Effective Valence Quark Model and a Possible Dip in $dBr(B \to K\ell\bar{\ell})/dq^2$

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Abstract

In rare B meson decays $B \to K\ell^+\ell^-$, a possible contribution of $\ell^+\ell^-$ emission via photon from the "spectator" quark q $(q = u, d)$ in the B meson $(q\bar{b})$ is investigated in addition to the conventional one $\bar{b} \to \bar{s}+\gamma \to \bar{s}+\ell^+ +\ell^-$. As a result of the interference between the conventional one and a new one, a dip appears in $d\Gamma(B \to K\ell^+\ell^-)/dq^2$ at a small region of $q^2 \equiv m^2_{\ell\ell}$. The interference effect in the B^0 decay will be observed differently from that in the B^+ decay.

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1. INTRODUCTION

Recent observations [1, 2] of the bottom meson decays $B \to K\ell^+\ell^-$ seem to reveal an interesting feature: the observed q^2 dependence [1, 2] of the differential branching fraction, $dBr(B \to K\ell^+\ell^-)/dq^2$, seems to have a dip at a small value of q^2 ($\equiv m_{\ell\ell}^2$), i.e. $q^2 \sim 1$ GeV^2. Of course, we cannot deduce such the existence of a dip only from the present B decay data, because the amount of the data is still not sufficient. However, if this prospect is the fact, it means that there is a new contribution to the decays $B \to K\ell^+\ell^-$ in addition to the conventional electroweak penguin decay [3],

$$\mathcal{H}^{eff} = G_{EW}^{eff}\frac{1}{e}(\bar{s}\sigma_{\mu\nu}b_R)F^{\mu\nu}\,, \tag{1.1}$$

where

$$G_{EW}^{eff} = \frac{G_F}{\sqrt{2}}\frac{\alpha}{\pi}V_{ts}^*V_{tb}\,2m_b\,, \tag{1.2}$$

and, for simplicity, we have dropped contribution from b_L. In the conventional analysis [3], they use effective Hamiltonian to perform this transition (see for a review [4]). Although we have certainly q^2 dependence in their Hamiltonian, we omit such term due to smallness of its Wilson coefficient. As a result, the differential branching fraction does not have the q^2 pole and it cannot also explain the dip at small q^2 region. On the other hand, in the recent analysis [5], they have focused on the low recoil region, that is the large $\sqrt{q^2}$ of the order of the b-quark mass, and improved how to analyze.

The purpose of the present paper is not to discuss the absolute value of $Br(B \to K\ell^+\ell^-)$ quantitatively, but to discuss the shape of $dBr(B \to K\ell^+\ell^-)/dq^2$ qualitatively. Therefore, for simplicity, we do not pay attention to the QCD corrections of the effective coupling constant G_{EW}^{eff} and so on. Although we discuss a new effect of a family gauge boson in this paper, our interest is not even an effect due to by a new particle as we state in the final section. In this paper, we will point out that there is a contribution to B meson decay which has neglected in the conventional operator expansion approach. We will speculate that if the observed dip in the q^2 distribution of $B \to K\ell^+\ell^-$ is true, a contribution due to photon emission from the "spectator" quark is important. (The terminology "spectator" quark is somewhat misleading: In this case, the "spectator" quark means $q = u, d$ in the B meson $(q\bar{b})$. In the conventional model, a lepton pair via photon is emitted by the effective interaction (1.1), $\bar{b} \to \bar{s} + \gamma$. However, in the present paper, we discuss a case in which a lepton pair via photon is emitted from the "spectator" quark $q = u, d$ when the $b \to s$ transition happens in the opposite side of the $q = u, d$. Nevertheless, we will use the terminology "spectator" quark for $q = u, d$ in the B meson $(q\bar{b})$ for convenience.

Usually, the emission of photon from quarks is considered as that from the transition $b \to s$ (1.1), so that the decay amplitude has no q^2 pole. The interaction gives a decay amplitude of $B \to K\ell^+\ell^-$

$$\mathcal{M} = G_{EW}^{eff} \frac{f_T(q^2)}{M_B + M_K} (P_B + P_K)^\mu [\bar{v}_\ell(k_2)\gamma_\mu u_\ell(k_1)] , \tag{1.3}$$

where $f_T(q^2)$ is a form factor in the meson currents for the effective quark interaction (1.1). However, if the photon can be emitted from the "spectator" quark line, the decay amplitude will have a factor $1/q^2$ differently from the effective interaction (1.1). In this paper, we consider a possibility that photon can be emitted from the "spectator" quark line as shown in Fig.1. (Of course, we will take other three diagrams similar to Fig.1 into consideration as discussed later.) Here A_2^3 is a family gauge boson which changes family number from "2" to "3", and the interaction is given by

$$\mathcal{H}_{fam} = g_F \left[(\bar{e}_i \gamma_\mu e_j) + (\bar{\nu}_i \gamma_\mu \nu_j) + U_{ik}^{*d} U_{jl}^d (\bar{d}_k \gamma_\mu d_l) + U_{ik}^{*u} U_{jl}^u (\bar{u}_k \gamma_\mu u_l) \right] (A_i^j)^\mu . \tag{1.4}$$

Although a formulation based on a family gauge boson is only an example to discuss the photon emission from the "spectator" quark inside meson, for convenience, we will give a brief review of a family gauge boson which plays a role in the present scenario. Conventionally, we have considered that family gauge bosons have extremely large masses in order to avoid the constraints from the data of K^0-$\bar{K}^0$ mixing. However, in this paper, we assume a gauge boson model with an inverse mass hierarchy [6], so that we suppose a mass of A_2^3 of an order of $1-10$ TeV [7]. The model is an extended version of the Sumino model [8]: Sumino has introduced family gauge bosons in order to understand why the charged lepton mass formula [9] $K \equiv (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$ is so remarkably satisfied with the pole masses: $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$, while if we take the running masses, the ratio becomes $K(\mu) = (2/3) \times (1.00189 \pm 0.000002)$, for example, at $\mu = m_Z$. The deviation comes from a $\log m_{ei}^2$ term in the QED radiative correction [10]. In the Sumino model, the factor $\log m_{ei}^2$ due to QED correction is canceled by a contribution from family gauge bosons whose masses are given by $m(A_i^j) \equiv M_{ij}^2 = k(m_{ei} + m_{ej})$. Note that in this cancellation mechanism, it is essential that the mass eigenstates of the family gauge bosons exactly correspond to the mass eigenstates of the charged leptons. In order to realize this cancellation, he has assigned the left- and right-handed charged leptons (e_L, e_R) to $(\mathbf{3}, \mathbf{3}^*)$ of a U(3) family symmetry, so that the family gauge boson currents have a structure of $(V-A) \otimes (V+A)$. However, this assignment is somewhat troublesome phenomenologically. In the modified version [6], for the purpose of making the conventional assignment $(e_L, e_R) = (\mathbf{3}, \mathbf{3})$ possible, a family gauge boson model with an inverted mass hierarchy has been proposed. The model has the following characteristics: (i) We assume a U(3) family symmetry [not SU(3)], so that we have nine family gauge bosons (not eight those). (ii) The masses M_{ij} of the gauge bosons A_i^j given by $M_{ij}^2 = k(m_{ei}^{-1} + m_{ej}^{-1})$, are different from those in the Sumino model $M_{ij}^2 = k(m_{ei} + m_{ej})$. (iii) The family gauge bosons couple to quarks and leptons with a pure vector type. (iv) The gauge coupling constant g_F is not free parameter because of the Sumino's cancellation mechanism as well as in the Sumino model. (v) The family gauge bosons are in the mass-eigenstates on the flavor basis in which the charged lepton mass matrix is diagonal. Therefore, a lepton number violating process never occurs at the tree level of the current-current interaction in the charged leptons, while quarks can mix among them via quark mixing matrices U^u and U^d, so that a family number violated processes can occur at the tree level, as we have shown in Eq.(1.4).

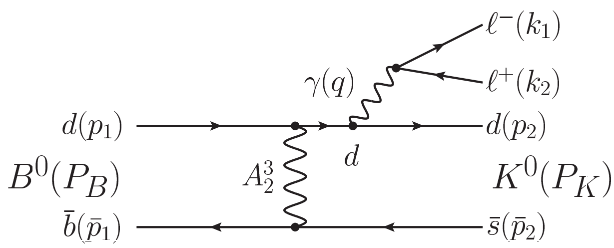


FIG. 1: Feynman diagram for $B^0 \to K^0\ell^+\ell^-$ and the momentum assignments.

In the present paper, for the time being, we assume the mixing among up-quarks is negligibly small compared with that among down-quarks, i.e. $|U_{ij}^u|^2 \ll |U_{ij}^d|^2$, so that we discuss only the case of $B^0 \to K^0 + \ell^+ + \ell^-$:

$$B^0(P_B) \to K^0(P_K) + \ell^-(k_1) + \ell^+(k_2)\,. \tag{1.5}$$

We define momenta of quarks $\bar{b}$ and d inside the bottom meson B^0 as $\bar{p}_1$ and p_1, respectively, and $\bar{s}$ and d inside kaon K^0 as $\bar{p}_2$ and p_2, respectively as shown in Fig.1. We also define the momentum of photon as q in the decay $B^0(P_B) \to K^0(P_K) + \ell^-(k_1) + \ell^+(k_2)$, i.e.

$$q \equiv k_1 + k_2 = P_B - P_K\,. \tag{1.6}$$

In order to know the momenta $\bar{p}_1$, p_1, $\bar{p}_2$ and p_2, we must know dynamical structures of the mesons. In this paper, in order to calculate such diagrams of a new type, we propose an approach as a kind of the effective theory for valence quark diagrams. In the next section, we represent those momenta $\bar{p}_1$, p_1, $\bar{p}_2$ and p_2 in terms of P_B and P_K with the help of an "on-shell quark" assumption. Thereby, we will estimate such diagrams given in Fig.1. Under this prescription, we will find that it is possible for photon to be emitted from d quark.

In Sec.3, we give a form factor-like function $f_+(q^2)$ which gives contribution of photon emission from quarks, and in Sec.4, we put an assumption in order to calculate the function

$f_+(q^2)$ simply. The numerical results are given in Sec.5. Finally, Sec.6 is devoted to the concluding remarks.

2. EFFECTIVE VALENCE QUARK MODEL

In the present paper, we denote momenta of B^0, K^0, $\bar{b}$ and d in the B^0 meson, s and d in the kaon K^0 as P_B, P_K, $\bar{p}_1$ and p_1, $\bar{p}_2$ and p_2, respectively. Our assumption of "on-shell quark" demands that quark masses are given by

$$\bar{p}_1^2 = m_b^2 \,, \quad p_1^2 = m_{d1}^2 \,, \quad \bar{p}_2^2 = m_s^2 \,, \quad p_2^2 = m_{d2}^2 \,, \tag{2.1}$$

where we have left a possibility that the mass of the d quark in the bottom meson can be different from that of the d quark in the kaon, so that we have denoted those as m_{d1} and m_{d2}, respectively. Here, it is our essential assumption that these quark masses are almost constant for q^2, although those are still dependent on the energy scale μ of the system.

We assume that P_B and P_K are given by

$$x_1 P_B = \bar{p}_1 + p_1 \,, \qquad x_2 P_K = \bar{p}_2 + p_2 \,, \tag{2.2}$$

where x_1 (x_2) is a fraction of momenta p_1 and $\bar{p}_1$ (p_2 and $\bar{p}_2$) of the valence quarks d and $\bar{b}$ (d and $\bar{b}$) versus the meson momentum P_B (P_K). From the constraint (2.2), we have the following relations

$$x_1^2 M_B^2 + m_{d1}^2 - 2x_1(p_1 P_B) = m_b^2 \,, \qquad x_2^2 M_K^2 + m_{d2}^2 - 2x_2(p_2 P_K) = m_s^2 \,, \tag{2.3}$$

respectively.

Under the on-shell assumption, the quark momenta p_1 and p_2 can be expressed in terms of P_B and P_K:

$$\begin{aligned} p_1^\mu &= a_1(P_B + P_K)^\mu + b_1(P_B - P_K)^\mu \,, \\ p_2^\mu &= a_2(P_B + P_K)^\mu + b_2(P_K - P_B)^\mu \,, \end{aligned} \tag{2.4}$$

where the coefficients a_1, b_1, a_2 and b_2 can in general, be functions of q^2. Then, we can obtain relations

$$m_{d1}^2 = a_1^2[2(M_B^2 + M_K^2) - q^2] + b_1^2\, q^2 + 2a_1 b_1 \Delta_{BK}^2 \,, \tag{2.5}$$

$$m_{d2}^2 = a_2^2[2(M_B^2 + M_K^2) - q^2] + b_2^2\, q^2 - 2a_2 b_2 \Delta_{BK}^2 \,, \tag{2.6}$$

from Eq.(2.1), and

$$x_1^2 M_B^2 + m_{d1}^2 - m_b^2 = x_1 a_1 \left[2(M_B^2 + M_K^2) + \Delta_{BK}^2 - q^2\right] + x_1 b_1 \left(\Delta_{BK}^2 + q^2\right) \,, \tag{2.7}$$

$$x_2^2 M_K^2 + m_{d2}^2 - m_s^2 = x_2 a_2 \left[2(M_B^2 + M_K^2) - \Delta_{BK}^2 - q^2\right] + x_2 b_2 \left(-\Delta_{BK}^2 + q^2\right) \,, \tag{2.8}$$

from Eq.(2.4), where

$$\Delta_{BK}^2 \equiv M_B^2 - M_K^2 \,. \tag{2.9}$$

Thus, if we give values of x_1 and x_2, we can completely determine the coefficients (a_1, b_1) from the two relations (2.5) and (2.7), and (a_2, b_2) from the two relations (2.6) and (2.8), respectively. Here, note that the replacement $(M_B, m_b, m_{d1}) \to (M_K, m_s, m_{d2})$ gives $(a_1, b_1) \to (a_2, b_2)$. Therefore, hereafter, we will discuss only the relations as to (a_1, b_1).

The coefficients (a_1, b_1) can be obtained as follows. From Eq.(2.5), we obtain a relation between a_1 and b_1 (see Appendix C):

$$b_1 = \frac{1}{q^2}\left[-a_1\Delta_{BK}^2 \pm \sqrt{Da_1^2 + m_{d1}^2 q^2}\right] , \tag{2.10}$$

where

$$D = \left[(M_B - M_K)^2 - q^2\right]\left[(M_B + M_K)^2 - q^2\right] . \tag{2.11}$$

By substituting Eq.(2.10) into Eq.(2.5), we obtain a relation for a_1

$$x_1^2 M_B^2 + m_{d1}^2 - m_b^2 = \frac{x_1}{q^2}\left[-a_1 D \pm (\Delta_{BK}^2 + q^2)\sqrt{a_1^2 D + m_{d1}^2 q^2}\right] . \tag{2.12}$$

The parameter a_1 can be obtained by solving Eq.(2.12) for a_1.

The relation (2.12) brings a new constraint to the model: Let us consider a limit of $q^2 = q^2_{max}$, where

$$q^2_{max} \equiv (M_B - M_K)^2 \,, \tag{2.13}$$

and it gives

$$D(q^2)|_{q^2 = q^2_{max}} = 0 \,. \tag{2.14}$$

Therefore, the relation (2.12) at a limit of $q^2 = q^2_{max}$ leads to a constraint

$$x_1 M_B = m_b \pm m_{d1} \,. \tag{2.15}$$

Similarly, we obtain a constraint

$$x_2 M_K = m_s \pm m_{d2} \,. \tag{2.16}$$

(Note that the sign $\pm$ in Eq.(2.15) corresponds to the sign $\pm$ in the relation (2.12), but the sign $\pm$ in Eq.(2.15) and $\pm$ in Eq.(2.16) are independent each other.)

Quark masses m_b, m_s and m_d are function of the energy scale μ, but this does not always mean those are functions of q^2 directly. We consider that the quark mass values m_b, m_s and m_d in the $B \to K\ell^+\ell^-$ decays are described by those at $\mu \sim M_B$. Then, we can estimate the values of x_1 and x_2 from Eqs.(2.15) and (2.17), so that we can also estimate the values of (a_1, b_1) and (a_2, b_2).

More discussions from a phenomenological point of view will be given in Sec.4.

3. CONTRIBUTION FROM THE FAMILY GAUGE BOSON A_2^3

We assume the following interactions for $B \to K\ell^+\ell^-$ in addition to the conventional $b \to s + \gamma$ interaction (1.1):

$$\mathcal{H} = \sum_{q=u,d,b,s} e_q(\bar{q}\gamma_\mu q)A^\mu - \sum_{\ell=e,\mu} e(\bar{\ell}\gamma_\mu \ell)A^\mu + \sum_{q=u,d} G^q_{fam}(\bar{b}\gamma_\rho s)(\bar{q}\gamma^\rho q)\,, \tag{3.1}$$

where $e_d = e_s = e_b = -e/3$, $e_u = 2e/3$, and

$$G^q_{fam} = \frac{g_F^2}{M_{23}^2} U^{*d}_{33} U^d_{22} U^{*q}_{21} U^q_{31}\,. \tag{3.2}$$

Here, U^q_{ij} are mixing matrix elements among quarks $q = (q_1, q_2, q_3)$, and M_{23} is a mass of a family gauge boson A_2^3. Based on the interactions (3.1), we calculate the following four diagrams for $B^0 \to K^0\ell^+\ell^-$ as shown in Fig.2. Hereafter, since we are interested in a case $B^0 \to K^0\ell^+\ell^-$, we will calculate only the case. Another case $B^+ \to K^+\ell^+\ell^-$ can easily be obtained by replacing $e_d \to e_u$ and $U^d \to U^u$.

Denominators of the propagators with momenta ℓ shown in Figs.2 (a), (b), (c) and (d) are given as follows:

$$\Delta_a \equiv \ell_{(a)}^2 - m_b^2 = q^2 - 2\bar{p}_1 q\,, \tag{3.3}$$
$$\Delta_b \equiv \ell_{(b)}^2 - m_s^2 = q^2 + 2\bar{p}_2 q\,, \tag{3.4}$$
$$\Delta_c \equiv \ell_{(c)}^2 - m_{d1}^2 = q^2 - 2p_1 q\,, \tag{3.5}$$
$$\Delta_d \equiv \ell_{(d)}^2 - m_{d2}^2 = q^2 + 2p_2 q\,, \tag{3.6}$$

respectively. By using the coefficients defined by Eq.(2.4), the expressions (3.3) - (3.6) are rewritten as follows:

$$\Delta_a = (2a_1 - x_1)\Delta_{BK}^2 + (1 - x_1 + 2b_1)q^2\,, \tag{3.7}$$
$$\Delta_b = -(2a_2 - x_2)\Delta_{BK}^2 + (1 - x_2 + 2b_2)q^2\,, \tag{3.8}$$
$$\Delta_c = -2a_1\Delta_{BK}^2 + (1 - 2b_1)q^2\,, \tag{3.9}$$
$$\Delta_d = 2a_2\Delta_{BK}^2 + (1 - 2b_2)q^2\,. \tag{3.10}$$

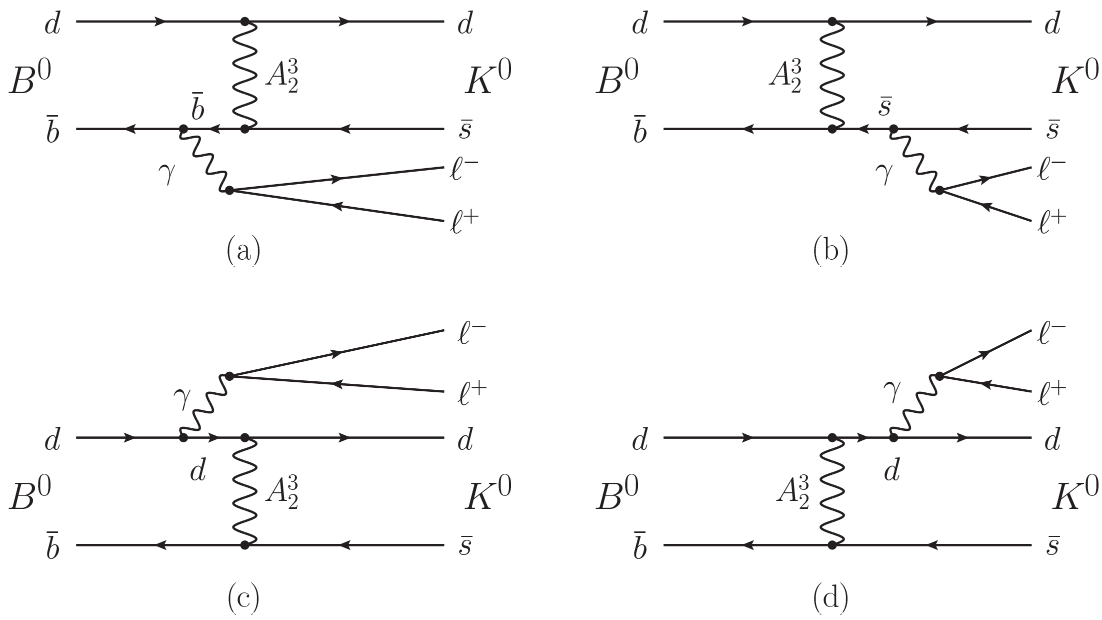


FIG. 2: Feynman diagrams for $B^0 \to K^0\ell^+\ell^-$.

In order to translate effective interactions among quarks into hadronic fields, we use

$$\langle 0|(\bar{b}\gamma^\mu\gamma_5 d)|B^0(P_B)\rangle = -iP_B^\mu f_B\,, \tag{3.11}$$

and so on. The details to obtain amplitudes which correspond to the diagrams (a), (b), (c) and (d) in Fig.2 are given in Appendix A.

When we use the expression (2.4), we obtain the following form for the meson currents:

$$\mathcal{M} = i\frac{1}{6}\bar{e}_b e f_K f_B G_{fam}\frac{1}{2}\left[f_+(q^2)(P_B+P_K)_\mu + f_-(q^2)(P_B-P_K)_\mu\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \tag{3.12}$$

where G_{fam} is defined by Eq.(3.2) and we have dropped the index $q = d$ because it is obvious that we calculate a case of $B^0 \to K^0\ell^+\ell^-$.

The second term with $q_\mu = (P_B - P_K)_\mu$ in Eq.(3.12) does not contribute the decay amplitudes because of $q_\mu[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)] = 0$ for $m_{\ell 1} = m_{\ell 2}$. For the expression $f_+(q^2)$, we obtain

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) - f_+^c(q^2) - f_+^d(q^2)\,, \tag{3.13}$$

where

$$f_+^a(q^2) = \frac{(x_1 - 2a_1)M_K^2 + (1 - x_1 + a_1 + b_1)q^2}{-(x_1 - 2a_1)\Delta_{BK}^2 + (1 - x_1 + 2b_1)q^2}\,, \tag{3.14}$$

$$f_+^b(q^2) = \frac{(x_2 - 2a_2)M_B^2 + (1 - x_2 + a_2 + b_2)q^2}{(x_2 - 2a_2)\Delta_{BK}^2 + (1 - x_2 + 2b_2)q^2}\,, \tag{3.15}$$

$$f_+^c(q^2) = \frac{2a_1 M_K^2 + (1 - a_1 - b_1)q^2}{-2a_1\Delta_{BK}^2 + (1 - 2b_1)q^2}\,, \tag{3.16}$$

$$f_+^d(q^2) = \frac{2a_2 M_B^2 + (1 - a_2 - b_2)q^2}{2a_2\Delta_{BK}^2 + (1 - 2b_2)q^2}\,. \tag{3.17}$$

So far we have discussed the case of the decay mode $B^0 \to K^0\ell^+\ell^-$, because we have considered that up-quark mixing will be considerably small compared with down-quark mixing, $|U_{ij}^u|^2 \ll |U_{ij}^d|^2$. However, we can easily calculate the case $B^+ \to K^+\ell^+\ell^-$ similarly to the case $B^0 \to K^0\ell^+\ell^-$: A form of $f_+(q^2)$ for the decay $B^+ \to K^+\ell^+\ell^-$ can be obtained by replacing $e_d = -e/3 \to e_u = +2e/3$ in Eq.(3.12), i.e.

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) + 2f_+^c(q^2) + 2f_+^d(q^2)\,. \tag{3.18}$$

4. INTERFERENCE EFFECT IN $d\Gamma/dq^2$

The partial decay width $\Gamma(B \to K\ell^+\ell^-)$ is calculated from the matrix element

$$\mathcal{M} = G\left(1 + \xi\frac{f_+(q^2)}{q^2}\right)(P_B + P_K)_\mu[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \tag{4.1}$$

where

$$G = G_{EW}^{eff}\frac{2m_b f_T(0)}{M_B + M_K}\,, \tag{4.2}$$

and, for simplicity, we have neglected the q^2 dependence of the form factor $f_T(q^2)$ in the conventional model. The parameter ξ is defined by

$$\xi = \frac{g_{fam}^2}{g_w^2}\frac{8M_w^2}{M_{23}^2}\frac{U_{33}^{*d}U_{22}^d U_{21}^{*d}U_{31}^d}{V_{ts}^* V_{tb}}\frac{\pi^2}{9}\frac{M_B + M_K}{2m_b f_T(0)}f_K f_B\,, \tag{4.3}$$

but, at present, it is a free parameter whose value is phenomenologically determined by the observed q^2 dependence of dBr/dq^2. Let us define a function $F(q^2)$ as

$$G^2 F(q^2) \equiv \left. \frac{d\Gamma}{dq^2} \right|_{\xi=0} = \frac{1}{(2\pi)^3} \frac{1}{32 M_B^3} \int_{y_1}^{y_2} dy \, |\mathcal{M}|^2_{\xi=0} \, , \tag{4.4}$$

where $y \equiv m_{\ell K}^2 = (k_2 + P_K)^2$, and $y_1 = y_{min}$, $y_2 = y_{max}$. Then, $d\Gamma/dq^2$ is given by

$$\frac{d\Gamma}{dq^2}(B \to K \ell^+ \ell^-) = G^2 \left(1 + \xi \frac{f_+(q^2)}{q^2} \right)^2 F(q^2) \, . \tag{4.5}$$

The explicit form of $F(q^2)$ is given in Appendix B.

Now, we can numerically evaluate the function $f_+(q^2)$ and $d\Gamma/dq^2$ by using these formulas (4.1) - (4.5). First, we give quark mass values $m_b(\mu)$, $m_s(\mu)$, and $m_{d1}(\mu) = m_{d2}(\mu)$ at $\mu = M_B - M_K$. Then, we obtain the parameter values x_1 and x_2 by the relations (2.15) and (2.16). We assume that the quark mass values in this prescription are almost independent of q^2, and those are only dependent on the value μ. We assume that these quark mass values at $\mu = M_B - M_K$ are approximately not so deviated from those at $\mu = M_B$, so that we use the values which are determined by using (2.15) and (2.16). The coefficients a_1 (a_2) can be obtained by using Eq.(2.12) and then a_1 (a_2) can be get by using Eq.(2.10). We will obtain two solutions for Eq.(2.12). Note that the coefficients (a_1, b_1) and (a_2, b_2) are, in general, given as functions of q^2 which are complex form of q^2.

However, in order to give a more concise form of (a_1, b_1) and (a_2, b_2), let us put the following phenomenological assumption: the coefficients (a_1, b_1) and (a_2, b_2) have no q^2 dependence approximately. This requirement demands $a_1 = b_1$ ($a_2 = b_2$) as seen in Eqs.(2.5) and (2.7) [Eqs.(2.6) and (2.8)]. Then, we obtain concise forms

$$a_1 = b_1 = \pm \frac{m_{d1}}{2M_B} \, , \quad a_2 = b_2 = \pm \frac{m_{d2}}{2M_K} \, , \tag{4.6}$$

from Eq.(2.12). The sign $\pm$ in (4.6) corresponds to $\pm$ in Eq.(2.12), but the sign $\pm$ in $a_1 = b_1$ need not to correspond to that in $a_2 = b_2$. By using this solution (4.6), the expressions (3.14) - (3.17) are rewritten as follows:

$$f_+^a(q^2) = \frac{m_b M_K^2 + (M_B - m_b)q^2}{-m_b \Delta_{BK}^2 + (M_B - m_b)q^2}\,, \tag{4.7}$$

$$f_+^b(q^2) = \frac{m_s M_B^2 + (M_K - m_s)q^2}{-m_s \Delta_{BK}^2 + (M_K - m_s)q^2}\,, \tag{4.8}$$

$$f_+^c(q^2) = \frac{\pm m_{d1} M_K^2 + (M_B \mp m_{d1})q^2}{\mp m_{d1} \Delta_{BK}^2 + (M_B \mp m_{d1})q^2}\,, \tag{4.9}$$

$$f_+^d(q^2) = \frac{\pm m_{d2} M_B^2 + (M_K \mp m_{d2})q^2}{\pm m_{d2} \Delta_{BK}^2 + (M_K \mp m_{d2})q^2}\,. \tag{4.10}$$

Note that $f_+^a(q^2)$ and $f_+^b(q^2)$ are independent of the choices $\pm$ in Eq.(4.6), but $f_+^c(q^2)$ and $f_+^d(q^2)$ are dependent on the choices. If we take the sign $+$ for $a_1 = b_1$ in (4.6), then the function $f_+^c(q^2)$ will have a pole at $q^2 = m_{d1}\Delta_{BK}^2/(M_B - m_{d1})$. Also, if we take the sign $-$ in Eq.(4.6), then the function $f_+^d(q^2)$ will have a pole at $q^2 = m_{d2}\Delta_{BK}^2/(M_K + m_{d2})$. Therefore, in the numerical estimate of $d\Gamma/dq^2$, we take the signs in Eq.(4.6) as follows:

$$a_1 = b_1 = -\frac{m_{d1}}{2M_B}\,, \quad a_2 = b_2 = +\frac{m_{d2}}{2M_K}\,. \tag{4.11}$$

5. NUMERICAL RESULTS

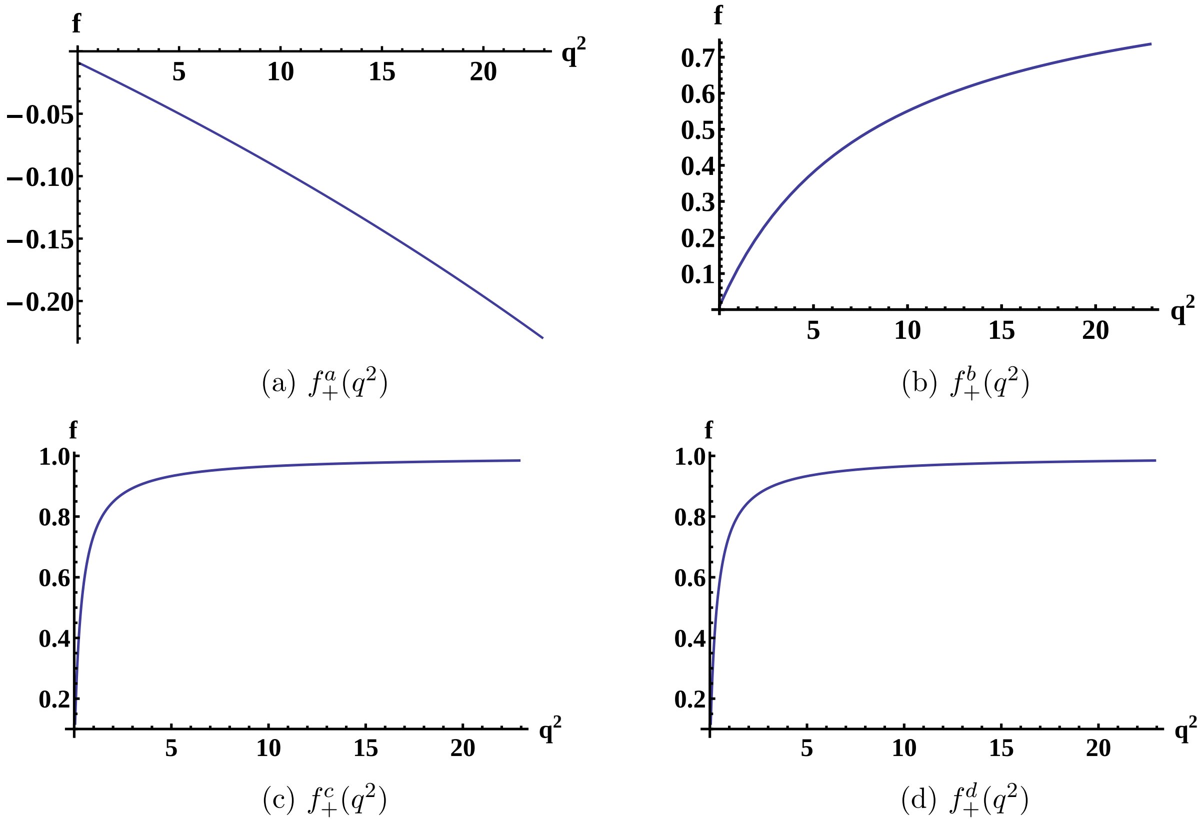
For numerical estimates, for convenience, we adopt quark mass values [11] at $\mu = m(m_b) = 4.34$ GeV in place of those at $\mu = M_B - M_K$:

$$m_b = 4.34 \text{ GeV}\,, \quad m_s = 0.126 \text{ GeV}\,, \quad m_d \equiv m_{d1} = m_{d2} = 0.00637 \text{ GeV}\,. \tag{5.1}$$

We may choose another quark values. However, numerical results are almost similar. Hereafter, we use the values (5.1) as typical values in our prescription.

First, in Fig.3, we show the behavior of the functions $f_+^a(q^2)$, $f_+^b(q^2)$, $f_+^c(q^2)$ and $f_+^d(q^2)$ which represent the contributions of photon emissions from b, s, d_1 and d_2 quarks, respectively. Note that although we have chosen the coefficients (a_1, b_1) and (a_2, b_2) so that those are independent of q^2, the functions $f_+^a(q^2)$, $f_+^b(q^2)$, $f_+^c(q^2)$ and $f_+^d(q^2)$ still depend on q^2. We find that $f_+^c(q^2) \simeq +1$ and $f_+^d(q^2) \simeq +1$ except for a small range of q^2. Also, we show the behavior of $f_+(q^2)$ in Fig.4. Note that $f_+(q^2) < 0$ over the whole physical region.

Also, we show the behavior of $dBr(B^0 \to K^0 \ell^+ \ell^-)/dq^2$ in the unit of G defined by Eq.(4.2) for typical values of the parameter ξ in Fig.5. We can obtain a reasonable dip at $q^2 \sim 1$ GeV.



(a) $f_+^a(q^2)$ (b) $f_+^b(q^2)$

(c) $f_+^c(q^2)$ (d) $f_+^d(q^2)$

FIG. 3: Contribution from each quark line to the factor $f_+(q^2)$. Figures are illustrated for a physical range $4m_\mu^2 < q^2 < (M_B - M_K)^2$.

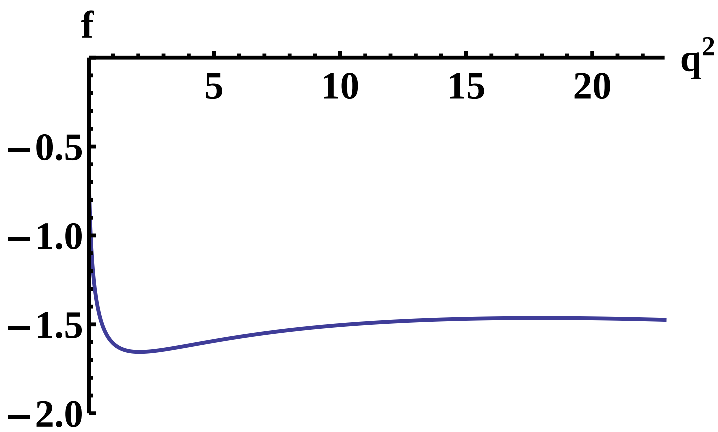


FIG. 4: Behavior of $f_+(q^2)$ in the neutral B meson decay $B^0 \to K^0\ell^+\ell^-$.

Similarly, we can demonstrate the case of $B^+ \to K^+\ell^+\ell^-$. The behaviors of $f_+(q^2)$ and dBr/dq^2 are illustrated in Figs. 6 and 7, respectively. If the up-quark mixing is sizable compared with the down-quark mixing, the case will be also visible. The shape of the dBr/dq^2 in $B^+ \to K^+\ell^+\ell^-$ is almost similar to that in $B^0 \to K^0\ell^+\ell^-$. However,

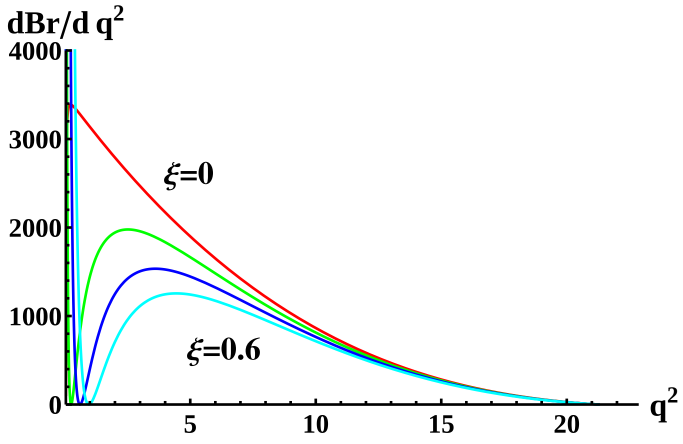


FIG. 5: Behavior of dBr/dq^2 in the decay $B^0 \to K^0\ell^+\ell^-$ in the unit of G defined by Eq.(4.1). Curves are lined up in order of the cases $\xi = 0$, 0.2, 0.4 and 0.6 in the unit of GeV^2 (the colors red, green, blue and cyan, respectively).

note that the dip in dBr/dq^2 appears for $\xi > 0$ in the case $B^0 \to K^0\ell^+\ell^-$, while the dip appears for $\xi < 0$ in the case $B^+ \to K^+\ell^+\ell^-$. It will be possible because $U^{*u}_{21}U^u_{31}$ takes an opposite sign to $U^{*d}_{21}U^d_{31}$.

However, we do not consider that the magnitude of $U^{*u}_{21}U^u_{31}$ is accidentally the same as that of $U^{*d}_{21}U^d_{31}$. We expect that the behavior of dBr/dq^2 in $B^+ \to K^+\ell^+\ell^-$ will be different from that in $B^0 \to K^0\ell^+\ell^-$. We hope data of dBr/dq^2 which are distinguished between $B^0 \to K^0\ell^+\ell^-$. and $B^+ \to K^+\ell^+\ell^-$.

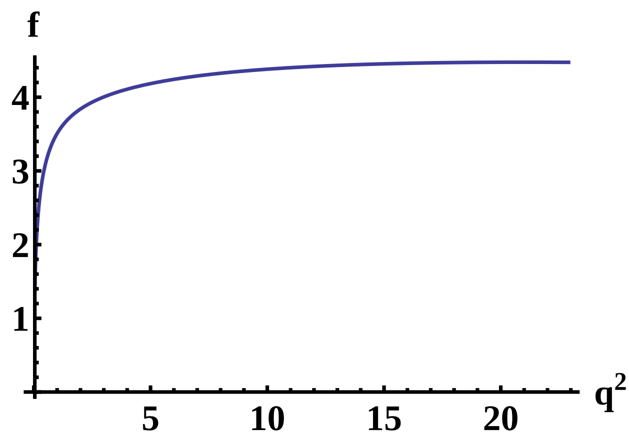


FIG. 6: Behavior of $f_+(q^2)$ in the charged B meson decay $B^+ \to K^+\ell^+\ell^-$.

Finally, we would like to give some comments on the predicted partial decay width.

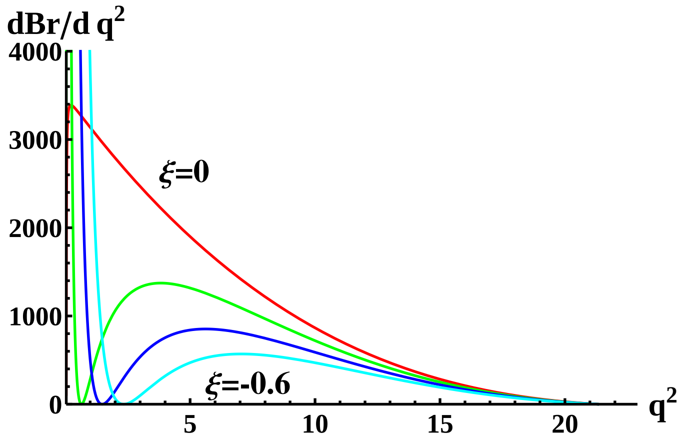


FIG. 7: Behavior of dBr/dq^2 in the decay $B^+ \to K^+\ell^+\ell^-$ in the unit of G^2 defined by Eq.(4.1). Curves are lined up in order of the cases $\xi = 0$, -0.2, -0.4 and -0.6 in the unit of GeV^2 (the colors red, green, blue and cyan, respectively).

The decay width is given by

$$\Gamma(B \to K\ell^+\ell^-) = G^2 \int_{q^2_{min}}^{q^2_{max}} dq^2 F(q^2)\,, \tag{5.2}$$

where the function $F(q^2)$ is defined by Eq.(4.1) and q^2_{min} is given by $q^2_{min} = 4m_\ell^2$. The numerical value is highly sensitive to whether $\ell = \mu$ or $\ell = e$, because the contribution becomes very large at $q^2 \simeq 0$. However, it seems to be impossible to measure accurately until $q^2 = 4m_e^2 = 1.044 \times 10^{-6}$ GeV^2. If we take $q^2_{min} = 4m_\mu^2 = 0.04465$ GeV^2 for the case of $\Gamma(B \to Ke^+e^-)$, too, we cannot find a significant difference between $\Gamma(B \to Ke^+e^-)$ and $\Gamma(B \to K\mu^+\mu^-)$. Another comment is as follows: The predicted decay width $\Gamma(B \to K\ell^+\ell^-)$ is dependent on the value of ξ. We illustrate the behavior $R(\xi) \equiv \Gamma(\xi)/\Gamma(0)$ in Fig.8. The present data [12] show $Br(B^+ \to K^+\ell^+\ell^-) = (5.1 \pm 0.5) \times 10^{-7}$, $Br(B^0 \to K^0\ell^+\ell^-) = (3.1^{+0.8}_{-0.7}) \times 10^{-7}$ and $\tau(B^+)/\tau(B^0) = 1.079 \pm 0.007$, so that we obtain

$$R_{+/0} \equiv \frac{\Gamma(B^+ \to K^+\ell^+\ell^-)}{\Gamma(B^0 \to K^0\ell^+\ell^-)} = 1.52^{+0.42}_{-0.38}\,. \tag{5.3}$$

Although the value has a large error, if we dare to take the center value in (5.3), A case of the value of ξ which gives $R_{+/0} \sim 1.5$ is only in the case $B^+ \to K^+\ell^+\ell^-$. The case with $\xi \sim 0.4$ GeV^2 can also give a reasonable shape of $d\Gamma/dq^2$ as seen in Fig.7. However, this view conflicts with our anticipation that $|U^{*u}_{21}U^u_{31}| \ll |U^{*d}_{21}U^d_{31}|$. We must wait future data of $B^0 \to K^0\ell^+\ell^-$ and $B^+ \to K^+\ell^+\ell^-$ separately.

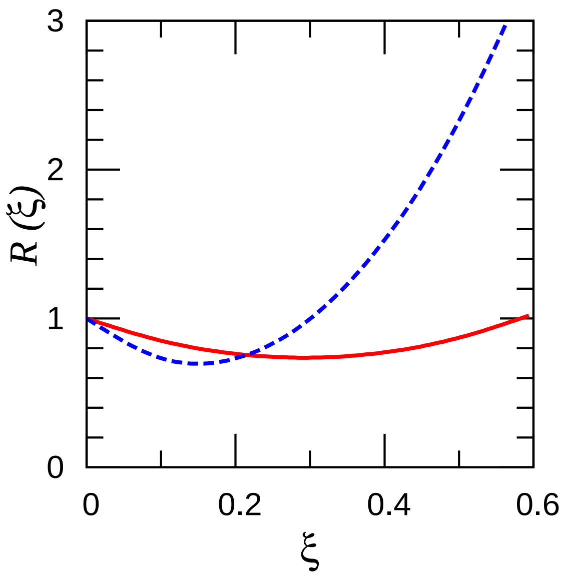


FIG. 8: Behaviors of $R(\xi) \equiv \Gamma(\xi)/\Gamma(0)$ in the decays $B^0 \to K^0\ell^+\ell^-$ (solid curve) and $B^+ \to K^+\ell^+\ell^-$ (dashed curve).

6. CONCLUDING REMARKS

In conclusion, we have discussed a contribution of photon emission from the "spectator" quark which is opposite the $b \to s$ transition in the B meson, and thereby we have obtained interesting results: (i) The contribution from the spectator is large, $f_+^c(q^2) \simeq 1$ and $f_+^d(q^2) \simeq 1$ in contrast to $f_+^a(q^2) \simeq 0$ and $f_+^b(q^2) \simeq 0$ - 0.7. (ii) For a sizable value of the parameter $|\xi|$, we can demonstrate a dip of $Br(B \to K\ell^+\ell^-)$ in the small q^2 region.

In the numerical analysis, since our interest is the difference between $dBr(B^0 \to K^0\ell^+\ell^-)/dq^2$ and $dBr(B^+ \to K^+\ell^+\ell^-)/dq^2$, for simplicity, we have neglected QCD effects and so on. For example, we have regarded the form factor $f_T(q^2)$ as a constant in respect to q^2. Therefore, the numerical results should be rigidly taken. However, we consider that the qualitative conclusions are reliable since we have treated only relative quantities (ratios and so on).

The most controversial point in the present work is as to the magnitude of the parameter ξ. The rough estimate of $|\xi|$ from Eq.(4.3) gives $|\xi| \sim 10^{-5}$ GeV^2 for $M_{23} \sim$ a few TeV [11], so that the value is too small in order that the effect is visible. We need some enhancement mechanism of the A_2^3 exchange diagrams. However, note that the $b \to s$ transition is not only due to the A_2^3 exchange. We have other diagrams, electroweak penguin, gluon penguin, and so on. Especially, so far, the gluon penguin has been neglected in the operator expansion approach. If we replace the family gauge boson A_2^3 with gluon

g from the gluon penguin, the value of ξ can be sizable. Then, the squared mass M_{23}^2 in our calculation must be replaced with $\bar{q}^2 = (\bar{p}_1^2 - \bar{p}_2)^2$. The value $\bar{q}^2$ is calculable in the present prescription. Since our parameters a_1, b_1, a_2 and b_2 are small, the value $\bar{q}^2$ is the order of q^2. Therefore, the q^2 dependence will be somewhat different from the present result based on the A_2^3 exchange.

If the case that the additional contributions due to A_2^3 shown in Fig.2 should be replaced with those due to gluon penguin, the decay widths of B^0 and B^+ decays are given by the same forms except for the factors $f_+(q^2)$. Since the parameters $\xi(B^0)$ and $\xi(B^+)$ are also given by the same value, the dip in $d\Gamma/dq^2$ can appear only in either B^0 or B^+ decay. (For the case of A_2^3 exchange, $\xi(B^0)$ and $\xi(B^+)$ can take opposite sign each other by supposing $U_{21}^{*u}U_{31}^u/U_{21}^{*d}U_{31}^d < 0$.)

Thus, it is our greatest concern whether the data show dip in $d\Gamma/dq^2$ both or either in B^0 and/or B^+ decays. We expect that such data will soon be reported.

Acknowledgments

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Appendix A

First, at quark level, we obtain the following amplitudes which correspond to the diagrams (a), (b), (c) and (d) in Fig.2:

$$\mathcal{M}_{(a)}^{eff} = i\frac{1}{6}\bar{e}_b e\,[\bar{u}_d(p_2)\,\Gamma\,v_s(\bar{p}_2)]\left[\bar{v}_b(\bar{p}_1)\,\gamma_\mu\,\frac{\not{\ell}_{(a)}+m_b}{\ell_{(a)}^2-m_b^2}\,\Gamma\,u_d(p_1)\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \quad \text{(A.1)}$$

$$\mathcal{M}_{(b)}^{eff} = i\frac{1}{6}\bar{e}_b e\left[\bar{u}_d(p_2)\,\Gamma\,\frac{\not{\ell}_{(b)}+m_s}{\ell_{(b)}^2-m_s^2}\,\gamma_\mu\,v_s(\bar{p}_2)\right][\bar{v}_b(\bar{p}_1)\,\Gamma\,u_d(p_1)]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \quad \text{(A.2)}$$

$$\mathcal{M}_{(c)}^{eff} = i\frac{1}{6}e_d e\,[\bar{u}_d(p_2)\,\Gamma\,v_s(\bar{p}_2)]\left[\bar{v}_b(\bar{p}_1)\,\Gamma\,\frac{\not{\ell}_{(c)}+m_d}{\ell_{(c)}^2-m_d^2}\,\gamma_\mu\,u_d(p_1)\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \quad \text{(A.3)}$$

$$\mathcal{M}_{(d)}^{eff} = i\frac{1}{6}e_d e\left[\bar{u}_d(p_2)\,\gamma_\mu\,\frac{\not{\ell}_{(d)}+m_d}{\ell_{(d)}^2-m_d^2}\,\Gamma\,v_s(\bar{p}_2)\right][\bar{v}_b(\bar{p}_1)\,\Gamma\,u_d(p_1)]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \quad \text{(A.4)}$$

where

$$\ell_{(a)} = \bar{p}_1 - q\,,\quad \ell_{(b)} = \bar{p}_2 + q\,,\quad \ell_{(c)} = p_1 - q\,,\quad \ell_{(d)} = p_2 + q\,, \quad \text{(A.5)}$$

and the common coefficient G_{fam}^{eff} has been dropped. Here, in order to provide for the next step in which we obtain hadronic current form from the quark current form, the expressions (A.1) - (A.4) have been given by using a Fierz transformation

$$(\bar{b}\gamma_\rho s)(\bar{d}\gamma^\rho d) \;\Rightarrow\; \sum_\Gamma \left[-\frac{1}{3}(\bar{d}\,\Gamma\, s)(\bar{b}\,\Gamma\, d) - \frac{1}{2}\sum_{a=1}^{8}(\bar{d}\,\Gamma\lambda_a\, s)(\bar{b}\,\Gamma\lambda_a\, d)\right], \tag{A.6}$$

where

$$\Gamma \otimes \Gamma = -\mathbf{1}\otimes\mathbf{1} + \gamma_5\otimes\gamma_5 + \frac{1}{2}\gamma_\rho\otimes\gamma^\rho + \frac{1}{2}\gamma_\rho\gamma_5\otimes\gamma^\rho\gamma_5\,. \tag{A.7}$$

Next, we must translate the amplitudes (A.1) - (A.4) in quark level into those in hadronic level. We use the prescription (3.11). We obtain the following decay amplitudes from (A.1) - (A.4):

$$\mathcal{M}_a = i\frac{e^2}{18}f_K f_B \frac{1}{\Delta_a}\left[(\bar{p}_1 - q)_\mu (P_B P_K) + P_{B\mu}(\bar{p}_1 - q)P_K - P_{K\mu}(\bar{p}_1 - q)P_B\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \tag{A.8}$$

$$\mathcal{M}_b = i\frac{e^2}{18}f_K f_B \frac{1}{\Delta_b}\left[(\bar{p}_2 + q)_\mu (P_B P_K) + P_{K\mu}(\bar{p}_2 + q)P_B - P_{B\mu}(\bar{p}_2 + q)P_K\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \tag{A.9}$$

$$\mathcal{M}_c = -i\frac{e^2}{18}f_K f_B \frac{1}{\Delta_c}\left[(p_1 - q)_\mu (P_B P_K) + P_{B\mu}(p_1 - q)P_K - P_{K\mu}(p_1 - q)P_B\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \tag{A.10}$$

$$\mathcal{M}_d = -i\frac{e^2}{18}f_K f_B \frac{1}{\Delta_d}\left[(p_2 + q)_\mu (P_B P_K) + P_{K\mu}(p_2 + q)P_B - P_{B\mu}(p_2 + q)P_K\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,, \tag{A.11}$$

When we use the expression (2.4), we obtain the following form for the meson currents:

$$\mathcal{M} = i\frac{e^2}{18}f_K f_B \frac{1}{2}\left[f_+(q^2)(P_B + P_K)_\mu + f_-(q^2)(P_B - P_K)_\mu\right]\frac{1}{q^2}[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)]\,. \tag{A.12}$$

The second term with $q_\mu = (P_B - P_K)_\mu$ in Eq.(A.12) does not contribute the decay amplitude because of $q_\mu[\bar{v}_\ell(k_2)\gamma^\mu u_\ell(k_1)] = 0$ for $m_{\ell 1} = m_{\ell 2}$. For the expression $f_+(q^2)$, we obtain

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) - f_+^c(q^2) - f_+^d(q^2)\,, \tag{A.13}$$

where

$$f_+^a(q^2) = \frac{(x_1 - 2a_1)M_K^2 + (1 - x_1 + a_1 + b_1)q^2}{-(x_1 - 2a_1)\Delta_{BK}^2 + (1 - x_1 + 2b_1)q^2}\,, \tag{A.14}$$

$$f_+^b(q^2) = \frac{(x_2 - 2a_2)M_K^2 + (1 - x_2 + a_2 + b_2)q^2}{(x_2 - 2a_2)\Delta_{BK}^2 + (1 - x_2 + 2b_2)q^2}\,, \tag{A.15}$$

$$f_+^c(q^2) = \frac{2a_1 M_K^2 + (1 - a_1 - b_1)q^2}{-2a_1\Delta_{BK}^2 + (1 - 2b_1)q^2}\,, \tag{A.16}$$

$$f_+^d(q^2) = \frac{2a_2 M_B^2 + (1 - a_2 - b_2)q^2}{2a_2\Delta_{BK}^2 + (1 - 2b_2)q^2}\,. \tag{A.17}$$

A form of $f_+(q^2)$ for the decay $B^+ \to K^+\ell^+\ell^-$ can be obtained by replacing $e_d = -e/3 \to e_u = +2e/3$ in (A.13):

$$f_+(q^2) = f_+^a(q^2) + f_+^b(q^2) + 2f_+^c(q^2) + 2f_+^d(q^2)\,. \tag{A.18}$$

Appendix B

The function $F(q^2)$ corresponds to $d\Gamma/dq^2$ for the conventional electroweak photon penguin, and it is calculated from the matrix element

$$\mathcal{M} = G(P_B + P_K)_\mu \bar{\ell}(k_2)\gamma^\mu \ell(k_1)\,, \tag{B.1}$$

where G is defined by Eq.(4.2). By defining a parameter $y \equiv m_{\ell K}^2 = (k_2 + P_K)^2$ together with $y_1 = y_{min}$ and $y_2 = y_{max}$, the form $F(q^2)$ is represented as

$$\begin{aligned} G^2 F(x) &\equiv \frac{1}{(2\pi)^3}\frac{1}{32M_B^3}\int_{y_1}^{y_2} dy\,|\mathcal{M}|^2 \\ &= -\frac{1}{(2\pi)^3}\frac{1}{32M_B^3}\left[\frac{1}{3}(y_2^3 - y_1^3) + \frac{1}{2}a(y_2^2 - y_1^2) + b(y_2 - y_1)\right]\,, \end{aligned} \tag{B.2}$$

where

$$a = q^2 - (M_B^2 + M_K^2 + 2m_\ell^2)\,, \tag{B.3}$$
$$b = (M_B^2 + M_K^2)(M_K^2 + m_\ell^2) - m_\ell^2 q^2\,. \tag{B.4}$$

Appendix C

The coefficients (a_1, b_1) can be obtained as follows. When we define

$$A = 2(M_B^2 + M_K^2) - q^2, \quad B = q^2, \quad C = \Delta_{BK}^2, \tag{C.1}$$

from Eq.(2.5), we obtain a relation between a_1 and b_1:

$$b_1 = \frac{1}{B}\left[-Ca_1 \pm \sqrt{Da_1^2 + Bm_{d1}^2}\right], \tag{C.2}$$

i.e.

$$b_1 = \frac{1}{q^2}\left[-a_1\Delta_{BK}^2 \pm \sqrt{Da_1^2 + m_{d1}^2 q^2}\right], \tag{C.3}$$

where

$$\begin{aligned} D \equiv C^2 - AB &= (\Delta_{BK}^2)^2 - q^2[2(M_B^2 + M_K^2) - q^2] \\ &= \left[(M_B - M_K)^2 - q^2\right]\left[(M_B + M_K)^2 - q^2\right] . \end{aligned} \tag{C.4}$$

By substituting Eq.(C.4) into Eq.(2.7), we obtain a relation for a_1

$$x_1^2 M_B^2 + m_{d1}^2 - m_b^2 = \frac{x_1}{q^2}\left[-a_1 D \pm (\Delta_{BK}^2 + q^2)\sqrt{a_1^2 D + m_{d1}^2 q^2}\right] . \tag{C.5}$$

The parameter a_1 can be obtained by solving Eq.(C.5) for a_1.

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