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Large θ_{13}^{ν} and Unified Description of Quark and Lepton Mixing Matrices

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Abstract

We present a revised version of the so-called "yukawaon model", which was proposed for the purpose of a unified description of the lepton mixing matrix U_{PMNS} and the quark mixing matrix V_{CKM}. It is assumed from a phenomenological point of view that the neutrino Dirac mass matrix M_D is given with a somewhat different structure from the charged lepton mass matrix M_e, although $M_D = M_e$ was assumed in the previous model. As a result, the revised model predicts a reasonable value $\sin^2 2\theta_{13} \sim 0.07$ with keeping successful results for other parameters in U_{PMNS} as well as V_{CKM} and quark and lepton mass ratios.

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1 Introduction

In a series of papers [1, 2, 3], the authors have investigated a unified description of the lepton mixing matrix [4] U_{PMNS} and the quark mixing matrix [5] V_{CKM}. The essential idea is as follows: (i) The Yukawa coupling constants Y_f ($f = u, d, e$, and so on) in the standard model are effectively given by vacuum expectation values (VEVs) of scalars ("yukawaon") Y_f with 3×3 components, i.e. by $\langle Y_f \rangle / \Lambda$. Here Λ is an energy scale of the effective theory. (The yukawaon model is a kind of the "flavon" model [6].) (ii) The model does not contain any coefficients which are dependent on the family numbers. The hierarchical structures of the effective Yukawa coupling constants originate only in a fundamental VEV matrix $\langle \Phi_0 \rangle$, whose hierarchical structure is ad hoc assumed at present and whose VEV values are fixed by the observed charged lepton masses. (iii) Relations among those VEV matrices are obtained from SUSY vacuum conditions for a given superpotential under family symmetries and R charges assumed. (Since we use the observed charged lepton mass values as the input values, it is a characteristic in the yukawaon model that adjustable parameters are quite few.)

In the previous model[1, 3], the quark and lepton mass matrices (charged lepton mass matrix M_e, Dirac neutrino mass matrix M_D, down-quark mass matrix M_d, neutrino mass matrix M_ν, and right-handed Majorana neutrino mass matrix M_R) are given as follows:

$$\begin{aligned}
&M_e = k_e \Phi_0 (\mathbf{1} + a_e X_3) \Phi_0, \\
&M_D = M_e, \\
&M_d = k_d \left[\Phi_0 (\mathbf{1} + a_d X_3) \Phi_0 + m_d^0 \mathbf{1} \right], \\
&M_u = k_u' \hat{M}_u \hat{M}_u, \\
&\hat{M}_u = k_u \Phi_0 (\mathbf{1} + a_u X_3) \Phi_0, \\
&M_\nu = M_D M_R^{-1} M_D^T, \\
&M_R = k_R (\hat{M}_u M_e + M_e \hat{M}_u) + \cdots ,
\end{aligned} \tag{1.1}$$

where M_e, Φ_0, X_3, $\cdots$ are 3×3 numerical matrices which result from VEV matrices of scalar fields. Here the VEV matrices Φ_0, X_3, and $\mathbf{1}$ have structures given by

$$\Phi_0 = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{1.2}$$

The coefficients a_f ($f =$ e,u,d) which are important parameters in the model play an essential role in the mass ratios and mixings. On the other hand, the family-number independent coefficients k_f and k_u' do not any role in predicting family mixings and mass ratios. The values of (x_1, x_2, x_3) with $x_1^2 + x_2^2 + x_3^2 = 1$ are fixed by the observed charged lepton mass values under the given value of a_e. (In an earlier model [7], the charged lepton mass matrix M_e was given by $M_e = k_e' \Phi_e \Phi_e$ and M_d and $\hat{M}_u$ are given by those in (1.1) with the replacement $\Phi_0 \to \Phi_e$. The structures with $(\mathbf{1} + a_f X_3)$ were suggested in a phenomenological model by Fusaoka and one of the authors [8].)

The previous models [1, 2, 3] have given almost successful unified description and predictions of U_{PMNS} and V_{CKM}. However, these models have failed to give the observed large mixing of θ_{13} in U_{PMNS}: the observed value is $\sin^2 2\theta_{13} \sim 0.09$ [10], while the model in Ref.[1] predicts $\sin^2 2\theta_{13} \sim 10^{-4}$. Even in a recent revised model [3], the predicted value was, at most, $\sin^2 2\theta_{13} \sim 0.03$. Since the model does not contain enough number of adjustable parameters as it is, it is hard to improve the prediction of $\sin^2 2\theta_{13}$ without the cost of other successful predictions. So, an interesting attempt of introducing the structure X_2 into the model has been done in Ref.[2]. In Ref.[2], the structure X_2 [see Eq.(1.44)] was introduced in M_e together with assumption $M_D = M_e$, but the predicted value of $\sin^2 2\theta_{13}$ was still small: $\sin^2 2\theta_{13} \sim 10^{-2}$. The V_{CKM} was not discussed in Ref.[2].

In the present paper, we revise the model given in (1.1) by changing the structure only for the neutrino Dirac mass matrix M_D as follows; the structure X_2 is introduced in M_D not in the charged lepton mass matrix M_e unlikely in Ref.[2], and also it is assumed from a phenomenological point of view that the M_D is given with a somewhat different coefficient from

M_e:

$$M_D = k_D \Phi_0 (\mathbf{1} + a_D X_2) \Phi_0, \tag{1.3}$$

where

$$X_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{1.4}$$

Using this form we shall discuss U_{PMNS} as well as V_{CKM} of the model. As to the structure X_2, we will discuss in Sec.2. When once we accept the form (1.3), we predict a reasonable value of $\sin^2 2\theta_{13} \sim 0.07$ together with reasonable other parameters of U_{PMNS}, V_{CKM} and quark and lepton mass ratios.

2 Model

We assume that a would-be Yukawa interaction is given as follows:

$$W_Y = \frac{y_e}{\Lambda} e_i^c Y_e^{ij} \ell_j H_d + \frac{y_\nu}{\Lambda} \nu_i^c Y_D^{ij} \ell_j H_u + \lambda_R \nu_i^c Y_R^{ij} \nu_j^c + \frac{y_d}{\Lambda} d_i^c Y_d^{ij} q_j H_d + \frac{y_u}{\Lambda} u_i^c Y_u^{ij} q_j H_u, \tag{2.1}$$

where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are SU(2)$_L$ doublets. Under this definition of Y_f, the CKM mixing matrix and the PMNS mixing matrix are given by $V_{CKM} = U_u^\dagger U_d$ and $U_{PMNS} = U_e^\dagger U_\nu$, respectively, where U_f are defined by $U_f^\dagger M_f^\dagger M_f U_f = D_f^2$ (D_f are diagonal). (Hereafter, for simplicity, we denote U_{PMNS} and V_{CKM} as U and V, respectively.) In order to distinguish each yukawaon from others, we assume that Y_f have different R charges from each other together with R charge conservation (a global U(1) symmetry in $N = 1$ supersymmetry; for example, see Ref.[9]). (Of course, the R charge conservation is broken at an energy scale Λ'.)

We assume the following superpotential for yukawaons by introducing fields Θ^f, P, E, $\bar{E}$, E', $\bar{E}''$, E'', $\bar{E}'$, ϕ_e, and ϕ_d:

$$W_e = \lambda_e \left\{ \phi_e Y_e^{ij} + \frac{1}{\Lambda} (\Phi_0)^{i\alpha} \left((E'')_{\alpha\beta} + a_e \frac{1}{\Lambda^2} X_{\alpha k} \bar{E}^{kl} X_{l\beta}^T \right) (\Phi_0^T)^{\beta j} \right\} \Theta_{ji}^e, \tag{2.2}$$

$$W_D = \frac{\lambda_D}{\Lambda} \left\{ (E')_i^\alpha Y_D^{ij} (E')_j^\beta + (\Phi_0^T)^{\alpha i} \left(E_{ij} + a_D \frac{1}{\Lambda^2} X_{j\gamma}^T (\bar{E}'')^{\gamma\delta} X_{\delta j} \right) (\Phi_0)^{j\beta} \right\} \Theta_{\beta\alpha}^D, \tag{2.3}$$

$$W_u = \frac{\lambda_u}{\Lambda} \left\{ P_{ik} Y_u^{kl} P_{lj} + \hat{Y}_{ik}^u \bar{E}^{kl} \hat{Y}_{lj}^u \right\} \Theta_u^{ji}, \tag{2.4}$$

$$W_u' = \frac{\lambda_u'}{\Lambda} \left\{ \bar{E}^{ik} \hat{Y}_{kl}^u \bar{E}^{lj} + (\Phi_0)^{i\alpha} \left((E'')_{\alpha\beta} + a_u \frac{1}{\Lambda^2} X_{\alpha k} \bar{E}^{kl} X_{l\beta}^T \right) (\Phi_0^T)^{\beta j} \right\} \hat{\Theta}_{ji}^u, \tag{2.5}$$

Table 1: Assignments of $\mathrm{SU}(2)_L\times\mathrm{SU}(3)_c\times\mathrm{U}(3)\times\mathrm{U}(3)'$ and R charges

	ℓ	e^c	ν^c	q	u^c	d^c	H_u	H_d
$\mathrm{SU}(2)_L$	**2**	**1**	**1**	**2**	**1**	**1**	**2**	**2**
$\mathrm{SU}(3)_c$	**1**	**1**	**1**	**3**	$\mathbf{3}^*$	$\mathbf{3}^*$	**1**	**1**
U(3)	**3**	**3**	**3**	**3**	**3**	**3**	**1**	**1**
U(3)$'$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
R	r_ℓ	r_{ec}	$r_{\nu c}$	r_q	r_{uc}	r_{dc}	r_{Hu}	r_{Hd}

	Y_e	Y_D	Y_R	Y_u	$\hat{Y}^u$	Y_d	Θ^e	Θ^D	Θ^R	Θ_u	$\hat{\Theta}^u$	Θ^d
$\mathrm{SU}(2)_L$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$\mathrm{SU}(3)_c$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
U(3)	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	**6**	**1**	$\mathbf{6}^*$	**6**	**6**	**6**
U(3)$'$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**6**	**1**	**1**	**1**	**1**
R	r_{Ye}	r_{YR}	r_{YR}	r_{Yu}	$\hat{r}_{Yu}$	r_{Yd}	$r_{\Theta e}$	$r_{\Theta D}$	$r_{\Theta R}$	$r_{\Theta u}$	$\hat{r}_{\Theta u}$	$r_{\Theta d}$

	Φ_0	X	E	$\bar{E}$	E'	$\bar{E}'$	E''	$\bar{E}''$	P	$\bar{P}$	ϕ_e	ϕ_d
$\mathrm{SU}(2)_L$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
$\mathrm{SU}(3)_c$	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**	**1**
U(3)	$\mathbf{3}^*$	**3**	**6**	$\mathbf{6}^*$	**3**	$\mathbf{3}^*$	**1**	**1**	**6**	$\mathbf{6}^*$	**1**	**1**
U(3)$'$	$\mathbf{3}^*$	**3**	**6**	$\mathbf{6}^*$	**3**	$\mathbf{3}^*$	**1**	**1**	**6**	$\mathbf{6}^*$	**1**	**1**
R	r_0	$\frac{1}{2}(r_E+r''_E-1)$	r_E	$1-r_E$	r'_E	$1-r'_E$	r''_E	$1-r''_E$	r_P	$1-r_P$	$r_{\phi e}$	$r_{\phi e}$

$$W_d = \lambda_d \left\{ \phi_d Y_d^{ij} + \frac{1}{\Lambda}\left[(\Phi_0)^{i\alpha}\left((E'')_{\alpha\beta} + a_d \frac{1}{\Lambda^2} X_{\alpha k}\bar{E}^{kl}X^T_{l\beta}\right)(\Phi_0^T)^{\beta j} + m_d^0 (\bar{E}')^i_\alpha (\bar{E}'')^{\alpha\beta}(\bar{E}')^j_\beta \right]\right\}\Theta^d_{ji}, \tag{2.6}$$

$$W_R = \left\{ \mu_R Y_R^{ij} + \frac{\lambda_R}{\Lambda}\left[Y_e^{ik}\hat{Y}^u_{kl}\bar{E}^{lj} + \bar{E}^{ik}\hat{Y}^u_{kl}Y_e^{lj} + \xi^0_\nu Y_D^{ik}E_{kl}Y_D^{lj}\right]\right\}\Theta^R_{ji}. \tag{2.7}$$

Here, we have assumed family symmetries U(3)×U(3)$'$. The fundamental yukawaon Φ_0 is assigned to (3, 3) of U(3)× U(3)$'$, although quarks and leptons are still assigned to (3, 1) and yukawaons Y_f are assigned to (6^*, 1) of U(3)× U(3)$'$. In order to distinguish R charges between Y_e and Y_d, we have introduced U(3)×U(3)$'$ singlet scalar fields ϕ_e and ϕ_d.

We list the assignments of $\mathrm{SU}(2)_L\times\mathrm{SU}(3)_c\times\mathrm{U}(3)\times\mathrm{U}(3)'$ and R charges for the fields in the present model in Table 1. The assignments of R charges are done so that the total R charge of the superpotential term is $R(W)=2$. The r parameters in Table 1 must satisfy the following relations: $r_{Hu} = 2 - r_\ell - r_D - r_{\nu c} - r_{Ye} = 2 - r_q - r_{uc} - r_{Yu}$, $r_{Hd} = 2 - r_\ell - r_{\nu c} - r_{Yd} = 2 - r_q - r_{uc} - r_{Yd}$, $r_{\Theta e} = 2 - r_{Ye} - r_{\phi e}$, $r_{\Theta_D} = 2 - r_{YD} - 2r'_E$, $r_{\Theta_R} = 2 - r_{YR}$, $r_{\Theta_u} = 2 - r_{Yu} - 2r_P$,

$\hat{r}_{\Theta_u} = 1 + r_E - \hat{r}_{Yu}$, and $r_{\Theta_d} = 2 - r_{Yd} - r_{\phi d}$. Here, the R charges of these fields must satisfy the following relations: $2r_0 + r''_E = r_{Ye} + r_{\phi e} = r_{Yd} + r_{\phi d} = \hat{r}_{Yu} + 1 - r_E$, $2r_0 + r_E = r_{YD} + 2r'_D$, and $r_{YR} = r_{Ye} + \hat{r}_{Yu} = 2r_{YD} + r_E$. Since we consider that family symmetries U(3) and U(3)$'$ are gauge symmetries, the model must be anomaly free. However, as seen in Table 1, the present model has anomaly coefficients $A(\mathrm{SU}(3)) = 9$ and $A(\mathrm{U}(3)') = 7$, so that we need further fields $(\mathbf{6}^* + \mathbf{3}^* + \mathbf{3}^*, \mathbf{1})$ and $(\mathbf{1}, \mathbf{6}^*)$ of U(3)×U(3)$'$. However, since roles of such additional fields in the present model are, at present, not clear, we do not discuss such fields.

From Eqs.(2.2) and (2.3) [and also (2.5) and (2.6)], we obtain

$$R(E) + R(\bar{E}) = R(E'') + R(\bar{E}''). \tag{2.8}$$

The VEVs of the introduced fields E, $\bar{E}$, P, and $\bar{P}$ are described by the following superpotential by assuming $R(E\bar{E}) = R(P\bar{P}) = 1$:

$$W_{E,P} = \frac{\lambda_1}{\Lambda}\mathrm{Tr}[\bar{E}E\bar{P}P] + \frac{\lambda_2}{\Lambda}\mathrm{Tr}[\bar{E}E]\mathrm{Tr}[\bar{P}P], \tag{2.9}$$

which leads to

$$\langle E\rangle\langle\bar{E}\rangle \propto \mathbf{1}, \quad \langle P\rangle\langle\bar{P}\rangle \propto \mathbf{1}. \tag{2.10}$$

We assume specific solutions of Eq.(2.10):

$$\frac{1}{v_E}\langle E\rangle = \frac{1}{\bar{v}_E}\langle\bar{E}\rangle = \mathbf{1}, \tag{2.11}$$

$$\frac{1}{v_P}\langle P\rangle = \frac{1}{\bar{v}_P^*}\langle\bar{P}\rangle^\dagger = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1), \tag{2.12}$$

as the explicit forms of $\langle E\rangle$, $\langle\bar{E}\rangle$, and $\langle\bar{P}\rangle$. We assume similar superpotential forms for E'' and $\bar{E}''$, and for E' and $\bar{E}'$.

From SUSY vacuum conditions $\partial W/\partial\Theta = 0$, we obtain the following relations:

$$\langle Y_e\rangle = k_e\langle\Phi_0\rangle\left(\mathbf{1} + a_e X X^T\right)\langle\Phi_0^T\rangle, \tag{2.13}$$

$$\langle Y_D\rangle = k_D\langle\Phi_0^T\rangle\left(\mathbf{1} + a_D X^T X\right)\langle\Phi_0\rangle, \tag{2.14}$$

$$\langle P\rangle\langle Y_u\rangle\langle P\rangle = k'_u\langle\hat{Y}^u\rangle\langle\hat{Y}^u\rangle, \tag{2.15}$$

$$\langle\hat{Y}^u\rangle = k_u\langle\Phi_0\rangle\left(\mathbf{1} + a_u X X^T\right)\langle\Phi_0^T\rangle, \tag{2.16}$$

$$\langle Y_d\rangle = k_d\left[\langle\Phi_0\rangle\left(\mathbf{1} + a_d X X^T\right)\langle\Phi_0^T\rangle + m_d^0\mathbf{1}\right], \tag{2.17}$$

$$\langle Y_R\rangle = k_R\left(\langle Y_e\rangle\langle\hat{Y}^u\rangle + \langle\hat{Y}^u\rangle\langle Y_e\rangle + \xi_\nu^0\langle Y_D\rangle\langle Y_D\rangle\right), \tag{2.18}$$

where, for convenience, we have already put $\langle E\rangle$ as $\mathbf{1}$, and so on. Here, since we have assumed that all Θ fields take $\langle\Theta\rangle = 0$, we do not need to consider vacuum conditions for other fields $\partial W/\partial Y_e = 0$, because those always contain $\langle\Theta\rangle$. Thus, mass matrices are given by $M_e = \langle Y_e\rangle$, $M_D = \langle Y_D\rangle$, $M_u = \hat{k}_u \hat{M}_u \hat{M}_u$, $\hat{M}_u = \langle\hat{Y}^u\rangle$, $M_d = \langle Y_d\rangle$, $M_\nu = M_D M_R^{-1} M_D^T$, and $M_R = \langle Y_R\rangle$.

The most curious assumption is to assume the VEV matrix form of the scalar X as

$$\frac{1}{v_X}\langle X\rangle_{\alpha i} = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}_{\alpha i} . \tag{2.19}$$

The form (2.19) leads to

$$\left(\langle X\rangle\langle X^T\rangle\right)_{\alpha\beta} = \frac{3}{2}(X_3)_{\alpha\beta}, \qquad \left(\langle X^T\rangle\langle X\rangle\right)_{ij} = \frac{3}{2}(X_2)_{ij}, \tag{2.20}$$

together with $\langle X\rangle\langle X\rangle = \langle X\rangle$, where X_3 and X_2 is defined by Eqs. (1.2) and (1.4), respectively, and, for simplicity, we have put $v_X = 1$ because we are interested only in the relative ratios among the family components.

At present, there is no idea for the origin of this form (2.19). We may speculate that this form is related to a breaking pattern of U(3)×U(3)′ (for example, discrete symmetries $S_2\times S_3$). In the present paper, the form (2.12) is only ad hoc assumption. However, as seen later, we can obtain a good fitting for the neutrino mixing angle $\sin^2 2\theta_{13}$ due to this assumption.

3 Parameter fitting

We summarize our mass matrices in the present model as follows:

$$M_e = k_e \Phi_0 (\mathbf{1} + a_e X_3) \Phi_0^T, \tag{3.1}$$

$$M_D = k_D \Phi_0^T (\mathbf{1} + a_D X_2) \Phi_0, \tag{3.2}$$

$$P M_u P = k_u' \hat{M}^u \hat{M}^u, \tag{3.3}$$

$$\hat{M}^u = k_u \Phi_0 \left(\mathbf{1} + a_u e^{i\alpha_u} X_3\right) \Phi_0^T, \tag{3.4}$$

$$M_d = k_d \left[\Phi_0 (\mathbf{1} + a_d X_3) \Phi_0^T + m_d^0 \mathbf{1}\right], \tag{3.5}$$

$$M_\nu = M_D M_R^{-1} M_D^T, \tag{3.6}$$

$$M_R = k_R \left(M_e \hat{M}^u + \hat{M}^u M_e + \xi_\nu^0 M_D M_D\right), \tag{3.7}$$

where, for convenience, we have dropped the notations "⟨" and "⟩". Since we are interested only in the mass ratios and mixings, hereafter, we will use dimensionless expressions $\Phi_0 = \mathrm{diag}(x_1, x_2, x_3)$, $P = \mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, 1)$, and $E = \mathrm{diag}(1, 1, 1)$. For simplicity, we have regarded

Table 2: Process for fitting parameters. Of course, since these parameters listed in each step can slightly affect predicted values listed in the other steps, we need fine tuning after the step 5th.

Step	Inputs	N_{inp}	Parameters	N_{par}	Predictions
1st	$\frac{m_e}{m_\mu}$, $\frac{m_\mu}{m_\tau}$ $\frac{m_u}{m_c}$, $\frac{m_c}{m_t}$ $\sin^2 2\theta_{23}$	5	$\frac{x_1}{x_2}$, $\frac{x_2}{x_3}$ a_e, a_u α_u	5	
2nd	$\sin^2 2\theta_{12}$ R_ν	2	a_D ξ_ν^0	2	$\sin^2 2\theta_{13}$, δ_{CP}^ℓ , 2 Majorana phases $\frac{m_{\nu 1}}{m_{\nu 2}}$, $\frac{m_{\nu 2}}{m_{\nu 3}}$
3rd	$\frac{m_s}{m_b}$	1	a_d	1	
4th	$\|V_{us}\|$, $\|V_{cb}\|$	2	(ϕ_1, ϕ_2)	2	$\|V_{ub}\|$, $\|V_{td}\|$, δ_{CP}^q
5th	$\frac{m_d}{m_s}$	1	m_d^0	1	not affect to other predictions
option	Δm_{atm}^2		$m_{\nu 3}$		$(m_{\nu 1}, m_{\nu 2}, m_{\nu 3})$, $\langle m\rangle$
$\sum N_{...}$		11		11	

the parameter a_d as real correspondingly to the parameter a_e. The parameters are re-refined by Eqs.(3.1)-(3.5).

In the present model, we have 9 adjustable parameters except for x_i [a_e, a_D, (a_u, α_u), a_d, (ϕ_1, ϕ_2), m_d^0, and ξ_ν^0] for the 16 observable quantities (6 mass ratios in the up-quark-, down-quark-, and neutrino-sectors, 4 CKM mixing parameters, and 4+2 PMNS mixing parameters). In order to fix these parameters, we use, as input values, the observed values for m_c/m_t, m_u/m_c, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, R_ν, m_d/m_s as shown later. The relative ratios of parameters x_i in Φ_0 are fixed by the ratios of the charged lepton masses m_e/m_μ and m_μ/m_τ. The process of fixing parameters are summarized in Table. 2.

Now let us present the details of parameter fitting. Since we do not change the mass matrix structures for M_e, M_u, and M_d from the previous paper [3], we use the following parameter values of a_e and (a_u, α_u)

$$(a_e, a_u, \alpha_u) \sim (8, -1.35, \pm 8^\circ), \tag{3.8}$$

which are fixed from the observed values of m_c/m_t, m_u/m_c, and $\sin^2 2\theta_{23}$:

$$r_{12}^u \equiv \sqrt{\frac{m_u}{m_c}} = 0.045^{+0.013}_{-0.010}, \qquad r_{23}^u \equiv \sqrt{\frac{m_c}{m_t}} = 0.060 \pm 0.005, \tag{3.9}$$

at $\mu = m_Z$ [11], and $\sin^2 2\theta_{23} > 0.95$ [12]. (These values will be fine-tuned in whole parameter fitting of U and V later.) Note that the neutrino sector of the model is different from the previous model, however the predicted values of $\sin^2 2\theta_{23}$ are almost the same before.

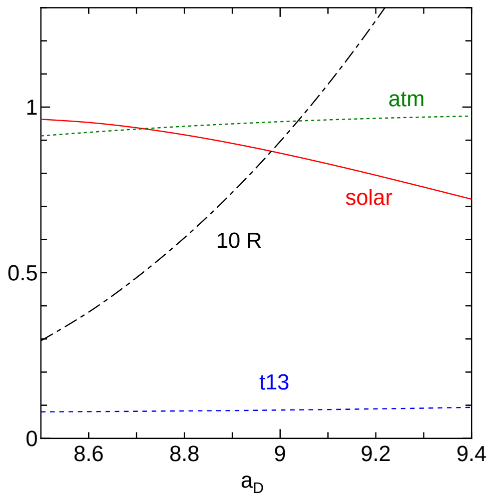

Figure 1: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio R_ν versus the parameter a_D. ("solar", "atm", "t13", and "10 R" denote curves of $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and $R_\nu \times 10$, respectively. Other parameter values are taken as $a_e = 7.5$, $a_u = -1.35$, and $\alpha_u = 7.6°$.

First, let us investigate lepton sector. In the revised model, a new parameter a_D is added. We illustrate the behaviors of Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio R_ν versus the parameter a_D for a case of $\xi_\nu^0 = 0$. As seen in Fig.1, the parameter a_D does not change the prediction $\sin^2 2\theta_{23} \sim 1$ in the previous model. Also, note that the prediction of $\sin^2 2\theta_{13}$ is insensitive to the parameter a_D, i.e. $\sin^2 2\theta_{13} \sim 0.08$. Only the predictions of $\sin^2 2\theta_{12}$ and $R_\nu = (m_{\nu 2}^2 - m_{\nu 1}^2)/(m_{\nu 3}^2 - m_{\nu 2}^2)$ are sensitive to the parameter a_D. In order to fit the observed value [12] $\sin^2 2\theta_{12} = 0.857 \pm 0.024$, we take $a_D = 9.01$. However, in the model with $\xi_\nu^0 = 0$, the value $a_D = 9.01$ cannot fit the observed value [12] of R_ν,

$$R_\nu \equiv \frac{\Delta m_{21}^2}{\Delta m_{32}^2} = \frac{(7.50 \pm 0.20) \times 10^{-5}\ \mathrm{eV}^2}{(2.32^{+0.12}_{-0.08}) \times 10^{-3}\ \mathrm{eV}^2} = (3.23^{+0.14}_{-0.19}) \times 10^{-2}. \tag{3.10}$$

The non-zero parameter ξ_ν^0 has phenomenologically been brought in order to adjust the predicted value of R_ν. The predicted values of $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{12}$, and $\sin^2 2\theta_{13}$ are almost unchanged against the parameter ξ_ν^0. In order to fit the neutrino mass ratio R_ν, we take $\xi_\nu^0 = -0.78$.

Next, we discuss quark sector. Since we have fixed the five parameters a_e, a_u, α_u, a_D, and ξ_ν^0, we have remaining four parameters for six observables (2 down-quark mass ratios and 4 CKM mixing parameters). The parameters a_d and m_d^0 are used to fit the observed down-quark mass ratios [11]

$$r_{23}^d \equiv \frac{m_s}{m_b} = 0.019^{+0.006}_{-0.006}, \quad r_{12}^d \equiv \frac{m_d}{m_s} = 0.053^{+0.005}_{-0.003}, \tag{3.11}$$

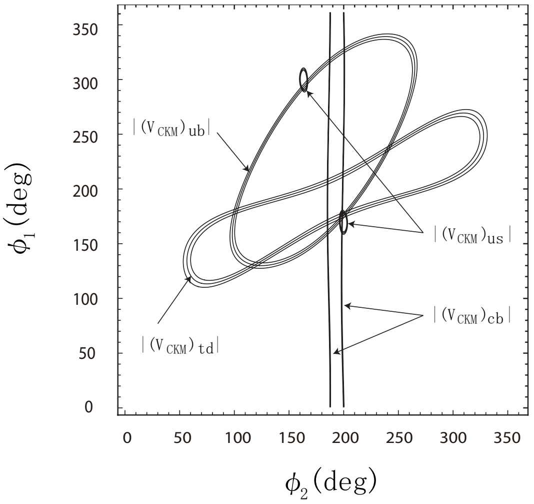

Figure 2: Contour plots in the (ϕ_1, ϕ_2) parameter plane, which are shown by using experimental constraints on CKM mixing matrix elements: $|V_{us}| = 0.2252 \pm 0.0009$, $|V_{cb}| = 0.0406 \pm 0.0013$, $|V_{ub}| = 0.00389 \pm 0.00044$, and $|V_{td}| = 0.0084 \pm 0.0006$. The CKM elements depends on only the parameter set of [a_e, (a_u, α_u), a_d, m_d^0, ϕ_1, and ϕ_2]. Here we present contour plots of the CKM elements in the (ϕ_1, ϕ_2) parameter plane by taking the values of other parameters as $a_e = 7.5$, $a_u = -1.35$, $\alpha_u = -7.6°$, $a_d = 25$, and $m_d^0 = 0.0115$. We find that (ϕ_1, ϕ_2)=(177.0°, 197.4°) is consistent with all the CKM constraints.

respectively. Therefore, the four CKM mixing parameters are described only by two parameters (ϕ_1, ϕ_2). As shown in Fig. 2, all the experimental constraints on CKM mixing matrix elements are satisfied by fine tuning with use of only two parameters (ϕ_1, ϕ_2).

Finally, we do fine-tuning of whole parameter values in order to give more improved fitting with the whole data. Our final result is as follows: under the parameter values

$$a_e = 7.5, \ a_D = 9.01, \ (a_u, \alpha_u) = (-1.35, -7.6°), \ a_d = 25,$$
$$m_d^0 = 0.0115, \quad \xi_\nu^0 = -0.78, \ (\phi_1, \phi_2) = (177.0°, 197.4°), \tag{3.12}$$

we obtain

$$r_{12}^u = 0.0465, \quad r_{23}^u = 0.0614, \quad r_{12}^d = 0.0569, \quad r_{23}^d = 0.0240, \tag{3.13}$$

$$\sin^2 2\theta_{23} = 0.969, \quad \sin^2 2\theta_{12} = 0.860, \quad \sin^2 2\theta_{13} = 0.0711, \quad R_\nu = 0.0324, \tag{3.14}$$

$$\delta_{CP}^\ell = -131° \quad (J^\ell = -2.3 \times 10^{-2}), \tag{3.15}$$

$$|V_{us}| = 0.2271, \quad |V_{cb}| = 0.0394, \quad |V_{ub}| = 0.00347, \quad |V_{td}| = 0.00780, \tag{3.16}$$

$$\delta_{CP}^q = 59.6° \quad (J^q = 2.6 \times 10^{-5}). \tag{3.17}$$

Here, δ_{CP}^ℓ and δ_{CP}^q are Dirac CP violating phases in the standard conventions of U and V, respectively.

Even if we choose a value of ξ_ν^0 which gives a value of R_ν within 1σ given in Eq.(3.10), our predicted value of $\sin^2 2\theta_{13}$ is $\sin^2 2\theta_{13} = 0.0711^{+0.003}_{-0.004}$, which is still somewhat small compared with the observed value $\sin^2 2\theta_{13} = 0.098 \pm 0.013$ [12]. So far, we have assumed that the parameter ξ_ν^0 is real. If we consider that the parameter ξ_ν^0 is complex, $\xi_\nu^0 \to \xi_\nu^0 e^{i\alpha_\nu}$, we can adjust the value of $\sin^2 2\theta_{13}$ without changing other predicted values as seen in Fig.3. However, such a modification by the parameter α_ν is not essential in the present model, so that we will regard that the parameter ξ_ν^0 is real.

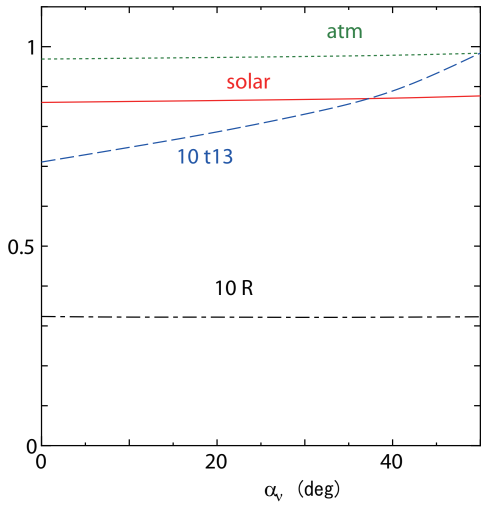

Figure 3: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio R_ν versus the phase parameter α_ν. ("solar", "atm", "10 t13", and "10 R" denote curves of $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13} \times 10$, and $R_\nu \times 10$, respectively. Here the α_ν dependence is presented under the parameter values given by (3.12).

We can also predict neutrino masses, for the parameters given (3.12) with real ξ_ν^0,

$$m_{\nu 1} \simeq 0.0048 \text{ eV}, \quad m_{\nu 2} \simeq 0.0101 \text{ eV}, \quad m_{\nu 3} \simeq 0.0503 \text{ eV}, \tag{3.18}$$

by using the input value [13] $\Delta m^2_{32} \simeq 0.00243$ eV^2. We also predict the effective Majorana neutrino mass [14] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = \left| m_{\nu 1} (U_{e1})^2 + m_{\nu 2} (U_{e2})^2 + m_{\nu 3} (U_{e3})^2 \right| \simeq 7.3 \times 10^{-4} \text{ eV}. \tag{3.19}$$

Finally, let us comment on sensitivity of the predicted values Eq.(3.14) to the input parameter values Eq.(3.12). For simplicity, we show the sensitivity of only the lepton mixing and up-quark mass ratios to the input parameters a_D, a_u and α_u in Table 3. (We do not show sensitivity for the predicted CKM parameters, because it can be easily seen in Fig. 3.) In Table 3, values Δx ($x = a_D$, a_u and α_u) for the parameter values x are taken such as $(\Delta x)/x = 0.05$,

Table 3: Sensitivity of the predicted values to the input parameter values. In the table, values Δx ($x = a_D$, a_u and α_u) for the parameter values x are taken such as $(\Delta x)/x = 0.05$, where the values x are given in Eq.(3.12). Note that r^u_{12} and r^u_{23} are independent of the parameter a_D.

	$\Delta a_D = \pm 0.451$	$\Delta a_u = \pm 0.068$	$\Delta \alpha_u = \pm 0.38$
r^u_{12}	0.0465	$0.0465^{+0.0239}_{-0.0179}$	$0.0465^{+0.0023}_{-0.0022}$
r^u_{23}	0.0614	$0.0614^{+0.0075}_{-0.0054}$	$0.0614^{+0.0017}_{-0.0016}$
$\sin^2 2\theta_{12}$	$0.860^{+0.092}_{-0.149}$	$0.860^{+0.036}_{-0.045}$	$0.860^{+0.004}_{-0.004}$
$\sin^2 2\theta_{23}$	$0.969^{+0.021}_{-0.040}$	$0.969^{+0.023}_{-0.034}$	$0.969^{+0.002}_{-0.002}$
$\sin^2 2\theta_{13}$	$0.0711^{+0.0012}_{-0.0016}$	$0.0711^{+0.0094}_{-0.0091}$	$0.0711^{+0.0001}_{-0.0001}$

where the values x are given in Eq.(3.12). Here, we consider no change of values for the parameters a_e and x_i ($i = 1, 2, 3$) because those have been fixed by the observed charged lepton masses with high accuracy. We also do not discuss the parameter dependence of R_ν and $r^d_{12} = m_d/m_s$, because those are freely adjustable by the parameters ξ^0_ν and m^0_d, respectively, without almost affecting other observables. As seen in Table 3, the predicted value $\sin^2 2\theta_{13}$ is sensitive to the parameter value a_u, so that we can take a value of a_u which gives $\sin^2 2\theta_{13} \simeq 0.08$ at the cost of other fitting. Also, we find that those predicted values are practically insensitive to the parameter value α_u.

6 Concluding remarks

In conclusion, by assuming VEV matrix forms of the yukawaons Eqs.(3.1) -(3.7), we have obtained reasonable CKM and PMNS mixing parameters together with quark and neutrino mass ratios. The major change from the previous yukawaon models is in the form of M_D. Although we give the form by assuming the VEV matrix $X_{\alpha i}$ which is given by Eq.(2.19), and by considering the mechanism $(XX^T)_{\alpha\beta} = (X_3)_{\alpha\beta}$ versus $(X^T X)_{ij} = (X_2)_{ij}$, it is still phenomenological and somewhat factitious. However, when once we accept the form of M_D, we can obtain $\sin^2 2\theta_{13} \sim 0.07$ whose value is not sensitive to the other parameters.

Note that the present model does not have any family-dependent parameters except for (x_1, x_2, x_3) in $\langle \Phi_0 \rangle$ and (ϕ_1, ϕ_2) in $\langle P \rangle$. The parameter values (x_1, x_2, x_3) have been fixed by the observed charged lepton masses. Therefore, the model have only 9 adjustable parameters for 16 observables. The 5 parameter values of 9 parameters, (a_e, a_D, (a_u, α_u), and ξ^0_ν), have been fixed by the observed values m_u/m_c, m_c/m_t, $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, and R_ν. Especially, the parameter a_D has been fixed the observed value $\sin^2 2\theta_{12}$. The parameter ξ^0_ν has been introduced only in order to adjust the ratio R_ν. (In other words, the value of ξ^0_ν has been fixed by R^{obs}_ν, the value (3.10).) Logically speaking, we need four observed values in order to fix the four parameters a_e, a_D, and (a_u, α_u). However, as seen in Fig.1, the values $\sin^2 2\theta_{23} \sim 0.9$ and $\sin^2 2\theta_{13} \sim 0.07$

are almost determined independently of the parameter a_D when we fix a_e and (a_u, α_u) from the observed up-quark mass ratios. Therefore, $\sin^2 2\theta_{23} \sim 0.9$ and $\sin^2 2\theta_{13} \sim 0.07$ can substantially be regarded as predictions in the present model. Of course, after we fix the 5 parameters, predictions are 6 quantities: $\sin^2 2\theta_{13}$, 2 neutrino mass ratios, CP-violating phase parameter, and 2 Majorana phases.

Of the remaining 4 parameters a_d, m_d^0, and (ϕ_1, ϕ_2), the parameters a_d and m_d^0 are determined by the down-quark mass ratios m_s/m_b and m_d/m_s, respectively. Therefore, the 4 CKM mixing parameters are predicted only by adjusting the two parameters (ϕ_1, ϕ_2). We can obtain reasonable predictions of the CKM mixing parameters.

The present model is still in a level of a phenomenological model. Nevertheless, it seems that the yukawaon model offers us a promising hint for a unified mass matrix model for quarks and leptons, i.e. it seems to suggest an idea that the observed family mixings and mass ratios of quarks and leptons are caused by a common origin.

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