a dip at a small value of q^2 ($\equiv m_{\ell\ell}^2$), i.e. $q^2 \sim 1 \text{ GeV}^2$. On the other hand, the LHCb experiments have reported a dip in $dBr(B^0 \rightarrow K^0\mu^+\mu^-)/dq^2$ [3] and no dip in $dBr(B^+ \rightarrow K^+\mu^+\mu^-)/dq^2$ [4]. As we emphasize in the end of the final section, these experimental results are very suggestive to us. Of course, we cannot deduce such the existence of a dip only from the current B decay data, because the amount of data is still not sufficient. Besides, we cannot see such a dip in the data of CDF [5]. Nevertheless, in this paper, we dare to investigate a possibility that a dip in dBr/dq^2 is true, because it means that there is a new contribution to the decays $B \rightarrow K\ell^+\ell^-$ in addition to the conventional electroweak penguin decay [6],

$$\mathcal{H}^{eff} = G_{EW}^{eff} \frac{1}{e} (\bar{s}\sigma_{\mu\nu}b_R) F^{\mu\nu}, \quad (1.1)$$

where

$$G_{EW}^{eff} = \frac{G_F \alpha}{\sqrt{2} \pi} V_{ts}^* V_{tb} 2m_b, \quad (1.2)$$

and, for simplicity, we have dropped contribution from b_L . In the conventional analysis [6], they use effective Hamiltonian to perform this transition (see for a review [7]). Although we have certainly q^2 dependence in their Hamiltonian, we omit such term due to smallness of its Wilson coefficient. As a result, the differential branching fraction does not have the q^2 pole and it cannot also explain the dip at small q^2 region. On the other hand, in the recent analysis [8]-[11], they have promoted to improve the analysis at the low recoil region, that is the large $\sqrt{q^2}$ of the order of the b -quark mass.

The purpose of the present paper is not to discuss the absolute value of $Br(B \rightarrow K\ell^+\ell^-)$ quantitatively, but to discuss the shape of $dBr(B \rightarrow K\ell^+\ell^-)/dq^2$ qualitatively. We will speculate that if the ‘‘observed’’ dip in the q^2 distribution of $B \rightarrow K\ell^+\ell^-$ is true, a contribution due to photon emission from the ‘‘spectator’’ quark¹ is important. The

¹ The terminology ‘‘spectator’’ quark is somewhat misleading : In this case, the ‘‘spectator’’ quark means $q = u, d$ in the B meson ($q\bar{b}$). In the conventional model, photon which produces a lepton pair is emitted via the effective interaction (1.1), $\bar{b} \rightarrow \bar{s} + \gamma$. However, in the present paper, we discuss a case in which such photon is emitted from the ‘‘spectator’’ quark $q = u, d$ as well as the $b \rightarrow s$ transition happens in the opposite side of the $q = u, d$. Nevertheless, we will use the terminology ‘‘spectator’’ quark for $q = u, d$ in the B meson ($q\bar{b}$) for convenience.

first analysis of the “spectator” quark contribution to $B \rightarrow K\ell^+\ell^-$ has been done by Beneke, Feldmann and Seidel [12]. (For a recent analysis, for example, see Ref.[13] and the references therein.) If contribution from the spectator quark to the $B \rightarrow K\ell^+\ell^-$ is sizable, the q^2 dependence of $dB(B \rightarrow K\ell^+\ell^-)/dq^2$ will be considerably different between B^0 and B^+ in so far as there is a dynamics which can distinguish the spectator quarks. Our interest is in this difference between B^0 and B^+ decays.

Usually, the emission of photon from quarks is considered as that from the transition $b \rightarrow s$, Eq.(1.1), so that the decay amplitude has no q^2 pole. The interaction gives a decay amplitude of $B \rightarrow K\ell^+\ell^-$

$$\mathcal{M} = G_{EW}^{eff} \frac{f_T(q^2)}{M_B + M_K} (P_B + P_K)^\mu [\bar{v}_\ell(k_2) \gamma_\mu u_\ell(k_1)], \quad (1.3)$$

where $f_T(q^2)$ is a form factor in the meson currents for the effective quark interaction (1.1). However, if the photon can be emitted from the “spectator” quark line as shown in Fig.1, the decay amplitude will have a factor $1/q^2$ differently from the effective interaction (1.1). In this paper, we consider a possibility that photon can be emitted from the “spectator” quark line, $d \rightarrow d\gamma$ as shown in Fig.1. (Of course, we will take other three diagrams similar to Fig.1 into consideration as discussed later.) The contribution (1.3) is not entire one in the current estimates of the $B \rightarrow K\ell\ell$ decays. There are actually many other contributions called weak annihilation [14], and so on. What we refer to the contribution (1.3) is for the purpose to compare our estimates of the spectator quark effects with the conventional one. Therefore, we will use (1.3) as the typical one of the conventional estimates with the knowledge that only such presentation is too oversimplified. In our calculations, for simplicity, we do not apply any QCD corrections, form factor effects, and so on, so that we will also neglect such corrections in the conventional contributions, too. These oversimplified treatments do not mean that those effects are not important. It is nothing but we dare to oversimplify in order to shed light on our calculation. For example, we illustrate q^2 dependence in dBr/dq^2 later in Figs. 6 and 8 by introducing a parameter ξ . Since we illustrate the standard model contribution by a curve with $\xi = 0$, we can easily see corrected curves with $\xi \neq 0$ by imaging the standard model contribution correctly for the curve with $\xi = 0$.

The purpose of the present paper is not to propose a new mechanism of the b - s transition. The purpose is to demonstrate sizable contribution of photon emission from the spectator quark. In the present model, the photon emission from the spectator quark d (u), $d \rightarrow d + \gamma$ ($u \rightarrow u + \gamma$) is independent of the b - s transition mechanism, and the characteristic results are due to a straightforward estimate of the quark propagator which cannot be incorporated into the factorization method. Therefore, in the present paper, the origin of the b - s transition is not essential. At present, the most likely candidate of

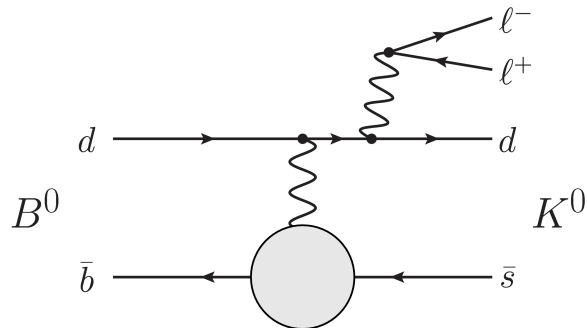


FIG. 1: Feynman diagram for $B^0 \rightarrow K^0 \ell^+ \ell^-$ due to photon emission from spectator quark.

such a b - s transition will be the so-called gluon-penguin contribution. However, since we want to demonstrate the contribution from the spectator quark effects schematically and distinctively, we neglect QCD corrections, form factor effects, and so on, in fear of making our characteristic results of spectator effects blurry by taking account of such effects. We will give only story line of the spectator contribution. Therefore, we demonstrate our calculation prescription for a case of an exchange of a family gauge boson A_2^3 , as shown in Fig.2 instead of the gluon penguin. Here A_2^3 is a family gauge boson which changes family number from “2” to “3”. The family gauge boson exchange is insensitive to gluon corrections. The family gauge boson interaction is given by

$$\mathcal{H}_{fam} = g_F [(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U_{ik}^{*d} U_{jl}^d (\bar{d}_k \gamma_\mu d_l) + U_{ik}^{*u} U_{jl}^u (\bar{u}_k \gamma_\mu u_l)] (A_i^j)^\mu. \quad (1.4)$$

In the present model, the family gauge boson mass matrix is diagonal on the basis in which the charged lepton mass matrix is diagonal, so that flavor-changing process appear only in the quark sector. K^0 - \bar{K}^0 , D^0 - \bar{D}^0 and B^0 - \bar{B}^0 mixings are caused only by A_1^1 , A_2^2 and A_3^3 exchanges under non-vanishing values of the non-diagonal elements of the quark mixing matrices U^u and U^d . By the similar reason, the family gauge boson A_2^3 cannot contribute to B_s - \bar{B}_s mixing because there is no mixing between A_2^2 and A_3^3 . The B_s - \bar{B}_s mixing, too, is caused only by A_1^1 , A_2^2 and A_3^3 exchanges under the down-quark mixing $U^d \neq \mathbf{1}$. Straightforwardly speaking, the mass of A_2^3 is independent of constraints from these ps-meson-anti-ps-meson mixings. Besides, we assume a gauge boson model with an inverted mass hierarchy [15], i.e. $m^2(A_1^1), m^2(A_2^2), m^2(A_3^3) \gg m^2(A_2^3) \gg m^2(A_3^2) \gg m^2(A_3^1)$, so that we may suppose a mass of A_2^3 of an order of 1 – 10 TeV [16]. The model has the following characteristics: (i) We assume a U(3) family symmetry [not SU(3)], so that we have nine family gauge bosons (not eight those). (ii) The masses M_{ij} of the gauge bosons A_i^j given by $M_{ij}^2 = k(m_{ei}^{-1} + m_{ej}^{-1})$. (iii) The family gauge bosons couple to quarks and leptons with a pure vector type. Therefore, for example, the annihilation type diagram of A_2^3 (A_2^3 -exchange diagram in s -channel) cannot contribute to the B^0 decay because the transition B^0 with $J^P = 0^-$ into A_2^3 with $J^P = 1^-$ is forbidden. (iv) The family gauge

bosons are in the mass-eigenstates on the flavor basis in which the charged lepton mass matrix is diagonal. Therefore, a lepton number violating process never occurs at the tree level of the current-current interaction in the charged leptons, while quarks can mix among them via quark mixing matrices U^u and U^d , so that a family number violated processes can occur at the tree level, as we have shown in Eq.(1.4). (Although a formulation based on a family gauge boson is only an example to discuss the photon emission from the “spectator” quark inside meson, for convenience, we will give a brief review of a family gauge boson in Appendix A.)

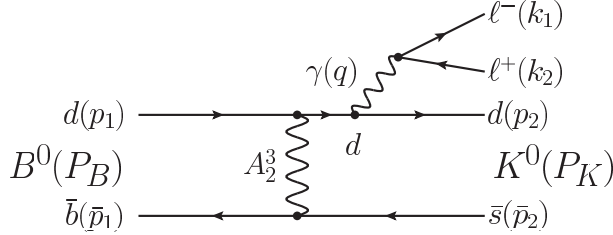


FIG. 2: Feynman diagram for $B^0 \rightarrow K^0 \ell^+ \ell^-$ and the momentum assignments.

In the present paper, for the time being, we assume the mixing among up-quarks is negligibly small compared with that among down-quarks, i.e. $|U_{ij}^u|^2 \ll |U_{ij}^d|^2$, so that we discuss only the case of neutral B meson decay:

$$B^0(P_B) \rightarrow K^0(P_K) + \ell^-(k_1) + \ell^+(k_2). \quad (1.5)$$

We define momenta of quarks \bar{b} and d inside the bottom meson B^0 as \bar{p}_1 and p_1 , respectively, and \bar{s} and d inside K^0 as \bar{p}_2 and p_2 , respectively as shown in Fig.2. We also define the momentum of photon as q in the decay $B^0(P_B) \rightarrow K^0(P_K) + \ell^-(k_1) + \ell^+(k_2)$, i.e.

$$q \equiv k_1 + k_2 = P_B - P_K. \quad (1.6)$$

In order to know the momenta \bar{p}_1 , p_1 , \bar{p}_2 and p_2 , we must reveal dynamical structures of the mesons. In this paper, in an effort to calculate such new type diagram, we propose an approach as a kind of the effective theory for valence quark diagrams. In the next section, we represent those momenta \bar{p}_1 , p_1 , \bar{p}_2 and p_2 in terms of P_B and P_K with the help of an “on-shell quark” assumption. Thereby, we will estimate such diagrams given in Fig.2. Under this prescription, we will find that it is possible for photon to be emitted from d quark.

In Sec.3, we give a form factor-like function $f_+(q^2)$ which gives contribution of photon emission from quarks. (However, as we emphasize in Sec.3, the factor $f_+(q^2)$ is not the so-called “form factor”. In the present prescription, we do not introduce any form factor. The factor $f_+(q^2)$ originates the existence of quark propagator seen in Fig.1.) In Sec.4, we

put an assumption in order to calculate the function $f_+(q^2)$ simply. One of the purpose of the present paper is to demonstrate such q^2 dependence of the factors $f_+(q^2)$ given in Eqs.(3.14)-(3.17) corresponding to four diagrams given in Fig.3. The numerical results are given by Fig.4 in Sec.5. Our purpose is to see the individual contribution from each quark to the photon emission as shown in Fig.4 (a) - (d), so that the standard model contributions are oversimplified as given in Eq.(1.3) and we do not take QCD corrections, form factor effects, and so on in to our naive results into consideration. Finally, Sec.6 is devoted to the concluding remarks. Our results are somewhat different from the conventional one. The reason of the difference is in that in the present calculation we straightforwardly calculate effects of the quark propagator between the gauge-boson mediated vertex and the emitted photon vertex. We will emphasize the meaning of our prescription.

2. EFFECTIVE VALENCE QUARK MODEL

In the present paper, we denote momenta of B^0 , K^0 , \bar{b} and d in the B^0 meson, s and d in the neutral kaon K^0 as P_B , P_K , \bar{p}_1 and p_1 , \bar{p}_2 and p_2 , respectively. Our assumption of “on-shell quark” demands that quark masses are given by

$$\bar{p}_1^2 = m_b^2, \quad p_1^2 = m_{d1}^2, \quad \bar{p}_2^2 = m_s^2, \quad p_2^2 = m_{d2}^2, \quad (2.1)$$

where we have left a possibility that the mass of the d quark in the bottom meson can be different from that of the d quark in the kaon, so that we have denoted those as m_{d1} and m_{d2} , respectively. Here, it is our essential assumption that these quark masses are almost constant for q^2 , although those are still dependent on the energy scale μ of the system.

If we want to calculate a meson decay into a meson and something, we must solve a composite state problem. For example, in the $\bar{b}(x_b)$ and $d(x_d)$ system for the $B^0(X)$ meson, two body bound state problem can be reduced into a one-body problem as to the relative coordinates $x = x_b - x_d$. The variables $x = x_b - x_d$ and $X_B = (x_b + x_d)/2$ corresponds to the momenta $p_b - p_d$ [$\bar{p}_1 - p_1$ in the notation in Eq.(2.1)] and $P_B = p_b + p_d$, respectively. However, in general, it is hard to solve such dynamics relativistically and exactly. Therefore, we usually use an easy method. For example, we can treat the system as a two-body system of quark and anti-quark system non-relativistically. Then, we must use effective quark masses (not the running quark masses $m_q(\mu)$) as masses of the constituents, in which all of the effects of gluons, sea-quarks, and so on are already taken into consideration. (For such a semi-classical approach to pseudo-scalar mesons, for example, see Ref.[17].) Another easy method is to use the running quark mass values for the valence quarks, but is to consider that the valence quarks in the meson carry only a part of the momentum of the meson. In this paper, we adopt the latter prescription.

We define the fraction parameters x_1 and x_2 as follows:

$$x_1 P_B = \bar{p}_1 + p_1, \quad x_2 P_K = \bar{p}_2 + p_2, \quad (2.2)$$

where x_1 (x_2) is a fraction of momenta p_1 and \bar{p}_1 (p_2 and \bar{p}_2) of the valence quarks d and \bar{b} (d and \bar{s}) versus the meson momentum P_B (P_K). Although the parameters x_1 and x_2 are analogous to x parameters in the high energy quark parton model in which x distributions of the quark partons are well known (for a review, for example, see [18]), in the present prescription, the parameters, x_1 and x_2 , defined by Eq.(2.2) are fixed to specific values because the kinematics of two quark are determined as we show in Eqs.(2.15) and (2.16) later. From the constraint (2.2), we have the following relations

$$x_1^2 M_B^2 + m_{d1}^2 - 2x_1(p_1 P_B) = m_b^2, \quad x_2^2 M_K^2 + m_{d2}^2 - 2x_2(p_2 P_K) = m_s^2, \quad (2.3)$$

respectively.

Under the on-shell assumption, the quark momenta p_1 and p_2 can be expressed in terms of P_B and P_K :

$$\begin{aligned} p_1^\mu &= a_1(P_B + P_K)^\mu + b_1(P_B - P_K)^\mu, \\ p_2^\mu &= a_2(P_B + P_K)^\mu + b_2(P_K - P_B)^\mu, \end{aligned} \quad (2.4)$$

where the coefficients a_1 , b_1 , a_2 and b_2 can, in general, be functions of q^2 . Then, we can obtain relations

$$m_{d1}^2 = a_1^2[2(M_B^2 + M_K^2) - q^2] + b_1^2 q^2 + 2a_1 b_1 \Delta_{BK}^2, \quad (2.5)$$

$$m_{d2}^2 = a_2^2[2(M_B^2 + M_K^2) - q^2] + b_2^2 q^2 - 2a_2 b_2 \Delta_{BK}^2, \quad (2.6)$$

from Eq.(2.1), and

$$x_1^2 M_B^2 + m_{d1}^2 - m_b^2 = x_1 a_1 [2(M_B^2 + M_K^2) + \Delta_{BK}^2 - q^2] + x_1 b_1 (\Delta_{BK}^2 + q^2), \quad (2.7)$$

$$x_2^2 M_K^2 + m_{d2}^2 - m_s^2 = x_2 a_2 [2(M_B^2 + M_K^2) - \Delta_{BK}^2 - q^2] + x_2 b_2 (-\Delta_{BK}^2 + q^2), \quad (2.8)$$

from Eq.(2.4), where

$$\Delta_{BK}^2 \equiv M_B^2 - M_K^2. \quad (2.9)$$

Thus, if we give values of x_1 and x_2 , we can completely determine the coefficients (a_1, b_1) from the two relations (2.5) and (2.7), and (a_2, b_2) from the two relations (2.6) and (2.8), respectively. Here, note that the replacement $(M_B, m_b, m_{d1}) \rightarrow (M_K, m_s, m_{d2})$ gives $(a_1, b_1) \rightarrow (a_2, b_2)$. Therefore, hereafter, we will discuss only the relations as to (a_1, b_1) .

The coefficients (a_1, b_1) can be obtained as follows. From Eq.(2.5), we obtain a relation between a_1 and b_1 (see Appendix D):

$$b_1 = \frac{1}{q^2} \left[-a_1 \Delta_{BK}^2 \pm \sqrt{D a_1^2 + m_{d1}^2 q^2} \right], \quad (2.10)$$

where

$$D = [(M_B - M_K)^2 - q^2] [(M_B + M_K)^2 - q^2]. \quad (2.11)$$

By substituting Eq.(2.10) into Eq.(2.5), we obtain a relation for a_1

$$x_1^2 M_B^2 + m_{d1}^2 - m_b^2 = \frac{x_1}{q^2} \left[-a_1 D \pm (\Delta_{BK}^2 + q^2) \sqrt{a_1^2 D + m_{d1}^2 q^2} \right]. \quad (2.12)$$

The parameter a_1 can be obtained by solving Eq.(2.12) for a_1 .

The relation (2.12) brings a new constraint to the model: Let us consider a limit of $q^2 = q_{max}^2$, where

$$q_{max}^2 \equiv (M_B - M_K)^2, \quad (2.13)$$

and it gives

$$D(q^2)|_{q^2=q_{max}^2} = 0. \quad (2.14)$$

Therefore, the relation (2.12) at a limit of $q^2 = q_{max}^2$ leads to a constraint

$$x_1 M_B = m_b \pm m_{d1}. \quad (2.15)$$

Similarly, we obtain a constraint

$$x_2 M_K = m_s \pm m_{d2}. \quad (2.16)$$

Note that the sign \pm in Eq.(2.15) corresponds to the sign \pm in the relation (2.12), but the sign \pm in Eq.(2.15) and \pm in Eq.(2.16) are independent each other.

Quark masses m_b , m_s and m_d are function of the energy scale μ , but it does not mean that those are always function of q^2 directly. We consider that the quark mass values m_b , m_s and m_d in the $B \rightarrow K \ell^+ \ell^-$ decays are described by those at $\mu \sim M_B$. Then, we can estimate the values of x_1 and x_2 from Eqs.(2.15) and (2.16), so that we can also estimate the values of (a_1, b_1) and (a_2, b_2) .

More discussions from a phenomenological point of view will be given in Sec.4.

3. CONTRIBUTION FROM THE FAMILY GAUGE BOSON A_2^3

We assume the following interactions for $B \rightarrow K \ell^+ \ell^-$ in addition to the conventional $b \rightarrow s + \gamma$ interaction (1.1):

$$\mathcal{H} = \sum_{q=u,d,b,s} e_q (\bar{q} \gamma_\mu q) A^\mu - \sum_{\ell=e,\mu} e (\bar{\ell} \gamma_\mu \ell) A^\mu + \sum_{q=u,d} G_{fam}^q (\bar{b} \gamma_\rho s) (\bar{q} \gamma^\rho q), \quad (3.1)$$

where $e_d = e_s = e_b = -e/3$, $e_u = 2e/3$, and

$$G_{fam}^q = \frac{g_F^2}{M_{23}^2} U_{33}^{*d} U_{22}^d U_{21}^{*q} U_{31}^q. \quad (3.2)$$

Here, U_{ij}^q are mixing matrix elements among quarks $q = (q_1, q_2, q_3)$, and M_{23} is a mass of a family gauge boson A_2^3 . Based on the interactions (3.1), we calculate the following four diagrams for $B^0 \rightarrow K^0 \ell^+ \ell^-$ as shown in Fig.3.² Hereafter, since we are interested in a case $B^0 \rightarrow K^0 \ell^+ \ell^-$, we will calculate only the case. Another case $B^+ \rightarrow K^+ \ell^+ \ell^-$ can easily be obtained by replacing $e_d \rightarrow e_u$ and $U^d \rightarrow U^u$.

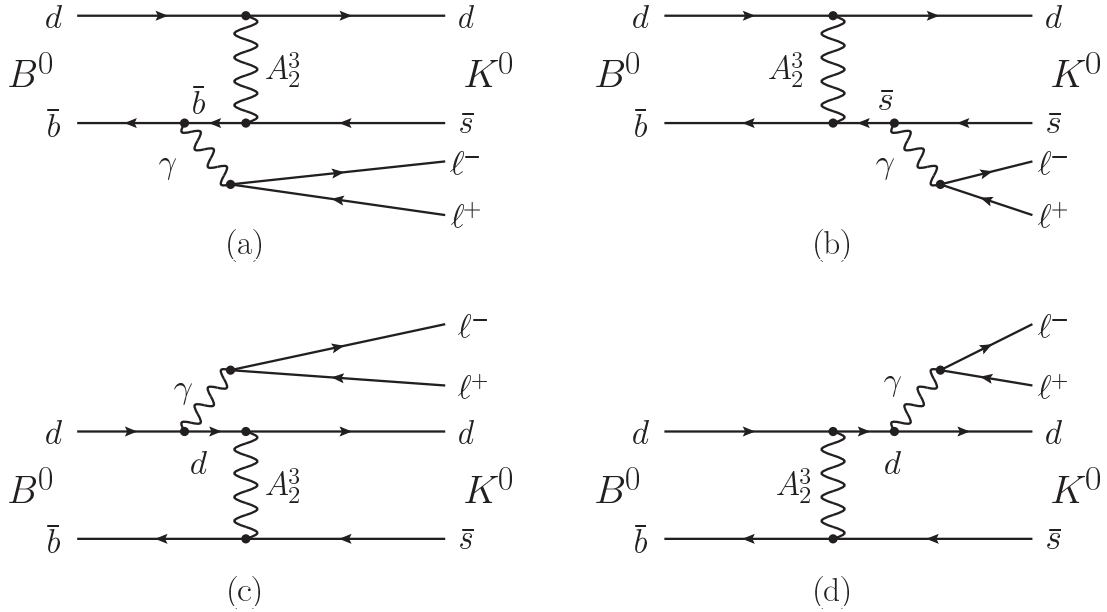


FIG. 3: Feynman diagrams for $B^0 \rightarrow K^0 \ell^+ \ell^-$.

Denominators of the propagators with momenta ℓ shown in Figs.3 (a), (b), (c) and (d) are given as follows:

$$\Delta_a \equiv \ell_{(a)}^2 - m_b^2 = q^2 - 2\bar{p}_1 q, \quad (3.3)$$

$$\Delta_b \equiv \ell_{(b)}^2 - m_s^2 = q^2 + 2\bar{p}_2 q, \quad (3.4)$$

$$\Delta_c \equiv \ell_{(c)}^2 - m_{d1}^2 = q^2 - 2p_1 q, \quad (3.5)$$

$$\Delta_d \equiv \ell_{(d)}^2 - m_{d2}^2 = q^2 + 2p_2 q, \quad (3.6)$$

² We may consider that the contributions given in Fig.3 (a) - (b) are already included in the standard model contributions for the case of the gluon-penguin instead of the A_2^3 exchange. However, we go on this prescription in order to see effects of photon emission from non-spectator quark.

respectively. By using the coefficients defined by Eq.(2.4), the expressions (3.3) - (3.6) are rewritten as follows:

$$\Delta_a = (2a_1 - x_1)\Delta_{BK}^2 + (1 - x_1 + 2b_1)q^2, \quad (3.7)$$

$$\Delta_b = -(2a_2 - x_2)\Delta_{BK}^2 + (1 - x_2 + 2b_2)q^2, \quad (3.8)$$

$$\Delta_c = -2a_1\Delta_{BK}^2 + (1 - 2b_1)q^2, \quad (3.9)$$

$$\Delta_d = 2a_2\Delta_{BK}^2 + (1 - 2b_2)q^2. \quad (3.10)$$

In order to translate effective interactions among quarks into hadronic fields, we use

$$\langle 0 | (\bar{b}\gamma^\mu\gamma_5 d) | B^0(P_B) \rangle = -iP_B^\mu f_B, \quad (3.11)$$

and so on. The details to obtain amplitudes which correspond to the diagrams (a), (b), (c) and (d) in Fig.3 are given in Appendix B.

When we use the expression (2.4), we obtain the following form for the meson currents:

$$\mathcal{M} = i\frac{1}{6}\bar{e}_b e f_K f_B G_{fam} \frac{1}{2} [f_+(q^2)(P_B + P_K)_\mu + f_-(q^2)(P_B - P_K)_\mu] \frac{1}{q^2} [\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)], \quad (3.12)$$

where G_{fam} is defined by Eq.(3.2) and we have dropped the index $q = d$ because it is obvious that we calculate a case of $B^0 \rightarrow K^0 \ell^+ \ell^-$.

The second term with $q_\mu = (P_B - P_K)_\mu$ in Eq.(3.12) does not contribute the decay amplitudes because of $q_\mu [\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)] = 0$ for $m_{\ell 1} = m_{\ell 2}$. For the expression $f_+(q^2)$, we obtain

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) - f_+^c(q^2) - f_+^d(q^2), \quad (3.13)$$

where

$$f_+^a(q^2) = \frac{(x_1 - 2a_1)M_K^2 + (1 - x_1 + a_1 + b_1)q^2}{-(x_1 - 2a_1)\Delta_{BK}^2 + (1 - x_1 + 2b_1)q^2}, \quad (3.14)$$

$$f_+^b(q^2) = \frac{(x_2 - 2a_2)M_B^2 + (1 - x_2 + a_2 + b_2)q^2}{(x_2 - 2a_2)\Delta_{BK}^2 + (1 - x_2 + 2b_2)q^2}, \quad (3.15)$$

$$f_+^c(q^2) = \frac{2a_1 M_K^2 + (1 - a_1 - b_1)q^2}{-2a_1 \Delta_{BK}^2 + (1 - 2b_1)q^2}, \quad (3.16)$$

$$f_+^d(q^2) = \frac{2a_2 M_B^2 + (1 - a_2 - b_2)q^2}{2a_2 \Delta_{BK}^2 + (1 - 2b_2)q^2}. \quad (3.17)$$

Note that these factors $f_+(q^2)$ given in Eqs.(3.14) - (3.17) are not the so-called ‘‘form factor’’ which denotes a quark structure. The functions, $f_+^a(q^2)$, $f_+^b(q^2)$, $f_+^c(q^2)$ and $f_+^d(q^2)$,

originate in the propagators shown in Fig.3 (a) - (d). Although we do not introduce any form factor since it is not a main story in the present prescription, this does not mean that we deny the existence of such form factors. It will become important to take such effects into consideration in an extended study in a future.

So far we have discussed the case of the decay mode $B^0 \rightarrow K^0 \ell^+ \ell^-$, because we have considered that up-quark mixing will be considerably small compared with down-quark mixing, $|U_{ij}^u|^2 \ll |U_{ij}^d|^2$. However, we can easily calculate the case $B^+ \rightarrow K^+ \ell^+ \ell^-$ similarly to the case $B^0 \rightarrow K^0 \ell^+ \ell^-$: A form of $f_+(q^2)$ for the decay $B^+ \rightarrow K^+ \ell^+ \ell^-$ can be obtained by replacing $e_d = -e/3 \rightarrow e_u = +2e/3$ in Eq.(3.12), i.e.

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) + 2f_+^c(q^2) + 2f_+^d(q^2). \quad (3.18)$$

4. INTERFERENCE EFFECT IN $d\Gamma/dq^2$

The partial decay width $\Gamma(B \rightarrow K \ell^+ \ell^-)$ is calculated from the matrix element

$$\mathcal{M} = G \left(1 + \xi \frac{f_+(q^2)}{q^2} \right) (P_B + P_K)_\mu [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (4.1)$$

where

$$G = G_{EW}^{eff} \frac{2m_b f_T(0)}{M_B + M_K}. \quad (4.2)$$

Here, for simplicity, we have neglected the q^2 dependence of the form factor $f_T(q^2)$ in the conventional model. (The numerical results are not almost change even if we take the q^2 dependence of $f_T(q^2)$ into consideration. We will demonstrate it in Appendix E.) The parameter ξ is defined by

$$\xi = \frac{g_{fam}^2}{g_w^2} \frac{8M_w^2}{M_{23}^2} \frac{U_{33}^{*d} U_{22}^d U_{21}^{*d} U_{31}^d}{V_{ts}^* V_{tb}} \frac{\pi^2}{9} \frac{M_B + M_K}{2m_b f_T(0)} f_K f_B, \quad (4.3)$$

but, at present, we regard this parameter ξ as a free parameter whose value is phenomenologically determined by the observed q^2 dependence of dBr/dq^2 . Let us define a function $F(q^2)$ as

$$G^2 F(q^2) \equiv \left. \frac{d\Gamma}{dq^2} \right|_{\xi=0} = \frac{1}{(2\pi)^3} \frac{1}{32M_B^3} \int_{y_1}^{y_2} dy |\mathcal{M}|_{\xi=0}^2, \quad (4.4)$$

where $y \equiv m_{\ell K}^2 = (k_2 + P_K)^2$, and $y_1 = y_{min}$, $y_2 = y_{max}$. Then, $d\Gamma/dq^2$ is given by

$$\frac{d\Gamma}{dq^2}(B \rightarrow K\ell^+\ell^-) = G^2 \left(1 + \xi \frac{f_+(q^2)}{q^2} \right)^2 F(q^2). \quad (4.5)$$

The explicit form of $F(q^2)$ is appeared in Appendix C.

Now, we can numerically evaluate the function $f_+(q^2)$ and $d\Gamma/dq^2$ by using these formulas (4.1) - (4.5). First, we give quark mass values $m_b(\mu)$, $m_s(\mu)$, and $m_{d1}(\mu) = m_{d2}(\mu)$ at $\mu = M_B - M_K$. Then, we obtain the values, x_1 and x_2 , by the relations (2.15) and (2.16). We assume that the quark mass values in this prescription are almost independent of q^2 , and those are only dependent on the value μ . We assume that these quark masses at $\mu = M_B - M_K$ are approximately not so deviated from those at $\mu = M_B$, so that we use the values which are determined by using (2.15) and (2.16). The coefficients a_1 (a_2) can be obtained by using Eq.(2.12) and then a_1 (a_2) can be get by using Eq.(2.10). We will obtain two solutions for Eq.(2.12). Note that the coefficients (a_1, b_1) and (a_2, b_2) are, in general, given as functions of q^2 .

However, in order to give a more concise form of (a_1, b_1) and (a_2, b_2) , let us put the following assumption from phenomenological point of view: These coefficients have no q^2 dependence approximately. This requirement demands $a_1 = b_1$ ($a_2 = b_2$) as seen in Eqs.(2.5) and (2.7) [Eqs.(2.6) and (2.8)]. Then, we obtain concise forms

$$a_1 = b_1 = \pm \frac{m_{d1}}{2M_B}, \quad a_2 = b_2 = \pm \frac{m_{d2}}{2M_K}, \quad (4.6)$$

from Eq.(2.12). The sign \pm in (4.6) corresponds to \pm in Eq.(2.12), but the sign \pm in $a_1 = b_1$ need not to correspond to that in $a_2 = b_2$. By using these solutions in Eq. (4.6), the expressions (3.14) - (3.17) are rewritten as follows:

$$f_+^a(q^2) = \frac{m_b M_K^2 + (M_B - m_b)q^2}{-m_b \Delta_{BK}^2 + (M_B - m_b)q^2}, \quad (4.7)$$

$$f_+^b(q^2) = \frac{m_s M_B^2 + (M_K - m_s)q^2}{m_s \Delta_{BK}^2 + (M_K - m_s)q^2}, \quad (4.8)$$

$$f_+^c(q^2) = \frac{\pm m_{d1} M_K^2 + (M_B \mp m_{d1})q^2}{\mp m_{d1} \Delta_{BK}^2 + (M_B \mp m_{d1})q^2}, \quad (4.9)$$

$$f_+^d(q^2) = \frac{\pm m_{d2} M_B^2 + (M_K \mp m_{d2})q^2}{\pm m_{d2} \Delta_{BK}^2 + (M_K \mp m_{d2})q^2}. \quad (4.10)$$

Note that $f_+^a(q^2)$ and $f_+^b(q^2)$ are independent of the choices \pm in Eq.(4.6), but $f_+^c(q^2)$ and $f_+^d(q^2)$ are dependent on the choices. If we take the positive sign for $a_1 = b_1$ in

(4.6), then the function $f_+^c(q^2)$ will have a pole at $q^2 = m_{d1}\Delta_{BK}^2/(M_B - m_{d1})$. Also, if we take the negative sign in Eq.(4.6), then the function $f_+^d(q^2)$ will have a pole at $q^2 = m_{d2}\Delta_{BK}^2/(M_K + m_{d2})$. Therefore, in the numerical estimate of $d\Gamma/dq^2$, we take the signs in Eq.(4.6) as follows:

$$a_1 = b_1 = -\frac{m_{d1}}{2M_B}, \quad a_2 = b_2 = +\frac{m_{d2}}{2M_K}. \quad (4.11)$$

Then, the propagator effects at $q^2 = 0$ are given by

$$f_+^a(0) = f_+^c(0) = -\frac{M_K^2}{M_B^2 - M_K^2}, \quad f_+^b(0) = f_+^d(0) = +\frac{M_B^2}{M_B^2 - M_K^2}. \quad (4.12)$$

5. NUMERICAL RESULTS

For numerical estimates, for convenience, we adopt quark mass values [19] at $\mu = m(m_b) = 4.34$ GeV in place of those at $\mu = M_B - M_K$:

$$m_b = 4.34 \text{ GeV}, \quad m_s = 0.127 \text{ GeV}, \quad m_d \equiv m_{d1} = m_{d2} = 0.00637 \text{ GeV}. \quad (5.1)$$

The input values (5.1) leads to the following values of the fraction factors x_1 and x_2 :

$$x_1(B) = 0.821, \quad x_2(K) = 0.244, \quad (5.2)$$

from Eqs.(2.15) and (2.16), respectively. We may choose another quark values. However, numerical results are almost similar. Hereafter, we use the values (5.1) as typical values in our prescription.

The values (5.2) mean that the valence quarks b and d are almost dominant in the B meson, while the valence quarks s and d carry only 25% of the momentum of the kaon in the final state. However, the value $x_2(K) = 0.244$ is not common in the all kaon processes, but the value $x_2(K) = 0.244$ is one only in the case of $B \rightarrow K\ell^+\ell^-$. For example, in a kaon decay (note that the value is not x_2 , but it is x_1 because K is one in the initial state), we will again obtain a value near to $x_1(K) \simeq 1$, because in this times we will use quark mass values $m_s(\mu)$ and $m_d(\mu)$ at $\mu = M_K$ (not $\mu = M_B$).

We may choose another possibility for quark masses. However, numerical results are almost similar. Hereafter, we use the values (5.1) as typical values in our prescription.

First, in Fig.4, we show the behavior of the functions $f_+^a(q^2)$, $f_+^b(q^2)$, $f_+^c(q^2)$ and $f_+^d(q^2)$ which represent the contributions of photon emissions from b , s , d_1 and d_2 quarks, respectively, and which are due to quark propagator effects. Note that although we have chosen the coefficients (a_1, b_1) and (a_2, b_2) so that those are independent of q^2 , the functions $f_+^a(q^2)$, $f_+^b(q^2)$, $f_+^c(q^2)$ and $f_+^d(q^2)$ still depend on q^2 . We find that $f_+^c(q^2) \simeq +1$

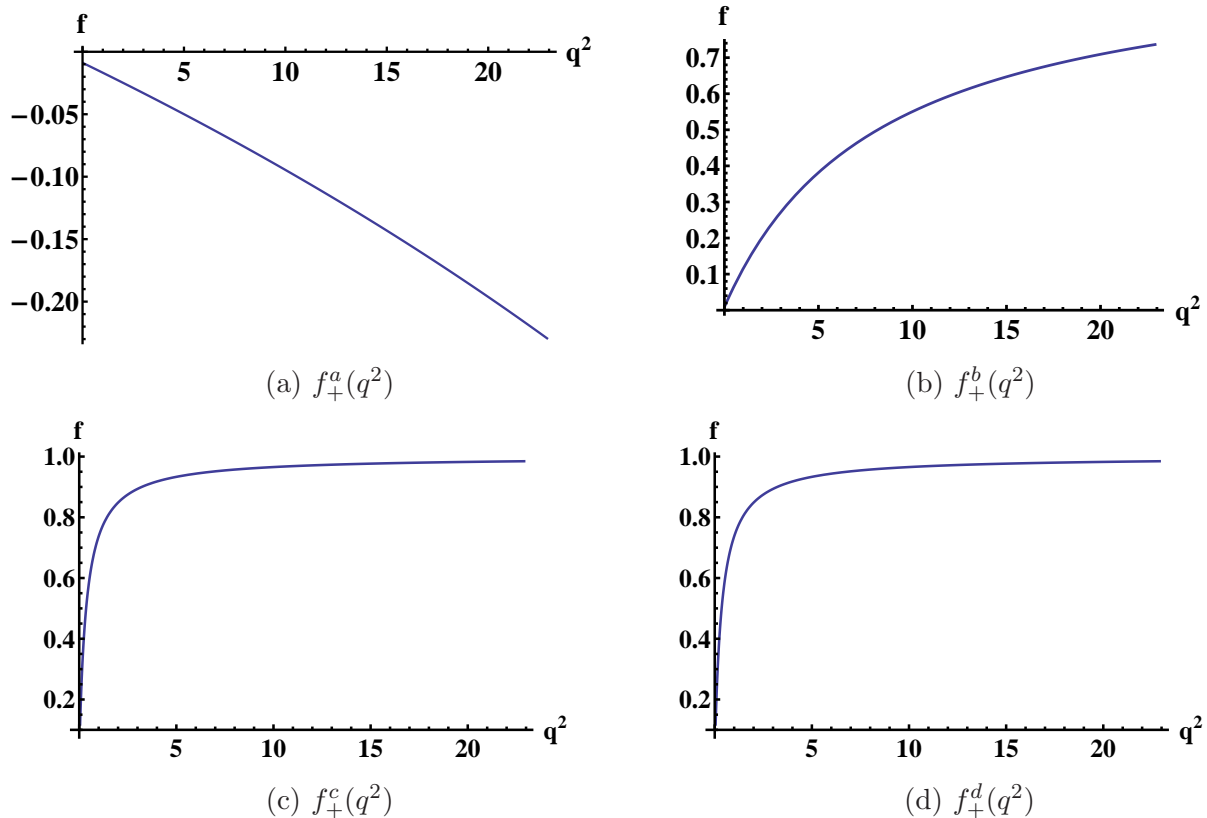


FIG. 4: Contribution from each quark line to the functions $f_+(q^2)$. Figures are illustrated for a physical range $4m_\mu^2 < q^2 < (M_B - M_K)^2$.

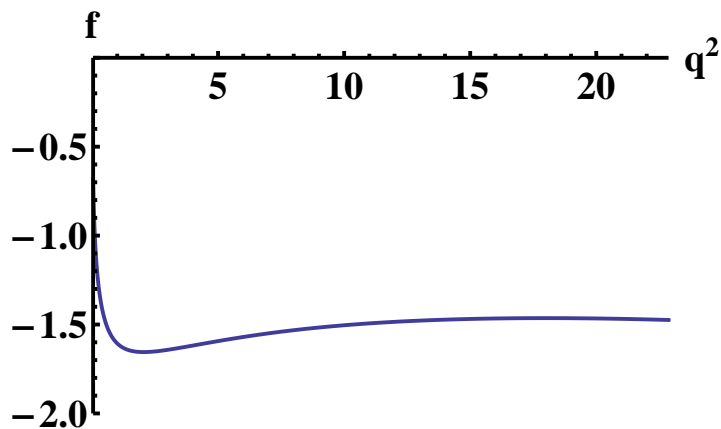


FIG. 5: Behavior of $f_+(q^2)$ in the neutral B meson decay $B^0 \rightarrow K^0 \ell^+ \ell^-$.

and $f_+^d(q^2) \simeq +1$ except for a small range of q^2 . Also, we show the behavior of $f_+(q^2)$ in Fig.5. Note that $f_+(q^2) < 0$ over the whole physical region.

Also, we show the behavior of $dBr(B^0 \rightarrow K^0 \ell^+ \ell^-)/dq^2$ in the unit of G defined by Eq.(4.2) for typical values of the parameter ξ in Fig.6. We can obtain a reasonable dip at

$q^2 \sim 1 \text{ GeV}$ with $\xi = 0.6$.

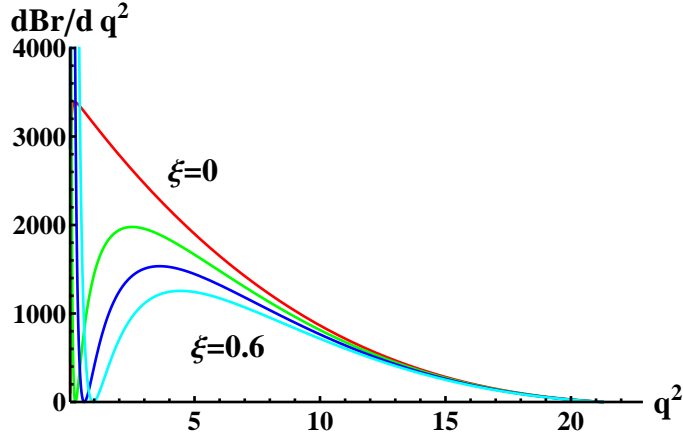


FIG. 6: Behavior of dBr/dq^2 in the decay $B^0 \rightarrow K^0 \ell^+ \ell^-$ in the unit of G defined by Eq.(4.1). Curves are lined up in order of the cases $\xi = 0, 0.2, 0.4$ and 0.6 in the unit of GeV^2 (the colors red, green, blue and cyan, respectively).

Similarly, we can demonstrate the case of $B^+ \rightarrow K^+ \ell^+ \ell^-$. The behaviors of $f_+(q^2)$ and dBr/dq^2 are illustrated in Figs.7 and 8, respectively. (Here, for convenience, we have used the same value of G defined by Eq.(4.2), although a weak annihilation diagram effect [14] should be added in the case of $B^+ \rightarrow K^+ \ell^+ \ell^-$.) If the up-quark mixing is sizable compared with the down-quark mixing, the case will be also visible. The shape of the dBr/dq^2 in $B^+ \rightarrow K^+ \ell^+ \ell^-$ is almost similar to that in $B^0 \rightarrow K^0 \ell^+ \ell^-$. However, note that the dip in dBr/dq^2 appears for $\xi > 0$ in the case $B^0 \rightarrow K^0 \ell^+ \ell^-$, while the dip appears for $\xi < 0$ in the case $B^+ \rightarrow K^+ \ell^+ \ell^-$. It will be possible because $U_{21}^{*u} U_{31}^u$ takes an opposite sign to $U_{21}^{*d} U_{31}^d$. Moreover, the position of the dip is slightly shifted to the larger value of q^2 than the case of neutral B meson.

However, we do not consider that the magnitude of $U_{21}^{*u} U_{31}^u$ is accidentally the same as that of $U_{21}^{*d} U_{31}^d$. We expect that the behavior of dBr/dq^2 in $B^+ \rightarrow K^+ \ell^+ \ell^-$ will be different from that in $B^0 \rightarrow K^0 \ell^+ \ell^-$. We hope data of dBr/dq^2 will be able to distinguish between $B^0 \rightarrow K^0 \ell^+ \ell^-$ and $B^+ \rightarrow K^+ \ell^+ \ell^-$.

Finally, we would like to give some comments on the predicted partial decay width. The decay width is given by

$$\Gamma(B \rightarrow K \ell^+ \ell^-) = G^2 \int_{q_{min}^2}^{q_{max}^2} dq^2 F(q^2), \quad (5.3)$$

where the function $F(q^2)$ is defined by Eq.(4.1) and q_{min}^2 is given by $q_{min}^2 = 4m_\ell^2$. The numerical value is highly sensitive to whether $\ell = \mu$ or $\ell = e$, because the contribution

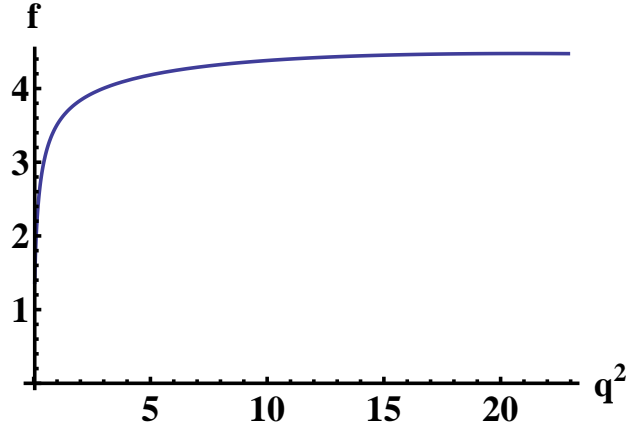


FIG. 7: Behavior of $f_+(q^2)$ in the charged B meson decay $B^+ \rightarrow K^+\ell^+\ell^-$.

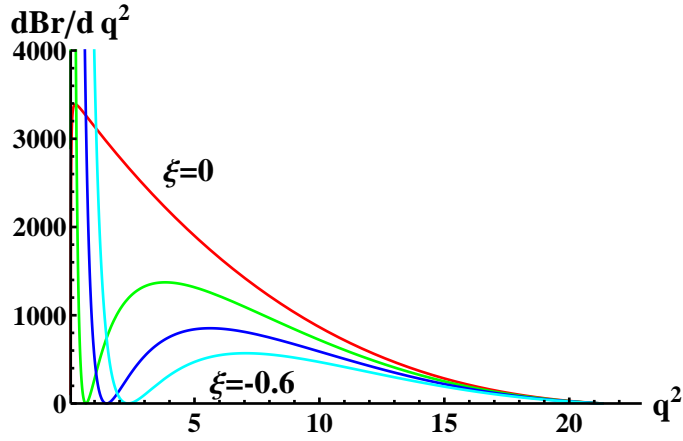


FIG. 8: Behavior of dBr/dq^2 in the decay $B^+ \rightarrow K^+\ell^+\ell^-$ in the unit of G^2 defined by Eq.(4.1). Curves are lined up in order of the cases $\xi = 0, -0.2, -0.4$ and -0.6 in the unit of GeV^2 (the colors red, green, blue and cyan, respectively).

becomes very large at $q^2 \simeq 0$. However, it seems to be impossible to measure accurately until $q^2 = 4m_e^2 = 1.044 \times 10^{-6} \text{ GeV}^2$. If we take $q_{min}^2 = 4m_\mu^2 = 0.04465 \text{ GeV}^2$ for the case of $\Gamma(B \rightarrow Ke^+e^-)$, too, we cannot find a significant difference between $\Gamma(B \rightarrow Ke^+e^-)$ and $\Gamma(B \rightarrow K\mu^+\mu^-)$. Another comment is as follows: The predicted decay width $\Gamma(B \rightarrow K\ell^+\ell^-)$ is dependent on the value of ξ . We illustrate the behavior $R(\xi) \equiv \Gamma(\xi)/\Gamma(0)$ in Fig.9. The present data [20] show $Br(B^+ \rightarrow K^+\ell^+\ell^-) = (5.1 \pm 0.5) \times 10^{-7}$, $Br(B^0 \rightarrow K^0\ell^+\ell^-) = (3.1^{+0.8}_{-0.7}) \times 10^{-7}$ and $\tau(B^+)/\tau(B^0) = 1.079 \pm 0.007$, so that we obtain

$$R_{+/0} \equiv \frac{\Gamma(B^+ \rightarrow K^+\ell^+\ell^-)}{\Gamma(B^0 \rightarrow K^0\ell^+\ell^-)} = 1.52^{+0.42}_{-0.38}. \quad (5.4)$$

Although the value has a large error, if we dare to take the center value in (5.3), a case of the value of ξ which gives $R_{+/0} \sim 1.5$ is only in the case $B^+ \rightarrow K^+\ell^+\ell^-$. The case with $\xi \sim 0.4 \text{ GeV}^2$ can also give a reasonable shape of $d\Gamma/dq^2$ as seen in Fig.8. However, this view conflicts with our anticipation that $|U_{21}^{*u}U_{31}^u| \ll |U_{21}^{*d}U_{31}^d|$. We must wait individual future data of $B^0 \rightarrow K^0\ell^+\ell^-$ and $B^+ \rightarrow K^+\ell^+\ell^-$.

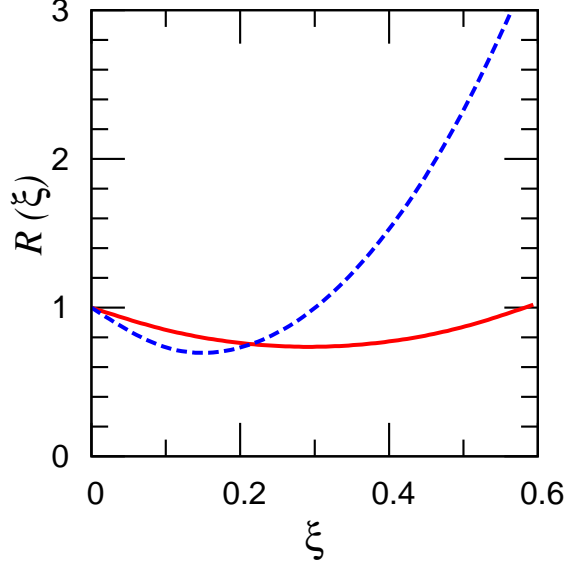


FIG. 9: Behaviors of $R(\xi) \equiv \Gamma(\xi)/\Gamma(0)$ in the decays $B^0 \rightarrow K^0\ell^+\ell^-$ (solid curve) and $B^+ \rightarrow K^+\ell^+\ell^-$ (dashed curve).

6. CONCLUDING REMARKS

In conclusion, we have investigated a contribution of photon emission from the “spectator” quark $d \rightarrow d + \gamma$ ($u \rightarrow u + \gamma$) in the B^0 (B^+) meson, and thereby we have obtained interesting results: (i) The contribution from the spectator quark is large, $f_+^c(q^2) \simeq 1$ and $f_+^d(q^2) \simeq 1$ in contrast to the contribution from \bar{b} quark, $f_+^a(q^2) \simeq 0$, and that from \bar{s} quark, $f_+^b(q^2) \simeq 0 - 0.7$. (ii) For a sizable value of the parameter $|\xi|$, we can demonstrate a dip of $Br(B \rightarrow K\ell^+\ell^-)$ in the small q^2 region. However, in order to obtain such a dip in both decay modes, $B^0 \rightarrow K^0\ell^+\ell^-$ and $B^+ \rightarrow K^+\ell^+\ell^-$, simultaneously, the sign of ξ parameter must be opposite each other.

However, note that the standard model contribution (1.3) is oversimplified to regard it as the conventional estimates in the decay $B \rightarrow K\ell^+\ell^-$. Although the results $f_+(q^2)$ given in Fig.4 are independent of the form $F(q^2)$ defined by Eq.(4.4), the q^2 dependence of dBr/dq^2 is correlated with the form $F(q^2)$. In the present analysis, we have not taken

QCD corrections, form factor effects, and so on, in order to demonstrate photon emission from the spectator quark straightforwardly. Therefore, correspondingly to the treatments, we also simplify the conventional standard model contributions, too. The purpose of the present paper is to demonstrate the spectator quark effects qualitatively, and not to estimate the spectator quark effects quantitatively. In the numerical analysis, since our interest is the difference between $dBr(B^0 \rightarrow K^0 \ell^+ \ell^-)/dq^2$ and $dBr(B^+ \rightarrow K^+ \ell^+ \ell^-)/dq^2$, for simplicity, we have neglected QCD effects and so on. For example, we have regarded the form factor $f_T(q^2)$ as a constant in respect to q^2 . Therefore, the numerical results should be rigidly taken. However, we consider that the qualitative conclusions are reliable since we have treated only relative quantities (ratios and so on).

In the present paper, the origin of b - s transition is not specified, although, for convenience, the formulation has been given for the case of a family gauge boson A_2^3 exchange. In the present paper, the b - s transition is given in the Eq.(3.1), but the definition (3.2) of the coupling constant G_{fam}^q is nothing but an example. The parameter ξ given in Eq.(4.5) is a phenomenological one, at present. The value of ξ is one which should be determined by experiments.

If we consider that the origin is due to the exchange of A_2^3 , the rough estimate of $|\xi|$ from Eq.(4.3) gives $|\xi| \sim 10^{-5} \text{ GeV}^2$ for $M_{23} \sim$ a few TeV, hence such contribution cannot become visible in the family gauge boson model with an inverted mass hierarchy [15] (also in a revised model [16]). We need some enhancement mechanism of the A_2^3 exchange diagrams. On the other hand, we have other diagrams for the source of b - s transition, electroweak penguin, gluon penguin, and so on. Especially, so far, the gluon penguin has been neglected in the operator expansion approach. If we replace the family gauge boson A_2^3 with gluon g from the gluon penguin, the value of ξ can be sizable. Therefore, we may rather regard the parameter ξ defined by Eq.(4.1) as a phenomenological one, discarding Eq.(4.3). Then, the squared mass M_{23}^2 in Eq.(3.2) must be replaced with $\bar{q}^2 = (\bar{p}_1^2 - \bar{p}_2)^2$. The value \bar{q}^2 is calculable in the present prescription. Since our parameters a_1 , b_1 , a_2 and b_2 are small, the value \bar{q}^2 is the order of q^2 . Therefore, the q^2 dependence will be somewhat different from the present result based on the A_2^3 exchange.

If the case that the additional contributions due to A_2^3 shown in Fig.3 should be replaced with those due to gluon penguin, the decay widths of B^0 and B^+ decays are given by the same forms except for the factors $f_+(q^2)$. Since the parameters $\xi(B^0)$ and $\xi(B^+)$ are also given by the same value, the dip in $d\Gamma/dq^2$ can appear only in either B^0 or B^+ decay. (For the case of A_2^3 exchange, $\xi(B^0)$ and $\xi(B^+)$ can take opposite sign each other by supposing $U_{21}^{*u}U_{31}^u/U_{21}^{*d}U_{31}^d < 0$.) At present, the data by Belle [1] and BABAR [2] have shown a possibility that there is a dip in $d\Gamma/dq^2$, but data are not separated between B^0 and B^+ . In the LHCb, we can see a possibility of a dip in the B^0 decay [3], but we cannot see such a dip in the B^+ decay [4]. It seems that this is favor of the gluon penguin model.

Thus, it is our greatest concern whether the data show a dip in dBr/dq^2 both or either in B^0 and/or B^+ decays. We expect that such data will soon be reported.

The present results highly depend on our treatment for the quark-anti-quark bound system. In our prescription, the existence of the quark propagator, which cannot be incorporated into the factorization method, has played an essential role. We have straightforwardly and faithfully calculated the effects based on the effective valence quark model. We think that the present prescription should be worthwhile to be tested by future experimental data.

Acknowledgments

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Appendix A

Although a formulation based on a family gauge boson is only an example to discuss the photon emission from the “spectator” quark inside meson, for convenience, we will give a brief review of a family gauge boson which plays a role in the present scenario. Conventionally, we have considered that family gauge bosons have extremely large masses in order to avoid the constraints from the data of K^0 - \bar{K}^0 mixing. However, in this paper, we assume a gauge boson model with an inverted mass hierarchy [15], i.e. $m^2(A_1^1), m^2(A_2^1), m^2(A_3^1) \gg m^2(A_2^2) \gg m^2(A_2^3) \gg m^2(A_3^3)$, so that we suppose a mass of A_2^3 of an order of 1 – 10 TeV [16]. The model is an extended version of the Sumino model [21]: Sumino has introduced family gauge bosons in order to understand why the charged lepton mass formula [22]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \quad (\text{A.1})$$

is so remarkably satisfied with the pole masses: $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$, while if we take the running masses, the ratio becomes $K(\mu) = (2/3) \times (1.00189 \pm 0.000002)$, for example, at $\mu = m_Z$. The deviation comes from a $\log m_{ei}^2$ term in the QED radiative correction [23]. In the Sumino model, the factor $\log m_{ei}^2$ due to QED correction is canceled by a contribution from family gauge bosons whose masses are given by $m(A_i^j) \equiv M_{ij}^2 = k(m_{ei} + m_{ej})$. Note that in this cancellation mechanism, it is essential

that the mass eigenstates of the family gauge bosons exactly correspond to the mass eigenstates of the charged leptons. In order to realize this cancellation, he has assigned the left- and right-handed charged leptons (e_L, e_R) to $(\mathbf{3}, \mathbf{3}^*)$ of a $U(3)$ family symmetry, so that the family gauge boson currents have a structure of $(V - A) \otimes (V + A)$. However, this assignment is somewhat troublesome phenomenologically. In the modified version [15], for the purpose of making the conventional assignment $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$ possible, a family gauge boson model with an inverted mass hierarchy has been proposed. The model has the following characteristics: (i) We assume a $U(3)$ family symmetry [not $SU(3)$], so that we have nine family gauge bosons (not eight those). (ii) The masses M_{ij} of the gauge bosons A_i^j given by

$$M_{ij}^2 = k \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right), \quad (\text{A.2})$$

are different from those in the Sumino model, $M_{ij}^2 = k(m_{ei} + m_{ej})$. (iii) The family gauge bosons couple to quarks and leptons with a pure vector type. (iv) The gauge coupling constant g_F is not free parameter because of the Sumino's cancellation mechanism as well as in the Sumino model. (v) The family gauge bosons are in the mass-eigenstates on the flavor basis in which the charged lepton mass matrix is diagonal. Therefore, a lepton number violating process never occurs at the tree level of the current-current interaction in the charged leptons, while quarks can mix among them via quark mixing matrices U^u and U^d , so that a family number violated processes can occur at the tree level, as we have shown in Eq.(1.4).

Appendix B

First, at quark level, we obtain the following amplitudes which correspond to the diagrams (a), (b), (c) and (d) in Fig.3:

$$\mathcal{M}_{(a)}^{eff} = i\frac{1}{6}\bar{e}_b e [\bar{u}_d(p_2) \Gamma v_s(\bar{p}_2)] \left[\bar{v}_b(\bar{p}_1) \gamma_\mu \frac{\not{\ell}_{(a)} + m_b}{\ell_{(a)}^2 - m_b^2} \Gamma u_d(p_1) \right] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.1})$$

$$\mathcal{M}_{(b)}^{eff} = i\frac{1}{6}\bar{e}_b e \left[\bar{u}_d(p_2) \Gamma \frac{\not{\ell}_{(b)} + m_s}{\ell_{(b)}^2 - m_s^2} \gamma_\mu v_s(\bar{p}_2) \right] [\bar{v}_b(\bar{p}_1) \Gamma u_d(p_1)] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.2})$$

$$\mathcal{M}_{(c)}^{eff} = i\frac{1}{6}e_d e [\bar{u}_d(p_2) \Gamma v_s(\bar{p}_2)] \left[\bar{v}_b(\bar{p}_1) \Gamma \frac{\not{\ell}_{(c)} + m_d}{\ell_{(c)}^2 - m_d^2} \gamma_\mu u_d(p_1) \right] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.3})$$

$$\mathcal{M}_{(d)}^{eff} = i\frac{1}{6}e_d e \left[\bar{u}_d(p_2) \gamma_\mu \frac{\not{\ell}_{(d)} + m_d}{\ell_{(d)}^2 - m_d^2} \Gamma v_s(\bar{p}_2) \right] [\bar{v}_b(\bar{p}_1) \Gamma u_d(p_1)] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.4})$$

where

$$\ell_{(a)} = \bar{p}_1 - q, \quad \ell_{(b)} = \bar{p}_2 + q, \quad \ell_{(c)} = p_1 - q, \quad \ell_{(d)} = p_2 + q, \quad (\text{B.5})$$

and the common coefficient G_{fam}^{eff} has been dropped. Here, in order to provide for the next step in which we obtain hadronic current form from the quark current form, the expressions (B.1) - (B.4) have been given by using a Fierz transformation

$$(\bar{b}\gamma_\rho s)(\bar{d}\gamma^\rho d) \Rightarrow \sum_\Gamma \left[-\frac{1}{3}(\bar{d}\Gamma s)(\bar{b}\Gamma d) - \frac{1}{2} \sum_{a=1}^8 (\bar{d}\Gamma \lambda_a s)(\bar{b}\Gamma \lambda_a d) \right], \quad (\text{B.6})$$

where

$$\Gamma \otimes \Gamma = -\mathbf{1} \otimes \mathbf{1} + \gamma_5 \otimes \gamma_5 + \frac{1}{2}\gamma_\rho \otimes \gamma^\rho + \frac{1}{2}\gamma_\rho \gamma_5 \otimes \gamma^\rho \gamma_5. \quad (\text{B.7})$$

Next, we must translate the amplitudes (B.1) - (B.4) in quark level into those in hadronic level. We use the prescription (3.11). We obtain the following decay amplitudes

from (B.1) - (B.4):

$$\mathcal{M}_a = i \frac{e^2}{18} f_K f_B \frac{1}{\Delta_a} [(\bar{p}_1 - q)_\mu (P_B P_K) + P_{B\mu} (\bar{p}_1 - q) P_K - P_{K\mu} (\bar{p}_1 - q) P_B] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.8})$$

$$\mathcal{M}_b = i \frac{e^2}{18} f_K f_B \frac{1}{\Delta_b} [(\bar{p}_2 + q)_\mu (P_B P_K) + P_{K\mu} (\bar{p}_2 + q) P_B - P_{B\mu} (\bar{p}_2 + q) P_K] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.9})$$

$$\mathcal{M}_c = -i \frac{e^2}{18} f_K f_B \frac{1}{\Delta_c} [(p_1 - q)_\mu (P_B P_K) + P_{B\mu} (p_1 - q) P_K - P_{K\mu} (p_1 - q) P_B] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.10})$$

$$\mathcal{M}_d = -i \frac{e^2}{18} f_K f_B \frac{1}{\Delta_d} [(p_2 + q)_\mu (P_B P_K) + P_{K\mu} (p_2 + q) P_B - P_{B\mu} (p_2 + q) P_K] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{B.11})$$

When we use the expression (2.4), we obtain the following form for the meson currents:

$$\mathcal{M} = i \frac{e^2}{18} f_K f_B \frac{1}{2} [f_+(q^2) (P_B + P_K)_\mu + f_-(q^2) (P_B - P_K)_\mu] \frac{1}{q^2} [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)]. \quad (\text{B.12})$$

The second term with $q_\mu = (P_B - P_K)_\mu$ in Eq.(B.12) does not contribute the decay amplitude because of $q_\mu [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)] = 0$ for $m_{\ell 1} = m_{\ell 2}$. For the expression $f_+(q^2)$, we obtain

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) - f_+^c(q^2) - f_+^d(q^2), \quad (\text{B.13})$$

where

$$f_+^a(q^2) = \frac{(x_1 - 2a_1)M_K^2 + (1 - x_1 + a_1 + b_1)q^2}{-(x_1 - 2a_1)\Delta_{BK}^2 + (1 - x_1 + 2b_1)q^2}, \quad (\text{B.14})$$

$$f_+^b(q^2) = \frac{(x_2 - 2a_2)M_K^2 + (1 - x_2 + a_2 + b_2)q^2}{(x_2 - 2a_2)\Delta_{BK}^2 + (1 - x_2 + 2b_2)q^2}, \quad (\text{B.15})$$

$$f_+^c(q^2) = \frac{2a_1 M_K^2 + (1 - a_1 - b_1)q^2}{-2a_1 \Delta_{BK}^2 + (1 - 2b_1)q^2}, \quad (\text{B.16})$$

$$f_+^d(q^2) = \frac{2a_2 M_B^2 + (1 - a_2 - b_2)q^2}{2a_2 \Delta_{BK}^2 + (1 - 2b_2)q^2}. \quad (\text{B.17})$$

A form of $f_+(q^2)$ for the decay $B^+ \rightarrow K^+\ell^+\ell^-$ can be obtained by replacing $e_d = -e/3 \rightarrow e_u = +2e/3$ in (B.13):

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) + 2f_+^c(q^2) + 2f_+^d(q^2). \quad (\text{B.18})$$

Appendix C

The function $F(q^2)$ corresponds to $d\Gamma/dq^2$ for the conventional electroweak photon penguin, and it is calculated from the matrix element

$$\mathcal{M} = G(P_B + P_K)_\mu \bar{\ell}(k_2)\gamma^\mu \ell(k_1), \quad (\text{C.1})$$

where G is defined by Eq.(4.2). By defining a parameter $y \equiv m_{\ell K}^2 = (k_2 + P_K)^2$ together with $y_1 = y_{min}$ and $y_2 = y_{max}$, the form $F(q^2)$ is represented as

$$\begin{aligned} G^2 F(x) &\equiv \frac{1}{(2\pi)^3} \frac{1}{32M_B^3} \int_{y_1}^{y_2} dy |\mathcal{M}|^2 \\ &= -\frac{1}{(2\pi)^3} \frac{1}{32M_B^3} \left[\frac{1}{3}(y_2^3 - y_1^3) + \frac{1}{2}a(y_2^2 - y_1^2) + b(y_2 - y_1) \right], \end{aligned} \quad (\text{C.2})$$

where

$$a = q^2 - (M_B^2 + M_K^2 + 2m_\ell^2), \quad (\text{C.3})$$

$$b = (M_B^2 + M_K^2)(M_K^2 + m_\ell^2) - m_\ell^2 q^2. \quad (\text{C.4})$$

Appendix D

The coefficients (a_1, b_1) can be obtained as follows. When we define

$$A = 2(M_B^2 + M_K^2) - q^2, \quad B = q^2, \quad C = \Delta_{BK}^2, \quad (\text{D.1})$$

from Eq.(2.5), we obtain a relation between a_1 and b_1 :

$$b_1 = \frac{1}{B} \left[-Ca_1 \pm \sqrt{Da_1^2 + Bm_{d1}^2} \right], \quad (\text{D.2})$$

i.e.

$$b_1 = \frac{1}{q^2} \left[-a_1 \Delta_{BK}^2 \pm \sqrt{Da_1^2 + m_{d1}^2 q^2} \right], \quad (\text{D.3})$$

where

$$\begin{aligned} D &\equiv C^2 - AB = (\Delta_{BK}^2)^2 - q^2[2(M_B^2 + M_K^2) - q^2] \\ &= [(M_B - M_K)^2 - q^2] [(M_B + M_K)^2 - q^2]. \end{aligned} \quad (\text{D.4})$$

By substituting Eq.(D.4) into Eq.(2.7), we obtain a relation for a_1

$$x_1^2 M_B^2 + m_{d1}^2 - m_b^2 = \frac{x_1}{q^2} \left[-a_1 D \pm (\Delta_{BK}^2 + q^2) \sqrt{a_1^2 D + m_{d1}^2 q^2} \right]. \quad (\text{D.5})$$

The parameter a_1 can be obtained by solving Eq.(D.5) for a_1 .

Appendix E

More exactly speaking, the Eq.(4.1) should be replaced by

$$\mathcal{M} = G(q^2) \left(1 + \xi \frac{f_T(0)}{f_T(q^2)} \frac{f_+(q^2)}{q^2} \right) (P_B + P_K)_\mu [\bar{v}_\ell(k_2) \gamma^\mu u_\ell(k_1)], \quad (\text{E.1})$$

where

$$G(q^2) = G_{EW}^{eff} \frac{2m_b f_T(q^2)}{M_B + M_K}. \quad (\text{E.2})$$

The parameter ξ is defined by

$$\xi = \frac{g_{fam}^2}{g_w^2} \frac{8M_w^2}{M_{23}^2} \frac{U_{33}^{*d} U_{22}^d U_{21}^{*d} U_{31}^d}{V_{ts}^* V_{tb}} \frac{\pi^2}{9} \frac{M_B + M_K}{2m_b f_T(0)} f_K f_B, \quad (\text{E.3})$$

which is unchanged from Eq.(4.3). Then, $d\Gamma/dq^2$ is given by

$$\frac{d\Gamma}{dq^2}(B \rightarrow K \ell^+ \ell^-) = G^2(q^2) \left(1 + \xi \frac{f_T(0)}{f_T(q^2)} \frac{f_+(q^2)}{q^2} \right)^2 F(q^2). \quad (\text{E.4})$$

In order to compare with the over-simplified previous result Fig.6, we illustrate the behavior of dBr/dq^2 for the same value of ξ , where parameters of the form factor $f_T(q^2)$ have been quoted from Ref.[9]. We can see that the numerical results are almost not changed.

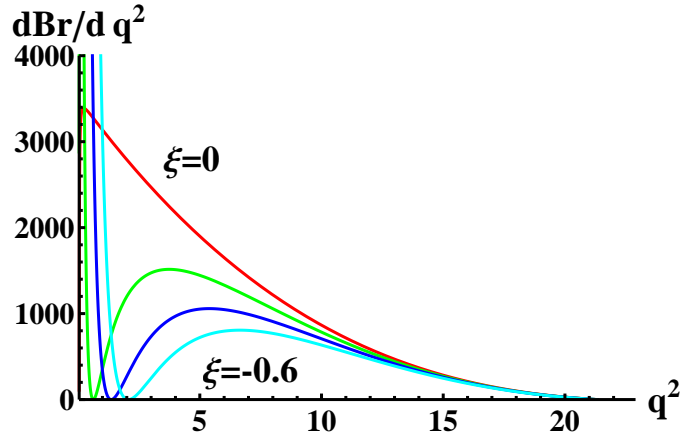


FIG. 10: Behavior of dBr/dq^2 in the decay $B^0 \rightarrow K^0 \ell^+ \ell^-$ in the unit of G defined by Eq.(E.1). Curves are lined up in order of the cases $\xi = 0, 0.2, 0.4$ and 0.6 in the unit of GeV^2 (the colors red, green, blue and cyan, respectively).

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