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Family Gauge Bosons with an Inverted Mass Hierarchy[1]

Yoshio Koide[a,b] and Toshifumi Yamashita[b]

[a] *Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan*
E-mail address: koide@kuno-g.phys.sci.osaka-u.ac.jp

[b] *MISC, Kyoto Sangyo University, Kyoto 603-8555, Japan*
E-mail address: tyamashi@cc.kyoto-su.ac.jp

Abstract

A model that gives family gauge bosons with an inverted mass hierarchy is proposed, stimulated by Sumino's cancellation mechanism for the QED radiative correction to the charged lepton masses. The Sumino mechanism cannot straightforwardly be applied to SUSY models because of the non-renormalization theorem. In this talk, an alternative model which is applicable to a SUSY model is proposed. It is essential that family gauge boson masses $m(A_i^j)$ in this model is given by an inverted mass hierarchy $m(A_i^i) \propto 1/\sqrt{m_{ei}}$, in contrast to $m(A_i^i) \propto \sqrt{m_{ei}}$ in the original Sumino model. Phenomenological meaning of the model is also investigated. In particular, we notice a deviation from the e-μ universality in the tau decays.

1 Motive to consider an inverted mass hierarchy

In this section, it is discussed why we consider a family gauge boson model with an inverted mass hierarchy. However, since our model has been stimulated by a Sumino mechanism, prior to giving our motive, we would like to give a brief review of the Sumino mechanism.

1.1 Why did Sumino need a family gauge symmetry?

In 2009, Sumino [1] has seriously taken why the mass formula [2]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}, \tag{1}$$

is so remarkably satisfied with the pole masses: $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$. However, in a mass matrix model, usually, "masses" do not mean "pole masses", but "running masses". The formula (1) is only valid with the order of 10^{-3} for the running masses, e.g. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$. The deviation of $K(\mu)$ from K^{pole} is caused by a logarithmic term $m_{ei} \log(\mu/m_{ei})$ in the QED radiative correction term [3]

$$m_{ei}(\mu) = m_{ei}^{pole} \left[1 - \frac{\alpha_{em}(\mu)}{\pi} \left(1 + \frac{3}{4} \log \frac{\mu^2}{m_{ei}^2(\mu)} \right) \right]. \tag{2}$$

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Meanwhile, under the transformation $m_i \to m_i(1+\varepsilon_0+\varepsilon_i)$, where ε_0 and ε_i are family-number independent and dependent factors, respectively, the ratio K is invariant if $\varepsilon_i = 0$. That is, ε_i term corresponds to the $\log m_{ei}$ term in (2). If we can remove the $\log m_{ei}$ term, we can build a model with $K^{pole} = K(\Lambda)$. Therefore, Sumino [1] has proposed an idea that the $\log m_{ei}$ term is canceled by a contribution from family gauge bosons as shown in Fig.1.

ψ_L $m_i(\mu)$ ψ_R m_i^{pole} $e^2 \log m_{ei}^2$ $-g_F^2 \log M_{ii}^2$

Figure 1: The Sumino mechansm

In order to work the Sumino mechanism correctly, the following conditions are essential: (i) the left- and right-handed charged leptons e_{Li} and e_{Ri} are assigned to **3** and **3*** of a U(3) family symmetry, respectively; (ii) masses of the gauge bosons A_i^j are given by

$$M_{ij} \equiv m(A_i^j) \propto \sqrt{m_{ei}+m_{ej}} \tag{3}$$

Thus, the contribution $-g_F^2 \log M_{ii}^2$ term from the family gauge bosons can cancel the $e^2 \log m_{ei}^2$ term.

Now, we want to apply this mechanism to a SUSY model. However, note that the contribution given in Fig. 1 (b) is zero in a SUSY model because it is a vertex correction type. Therefore, the Sumino mechanism cannot apply to a SUSY model.

1.2 Our motive

Here, we propose a Sumino-like mechanism which can work in a SUSY model. We notice the diagram (a) in Fig. 2.

$$\varepsilon_i = \rho\left(\log\frac{m_{ei}^2}{\mu^2} + \zeta\sum_j \log\frac{M_{ij}^2}{\mu^2}\right), \quad \rho = \frac{3}{4}\frac{\alpha_{em}}{\pi}, \quad \zeta = \frac{2}{3}\frac{\alpha_F}{\alpha_{em}}, \tag{4}$$

Figure 2: Contributions from family gauge bosons

In order that the cancellation works correctly, since $\zeta > 0$, we must consider

$$M_{ij}^2 \propto \frac{1}{m_{ei}} + \frac{1}{m_{ej}}, \tag{5}$$

differently from the Sumino's gauge boson mass relation (3).

Note that since the gauge bosons A_i^j with $j \neq i$ can contribute to the ε_i term, differently from the Sumino model, the QED $\log m_{ei}$ term cannot be canceled by the gauge boson terms exactly. However, practically, the cancellation $R-1$ ($R \equiv K(m_{ei})/K(m_{ei}^0)$) can be smaller than 10^{-5} at $\zeta = 7/4$.

2 Outline of the model

By introducing two types of scalars Φ_i^α (and $\bar{\Phi}_\alpha^i$) and Ψ_i^α (and $\bar{\Psi}_\alpha^i$) which are $(3, 3^*)$ $((3^*, 3))$ of family symmetries SU(3)×SU(3)′, we build our model as follows:

(i) The charged lepton mass matrix M_e is given by

$$M_e \propto \langle\bar{\Phi}\rangle\langle\Phi\rangle. \tag{6}$$

(ii) Family gauge boson masses M_{ij} are dominantly given by the VEV $\langle\Psi\rangle$. Therefore, we must show

$$\langle\Psi\rangle \gg \langle\Phi\rangle. \tag{7}$$

(iii) Gauge bosons take an inverted mass hierarchy: We must show

$$\langle\Psi\rangle\langle\Phi\rangle \propto \mathbf{1}. \tag{8}$$

The conditions (i), (ii) and (iii) are derived from the following superpotentials [4]

$$W_Y = y_\ell \ell_i \bar{\Phi}_\alpha^i \bar{L}^\alpha + y_{Hd} L_\alpha H_d E^{c\alpha} + y_e \bar{E}_\alpha^c \Phi_j^\alpha e^{cj} + M_E E^{c\alpha} \bar{E}_\alpha^c + M_L L_\alpha \bar{L}^\alpha, \tag{9}$$

$$\begin{aligned} W_\Phi &= \lambda_1 \Phi_i^\alpha \bar{\Phi}_\alpha^i \theta_\Phi - \lambda_2 S^2 \theta_\Phi, \\ W_{br} &= \mu_S S \theta_S - \varepsilon \mu_S^2 \theta_S, \end{aligned} \tag{10}$$

$$\begin{aligned} W_{\Phi\Psi} = \ & \left(\lambda_A \bar{\Psi}_\alpha^i \Phi_j^\alpha + \bar{\lambda}_A \bar{\Phi}_\alpha^i \Psi_j^\alpha\right) (\Theta_A)_i^j + \left(\lambda'_A \bar{\Psi}_\alpha^i \Phi_i^\alpha + \bar{\lambda}'_A \bar{\Phi}_\alpha^i \Psi_i^\alpha - \mu_A S\right) (\Theta_A)_j^j \\ & + \left(\lambda_B \Phi_i^\alpha \bar{\Psi}_\beta^i + \bar{\lambda}_B \Psi_i^\alpha \bar{\Phi}_\beta^i\right) (\Theta_B)_\alpha^\beta + \left(\lambda'_B \Phi_i^\alpha \bar{\Psi}_\alpha^i + \bar{\lambda}'_B \Psi_i^\alpha \bar{\Phi}_\alpha^i - \mu_B S\right) (\Theta_B)_\beta^\beta. \end{aligned} \tag{11}$$

respectively. As a result, we obtain

$$\langle\Phi\rangle = \langle\bar{\Phi}\rangle = v_\Phi Z, \quad \langle\Psi\rangle = \langle\bar{\Psi}\rangle = v_\Psi Z^{-1}, \tag{12}$$

where

$$Z = \mathrm{diag}(z_1, z_2, z_3), \tag{13}$$

and the parameter values of z_i are given by

$$z_i = \frac{\sqrt{m_{ei}}}{\sqrt{m_{e1} + m_{e2} + m_{e3}}}, \tag{14}$$

where $(m_{e1}, m_{e2}, m_{e3}) = (m_e, m_\mu, m_\tau)$. The explicit values of z_i are given by $(z_1, z_2, z_3) = (0.016473, 0.23688, 0.97140)$. Thus, we can approximately estimate the family gauge boson masses $m(A_j^i)$ as follows

$$M_{ij}^2 \equiv m^2(A_j^i) \simeq g_F^2 v_\Psi^2 \left(\frac{1}{z_i^2} + \frac{1}{z_j^2} \right) \propto \left(\frac{1}{m_{ei}} + \frac{1}{m_{ej}} \right), \tag{15}$$

if the mixing between the U(3) and U(3)$'$ gauge bosons can be neglected. This happens when the latter gauge bosons are sufficiently heavy.

However, for reasons of my talk time, I would like to skip the details of the model. For more details, see our recent paper [4].

3 How different from the Sumino model?

Let us give only a summary table for differences between Sumino model and ours in Table 1.

	Sumino model	Present model
	non-SUSY	SUSY
U(3) assignment of (e_L, e_R)	$\sim (3, 3^*)$	$\sim (3, 3)$
Anomaly	a model with anomaly	an anomaly-less model
Gauge boson masses	Normal	Inverted
	$M_{ij} \propto \sqrt{m_i + m_j}$	$M_{ij} \propto \sqrt{\frac{1}{m_i} + \frac{1}{m_j}}$
Family currents	$(J_\mu)_i^j = \bar{\psi}_L^j \gamma_\mu \psi_{Li} - \bar{\psi}_{Ri} \gamma_\mu \psi_R^j$	$(J_\mu)_i^j = \bar{\psi}^j \gamma_\mu \psi_i$
Effective $\Delta N_f = 2$ int.	appear even if $U_q = \mathbf{1}$	not appear in the limit of $U_q = \mathbf{1}$

Table 1: Comarison between the Sumino model and the present model.

4 How to observe the gauge boson effects

Note that the family number is defined on the diagonal basis of the charged lepton mass matrix M_e. Hadronic modes are in general dependent on the quark mixing matrices U_u and U_d. We know the observed values of $V_{CKM} = U_u^\dagger U_d$, but we do not know values of U_u and U_d individually. In the present model, we do not give an explicit model for quark mixings on the diagonal basis of M_e. A constraint from K^0-$\bar{K}^0$ mixing is highly dependent on a model of U_d. We will not discuss it in this talk.

4.1 Deviation from the e-μ universality in the tau decays

The pure-leptonic decays are independent of a model of the quark mixing. Therefore, first, we discuss the deviation from the e-μ universality in the tau decays.

We have family current-current interactions

$$\frac{G_{ij}}{\sqrt{2}}(\bar{\nu}_i \gamma_\mu \nu_j)(\bar{e}_j \gamma^\mu e_i), \tag{16}$$

where $G_{ij}/\sqrt{2} = g_F^2/2M_{ij}^2 \simeq z_j^2/2v_\Psi^2$, in addition to the conventional weak interactions

$$\frac{G_F}{\sqrt{2}}(\bar{e}_j \gamma_\mu (1-\gamma_5)\nu_j)(\bar{\nu}_i \gamma^\mu (1-\gamma_5) e_i), \tag{17}$$

where $G_F/\sqrt{2} = g_W^2/8M_W^2 = 1/2v_W^2$.

μ^- (e^-) ν_τ
τ^- $\bar{\nu}_\mu$ ($\bar{\nu}_e$) τ^- $\bar{\nu}_\mu$ ($\bar{\nu}_e$)
A_3^2 (A_3^1) W^-
ν_τ μ^- (e^-)

Figure 3: Deviation from e-μ universality in tau decays

We obtain a deviation from the e-μ universality as follows:

$$R_\tau \equiv \frac{1+\epsilon_\mu}{1+\epsilon_e} = \left[\frac{B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau)}{B(\tau^- \to e^- \bar{\nu}_e \nu_\tau)} \frac{f(m_e/m_\tau)}{f(m_\mu/m_\tau)}\right]^{1/2}, \tag{18}$$

where $f(m_e/m_\tau)/f(m_\mu/m_\tau) = 1.028215$ and

$$\epsilon_\mu \simeq \frac{1}{4} z_2^2 r^2 = 1.4 \times 10^{-2} r^2, \quad \epsilon_e \simeq \frac{1}{4} z_1^2 r^2 = 6.8 \times 10^{-5} r^2, \tag{19}$$

with $r = v_W/v_\Psi$ ($v_W = 246$ GeV). Present experimental values [5]

$$B(\tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau) = (17.39 \pm 0.04)\%, \quad B(\tau^- \to e^- \bar{\nu}_e \nu_\tau) = (17.82 \pm 0.04)\% \tag{20}$$

give

$$R_\tau^{exp} = 1.0017 \pm 0.0016, \quad \text{i.e. } \epsilon_\mu \simeq 0.0017 \pm 0.0016. \tag{21}$$

This result seems to be in favor of the inverted gauge boson mass hierarchy although it is just at 1 σ level. (If the gauge boson masses take a normal hierarchy, R_τ will show $R_\tau < 1$.) At present, we should not take the value (21) rigidly. If we dare to adopt the center value in (21), we obtain $r \sim 0.35$ ($v_\Psi \sim 0.7$ TeV). This value of v_Ψ seems to be somewhat small. We speculate $r \sim 10^{-1}$, i.e. $v_\Psi \sim$ a few TeV. The value will be confirmed by a tau factory in the near future.

Figure 4: Predicted branching ratios $B(P \to P' e_i^+ e_j^-)$ versus the VEV value $v_F \equiv v_\Psi$. The marks $\bullet$ and the dashed lines denote present lower limits of the observed branching ratios.

4.2 Family number conserved semileptonic decays of ps-mesons

Branching ratios of family number conserved semileptonic decays are not sensitive to explicit values of the quark mixings U_u and U_d. We predict those in the limit of $U_u = \mathbf{1}$ and $U_d = \mathbf{1}$:

$$\begin{array}{lll} B(K^+ \to \pi^+ \mu^+ e^-) & \sim 5 \times 10^{-12} & (< 1.3 \times 10^{-11}), \\ B(K_L \to \pi^0 \mu^\pm e^\mp) & \sim 1 \times 10^{-11} & (< 7.6 \times 10^{-11}), \\ B(D^+ \to \pi^+ \mu^- e^+) & \sim 6 \times 10^{-13} & (< 3.4 \times 10^{-5}), \\ B(D^0 \to \pi^0 \mu^- e^+) & \sim 1 \times 10^{-13} & (< 8.6 \times 10^{-5}), \\ B(B^+ \to K^+ \mu^- \tau^+) & \sim 2 \times 10^{-6} & (< 7.7 \times 10^{-5}), \\ B(B^0 \to K^0 \mu^- \tau^+) & \sim 2 \times 10^{-6} & \text{(no data)}. \end{array} \tag{22}$$

Here, the values in parentheses are present experimental upper limits [5]. For reference, we illustrate the predicted branching ratios in Fig. 4. As seen in Fig. 4, if v_Ψ is a few TeV, observations of the lepton-flavor violating K- and B-decays with $\Delta N_F = 0$ will be within our reach.

4.3 Direct searches at LHC and ILC

Since we speculate that v_Ψ is of the order of a few TeV, we may expect that the lightest family gauge boson A_3^3 is also of the order of a few TeV. The experimental search is practically the same as that [6] for Z' boson, but we will see only peak in $\tau^+\tau^-$ channel (no peaks in e^+e^- and $\mu^+\mu^-$ channels). Possible productions of A_3^3 at the LHC and ILC are illustrated in Figs. 5 and 6, respectively.

p b τ^- A_3^3 τ^+ $\bar{b}$ p W^- b τ^- τ^+ $\bar{b}$

Figure 5: A_3^3 production at the LHC

e^- $\bar{b}$ ($\bar{t}$) Z^*,γ^* A_3^3 τ^- b (t) τ^+ e^+ e^- $\bar{b}$ ($\bar{t}$) Z^*,γ^* A_3^3 τ^- τ^+ e^+ b (t)

Figure 6: A_3^3 production at the ILC

5 Summary

The Sumino mechanism does not work for a SUSY model. In order to work a Sumino-like mechanism for a SUSY model, we must consider a family gauge boson model with an inverted mass hierarchy. Whether the gauge boson masses are inverted or normal is confirmed by observing the deviation form the e-μ universality in the pure leptonic tau decays, i.e. $R_\tau > 1$ or $R_\tau < 1$. The present observed values show $R_\tau^{exp} = 1.0017 \pm 0.0016$ which is in favor of the inverted mass hierarchy. Since we speculate that the lightest gauge boson mass is a few TeV, we expect the deviation $\Delta R_\tau \equiv R_\tau - 1 \sim 10^{-4}$. A tau factory in the near future will confirm this deviation. We also expect a direct observation of $\tau^+\tau^-$ in the LHC and the ILC.

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