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Yukawaon Model with U(3)×S$_3$ Family Symmetries

Yoshio Koide[a] **and Hiroyuki Nishiura**[b]

[a] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
E-mail address: koide@het.phys.sci.osaka-u.ac.jp

[b] *Faculty of Information Science and Technology, Osaka Institute of Technology, Hirakata, Osaka 573-0196, Japan*
E-mail address: nishiura@is.oit.ac.jp

Abstract

A new yukawaon model is investigated under a family symmetry U(3)×S$_3$. In this model, all vacuum expectation values (VEVs) of the yukawaons, $\langle Y_f \rangle$, are described in terms of a fundamental VEV matrix $\langle \Phi_0 \rangle$ as in the previous yukawaon model, but the assignments of quantum number for fields are different from the previous ones: the fundamental yukawaon Φ_0 is assigned to $(3,3)$ of U(3)×U(3), which is broken into $(3, 1+2)$ of U(3)×S$_3$, although quarks and leptons are still assigned to triplets of U(3) and yukawaons Y_f are assigned to $\mathbf{6}^*$ of U(3). Then, VEV relations among Yukawaons become more concise considerably than the previous yukawaon models. By adjusting parameters, we can fit not only quark mixing parameters but also lepton mixing parameters together with their mass ratios.

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1 Introduction

It is interesting to consider that the observed quark and lepton mass spectra and mixings can be understood from a "family symmetry". In the standard model (SM) of quarks and leptons, their mass spectra and mixings originate in the structures of the Yukawa coupling constants. If we intend to understand the observed mass spectra and mixings by adopting a non-Abelian gauge symmetry as a "family symmetry", the symmetry will be explicitly broken at the beginning because the Yukawa coupling constants have family indices. Therefore, most family symmetry models are based on a discrete symmetry. However, there is an easy way to consider non-Abelian gauge symmetry as the family symmetry: the Yukawa coupling constants are effective ones Y_f^{eff} $(f = u, d, e, \cdots)$ and those are given by vacuum expectation values (VEVs) of new scalars Y_f:

$$(Y_f^{eff})_{ij} = \frac{y_f}{\Lambda} \langle (Y_f)_{ij} \rangle. \tag{1.1}$$

The fields Y_f are called as "yukawaon" [1]. The yukawaon model is a kind of "flavon model" [2], but, differently from the conventional flavon model, the effective Yukawa coupling constant Y_f^{eff} is essentially given by a 3×3 VEV matrix of a single scalar Y_f. Here we do not consider that Y_f^{eff} is given by a linear combination of scalars Y_A, Y_B, $\cdots$, which belong to different representations of a family symmetry G, as $Y_f^{eff} = c_1 \langle Y_A \rangle + c_2 \langle Y_B \rangle + \cdots$.

In the previous yukawaon model, the most characteristic feature is that all VEVs $\langle Y_f \rangle$ are described in terms of only one fundamental VEV matrix $\langle \Phi_e \rangle$. For example, mass matrices for quarks and charged leptons are given as follows [3]:

$$\begin{array}{l} M_e \propto \langle Y_e \rangle \propto \langle \Phi_e \rangle \langle \Phi_e \rangle, \\ M_d \propto \langle Y_d \rangle \propto \langle \Phi_e \rangle (\mathbf{1} + a_d X) \langle \Phi_e \rangle, \\ M_u^{1/2} \propto \langle \Phi_u \rangle \propto \langle \Phi_e \rangle (\mathbf{1} + a_u X) \langle \Phi_e \rangle, \end{array} \tag{1.2}$$

where $\langle \Phi_e \rangle \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ and

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X \equiv \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{1.3}$$

For neutrino sector, we consider a seesaw type of neutrino mass generation $M_\nu = m_D M_R^{-1} m_D^T$, where the Dirac neutrino mass matrix m_D and the right-handed neutrino Majorana mass matrix M_R are given, respectively, by $m_D \propto M_e$ and

$$M_R \propto \langle Y_R \rangle \propto \langle Y_e \rangle \langle P_u \rangle \langle \Phi_u \rangle + \langle \Phi_u \rangle \langle P_u \rangle \langle Y_e \rangle + \cdots , \tag{1.4}$$

where $\langle P_u \rangle = \mathrm{diag}(+1, -1, +1)$ in the diagonal basis of the up-quark mass matrix M_u. The model [3] has roughly well described masses and mixings of the quarks and the leptons with a few parameters. However, the fitting of the quark mixing has been somewhat unsatisfactory compared with those of the neutrino mixing. (For more precise parameter fitting, see Ref.[4].)

In the present paper, we propose a new yukawaon model where all yukawaon VEVs are described in terms of a new fundamental VEV matrix $\langle \Phi_0 \rangle$ similar to the previous model. However, in contrast to the previous model (1.2), the charged lepton mass matrix M_e is given by the same structure as those in the down- and up-quark sectors:

$$\begin{array}{l} M_e \propto \langle Y_e \rangle \propto \langle \Phi_0 \rangle (\mathbf{1} + a_e X) \langle \Phi_0 \rangle, \\ M_d \propto \langle Y_d \rangle \propto \langle \Phi_0 \rangle (\mathbf{1} + a_d X) \langle \Phi_0 \rangle, \\ M_u^{1/2} \propto \langle \hat{Y}_u \rangle \propto \langle \Phi_0 \rangle (\mathbf{1} + a_u X) \langle \Phi_0 \rangle. \end{array} \tag{1.5}$$

[For more details, see Eqs.(4.9) - (4.14).] In the past yukawaon models, it has been assumed that the factor $(\mathbf{1} + a_q X)$ $(q = u, d)$ in Eq.(1.2) originates in VEV structures of additional fields. However, in the present paper, we assign the fundamental yukawaon Φ_0 to $(\mathbf{1} + \mathbf{2})$ of a permutation symmetry S_3 [5], and we consider that the factor $(\mathbf{1} + a_q X)$ is due to a coefficient of $(\mathbf{1}+\mathbf{2}) \times (\mathbf{1}+\mathbf{2}) \to \mathbf{1}$ under S_3. Therefore, it can naturally be understood that all the factors which are sandwiched by $\langle \Phi_0 \rangle$ are given by $(\mathbf{1} + a_f X)$ as shown in Eq.(1.5), instead of Eq.(1.2). The details will be discussed in the next section.

In Sec.3, we investigate possible superpotential forms for yukawaons, and in Sec.4, we summarize VEV relations in the present model. In Sec.5, we give parameter fittings for the observed lepton mixing (PMNS mixing [6]) and the observed quark mixing (CKM mixing [7]) together with their mass ratios. In the present model, we have 8 adjustable parameters a_e, $a_u e^{i\alpha_u}$, a_d, ξ_ν, m_d^0 and (ϕ_1, ϕ_2) for 16 observables (2 up-quark mass ratios, 2 down-quark mass ratios, 2 neutrino mass ratios, 4 CKM mixing matrix parameters, 4+2 PMNS matrix parameters). The final section 6 is devoted to summary and discussions.

2 Fundamental Yukawaon Φ_0 and a permutation symmetry S_3

First, we give a brief review of the factor $(\mathbf{1} + a_f X)$ in S_3 symmetry. When we denote a doublet (ψ_π, ψ_η) and a singlet ψ_σ in a permutation symmetry S_3 [5] as

$$\begin{pmatrix} \psi_\pi \\ \psi_\eta \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\psi_1 - \psi_2) \\ \frac{1}{\sqrt{6}}(\psi_1 + \psi_2 - 2\psi_3) \end{pmatrix}, \tag{2.1}$$

$$\psi_\sigma = \frac{1}{\sqrt{3}}(\psi_1 + \psi_2 + \psi_3), \tag{2.2}$$

the field $\psi = (\psi_1, \psi_2, \psi_3)$ is represented as

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \psi_\pi \\ \psi_\eta \\ \psi_\sigma \end{pmatrix}. \tag{2.3}$$

Then, a bilinear form $\psi\psi$ is invariant under the S_3 symmetry only when $\psi_a \xi_{ab} \psi_b$ is given by the form

$$\psi_a \xi_{ab} \psi_b = \psi_a (\mathbf{1} + a_f X)_{ab} \psi_b, \tag{2.4}$$

where a_f is a free parameter, and $\mathbf{1}$ and X are defined by Eq.(1.3). The appearance of a free parameter a_f is due to a reason that there are two singlets which are composed of ψ_a, i.e. $\psi_\sigma \psi_\sigma$ and $(\psi_\pi, \psi_\eta)^T (\psi_\pi, \psi_\eta)$.

As seen from Eq.(2.4), we can assume that the fundamental yukawaon Φ_0 is transformed as $(\mathbf{3}, \mathbf{2}+\mathbf{1})$ of U(3)$\times$S$_3$, i.e.

$$\Phi^{0\,T}_{ia} (\mathbf{1} + a_f X)^{ab} \Phi^0_{bj}. \tag{2.5}$$

However, since the bilinear form (2.5) cannot have quantum numbers which distinguish the sectors $f = u, d, e$ (a_f are merely free parameters), we modify the expression (2.5) into

$$\Phi^{0\,T}_{i\alpha} S_f^{\alpha\beta} \Phi^0_{\beta j}, \tag{2.6}$$

where S_f are fields and indices α, β are of another U(3) symmetry (we denote it as U(3)$'$). We assume that U(3)$'$ is broken into S_3 at an energy scale of Λ' by non-vanishing VEV $\langle S_f \rangle$ whose forms are given by

$$\langle S_f \rangle = v_{Sf} (\mathbf{1} + a_f X). \tag{2.7}$$

Here, we assume $\Lambda' \gg \Lambda$ (U(3) family symmetry is broken at Λ). Note that the indices α, β in the expression (2.6) are of U(3)$'$, while the indices a, b in the expression

$$(\Phi_0)^T_{ia} \langle S_f^{ab} \rangle (\Phi_0)_{bj} \tag{2.8}$$

are of S_3. It is worthwhile to notice that in the conventional S_3 family model quarks and leptons are assigned to (singlet+ doublet)'s of S_3, while in the present model quarks and leptons are assigned to triplets of the U(3) family symmetry, and only the fundamental yukawaon Φ_0 is assigned to $(\mathbf{3}, \mathbf{1}+\mathbf{2})$ of U(3)×S_3.

3 Superpotential

In the present model, would-be Yukawa interactions are given as follows:

$$W_Y = \frac{y_e}{\Lambda} e_i^c Y_e^{ij} \ell_j H_d + \frac{y_\nu}{\Lambda} \nu_i^c Y_e^{ij} \ell_j H_u + \lambda_R \nu_i^c Y_R^{ij} \nu_j^c + \frac{y_d}{\Lambda} d_i^c Y_d^{ij} q_j H_d + \frac{y_u}{\Lambda} u_i^c Y_u^{ij} q_j H_u, \tag{3.1}$$

where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are $SU(2)_L$ doublets. Under this definition of Y_f, the CKM mixing matrix and the PMNS mixing matrix are given by $V_{CKM} = U_u^\dagger U_d$ and $U_{PMNS} = U_e^\dagger U_\nu$, respectively, where U_f are defined by $U_f^\dagger M_f^\dagger M_f U_f = D_f^2$ (D_f are diagonal). In order to distinguish each yukawaon from others, we assume that Y_f have different R charges from each other together with R charge conservation. (Of course, the R charge conservation is broken at the energy scale Λ'.) Here, we have assumed that the R charge of the Dirac neutrino yukawaon Y_ν is identical with that of the charged lepton yukawaon Y_e, so that Y_ν is replaced with Y_e for an efficient use of fields.

We assume the following superpotential for yukawaons:

$$W_e = \left[\mu_e Y_e^{ij} + \frac{\lambda_e}{\Lambda} \bar{P}^{ik} \hat{Y}^e_{kl} \bar{P}^{lj}\right] \Theta^e_{ji} + \left[\mu'_e \hat{Y}^e_{ij} + \frac{\lambda'_e}{\Lambda} (\Phi_0)^T_{ia} S_e^{ab} (\Phi_0)_{bj}\right] \bar{\Theta}_e^{ji}, \tag{3.2}$$

$$W_d = \left[\mu_d Y_d^{ij} + \frac{\lambda_d}{\Lambda} \bar{P}^{ik} \hat{Y}^d_{kl} \bar{P}^{lj}\right] \Theta^d_{ji} + \left[\mu'_d \hat{Y}^d_{ij} + \frac{\lambda'_d}{\Lambda} (\Phi_0)^T_{ia} S_d^{ab} (\Phi_0)_{bj} + m_d^0 E_{ij}\right] \bar{\Theta}_d^{ji}, \tag{3.3}$$

$$W_u = \frac{1}{\Lambda} \left[\lambda_u E_{ik} Y_u^{kl} E_{lj} + \lambda'_u \hat{Y}^u_{ik} E^{kl} \hat{Y}^u_{lj}\right] \bar{\Theta}_u^{ji} + \left[\mu'_u \hat{Y}^u_{ij} + \frac{\lambda''_u}{\Lambda} (\Phi_0)^T_{ia} S_e^{ab} (\Phi_0)_{bj}\right] \bar{\Theta}'^{ji}_u, \tag{3.4}$$

$$W_R = \frac{1}{\Lambda} \left\{\lambda_R E_{ik} Y_R^{kl} E_{lj} + \lambda'_R \left[\hat{Y}^u_{ik} \bar{E}^{kl} \hat{Y}^e_{lj} + \hat{Y}^e_{ik} \bar{E}^{kl} \hat{Y}^u_{kj} + \xi_\nu \left(\mathrm{Tr}[\hat{Y}_u \bar{E}] \hat{Y}^e_{ij} + \mathrm{Tr}[\bar{E} \hat{Y}^e] \hat{Y}^u_{ij}\right)\right]\right\} \bar{\Theta}_R^{ji}. \tag{3.5}$$

(In this paper, we use $\hat{Y}^u$ instead of Φ_u unlike the previous papers, since we treat the up- and down-quark sectors in the same way $\hat{Y}^u \leftrightarrow \hat{Y}^d$.) Here we have assumed that the R charges satisfy the following relation

$$R(Y_e) - R(Y_d) = R(\hat{Y}^e) - R(\hat{Y}^d) = R(S_e) - R(S_d). \tag{3.6}$$

The VEVs of the introduced fields E, $\bar{E}$ and $\bar{P}$ are described by the following superpotential

$$W_{E,P} = \frac{\lambda_1}{\Lambda} \mathrm{Tr}[\bar{E} E \bar{P} P] + \frac{\lambda_2}{\Lambda} \mathrm{Tr}[\bar{E} E] \mathrm{Tr}[\bar{P} P], \tag{3.7}$$

which leads to

$$\langle E \rangle \langle \bar{E} \rangle \propto \mathbf{1}, \quad \langle P \rangle \langle \bar{P} \rangle \propto \mathbf{1}. \tag{3.8}$$

	ℓ_i	e^c_i	ν^c_i	q_i	u^c_i	d^c_i	H_u	H_d	Φ^0_{ia}
U(3)	**3**	**3**	**3**	**3**	**3**	**3**	**1**	**1**	**3**
U(3)′	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**3**

	Y_e^{ij}	Θ^e_{ij}	$\hat{Y}^e_{ij}$	$\bar{\Theta}_e^{ij}$	S_e^{ab}	Y_d^{ij}	Θ^d_{ij}	$\hat{Y}^d_{ij}$	$\bar{\Theta}_d^{ij}$	S_d^{ab}
U(3)	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{1}$
U(3)′	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$

	Y_u^{ij}	$\bar{\Theta}_u^{ij}$	$\hat{Y}^u_{ij}$	$\bar{\Theta}'^{ij}_u$	S_u^{ab}	Y_R^{ij}	$\bar{\Theta}_R^{ij}$	$\bar{P}^{ij}$	P_{ij}	E_{ij}	$\bar{E}^{ij}$
U(3)	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}^*$
U(3)′	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Table 1: Quantum numbers of the fields

We take specific solutions of Eq.(3.8):

$$\frac{1}{v_E}\langle E\rangle = \frac{1}{\bar{v}_E}\langle \bar{E}\rangle = \mathbf{1}, \tag{3.9}$$

$$\frac{1}{v_P}\langle P\rangle = \frac{1}{\bar{v}_P^*}\langle \bar{P}\rangle^\dagger = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1), \tag{3.10}$$

as the explicit forms of $\langle E\rangle$, $\langle \bar{E}\rangle$ and $\langle \bar{P}\rangle$. Here, we have assigned R charges for those fields as

$$R(\bar{E}) + R(E) + R(\bar{P}) + R(P) = 2. \tag{3.11}$$

Since we consider $R(\bar{E}) + R(E) \neq R(\bar{P}) + R(P)$, terms $\mathrm{Tr}[\bar{E}E\bar{E}E]$, $\mathrm{Tr}[\bar{P}P\bar{P}P]$, and so on are forbidden by the R charge conservation. However, the absence of the term $\mathrm{Tr}[\bar{E}P]\mathrm{Tr}[\bar{P}E]$ in Eq.(3.7) must be assumed ad hoc.

In the superpotential W_d, Eq.(3.3), we have taken

$$R(E) = R(\hat{Y}^d), \tag{3.12}$$

and we have added the $m_0^d E_{ij}$ term to $\hat{Y}^d_{ij}$. The relation (3.12) has been ad hoc assumed in order to adjust the quark mass ratio m_{d1}/m_{d2}. Such the assignment is not contradict with whole R charge assignments of the fields. However, once the assignment (3.12) is done, then, we cannot add E term to $\hat{Y}^e$ and/or $\hat{Y}^u$ because of the R charge conservation.

We list whole fields in the present model in Table 1. As seen in Table 1, the sum of the anomaly coefficients A is $\sum A = -9$. In order to be $\sum A = 0$, we need further three fields, T^1_{ia}, T^2_{ia} and T^3_{ia}. For the time being, we do not specify the roles of these fields in this model.

4 VEV relations

We assume that our vacuum always takes $\langle\theta_A\rangle = 0$ $(A = e, u, d, \cdots)$. Therefore, VEV relations are obtained from the SUSY vacuum conditions $\partial W/\partial\Theta_A = 0$. Since relations from another SUSY vacuum conditions $\partial W/\partial Y_e = 0$ and so on always include one of $\langle\theta_A\rangle$, such the relations do not have meaning except for Eq.(3.7). Since we assume that the SUSY breaking is caused by gauge mediation (except for family gauge symmetries), we consider that our VEV relations in the yukawaon sector are satisfied until a low energy scale.

From the superpotential terms (3.2)-(3.5), we obtain the following VEV relations among the yukawaons:

$$\langle Y_e^{ij}\rangle = -\frac{\lambda_e}{\mu_e \Lambda}\langle \bar{P}^{ik}\rangle\langle \hat{Y}^e_{kl}\rangle\langle \bar{P}^{lj}\rangle, \tag{4.1}$$

$$\langle \hat{Y}^e_{ij}\rangle = -\frac{\lambda'_e}{\mu'_e \Lambda}\langle (\Phi_0)^T_{ia}\rangle\langle S_e^{ab}\rangle\langle (\Phi_0)_{bj}\rangle, \tag{4.2}$$

$$\langle Y_d^{ij}\rangle = -\frac{\lambda_d}{\mu_d \Lambda}\langle \bar{P}^{ik}\rangle\langle \hat{Y}^d_{kl}\rangle\langle \bar{P}^{lj}\rangle, \tag{4.3}$$

$$\langle \hat{Y}^d_{ij}\rangle = -\frac{\lambda'_d}{\mu'_d \Lambda}\langle (\Phi_0)^T_{ia}\rangle\langle S_d^{ab}\rangle\langle (\Phi_0)_{bj}\rangle + \frac{m_d^0}{\mu'_d}\langle E_{ij}\rangle, \tag{4.4}$$

$$\langle Y_u^{ij}\rangle = -\frac{\lambda'_u}{\lambda_u}(\langle E^{-1}\rangle)^{ik}\langle \hat{Y}^u_{kl}\rangle\langle E^{lm}\rangle\langle \hat{Y}^u_{mn}\rangle(\langle E^{-1}\rangle)^{nj}, \tag{4.5}$$

$$\langle \hat{Y}^u_{ij}\rangle = -\frac{\lambda'_u}{\mu'_u \Lambda}\langle (\Phi_0)^T_{ia}\rangle\langle S_u^{ab}\rangle\langle (\Phi_0)_{bj}\rangle, \tag{4.6}$$

$$\langle Y_R^{ij}\rangle = -\frac{\lambda'_R}{\lambda_R}(\langle E^{-1}\rangle)^{ik}\left[\langle \hat{Y}^u_{kl}\rangle\langle \bar{E}^{lm}\rangle\langle \hat{Y}^e_{mn}\rangle + \langle \hat{Y}^e_{kl}\rangle\langle \bar{E}^{lm}\rangle\langle \hat{Y}^u_{mn}\rangle\right.$$

$$\left. +\xi_\nu\left(\mathrm{Tr}[\langle \hat{Y}_u\rangle\langle \bar{E}\rangle]\langle \hat{Y}^e_{kn}\rangle + \mathrm{Tr}[\langle \bar{E}\rangle\langle \hat{Y}^e\rangle]\langle \hat{Y}^u_{kn}\rangle\right)\right](\langle E^{-1}\rangle)^{nj}, \tag{4.7}$$

where $\langle S_f\rangle$, $\langle E\rangle$, $\langle \bar{E}\rangle$ and $\langle \bar{P}\rangle$ are given by Eqs.(2.7), (3.9) and (3.10), respectively. We have assumed that the VEV matrix $\langle \Phi_0\rangle$ is diagonal in the basis in which the VEV matrix $\langle S_f\rangle$ take the form $(\mathbf{1} + a_f X)$, i.e.

$$\langle \Phi_0\rangle = \mathrm{diag}(v_1, v_2, v_3). \tag{4.8}$$

In the present model, common coefficients are not important. Therefore, when we omit those coefficients, quark and lepton mass matrices are given as follows:

$$M_e = \langle \bar{P}\rangle \hat{M}_e\langle \bar{P}\rangle, \tag{4.9}$$

$$\hat{M}_e = \langle \Phi_0^T\rangle\langle S_e\rangle\langle \Phi_0\rangle. \tag{4.10}$$

$$M_d = \langle \bar{P}\rangle\left(\langle \Phi_0^T\rangle\langle S_d\rangle\langle \Phi_0\rangle + m_d^0\mathbf{1}\right)\langle \bar{P}\rangle, \tag{4.11}$$

$$M_u^{1/2} = \langle \Phi_0^T\rangle\langle S_u\rangle\langle \Phi_0\rangle, \tag{4.12}$$

$$M_\nu = M_e M_R^{-1} M_e^T, \tag{4.13}$$

$$M_R = M_u^{1/2}\hat{M}_e + \hat{M}_e M_u^{1/2} + +\xi_\nu\left(\mathrm{Tr}[M_u^{1/2}]\hat{M}_e + \mathrm{Tr}[\hat{M}^e]M_u^{1/2}\right). \tag{4.14}$$

For numerical calculations in the next section, we will use dimensionless expressions $\langle\Phi_0\rangle = \mathrm{diag}(x_1, x_2, x_3)$ with $x_1^2 + x_2^2 + x_3^2 = 1$, $\langle\bar{P}\rangle = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$ and $\langle S_f\rangle = \mathbf{1} + a_f X$ in Eqs.(4.9) - (4.14). (Therefore, m_d^0 in Eq.(4.11) is also a dimensionless parameter.)

5 Parameter fitting

For simplicity, we assume that the parameter a_d in the down-quark sector is real as well as a_e in the charged lepton sector, so that only a_u is complex. For the convenience, hereafter, we denote a_u anew as $a_u e^{i\alpha_u}$ (a_u and α_u are real). Then, in this model, we have 8 adjustable parameters a_e, $a_u e^{i\alpha_u}$, a_d, ξ_ν, m_d^0 and (ϕ_1, ϕ_2) for 16 observables [2 up-quark mass ratios, 2 down-quark mass ratios, 2 neutrino mass ratios, 4 CKM mixing matrix parameters, 4+2 PMNS matrix parameters]. Since the observed charged lepton mass ratios are used as input values, we do not count them as adjustable parameters. The parameter m_d^0 is used only in order to fit the down-quark mass ratio m_d/m_s. (In other words, other observables are insensitive to the value of m_d^0.) Therefore, if we do not count this parameter m_d^0, we have 7 parameters for 15 observables.

5.1 PMNS mixing

In order to predict the neutrino mixing (PMNS mixing) parameters, we have 6 parameters [a_e, ξ_ν, $a_u e^{i\alpha_u}$ and (ϕ_1, ϕ_2)]. At present, we know only 5 observed quantities [2 up-quark mass ratios, 1 neutrino mass ratio $R_\nu = \Delta m_{21}^2/\Delta m_{32}^2$, 3 neutrino mixing angles, $\sin^2 2\theta_{atm}$, $\tan^2\theta_{solar}$ and $\sin^2 2\theta_{13}$] among 10 observable quantities [2 up-quark mass ratios, 2 neutrino mass ratios, 4+2 PMNS mixing parameters]. Therefore, in principle, we cannot determine the parameter values from the observed quantities. Instead, we use the observables which are described by parameters as few as possible. To start with, let us use up-quark mass ratios $r_{12}^u = \sqrt{m_u/m_c}$ and $r_{23}^u = \sqrt{m_c/m_t}$ which are described by 3 parameters a_e and $a_u e^{i\alpha_u}$. The observed values of r_{12}^u and r_{23}^u are as follows [8]:

$$r_{12}^u \equiv \sqrt{\frac{m_u}{m_c}} = 0.045^{+0.013}_{-0.010}, \qquad r_{23}^u \equiv \sqrt{\frac{m_c}{m_t}} = 0.06 \pm 0.005, \tag{5.1}$$

at $\mu = m_Z$. We illustrate a relation (a_u, α_u) and a_e which satisfy $r_{12}^u = 0.045$ and $r_{23}^u = 0.060$ in Fig. 1. As seen in Fig. 1, the these parameters are bounded in $a_u = -(1.3 - 1.8)$, $a_u = 0^\circ - 10^\circ$ and $a_e = 0 - 10$. (Since the observed center values have large errors, these bounds should not be taken rigidly.)

Next, we focus on the neutrino mixing $\sin^2 2\theta_{atm}$. We find that it is insensitive to α_u and ξ_ν, although it is described by parameters a_e, $a_u e^{i\alpha_u}$, ξ_ν and (ϕ_1, ϕ_2) in our model. In Fig. 2, we show curves in the (a_u, a_e) plane which satisfy $\sin^2 2\theta_{atm} = 0.90$, 0.92, 0.94 and 0.96, respectively. Here, for convenience, we have fixed (ϕ_1, ϕ_2) as $(\phi_1, \phi_2) = (180^\circ, 180^\circ)$. (For a case of $(\phi_1, \phi_2) = (0^\circ, 0^\circ)$, we could not find reasonable parameter solutions at all.) Here, we have used $\xi_\nu = 0.0025$ and $\alpha_u = 8^\circ$ and 10°, and $(\phi_1, \phi_2) = (180^\circ, 180^\circ)$ tentatively. As seen in Fig. 2, two curves ($\alpha_u = 8^\circ$ and 10°) are almost degenerated. For reference, we also show a curve which satisfies reasonable up-quark mass ratios. Since we cannot take $a_u > -1.3$ as we have shown in Fig. 1, we cannot obtain a value more than $\sin^2 2\theta_{atm} = 0.97$. If we want to obtain a value of

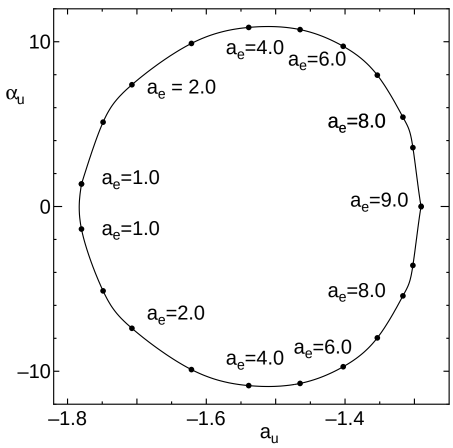

Figure 1: Parameter values (a_u, α_u) and a_e which satisfy the observed values of up-quark mass ratios $\sqrt{m_u/m_c} = 0.045$ and $\sqrt{m_c/m_t} = 0.060$

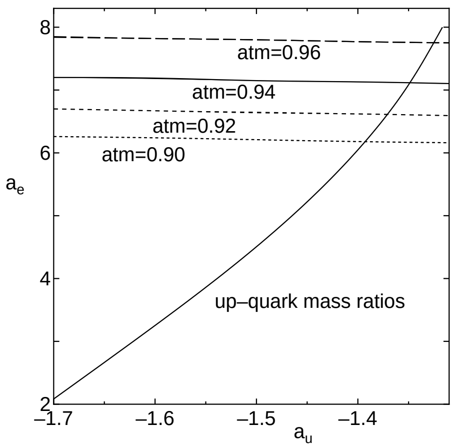

Figure 2: Parameter values (a_u, a_e) which satisfy $\sin^2 2\theta_{atm} = 0.90$, 0.92, 0.94, 0.96, respectively. (In the figure, "atm" means $\sin^2 2\theta_{atm}$.) For reference, the curve which satisfies up-quark mass ratios $\sqrt{m_u/m_c} = 0.045$ and $\sqrt{m_c/m_t} = 0.060$ are also shown.

$\sin^2 2\theta_{atm}$ as large as possible, we must take a parameter set

$$(a_e, a_u, \alpha_u) \sim (8, -1.35, \pm 6^\circ). \tag{5.2}$$

For this parameter set (5.2), we can obtain $\tan^2 \theta_{solar} \sim 0.5$ and $R \sim 0.04$. More accurate predicted values will be given after we discuss the CKM mixing values in the next subsection.

5.2 CKM mixing

The 4 observables in the CKM mixing matrix depend on 3 parameters a_d and (ϕ_1, ϕ_2) in addition to the parameter values (5.2). Although the parameter a_d can be determined by two down-quark mass rations [8]

$$r^d_{12} \equiv \frac{m_d}{m_s} = 0.053^{+0.005}_{-0.003}, \quad r^d_{23} \equiv \frac{m_s}{m_b} = 0.019^{+0.006}_{-0.006}, \tag{5.3}$$

we cannot give reasonable fitting for the CKM mixing and the down-quark mass ratios simul-

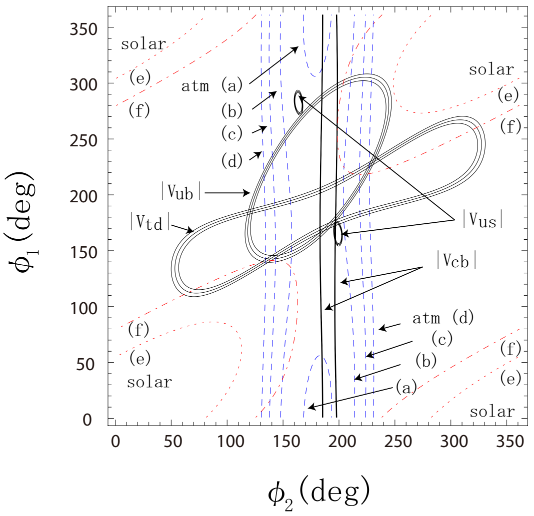

Figure 3: Contour Plots in the (ϕ_1, ϕ_2) parameter plane, which are shown by using experimental constraints on $|V_{us}| = 0.2252 \pm 0.0009$, $|V_{cb}| = 0.0406 \pm 0.0013$, $|V_{ub}| = 0.00389 \pm 0.00044$, and $|V_{td}| = 0.0084 \pm 0.0006$ and by taking some values for atm $\equiv \sin^2 2\theta_{atm}$ and $\equiv \tan^2\theta_{solar}$. (a): $\sin^2 2\theta_{atm}$=0.98, (b): $\sin^2 2\theta_{atm}$=0.96, (c): $\sin^2 2\theta_{atm}$=0.94, and (d): $\sin^2 2\theta_{atm}$=0.92, which are shown by dashed lines, and solar$\equiv \tan^2\theta_{solar}$ which is shown by dotted or dot-dashed line. (e): $\tan^2\theta_{solar} = 0.495$ (dotted line), and (f): $\tan^2\theta_{solar} = 0.515$ (dot-dashed line). Here we take the values for the parameter set of $[a_e, (a_u, \alpha_u), \xi_\nu, a_d, m_d^0]$ given in Eq. (5.10).

taneously. Therefore, we fit only the value of r_{23}^d by a_d, and the value r_{12}^d is adjusted by the parameter m_d^0 which does not affect the CKM mixing parameters.

Although we have roughly obtained the parameter values of (a_e, a_u, α_u) in Eq.(5.2) from r_{23}^u, r_{12}^u, and $\sin^2 2\theta_{atm}$, the predicted values for CKM mixing are also dependent on the parameter values (a_e, a_u, α_u). We consider that the observed values for the quark mass ratios are still controversial. Therefore, we search a set of reasonable parameter values for $[a_e, (a_u, \alpha_u), \xi_\nu, a_d, (\phi_1, \phi_2)]$ that can give the observed PMNS and CKM mixings within one sigma and quark mass ratios within two sigma, keeping the result (5.2) in mind. Namely, After searching a rough value of the parameter a_d which can give reasonable CKM mixings and the down-quark mass ratio r_{23}^d, we do a fine tuning of parameter values for $[a_e, (a_u, \alpha_u), a_d, (\phi_1, \phi_2)]$ with help of figures demonstrated in Fig. 3.

As a result, we obtain the following predictions:

$$\sin^2 2\theta_{atm} = 0.965, \quad \tan^2\theta_{solar} = 0.522, \quad \sin^2 2\theta_{13} = 0.027, \tag{5.4}$$

$$\delta_{CP}^{\ell} = -177^\circ \quad (J^\ell = -9.3 \times 10^{-4}), \tag{5.5}$$

$$R_\nu = \frac{\Delta m_{solar}^2}{\Delta m_{atm}^2} = 0.044, \tag{5.6}$$

$$|V_{us}| = 0.2240, \quad |V_{cb}| = 0.0404, \quad |V_{ub}| = 0.00409, \quad |V_{td}| = 0.00823, \tag{5.7}$$

$$\delta_{CP}^{q} = 65.3^\circ \quad (J^q = 3.3 \times 10^{-5}), \tag{5.8}$$

$$r^u_{12} = 0.0422, \quad r^u_{23} = 0.0659, \quad r^d_{12} = 0.0530, \quad r^d_{23} = 0.0360, \tag{5.9}$$

under the parameter values

$$a_e = 8.5,\ a_u = -1.32,\ \alpha_u = -6.5^\circ,\ a_d = 17,\ m^0_d = 0.0113,$$
$$\xi_\nu = 0.0019,\ (\phi_1, \phi_2) = (174.5^\circ, 195.9^\circ). \tag{5.10}$$

Although the predicted value $\tan^2\theta_{solar} = 0.522$ is somewhat large compared with the observed value [9] $\tan^2\theta_{solar} = 0.468^{+0.048}_{-0.029}$, it is consistent with the KamLAND data [10] $\tan^2\theta_{solar} = 0.56^{+0.10}_{-0.07}(\mathrm{stat})^{+0.10}_{-0.06}(\mathrm{syst})$. The predicted value $\sin^2 2\theta_{13} = 0.027$ is small compared with the T2K data [11] $0.03 < \sin^2 2\theta_{13} < 0.28$ for $\delta^\ell_{CP} = 0$, but it is still not ruled out because our model predicts $\delta^\ell_{CP} = -177^\circ$. Also, the predicted value $r^d_{23} = 0.0352$ is larger than the center value given in Eq.(5.3) by 3σ. Rather, the predicted value is near to the old value $r^d_{23} = 0.031 \pm 0.005$ given in the second literature in Ref.[8]. We consider that the value of m_s is still controversial.

We can also predict neutrino masses

$$m_{\nu 1} \simeq 0.0056 \text{ eV}, \quad m_{\nu 2} \simeq 0.0118 \text{ eV}, \quad m_{\nu 3} \simeq 0.0507 \text{ eV}, \tag{5.11}$$

by using the input value [12] $\Delta m^2_{32} \simeq 0.00243$ eV^2. We also predict the effective Majorana neutrino mass [13] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = \left| m_1 U^2_{e1} + m_2 U^2_{e2} + m_3 U^2_{e3} \right| \simeq 3.2 \times 10^{-4} \text{ eV}. \tag{5.12}$$

6 Concluding remarks

In conclusion, by assuming U(3)×S$_3$ family symmetries, we have proposed a new yukawaon model in which not only the quark mass matrices $M_u^{1/2}$ and M_d but also the charged lepton mass matrix M_e is given by a form $\langle\Phi_0\rangle(\mathbf{1}+a_f X)\langle\Phi_0\rangle$ (Φ_0 is a fundamental yukawaon) as shown in Eqs.(4.9) - (4.14). The model have only 7 parameters for 15 observables. However, we have obtained reasonable predictions.

Since all the mass matrices for quarks and charged leptons are given by the same form $\langle\Phi_0\rangle(\mathbf{1}+a_f X)\langle\Phi_0\rangle$, we can consider a possibility that if we define $\langle Y_0(a_f)\rangle = \langle\Phi_0\rangle(\mathbf{1}+a_f X)\langle\Phi_0\rangle$ by introducing a yukawaon Y_0, all the effective Yukawa coupling constants Y^{eff}_f can be given by a single yukawaon VEV $\langle Y_0(a_f)\rangle$. However, this idea runs into a stone wall, because we cannot distinguish $Y_0(a_d)$ from $Y_0(a_e)$ in the expression of $\langle Y_R\rangle$, Eq.(4.14). Besides, the VEV of the "single" yukawaon Y_0 is dependent on the parameter a_f. This is contradictory to the idea in the original yukawaon model that Y^{eff}_f is given by a single yukawaon VEV without including a free parameter. Therefore, in the present paper, we have considered that the yukawaons Y_f are distinguished by the fields S_f in $\Phi_0 S_f \Phi_0$ as shown in Eq.(2.8).

One of motivations of the yukawaon model [1] was to understand a charged lepton mass relation [14] $m_e + m_\mu + m_\tau = (2/3)(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2$ by considering $\langle Y_e\rangle \propto \langle\Phi_e\rangle\langle\Phi_e\rangle$. However, in the present model, since $\langle Y_e\rangle$ is given by $\langle\Phi_0\rangle(\mathbf{1}+a_e X)\langle\Phi_0\rangle$, the scenario for the charged lepton mass relation must be abandoned, and we have to search an alternative scenario for the charged lepton mass relation. Nevertheless, apart from this problem, it seems that the present new yukawaon model offers us a promising hint for a unified mass matrix model for quarks and leptons.

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