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Neutrino Mass Matrix Model with a Double Seesaw Form

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Abstract

Within the framework of the so-called yukawaon model, which has been proposed for the purpose of a unified description of the lepton mixing matrix U_{PMNS} and the quark mixing matrix V_{CKM}, a neutrino mass matrix model with a double seesaw form $M_\nu = k_\nu (m_D M_R^{-1} m_D^T)^2$ is proposed. The model has only two adjustable parameters for the PMNS mixing and neutrino mass ratios. (Other parameters are fixed from the observed quark and charged lepton mass ratios and CKM mixing.) The model can give reasonable values $\sin^2 2\theta_{12} \simeq 0.85$ and $\sin^2 2\theta_{23} \sim 1$ and $\sin^2 2\theta_{13} \sim 0.09$ together with $R_\nu \equiv \Delta m^2_{21}/\Delta m^2_{32} \sim 0.03$. Our prediction of the effective neutrino mass $\langle m \rangle$ in the neutrinoless double beta decay takes a sizable value $\langle m \rangle \simeq 0.0034$ eV.

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1 Introduction

Most particle physicists think that the mass spectra and mixing patterns of quarks and leptons will be understood by a unified description. As one of such models, "yukawaon" model [1, 2, 3, 4] has been proposed. The model, which is a kind of "flavon" model [5], has the following characteristics: (i) Yukawa coupling constants Y_f $(f = u, d, e, \cdots)$ in the standard model are understood as vacuum expectation values (VEVs) of scalars ("yukawaon") with 3×3 components, i.e. by $y_f \langle Y_f \rangle / \Lambda$, where Λ is an energy scale of the effective theory. (ii) The hierarchical structures of the effective Yukawa coupling constants originate only in a fundamental VEV matrix $\langle \Phi_0 \rangle$, whose hierarchical structure is ad hoc assumed and whose VEVs are fixed by the observed charged lepton masses. (Of course, the goal in the yukawaon model is to understand the VEVs of the fundamental scalar Φ_0 itself on the basis of a dynamical consideration.) In the yukawaon model, in principle, there are no family-number-dependent parameters except for $\langle \Phi_0 \rangle$. (Regrettably, in order to obtain reasonable values of quark mixing matrics [6] V_{CKM}, at present, we need a phase matrix P_u (or P_d) with two phase parameters [3, 4]. The final goal of our model is also to remove such family dependent parameters.) (iii) Relations among VEV matrices are obtained from SUSY vacuum conditions for a given superpotential under family symmetries and R charges assumed. (Since we use the observed charged lepton mass values as the input values, it is a characteristic in the yukawaon model that adjustable parameters are quite few.)

In a series of yukawaon models [2, 3, 4], mass matrices of quarks and leptons, (M_u, M_d) and (M_ν, M_e), have been given as follows:

$$\begin{aligned} &M_e = k_e \Phi_0 (\mathbf{1} + a_e X_3) \Phi_0, \\ &M_\nu = M_D M_R^{-1} M_D^T, \\ &M_D = M_e, \\ &M_R = k_R (\hat{M}_u M_e + M_e \hat{M}_u) + \cdots, \\ &P_u M_u P_u = k'_u \hat{M}_u \hat{M}_u, \\ &\hat{M}_u = k_u \Phi_0 (\mathbf{1} + a_u X_3) \Phi_0, \\ &M_d = k_d \left[\Phi_0 (\mathbf{1} + a_d X_3) \Phi_0 + m_d^0 \mathbf{1} \right]. \end{aligned} \tag{1.1}$$

Here, for convenience, we have denote VEV relations, which are obtained from SUSY vacuum conditions, by using the conventional notations for the quark mass matrices (M_u, M_d) and so on instead of those for VEV matrices $(\langle Y_u \rangle, \langle Y_d \rangle)$ and so on. In Eq.(1.1), the matrices Φ_0, X_3, $\mathbf{1}$ and P_u have structures given by

$$\Phi_0 = \begin{pmatrix} x_1 & 0 & 0 \\ 0 & x_2 & 0 \\ 0 & 0 & x_3 \end{pmatrix}, \quad X_3 = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{1.2}$$

and $P_u = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$, respectively. The coefficients a_f play an essential role in obtaining the mass ratios and mixings, while the family-number independent coefficients k_f and k'_u do not. The values of (x_1, x_2, x_3) with $x_1^2 + x_2^2 + x_3^2 = 1$ are fixed by the observed charged lepton mass values under the given value of a_e.

Although the model can give successful predictions, the model has regrettably failed to obtain the observed value [8] $\sin^2 2\theta_{13} \sim 0.09$. Therefore, from the phenomenological point of view, the authors have recently proposed a new yukawaon model [7] by changing a structure of the Dirac neutrino mass matrix M_D :

$$M_\nu = M_D M_R^{-1} M_D^T, \tag{1.3}$$

$$M_D = k_D \Phi_0^T (\mathbf{1} + a_D X_2) \Phi_0, \tag{1.4}$$

$$M_R = k_R \left(\hat{M}_u M_e + M_e \hat{M}_u + \xi_0^\nu M_D M_D \right), \tag{1.5}$$

where X_2 is given by

$$X_2 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{1.6}$$

The model with the parameter values $a_e = 7.5$, $a_D = 9.01$, $(a_u, \alpha_u) = (-1.35, -7.6^\circ)$ and $\xi_\nu^0 = -0.78$ can give reasonable fitting $\sin^2 2\theta_{12} = 0.860$, $\sin^2 2\theta_{23} = 0.969$, $\sin^2 2\theta_{13} = 0.0711$, and $R_\nu \equiv \Delta m_{21}^2/\Delta n_{32}^2 = 0.0324$ together with reasonable up-quark mass ratios. More explicitly speaking, the values of (a_u, α_u) have been fixed by the observed up-quark mass ratios. The value of ξ_0^ν has been fixed by the observed value of R_ν. (The term ξ_ν^0 plays as if it is a constant term $\xi_0^\nu \mathbf{1}$ in the neutrino mass matrix $M_\nu = M_D M_R^{-1} M_D^T$, so that the value of ξ_0^ν is used only to adjust the value of R_ν without changing other predicted values except for $\sin^2 2\theta_{13}$.) However, the existence of the ξ_ν^0 term is somewhat artificial. Besides, the non-vanishing value of ξ_ν^0 slightly lowers a predicted value of $\sin^2 2\theta_{13}$. For example, if we take $\xi_\nu^0 = 0$ keeping the values for the other parameters, we can obtain $\sin^2 2\theta_{13} = 0.086$, while we obtain an unwelcome value $R_\nu = 0.091$ at the same time.

Now, we notice that the value of $(R_\nu)^2 \sim (10^{-1})^2$ at $\xi_0^\nu = 0$ is an order of the observed value of R_ν. This suggests a possibility that if we modify the neutrino sector as follows:

$$M_\nu = k_\nu \hat{M}_\nu \hat{M}_\nu, \tag{1.7}$$

$$\hat{M}_\nu = M_D M_R^{-1} M_D^T, \tag{1.8}$$

$$M_R = k_R(\hat{M}_u M_e + M_e \hat{M}_u), \tag{1.9}$$

we can obtain reasonable solutions for all lepton mixing parameters. Note that there is no longer the ξ_ν^0-term. In this model, the parameters in the neutrino sector are only a_e, a_D and a_u. As we denote later, the parameters a_e and a_u are fixed from the observed CKM mixing and quark mass ratios, so that we have only a parameter a_D (complex) in the PMNS mixing and neutrino mass rations.

In the next section, we give a model for quark and lepton mixings and their mass rations on the basis of a revised yukawaon model. In Sec.3, as far as parameter fitting for observed values is concered, since we change only the neutrino sector, we discuss only the PMNS mixing and neutrino mass ratios. The parameter values in the down-quark sector are effectively unchanged, so that we can obtain the same predictions for the down-quark mass ratios and CKM matrix parameters without changing the successful results in the previous paper [7].

2 Model

We assume that a would-be Yukawa interaction is given as follows:

$$W_Y = \frac{y_e}{\Lambda} e_i^c \bar{Y}_e^{ij} \ell_j H_d + \frac{y_\nu}{\Lambda^2} (\ell_i H_u) \bar{Y}_\nu^{ij} (\ell_j H_u) + \frac{y_d}{\Lambda} d^{ci} Y_{ij}^d q^j H_d + \frac{y_u}{\Lambda} u^{ci} Y_{ij}^u q^j H_u, \tag{2.1}$$

where $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$ are SU(2)$_L$ doublets. Note that in Eq.(2.1) there are no SU(2)$_L$ singlet neutrinos. We have straightforwardly defined the neutrino mass matrix M_ν by the second term in Eq.(2.1). Although we denoted in Eq.(1.3) as if the matrix M_D is a Dirac neutrino mass matrix, the matrix M_D does not have a meaning of the Dirac mass matrix [see Eq.(2.3) later]. Under the definition of $\bar{Y}_\ell$ (Y^q) in Eq.(2.1), the quark mixing matrix (the Cabibbo-Kobayashi-Maskawa matrix [6] and the lepton mixing mixing matrix (the Pontecorvo-Maki-Nakagawa-Sakata matrix [9]) are given by $V_{CKM} = U_u^\dagger U_d$ and $U_{PMNS} = U_e^\dagger U_\nu$, respectively, where U_f are defined by $U_f^\dagger M_f^\dagger M_f U_f = D_f^2$ (D_f are diagonal). (Here and hereafter, sometimes, we denote $\bar{Y}_\ell$ and Y^q as Y_f for simplify. In order to distinguish each yukawaon from others, we assume that Y_f have different R charges from each other together with R charge conservation. (Of course, the R charge conservation is broken at the energy scale Λ'.)

We assume the following superpotential for yukawaons:

$$W_e = \left\{ \mu_e \bar{Y}_e^{ij} + \frac{\lambda_e}{\Lambda} (\bar{\Phi}_0)^{i\alpha} \left(E''_{\alpha\beta} + \frac{a_e}{\Lambda^2} X_{\alpha k} \bar{E}^{kl} X^T_{l\beta} \right) (\bar{\Phi}_0^T)^{\beta j} \right\} \Theta^e_{ji}, \tag{2.2}$$

$$W_\nu = \left[\mu_\nu \bar{Y}_\nu^{ij} + \frac{\lambda_\nu}{\Lambda} (\hat{Y}_\nu^T)^{i\alpha} E''_{\alpha\beta} (\hat{Y}_\nu)^{\beta j} \right] \Theta^\nu_{ji} + \left[\mu'_\nu \hat{Y}_\nu^{\alpha i} + \frac{\lambda'_\nu}{\Lambda} \bar{Y}_D^{\alpha k} \hat{Y}^R_{k\beta} \bar{Y}_D^{\beta i} \right] \Theta'^\nu_{i\alpha}, \tag{2.3}$$

$$W_D = \left[\lambda_D \bar{Y}_D^{\alpha k} (E')_k^\beta + \frac{\lambda'_D}{\Lambda} (\bar{\Phi}_0^T)^{\alpha k} \left(E_{kl} + \frac{\lambda''_D}{\Lambda^2} X^T_{k\beta} (\bar{E}'')^{\beta\gamma} X_{\gamma l} \right) \bar{\Phi}_0^{l\beta} \right] \Theta^D_{\beta\alpha}, \tag{2.4}$$

$$W_R = \lambda_R \left[\bar{Y}_R^{i\alpha} \hat{Y}^R_{\alpha j} + \mu_R (E')^i_j \right] (\Theta_R)^j_i$$

$$+ \left[(\bar{E}')^i_\beta \bar{Y}_R^{\beta j} + \bar{Y}_R^{Ti\beta} (\bar{E}'^T)^j_\beta + \frac{\lambda'_R}{\Lambda} \left(\bar{Y}_e^{ik} \hat{Y}^u_{kl} \bar{E}^{lj} + \bar{E}^{ik} \hat{Y}^u_{kl} \bar{Y}_e^{lj} \right) \right] \Theta^R_{ji}, \tag{2.5}$$

$$W_u = \left(\mu_u Y^u_{ij} + \frac{\lambda_u}{\Lambda} \hat{Y}^u_{ik} \bar{E}^{kl} \hat{Y}^u_{lj} \right) \bar{\Theta}^{ji}_u$$

$$+ \frac{1}{\Lambda} \left[\lambda'_u \bar{E}^{ik} \hat{Y}^u_{kl} \bar{E}^{lj} + \lambda''_u (\bar{\Phi}_0)^{i\alpha} \left(E''_{\alpha\beta} + \frac{a_u}{\Lambda^2} X_{\alpha k} \bar{E}^{kl} X^T_{l\beta} \right) (\bar{\Phi}_0^T)^{\beta j} \right] \Theta^{u\prime}_{ji}, \tag{2.6}$$

$$W_d = \left[\lambda_d \bar{P}_d^{ik} \left(Y^d_{kl} + \frac{\xi_0^d}{\Lambda^2} P^d_{km} \bar{E}^{mn} P^d_{nl} \right) \bar{P}_d^{lj} + \lambda'_d (\bar{\Phi}_0)^{i\alpha} \left(E''_{\alpha\beta} + \frac{a_d}{\Lambda^2} X_{\alpha k} \bar{E}^{kl} X^T_{l\beta} \right) (\bar{\Phi}_0^T)^{\beta j} \right] \Theta^d_{ji}. \tag{2.7}$$

Here, we have assumed family symmetries U(3)×U(3)$'$.

From Eqs(2.2) and (2.3), we obtain $R(E'') - R(\bar{E}) = R(E) - R(\bar{E}'') = 2R(X)$, i.e. $R(E'') + R(\bar{E}'') = R(E) + R(\bar{E})$. When we take $R(E'') + R(\bar{E}'') = R(E) + R(\bar{E}) = R(P^d) + R(\bar{P}_d) = 1$, we can write the following superpotential:

$$W_{E,P} = \frac{\lambda_1}{\Lambda}\mathrm{Tr}[\bar{E}E\bar{P}_d P_d] + \frac{\lambda_2}{\Lambda}\mathrm{Tr}[\bar{E}E]\mathrm{Tr}[\bar{P}_d P_d]. \tag{2.8}$$

From the superpotential (2.8), we obtain $\langle E\rangle\langle\bar{E}\rangle \propto \mathbf{1}$ and $\langle P_d\rangle\langle\bar{P}_d\rangle \propto \mathbf{1}$. We assume specific solutions from those solutions for $\langle E\rangle$ and $\langle P\rangle$:

$$\frac{1}{v_E}\langle E\rangle = \frac{1}{\bar{v}_E}\langle\bar{E}\rangle = \mathbf{1}, \tag{2.9}$$

$$\frac{1}{v_P}\langle P_d\rangle^\dagger = \frac{1}{\bar{v}_P^*}\langle\bar{P}_d\rangle = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1), \tag{2.10}$$

as the explicit forms of $\langle E\rangle$, $\langle\bar{E}\rangle$ and $\langle\bar{P}_d\rangle$. We assume similar superpotential forms for $(E'', \bar{E}'')$ and $(E', \bar{E}')$.

The term $P^d\bar{E}P^d$ in Eq.(2.7) is introduced in order to adjust the down-quark mass ratio m_d/m_s as see in the next section. If $R(\bar{E}) = R(\bar{Y}_e)$, such a $\bar{E}$ term also appears in Eq.(2.2). However, we obtain $R(Y^d) = R(\bar{E}) + 2R(P^d)$ from Eq.(2.7), and $R(\bar{Y}_e) = R(Y^d) + 2R(\bar{P}_d)$ from Eqs.(2.2) and (2.7), we see $R(\bar{Y}_e) = R(\bar{E}) + 2$, i.e. we obtain $R(\bar{Y}_e) \neq R(\bar{E})$

Since we assume that all Θ fields take $\langle\Theta\rangle = 0$, the superpotential terms (2.2) - (2.7) lead to relations

$$\langle\bar{Y}_e\rangle = k_e\langle\bar{\Phi}_0\rangle\left(\mathbf{1} + a_e X X^T\right)\langle\bar{\Phi}_0^T\rangle, \tag{2.11}$$

$$\langle\bar{Y}_\nu\rangle = k_\nu\langle\hat{Y}^\nu\rangle\langle\hat{Y}^\nu\rangle, \tag{2.12}$$

$$\langle\hat{Y}^\nu\rangle = k'_\nu\langle\bar{Y}_D\rangle\langle\hat{Y}_R\rangle\langle\bar{Y}_D\rangle = k''_\nu\langle\bar{Y}_D\rangle(\langle\bar{Y}_R\rangle)^{-1}\langle\bar{Y}_D\rangle, \tag{2.13}$$

$$\langle\bar{Y}_D\rangle = k_D\langle\bar{\Phi}_0^T\rangle\left(\mathbf{1} + a_D X^T X\right)\langle\bar{\Phi}_0\rangle, \tag{2.14}$$

$$\langle\bar{Y}_R\rangle = k_R\left(\langle\bar{Y}_e\rangle\langle\hat{Y}^u\rangle + \langle\hat{Y}^u\rangle\langle\bar{Y}_e\rangle\right), \tag{2.15}$$

$$\langle Y^u\rangle = k_u\langle\hat{Y}^u\rangle\langle\hat{Y}^u\rangle, \tag{2.16}$$

$$\langle\hat{Y}^u\rangle = k'_u\langle\bar{\Phi}_0\rangle\left(\mathbf{1} + a_u X X^T\right)\langle\bar{\Phi}_0^T\rangle, \tag{2.17}$$

$$\langle\bar{P}_d\rangle\langle Y^d\rangle\langle\bar{P}_d\rangle = k_d\langle\bar{\Phi}_0\rangle\left(\mathbf{1} + a_d X X^T\right)\langle\bar{\Phi}_0^T\rangle + \xi_0^d\mathbf{1}, \tag{2.18}$$

where X has phenomenologically been introduced in the previous model [7] and it has the form

$$X = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{pmatrix}. \tag{2.19}$$

The form (2.19) leads to

$$XX^T = \frac{3}{2}X_3, \quad X^T X = \frac{3}{2}X_2, \tag{2.20}$$

together with $XX = X$, where X_3 and X_2 is defined by Eqs. (1.2) and (1.6), respectively, and, for simplicity. For convenience, we have already put $\langle E \rangle$ as $\mathbf{1}$, and so on.

3 Parameter fitting

We again summarize our mass matrix model as follows:

$$\bar{Y}_e = k_e \bar{\Phi}_0 (\mathbf{1} + a_e X_3) \bar{\Phi}_0^T, \tag{3.1}$$

$$\bar{Y}_\nu = k_\nu \hat{Y}_\nu \hat{Y}_\nu, \tag{3.2}$$

$$\hat{Y}_\nu = k''_\nu \bar{Y}_D \bar{Y}_R^{-1} \bar{Y}_D, \tag{3.3}$$

$$\bar{Y}_D = k_D \bar{\Phi}_0^T (\mathbf{1} + a_D e^{i\alpha_D} X_2) \bar{\Phi}_0, \tag{3.4}$$

$$\bar{Y}_R = k_R \left(\bar{Y}_e \hat{Y}^u + \hat{Y}^u \bar{Y}_e \right), \tag{3.5}$$

$$Y^u = k_u \hat{Y}^u \hat{Y}^u, \tag{3.6}$$

$$\hat{Y}^u = k'_u \bar{\Phi}_0 \left(\mathbf{1} + a_u e^{i\alpha_u} X_3 \right) \bar{\Phi}_0^T, \tag{3.7}$$

$$\bar{P}_d Y^d \bar{P}_d = k_d \bar{\Phi}_0 (\mathbf{1} + a_d X_3) \bar{\Phi}_0^T + \xi_0^d \mathbf{1}, \tag{3.8}$$

where, for convenience, we have dropped the notations "$\langle$" and "$\rangle$". In numerical calculations, we use dimensionless expressions $\bar{\Phi}_0 = \mathrm{diag}(x_1, x_2, x_3)$ (with $x_1^2 + x_2^2 + x_3^2 = 1$), $\bar{P}_d = \mathrm{diag}(e^{-i\phi_1}, e^{-i\phi_2}, 1)$, and $E = \mathrm{diag}(1, 1, 1)$. The parameters are re-refined by Eqs.(3.1)-(3.8). In Eqs.(3.4) and (3.7), since we assume that the parameters a_e and a_d are real, while a_u and a_D are complex, we have denoted a_u and a_D as $a_u e^{i\alpha_u}$ and $a_D e^{i\alpha_D}$, respectively.

In this model, we have 2 parameters (a_D, α_D) for neutrino sector, 4 parameters a_D, ξ_0^d and (ϕ_1, ϕ_2) for down-quark mass ratios and V_{CKM}, and 3 parameters a_e, (a_u, α_u) for charged lepton mass ratios and up-quark mass ratios. Especially, it is worthwhile noticing that the neutrino mass ratios and U_{PMNS} are described only two parameters after a_e and (a_u, α_u) have been fixed from the observed CKM mixing and up-quark mass ratios. Since we do effectively not change the mass matrix structures except for Y_ν from the previous paper [7], we can use the same parameter values as given in the previous study:

$$a_e = 7.5, \quad (a_u, \alpha_u) = (-1.35, 7.6^\circ), \tag{3.9}$$

in the present model, too. Therefore, as far as PMNS mixing and neutrino mass ratios are concerned, we have only two free parameters (a_D, α_D) in the present neutrino mass matrix model.

At present, the observed values [10] are as follows:

$$\sin^2 2\theta_{12} = 0.857 \pm 0.024, \quad \sin^2 2\theta_{23} > 0.95, \quad \sin^2 2\theta_{13} = 0.098 \pm 0.013, \tag{3.10}$$

$$R_\nu \equiv \frac{\Delta m^2_{21}}{\Delta m^2_{32}} = \frac{(7.50 \pm 0.20) \times 10^{-5}\ \mathrm{eV}^2}{(2.32^{+0.12}_{-0.08}) \times 10^{-3}\ \mathrm{eV}^2} = (3.23^{+0.14}_{-0.19}) \times 10^{-2}. \tag{3.11}$$

Since the parameters (a_D, α_D) are sensitive to the observables $\sin^2 2\theta_{12}$ and R_ν, we use the observed values $\sin^2 2\theta_{12}$ and R_ν in order to fix our parameter values (a_D, α_D). In Fig.1, we illustrate an allowed parameter region of (a_D, α_D) for the observed values [10] $\sin^2 2\theta_{12}^{obs} = 0.875 \pm 0.024$ and $R_\nu^{obs} \times 10 = 0.324^{+0.014}_{-0.019}$. As seen in Fig.1, the observed values uniquely fix the parameter values (a_D, α_D) as

$$(a_D, \alpha_D) = (8.7, 12^\circ). \tag{3.12}$$

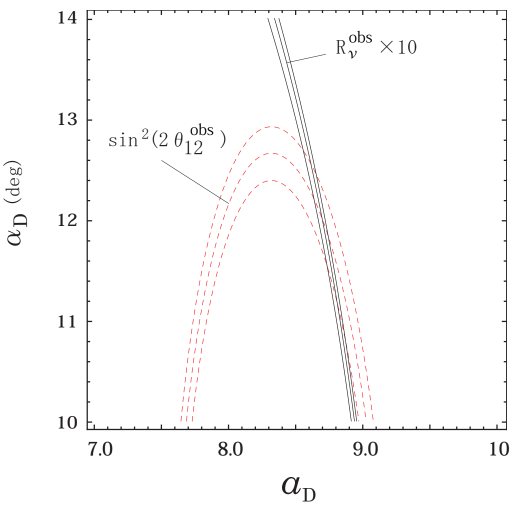

Figure 1: Allowed parameter region in (a_D, α_D) plane. The solid and dashed lines indicate the border and center curves of the allowed region which are obtained from the observe values $\sin^2 2\theta_{12}^{obs} = 0.875 \pm 0.024$ and $R_\nu^{obs} \times 10 = 0.324^{+0.014}_{-0.019}$, respectively.

For reference, in Fig.2, we illustrate behaviors of $\sin^2 2\theta_{12}$ and R_ν versus α_D for the case of $a_D = 8.7$. The choice $\alpha_D = 12^\circ$ gives excellent fittings to the observed values of $\sin^2 2\theta_{12}$ and R_ν simultaneously:

$$\sin^2 2\theta_{12} = 0.8544, \quad R_\nu = 0.0331. \tag{3.13}$$

Then, we obtain our predictions for $\sin^2 2\theta_{23}$ and $\sin^2 2\theta_{13}$ using (3.12) as follows:

$$\sin^2 2\theta_{23} = 0.9962, \quad \sin^2 2\theta_{13} = 0.0907, \tag{3.14}$$

which are in excellent agreement with the observed values in given in Eq.(3.10).

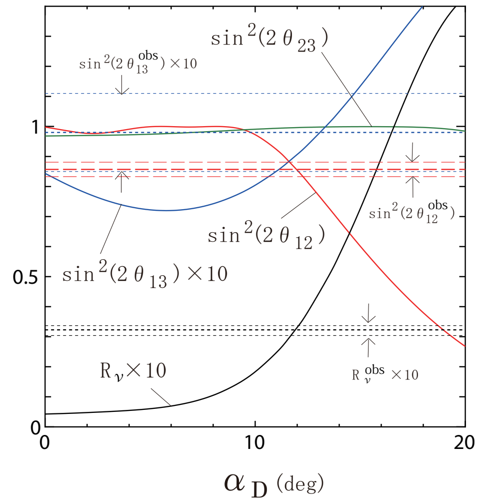

Figure 2: Lepton mixing parameters $\sin^2 2\theta_{12}$, $\sin^2 2\theta_{23}$, $\sin^2 2\theta_{13}$, and the neutrino mass squared ratio R_ν versus the phase parameter α_D for $a_D = 8.7$. The horizontal lines denote observed values $\sin^2 2\theta_{12}^{obs} = 0.875 \pm 0.024$, $\sin^2 2\theta_{13}^{obs} \times 10 = 0.98 \pm 0.13$ and $R_\nu^{obs} \times 10 = 0.324^{+0.014}_{-0.019}$. Our predicted value for $\sin^2 2\theta_{23}$ is well satisfied the obtained experimental bound $\sin^2 2\theta_{23}^{obs} > 0.95$.

The fixing of the parameters (a_D, α_D), Eq.(3.12), make the prediction of the CP violating phase parameter in the lepton sector possible too:

$$\delta_{CP}^{\ell} = 127^\circ \quad (J^\ell = 2.74 \times 10^{-2}). \tag{3.15}$$

We can also predict neutrino masses:

$$m_{\nu 1} = 0.00061\text{eV}, \quad m_{\nu 2} = 0.00899 \text{ eV}, \quad m_{\nu 3} = 0.05011 \text{ eV}, \tag{3.16}$$

by using the input value [11] $\Delta m_{32}^2 = 0.00243$ eV^2. We also predict the effective Majorana neutrino mass [12] $\langle m \rangle$ in the neutrinoless double beta decay as

$$\langle m \rangle = \left| m_1 U_{e1}^2 + m_2 U_{e2}^2 + m_3 U_{e3}^2 \right| = 0.0034 \text{ eV}. \tag{3.17}$$

This value is a magnitude which is within our reach to observe in a future neutrinoless double beta decay experiments.

Finally, we list the predicted values of the CKM mixing parameters and down-quark mass ratios, although they are essentially the same as those in the previous model [7]:

$$|V_{us}| = 0.2271, \quad |V_{cb}| = 0.0394, \quad |V_{ub}| = 0.00347, \quad |V_{td}| = 0.00780, \tag{3.18}$$

$$\delta_{CP}^{q} = 59.6^\circ \quad (J^q = 2.6 \times 10^{-5}), \tag{3.19}$$

$$r_{12}^u = \sqrt{\frac{m_d}{m_s}} = 0.00465, \qquad r_{23}^u = \sqrt{\frac{m_d}{m_b}} = 0.0614. \tag{3.20}$$

$$r_{12}^d = \frac{m_d}{m_s} = 0.0569, \quad r_{23}^d = \frac{m_d}{m_b} = 0.0240. \tag{3.21}$$

Here, we have used $a_d = 25$, $\xi_0^d = 0.0115$, and $(\phi_1, \phi_2) = (177.0°, 197.4°)$. The observed values are as follows: $|V_{us}| = 0.2252 \pm 0.0009$, $|V_{cb}| = 0.0409 \pm 0.0011$, $|V_{ub}| = 0.00415 \pm 0.00049$, $|V_{td}| = 0.0084 \pm 0.0006$, $J = (2.96^{+0.20}_{-0.16}) \times 10^{-5}$ [10], and $r_{12}^u = 0.045^{+0.013}_{-0.010}$, $r_{23}^u = 0.060 \pm 0.005$, $r_{12}^d = 0.053^{+0.005}_{-0.003}$, $r_{23}^d = 0.019 \pm 0.006$ [13].

4 Concluding remarks

In conclusion, we have proposed a new neutrino mass matrix form within the framework of the yukawaon model, in which we have only two adjustable parameters, (a_D, α_D), for PMNS mixing and neutrino mass ratios. We obtain reasonable results for PMNS mixing and neutrino mass ratios as shown in Eqs.(3.13) - (3.17) for the parameter values $(a_D, \alpha_D) = (8.7, 12°)$. As seen in Fig.2, it is worthwhile noticing that only when we choose a reasonable value of $R_\nu \simeq 0.033$, we obtain a reasonable value of $\sin^2 2\theta_{13} \simeq 0.09$. Also, we would like to emphasize that our prediction gives a sizable value of $\langle m \rangle \simeq 0.0034$ eV in spite of the normal mass hierarchy model (in spite of $m_{\nu 1} \simeq 0.0006$ eV, $m_{\nu 2} \simeq 0.00899$ eV and $m_{\nu 3} \simeq 0.05011$ eV).

Of course, we have also obtained reasonable results for CKM mixing and quark mass ratios as same as those in the previous paper [7].

The present model gives successful results from the phenomenological point of view. However, we still have some of theoretical problems. One of the major problems is why only Y_D takes the mass matrix form with X_2 (not X_3). In Ref.[4], the form X_3 has been understood by a symmetry breakdown U(3)×U(3)′ → U(3)×S$_3$. However, for the form X_2, the model is still in a phenomenological level. We have been able to remove the unnatural term [ξ_ν^0 term in Eq.(1.5)], but we still have ξ_d^0 term in the Y_d sector. These problems are our future tasks.

References

[1] Y. Koide, Phys. Lett. **B 680** (2009) 76.

[2] H. Nishiura and Y. Koide, Phys. Rev. **D 83** (2011) 035010.

[3] Y. Koide and H. Nishiura, Euro. Phys. J. **C 72** (2012) 1933.

[4] Y. Koide and H. Nishiura, Phys. Lett. **B 712** (2012) 396.

[5] C. D. Froggatt and H. B. Nelsen, Nucl. Phys. **B 147** (1979) 277. For recent works, for instance, see R. N. Mohapatra, AIP Conf. Proc. **1467** (2012) 7; A. J. Burasu *et al*, JHEP **1203** (2012) 088.

[6] N Cabibbo, Phys. Rev. Lett. **10** (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.

[7] Y. Koide and H. Nishiura, arXiv: 1209.1275 [hep-ph].

[8] K. Abe *et al.* (T2K collaboration), Phys. Rev. Lett. **107** (2011) 041801; MINOS collaboration, P. Adamson et. al., Phys. Rev. Lett. **107** (2011) 181802; Y. Abe *et al.* (DOUBLE-CHOOZ Collaboration), Phys. Rev. Lett. **108** (2012) 131801; F. P. An, *et al.* (Daya-Bay collaboration), Phys. Rev. Lett. **108** (2012) 171803; J. K. Ahn, *et al.* (RENO collaboration), Phys. Rev. Lett. **108** (2012) 191802.

[9] B. Pontecorvo, Zh. Eksp. Teor. Fiz. **33**, 549 (1957) and **34** (1957) 247; Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. **28** (1962) 870.

[10] J. Beringer *et al.*, Particle Data Group, Phys. Rev. D **86** (2012), 0100001.

[11] P. Adamson *et al.*, MINOS collaboration, Phys. Rev. Lett. **101** (2008) 131802.

[12] M. Doi, T. Kotani, H. Nishiura, K. Okuda, and E. Takasugi, Phys. Lett. **B103** (1981) 219; ibid. **B113** (1982) 513.

[13] Z.-z. Xing, H. Zhang, and S. Zhou, Phys. Rev. **D 77** (2008) 113016. And also see, H. Fusaoka and Y. Koide, Phys. Rev. **D 57** (1998) 3986.