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Yukawaon Model with U(3)×O(3) Family Symmetries

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Abstract

A quark and lepton mass matrix model with family symmetries U(3)×O(3) is investigated on the basis of the so-called yukawaon model. In the present model, quarks and leptons are assigned to $(\ell, e^c, u^c) \sim (\mathbf{3}, \mathbf{3}, \mathbf{3}^*)$ of U(3) and $(q, d^c, \nu^c) \sim (\mathbf{3}, \mathbf{3}, \mathbf{3})$ of O(3). Then, the neutrino mass matrix is given by $M_\nu = m_D M_R^{-1} m_D^T$ with $m_D \propto \langle \Phi_e \rangle$, where the charged lepton mass matrix M_e is given by $M_e = k \langle \Phi_e \rangle \langle \Phi_e^T \rangle$. A merit in considering U(3)×O(3) lies in that we can lower the cutoff scale Λ in the yukawaon model.

1. Introduction

One of the most challenging problems in contemporary particle physics is to clarify the origin of flavors. For such a purpose, it is interesting to investigate whether the observed flavor physics phenomena can be understood from a concept of a family gauge symmetry or not. The present data [1] suggest that numbers of lepton- and quark-families are both three. Then, from a point of view of a unification model of quarks and leptons, it will be natural to consider that the quarks and leptons obey the same family symmetry. However, at present, this is experimentally not yet confirmed. Can quarks and leptons be described by a sole family symmetry? In this paper, we investigate a possibility that quarks and leptons obey different family symmetries from each other.

In the present paper, by assuming family symmetries U(3)×O(3), we will propose a new version of the so-called "supersymmetric yukawaon" model [2, 3] (a kind of "flavon" model [4]). In the yukawaon model, all effective Yukawa coupling constants Y_f^{eff} $(f = u, d, e, \cdots)$ are given by vacuum expectation values (VEVs) of "yukawaons" Y_f as

$$Y_f^{eff} = y_f \frac{\langle Y_f \rangle}{\Lambda}. \tag{1.1}$$

The yukawaons Y_f are singlets under the conventional gauge symmetries $\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y$, and have only family indices. For $\mu < \Lambda$, the effective Yukawa coupling constants Y_f^{eff} evolve as those in the standard model. The effective Lagrangian is practically identical with the minimal SUSY standard model (MSSM) [5] except for that Y_f are not constants, but superfields. (A brief review of the yukawaon model is given in Sec.2.)

In the present model with family symmetries U(3)×O(3), we assume the following would-be Yukawa interaction terms:

$$W_Y = \frac{y_e}{\Lambda} \ell_i Y_e^{ij} e_j^c H_d + \frac{y_\nu}{\Lambda} \ell_i \Phi_e^{i\alpha} \nu_\alpha^c H_u + \lambda_R \nu_\alpha^c Y_R^{\alpha\beta} \nu_\beta^c + \frac{y_u}{\Lambda} u^{ci} Y_{i\beta}^u q_\beta H_u + \frac{y_d}{\Lambda} d_\alpha^c Y_{\alpha\beta}^d q_\beta H_d, \tag{1.2}$$

together with the conventional Higgs field term $W_H = \mu_H H_u H_d$. Here, ℓ and q are defined by $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$, and indexes $i, j, \cdots$ and $\alpha, \beta, \cdots$ denote those of U(3) and O(3), respectively. Note that, as seen in Eq.(1.2), the yukawaon Φ_e plays a role of the substitute for a yukawaon Y_ν. This is the most characteristic in the present model. The neutrino Dirac mass matrix m_D is given by

$$m_D = \frac{y_\nu}{\Lambda} \langle \Phi_e \rangle \langle H_u^0 \rangle. \tag{1.3}$$

As we show later, fields Y_e, Y_u, $\cdots$ satisfy the following VEV relations:

$$\langle Y_e^{ij} \rangle \propto \langle \Phi_e^{i\alpha} \rangle \langle \Phi_e^{T\alpha j} \rangle, \quad \langle Y_u^{i\alpha} \rangle \propto \langle \Phi_{i\beta}^u \rangle \langle \bar{E}^{T\beta j} \rangle \langle \Phi_{j\alpha}^u \rangle, \tag{1.4}$$

where $\langle \bar{E}^{i\alpha} \rangle = v_E \delta^{i\alpha}$.

So far, in a series of yukawaon models, the flavor symmetry was either U(3) [6] or O(3) [7, 3], and it was a global symmetry. In general, when the family symmetry is global, unwelcome massless scalars appear in the model. Therefore, in this paper, we want to consider that the family symmetry is local. However, since there are many family symmetry non-singlet fields in the yukawaon model, the family gauge symmetry cannot be asymptotic free, so that it is feared that the gauge coupling constant bursts at $\mu = \Lambda$. (Therefore, so far, we have not consider a possibility that the family symmetry in the yukawaon model is local.) If we consider a model with different family symmetries for quarks and leptons, we will be able to soften such a trouble.

The present model with U(3)×O(3) symmetries has received a hint from a charged lepton mass matrix model with U(3)×O(3) symmetries which has recently been proposed by Sumino [8, 9]. In the Sumino model, the charged lepton mass term is generated by a would-be Yukawa interaction

$$H_e = \frac{y_e}{\Lambda^2} \bar{\ell}_L^i \Phi_{i\alpha}^e \Phi_{\alpha j}^{eT} e_R^j H, \tag{1.5}$$

where H is the Higgs scalar in the standard non-SUSY model. (Sumino' model has not been based on a SUSY scenario.) The charged lepton masses m_{e_i} are acquired from the vacuum expectation value (VEV) of the scalar Φ^e [10], i.e. they are given by $m_{ei} = (y_e/\Lambda^2) \langle (\Phi^e)_{i\alpha} \rangle \langle (\Phi^{eT})_{\alpha i} \rangle \langle H^0 \rangle$. In other words, the VEV of Φ^e has a form $\langle \Phi^e \rangle_e \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, where the suffix "$e$" denotes that a VEV matrix $\langle A \rangle$ takes a form $\langle A \rangle_e$ in a flavor basis in which the charged lepton mass matrix M_e is diagonal. However, in his model, O(3) is not a family symmetry. Besides, Sumino has mentioned nothing about quark and neutrino family assignments explicitly. In this paper, we regard the O(3) symmetry as another family symmetry which is related to quarks and neutrinos.

In the Sumino model, it is essential that the left- and right-handed charged leptons e_{Li} and e_{Ri} are assigned to $\mathbf{3}$ and $\mathbf{3}^*$ of U(3) family symmetry, respectively. (A similar fermion assignment has been proposed by Applequist, Bai and Piai [11].) The reason for this assignment is as follow: Sumino's interest is in the charged lepton mass relation [12]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}. \tag{1.6}$$

The relation $K = 2/3$ is satisfied with the order of 10^{-5} for the pole masses, i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ [1], while it is only valid with the order of 10^{-3} for the running masses, e.g. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$. However, in conventional mass matrix models, "mass" means not "pole mass" but "running mass." Sumino has seriously taken why the mass formula $K = 2/3$ is so remarkably satisfied with the pole masses. The deviation of $K(\mu)$ from K^{pole} is caused by a logarithmic term $m_{ei} \log(\mu/m_{ei})$ in the radiative correction term [13] due to photon

$$m_{ei}(\mu) = m_{ei}^{pole} \left[1 - \frac{\alpha(\mu)}{\pi} \left(1 + \frac{3}{4} \log \frac{\mu}{m_{ei}(\mu)} \right) \right]. \tag{1.7}$$

Therefore, he assumed that a family symmetry is local, and that the logarithmic term in the radiative correction due to photon is canceled by that due to family gauge bosons. As a result, we can obtain $K(\mu) = K^{pole}$. (However, it does not mean $m_{ei}(\mu) = m_{ei}^{pole}$. The cancellation takes place only for the term with $\log m_{ei}$).

In this paper, stimulated by Sumino's idea, we consider a model with family symmetries U(3)×O(3) and with the assignments $\ell \sim \mathbf{3}$ and $e^c \sim \mathbf{3}$ of U(3). However, the purpose of the present paper is not to give the observed charged lepton mass spectra, but to describe a unified description of the quark and neutrino mass spectra and mixings by using the observed charged lepton mass spectrum as the input parameters. For convenience, we will use the charged lepton mass values at the $\mu = m_Z$ as the input values, because it is not essential for numerical predictions in quark and neutrino sectors.

In this paper, according to Sumino's suggestion, we assume that O(3) is already completely broken at an energy scale $\mu = \Lambda$, so that all the O(3) gauge bosons become massive and decouple from the present effective theory below Λ.

In the conventional yukawaon model, the neutrino Dirac mass matrix m_D was ad hoc given by $m_D \propto M_e$ (i.e. $\langle Y_\nu \rangle \propto \langle Y_e \rangle$) from a phenomenological point of view (see Sec.2). Therefore, we are obliged to accept a cutoff energy scale $\Lambda \sim 10^{12}$ GeV from the neutrino phenomenology (see Sec.5), so that most of new physics as to the yukawaons become invisible. In the present model, the yukawaon Y_e in the charged lepton sector is given by Y_e^{ij} [$\mathbf{6}^*$ of U(3)], while a yukawaon in the neutrino (Dirac) sector is given by $Y_\nu^{i\alpha}$ [($\mathbf{3}^*, \mathbf{3}$) of U(3)×O(3)]. Therefore, we can regard the field $\Phi_e^{i\alpha}$ in the Sumino model as $Y_\nu^{i\alpha}$. Then, as we state in Sec.5, in the present model with $m_D \propto \langle \Phi_e \rangle$, we can have a possibility that Λ takes a considerably low value (e.g. $\Lambda \sim 10^8$ GeV), and the lightest family gauge boson A_1^1 can have a mass of a few TeV. This is the greatest merit in considering U(3)×O(3) family symmetries in the yukawaon model.

As we see later, since we want that the phenomenological success in the previous yukawaon model [3] is inherited in the present model, as far as numerical results are concerned, most of the numerical results in the present model will be the same as those in the old model and not new. The differences of the present U(3)×O(3) yukawaon model from the previous O(3) yukawaon model will be summarized in the end of the next section.

2. Brief review of the yukawaon model

Although the yukawaon model is a kind of the flavon model [4], differently from the conventional flavon models, the quarks and leptons are assigned to "triplets" (and/or "anti-triplets") of a non-Abelian group G, e.g. not to $\mathbf{2} + \mathbf{1}$ of SU(2), $\mathbf{1} + \mathbf{1}' + \mathbf{1}''$ of U(1)^3, and so on. The

VEV values of yukawaons Y_f with 3×3 ($3 \times 3^*$) of G are directly determined by a structure of a scalar potential which is invariant under the symmetry G.

The yukawaon model intends to describe all quark and lepton mass matrices based on only a fundamental VEV matrix $\langle \Phi_e \rangle$. In the supersymmetric yukawaon model, the VEV matrices $\langle Y_f \rangle$ are related to the fundamental VEV matrix $\langle \Phi_e \rangle$ by using SUSY vacuum conditions. We cannot always uniquely determine a superpotential form from a flavor symmetry alone. In the previous yukawaon model, in order to distinguish a yukawaon Y_f from other yukawaons $Y_{f'}$, we assigned "sector" charges (U$(1)_X$ charges) by hand. (For example, we assign the sector charges as $Q_X(Y_e) = x_e$, $Q_X(e^c) = -x_e$, $Q_X(Y_u) = x_u$, $Q_X(u^c) = -x_u$, and so on, in each sector $f = e, \nu, u, d$ and $Q_X(Y_R) = 2x_\nu$. We assign $Q_X = 0$ to the SU(2) doublet fields.) In contrast to the previous model, in the present model, we do not need such a sector charge. We can distinguish the yukawaons by U(3)×O(3) assignments and R charges. Besides, in order to forbid unwelcome terms with Λ^{-n} ($n \gg 1$), we need R charge assingments.

The superpotential for yukawaons is usually given by a form

$$W = \sum_A f_A(Y_f, Y_{f'}, \cdots)\Theta_A, \tag{2.1}$$

where Θ_A is an auxiliary superfield. Therefore, a SUSY vacuum condition $\partial W/\partial \Theta_A = 0$ leads to a VEV relation $f_A(\langle Y_f \rangle, \langle Y_{f'} \rangle, \cdots) = 0$. We assume that our vacuum always takes $\langle \Theta_A \rangle = 0$. Then, other vacuum conditions $\partial W/\partial Y_f = 0$ do not give any VEV relation, because each term in those equations always contains one Θ field. For example, the VEV relation $\langle Y_e \rangle \propto \langle \Phi_e \rangle \langle \Phi_e^T \rangle$ in Eq.(1.4) is derived from a superpotential

$$W_e = (\mu_e Y_e^{ij} + \lambda_e \Phi_e^{i\alpha} \Phi_e^{T\alpha j})\Theta^e_{ji}. \tag{2.2}$$

Therefore, a SUSY vacuum condition $\partial W/\partial \Theta^e = 0$ ($W = W_e + \cdots$) leads to

$$\langle Y_e \rangle = -\frac{\lambda_e}{\mu_e} \langle \Phi_e \rangle \langle \Phi_e^T \rangle. \tag{2.3}$$

(The bilinear form (2.3) for the charged lepton mass matrix is needed for an explanation of the charged lepton mass relation $K = 2/3$ [10].) Since we take a vacuum with $\langle \Theta^e \rangle = 0$, other conditions $\partial W/\partial Y_e = \mu_e \Theta^e + \cdots = 0$ and $\partial W/\partial \Phi_e = \lambda_e(\Phi_e^T \Theta^e + \Theta^{eT} \Phi_e) + \cdots = 0$ do not affect the relation (2.3). In the present model, although the model is supersymmetric, "SUSY" plays only a role in obtaining VEV relations among yukawaons. What we practically investigate are only quarks and leptons as fermions and yukawaons as scalars. Nevertheless, we cannot dispense with SUSY, because, in a non-SUSY model, we cannot have such the convenient prescription with Θ fields given in Eq.(2.1).

For the time being, we assume that the observed supersymmetry breaking is induced by a gauge mediation mechanism (not including family gauge symmetries), so that our VEV relations among yukawaons are still valid after the SUSY was broken in the quark and lepton sectors.

In the previous yukawaon mode [3] (we refer to it as the O(3) model), the family symmetry O(3) was global. By using SUSY vacuum conditions, we could successfully obtain reasonable quark and lepton mass matrices, especially excellent predictions for up-quark mass ratios and neutrino mixing parameters by adjusting only two parameters. In contrast to the O(3) model, in the present paper, since we assume U(3)×O(3) as family symmetries, the theoretical framework is changed. However, we want to inherit the phenomenological success from the old yukawaon model.

In the O(3) model, a neutrino mass matrix M_ν is given by a seesaw-type mass matrix $M_\nu = m_D M_R^{-1} m_D^T$, where the Dirac and Majorana mass matrices m_D and M_R are given by

$$m_D \propto M_e \propto \langle Y_e \rangle, \tag{2.4}$$

$$\begin{aligned} M_R \propto \langle \Phi_u \rangle \langle P_u \rangle \langle Y_e \rangle + \langle Y_e \rangle \langle P_u \rangle \langle \Phi_u \rangle \\ +\xi_\nu (\langle \Phi_u \rangle \langle Y_e \rangle \langle P_u \rangle + \langle P_u \rangle \langle Y_e \rangle \langle \Phi_u \rangle) + \xi_0 \Lambda \langle Y_e \rangle \langle Y_e \rangle, \end{aligned} \tag{2.5}$$

respectively. Here, the last term (ξ_0 term) has been added in order to adjust the ratio $R_\nu \equiv \Delta m^2_{solar}/\Delta m^2_{atm}$ without affecting neutrino mixing parameters, because $M_\nu \propto \langle Y_e \rangle [(\cdots) + \xi_0 \langle Y_e \rangle \langle Y_e \rangle]^{-1} \langle Y_e \rangle = [\langle Y_e \rangle^{-1} (\cdots) \langle Y_e \rangle^{-1} + \xi_0 \mathbf{1}]^{-1}$ and the mixing matrix for M_ν is identical to that for M_ν^{-1} except for the phase factors [14]. (In the O(3) model, the yukawaon Y_R has the same U(1)$_X$ charge as $Y_e Y_e$, i.e. $Q_X(Y_R) = 2Q_X(Y_e)$.) The charged lepton mass matrix M_e is given by $M_e \propto \langle Y_e \rangle = k_e \langle \Phi_e \rangle \langle \Phi_e \rangle^T$ ($k_e = -\lambda_e/\mu_e$), while the quark mass matrices M_u and M_d are given by

$$M_u^{1/2} \propto \langle \Phi_u \rangle = k_u \langle \Phi_e \rangle (\mathbf{1} + a_u X) \langle \Phi_e \rangle, \quad M_d \propto \langle Y_d \rangle = k_d \langle \Phi_e \rangle (\mathbf{1} + a_d X) \langle \Phi_e \rangle, \tag{2.6}$$

where

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \tag{2.7}$$

and P_u is defined as a field with a VEV matrix form

$$\langle P_u \rangle_u = v_P \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{2.8}$$

Here, in Eq.(2.8), the index "u" denotes that a VEV matrix $\langle A \rangle$ takes a form $\langle A \rangle_u$ at the diagonal basis of the up-quark mass matrix M_u. The reason of the existence of the matrix $\langle P_u \rangle_u$ is as follows: If we take a value $a_u \simeq -1.8$ in the up-quark mass matrix relation given in Eq.(2.6), we can give reasonable up-quark mass ratios, but signs of the eigenvalues of $\langle \Phi_u \rangle$ show $(+, -, +)$, i.e. $(M_u^{1/2})^{diag} \propto \mathrm{diag}(\sqrt{m_u}, -\sqrt{m_c}, \sqrt{m_t})$ for $a_u \simeq -1.8$. In order to give reasonable neutrino mixing parameters (especially the observed value [15] $\sin^2 2\theta_{atm} \simeq 1$), the existence of P_u in Eq.(2.5) is indispensable in the prediction of $\sin^2 2\theta_{atm}$. The ξ_ν term in Eq.(2.5) has been added from a reason that the O(3) model was based on an O(3) family symmetry, so that terms $ACB + BCA$ were also possible in addition to terms $ABC + CBA$. By adjusting parameters a_u,

a_d and ξ_ν, we have obtained [3] not only the observed nearly tribimaximal neutrino mixing [16], but also reasonable Cabibbo-Kobayashi-Maskawa (CKM) quark mixing (however, the fitting of CKM mixing is not so excellent compared with that of neutrino mixing).

In conclusion, we summarize the previous O(3) model as follow:
(i) The charged lepton mass matrix M_e is given by a bilinear form of a fundamental VEV matrix $\langle\Phi_e\rangle$, i.e. $(M_e)_{ij} \propto \langle(Y_e)_{ij}\rangle \propto \langle(\Phi_e)_{ik}\rangle\langle(\Phi_e)_{kj}\rangle$. This VEV matrix $\langle\Phi_e\rangle$ plays an important role in another mass matrices M_u, M_d and M_ν.
(ii) All yukawaons Y_f are singlets under the conventional gauge symmetries and they have the same O(3) assignments $(\mathbf{5}+\mathbf{1})$, so that we assume an additional U(1) symmetry (we called it U$(1)_X$ symmetry), and each yukawaon is distinguished from others by the U$(1)_X$ charge Q_X.
(iii) In order to build a model without a Dirac neutrino yukawaon Y_ν, we assume $Q_X(\nu^c) = Q_X(e^c)$, so that the yukawaon Y_e couple not only to the charged lepton sector, but also to the Dirac neutrino sector. As a result, the neutrino Majorana mass matrix M_ν is given by a form $M_\nu = m_D M_R^{-1} m_D^T \propto \langle Y_e\rangle M_R^{-1}\langle Y_e^T\rangle$.
(iv) We assume a form (2.5) as the form of M_R. Even when we take only the dominant term $\Phi_u P_u Y_e + Y_e P_u \Phi_u$ into consideration, we can obtain favorable results $\sin^2 2\theta_{atm} \simeq 1$ and $|U_{13}|^2 \simeq 0$, although the predicted value $\tan^2\theta_{solar} \simeq 0.7$ is somewhat large compared with the observed value [17] $\tan^2\theta_{solar} \simeq 0.45$. (See a case of $\xi_\nu = 0$ in Table 1 in Sec.4.) Only when we take the ξ_ν term in Eq.(2.5) into consideration, we can fit the predicted $\tan^2\theta_{solar}$ to the observed value without almost affecting the favorable results $\sin^2 2\theta_{atm} \simeq 1$ and $|U_{13}|^2 \simeq 0$. We can also fit the neutrino mass ratios R_ν by adjusting the parameter ξ_0 in Eq.(2.5) without affecting the neutrino mixing parameters.
(v) The CKM parameter fitting in the original O(3) model is not so excellent, although a rough tendency is fine. Somewhat improvements are needed. (For example, see Ref.[18].)
(vi) The model is based on an effective theory. The cutoff scale Λ is of the order to 10^{12} GeV. The scale is originated in the scale M_R in the neutrino seesaw model. As a result, it is hard to observe new physics which comes from the O(3) model.

Correspondingly to the item numbers (i) - (vi) in the O(3) model, the present U(3)×O(3) model have the following characteristics:
(i) By considering $(M_e)_{ij} \propto \langle(Y_e)^{ij}\rangle \propto \langle(\Phi_e)^{i\alpha}\rangle\langle(\Phi_e^T)^{\alpha j}\rangle$, we can consider a similar picture described in (i) of the O(3) model.
(ii) In the U(3)×O(3) model, the yukawaons are assigned as $Y_e \sim (\mathbf{6}^*, \mathbf{1})$, $Y_R \sim (\mathbf{1}, \mathbf{5}+\mathbf{1})$, $Y_u \sim (\mathbf{3}, \mathbf{3})$ and $Y_d \sim (\mathbf{1}, \mathbf{5}+\mathbf{1})$ of U(3)×O(3). Therefore, if we can assign R charges to Y_R and Y_d differently, we do not need such U$(1)_X$ charges in the O(3) model.
(iii) In the O(3) model, in order to build a model without Y_ν, we have assumed $Q_X(\nu^c) = Q_X(e^c)$ by hand. In the present model, $Y_e^{i\alpha}$ couples to $\ell_i e_j^c$ as well as the O(3) model, while $(\Phi_e)^{i\alpha}$ couples only to $\ell_i \nu_\alpha^c$ without such a phenomenological assignment of the U(1) charge. As a result, the neutrino Majorana mass matrix M_ν is given by a form $M_\nu \propto \langle\Phi_e\rangle M_R^{-1}\langle\Phi_e^T\rangle$.
(iv) By replacing Y_R in the form (2.5) with $\Phi_e Y_R \Phi_e^T$, we can obtain a practically same result as the dominant term $\Phi_u P_u Y_e + Y_e P_u \Phi_u$ in the form (2.5). However, in the U(3)×O(3) model, we cannot write such a term as ξ_ν in the O(3) model. Therefore, in the present model, we assume an alternative term which is invariant under the U(3)×O(3) symmetries. In spite of the existence

of such a new ξ_ν term, as we show in Table 1 in Sec.4, the new ξ_ν term can also give reasonable fits for neutrino mixing parameters. We can also fit the neutrino mass ratios R_ν by introducing a similar term to the ξ_0 term in Eq.(2.5). However, we do not discuss the numerical results of R_ν in this paper, because it is not prediction, but it is a result of adjusting the parameter ξ_0.
(v) The numerical results for the CKM mixing parameters are identical with the results in the O(3) model, so that we do not show the results. In order to obtain more precise agreements of the CKM mixing parameters with the observed values, further investigation is needed for the present model, especially, for the structure of Y_d.
(vi) The greatest merit of the present U(3)×O(3) model is to lower the scale of Λ, e.g. $\Lambda \sim 10^8$ GeV, although $\Lambda \sim 10^{12}$ GeV in the O(3) model. The details are discussed in Sec.5.

3. Model for quark sector

First, we investigate possible superpotential forms in the quark sector. Correspondingly to the lepton sector with $(\ell_i, e_i^c, \nu_\alpha)$, we consider a model with $(q_\alpha, d_\alpha^c, u^{ci})$.

We have introduced a field $\Phi^u_{i\alpha}$ similar to $\Phi_e^{i\alpha}$ as shown in Eq.(1.4). However, we cannot identify $\Phi^u_{i\alpha}$ as $Y^d_{\alpha\beta}$ although we have regarded Φ_e as Y_ν^{Dirac} in the lepton sector. Note that in this model, the U(3) gauge bosons A_i^j which couple to charged lepton sector cannot couple to the down-quark sector. For yukawaons in the quark sector, we assume the following superpotential terms:[1]

$$W_u = \left(\mu_u Y^u_{i\alpha} + \frac{\lambda_u}{\Lambda} \Phi^u_{i\beta} \bar{E}_{u3}^{T\beta j} \Phi^u_{j\alpha} \right) \Theta_u^{\alpha i} + \frac{1}{\Lambda} \left(\lambda'_u \bar{E}_{u6}^{ik} \Phi^u_{k\alpha} \bar{E}_{u3}^{T\alpha j} + \lambda''_u \Phi_e^{i\alpha} (E_{\alpha\beta} + a_u S_{\alpha\beta}) \Phi_e^{T\beta j} \right) \Theta'^u_{ji}, \tag{3.1}$$

$$W_d = \frac{1}{\Lambda} \left(\lambda_d \bar{P}_d^{i\alpha} Y^d_{\alpha\beta} \bar{P}_d^{T\beta j} + \lambda'_d \Phi_e^{i\alpha} (E_{\alpha\beta} + a_u S_{\alpha\beta}) \Phi_e^{T\beta j} \right) \Theta^d_{ji}, \tag{3.2}$$

where the VEV forms of $\langle \bar{E}_{u3} \rangle$, $\langle \bar{E}_{u6} \rangle$, $\langle E \rangle$ and $\langle S \rangle$ are given by $\langle \bar{E}_{u3} \rangle = v_{Eu3} \mathbf{1}$, $\langle \bar{E}_{u6} \rangle = v_{Eu6} \mathbf{1}$, $\langle E \rangle = v_E \mathbf{1}$ and $\langle S \rangle = v_S X$ [X is defined by Eq.(2.7)]. These forms of the superpotential effectively lead to the quark mass matrices given in Eq.(2.6) in the O(3) model.

The form $\langle S \rangle = v_S X$ is given by the following superpotential term W_S:

$$W_S = \lambda_S \, \mathrm{det} S. \tag{3.3}$$

By using a formula for any 3×3 Hermitian matrix A

$$\mathrm{det} A = \frac{1}{3} \mathrm{Tr}[AAA] - \frac{1}{2} \mathrm{Tr}[AA] \mathrm{Tr}[A] + \frac{1}{6} (\mathrm{Tr}[A])^3, \tag{3.4}$$

we obtain

$$\frac{\partial W_S}{\partial S} = \lambda_S \left[SS - \frac{1}{2} \left(2S \mathrm{Tr}[S] + \mathbf{1} \mathrm{Tr}[SS] \right) + \frac{1}{2} \mathbf{1} (\mathrm{Tr}[S])^2 \right]. \tag{3.5}$$

Therefore, the SUSY vacuum condition $\partial W_S / \partial S = 0$ leads to a solution

$$\langle S \rangle \langle S \rangle = \langle S \rangle \mathrm{Tr}[\langle S \rangle]. \tag{3.6}$$

[1]In order to distinguish $\Phi^u_{i\alpha}$ from $Y^u_{i\alpha}$, $Y^d_{\alpha\beta}$ from $Y^R_{\alpha\beta}$, and so on, we must assume different R charges for those fields. For the R-charge assignments, see Table 2 in Sec.6.

By applying another formula for any 3×3 Hermitian matrix A

$$\mathbf{1}\det A = AAA - AA\mathrm{Tr}[A] + \frac{1}{2}A\left[(\mathrm{Tr}[A])^2 - \mathrm{Tr}[AA]\right], \tag{3.7}$$

to the solution (3.6), we obtain

$$\det\langle S\rangle = 0. \tag{3.8}$$

Therefore, from Eqs.(3.6) and (3.8), we choose a specific form

$$\langle S\rangle_e = v_S X = \frac{1}{3}v_S \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{3.9}$$

[Of course, the solution (3.9) is not a general solution. For example, $\langle S\rangle = v_S\,\mathrm{diag}(0,0,1)$ is also possible. We assume that $\langle S\rangle_e$ is given by the form (3.9) in the diagonal basis of the charged lepton mass matrix M_e (i.e. of $\langle\Phi_e\rangle_e$).] On the other hand, for the form $\langle E\rangle_e = v_E\,\mathbf{1}$, we consider a superpotential form

$$W_E = \lambda_E \mathrm{Tr}[EE]\mathrm{Tr}[E] + \lambda'_E(\mathrm{Tr}[E])^3. \tag{3.10}$$

A SUSY vacuum condition

$$\frac{\partial W_E}{\partial E} = 2\lambda_E E\,\mathrm{Tr}[E] + \mathbf{1}\left\{\lambda_E\mathrm{Tr}[EE] + 3\lambda'_E(\mathrm{Tr}[E])^2\right\} = 0, \tag{3.11}$$

leads to $\langle E\rangle = v_E\,\mathbf{1}$. Since we require the R-charge conservation, we have to assign $R = 2/3$ to the fields S and E.

In general, for fields $A_{i\alpha}$ and $\bar{A}^{\alpha i}$ with $R(A) + R(\bar{A}) = 1$, we can consider superpotential terms

$$W_A = \frac{1}{\Lambda}\left(\lambda_A\mathrm{Tr}[A\bar{A}A\bar{A}] + \lambda'_A\mathrm{Tr}[A\bar{A}]\mathrm{Tr}[A\bar{A}]\right). \tag{3.12}$$

SUSY vacuum conditions $\partial W_A/\partial A = 0$ and $\partial W_A/\partial \bar{A} = 0$ lead to

$$A\bar{A} = -\frac{\lambda'_A}{\lambda_A}\mathbf{1}\mathrm{Tr}[A\bar{A}]. \tag{3.13}$$

We assume that $\bar{E}_{u3}^{i\alpha}$, $\bar{E}_{u6}^{ij}$ and $\bar{P}_d^{i\alpha}$ are given by a superpotential form (3.12). We take a special solution

$$\langle A\rangle = a\,\mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3}), \tag{3.14}$$

in $\langle\bar{A}\rangle\langle A\rangle \propto \mathbf{1}$, i.e. $\langle E^{u3}\rangle = v_{Eu3}\mathbf{1}$, $\langle E^{u6}\rangle = v_{Eu6}\mathbf{1}$ and $\langle\bar{P}_d\rangle = v_{Pd}\,\mathrm{diag}(e^{i\phi_1}, e^{i\phi_2}, e^{i\phi_3})$. (In any case, we must assume a special form of $\langle\bar{A}\rangle$ in $\langle\bar{A}\rangle\langle A\rangle \propto \mathbf{1}$.)

In the O(3) model, the CKM parameter fitting is not so remarkable compared with those in the up-quark mass ratios and the neutrino mixing parameters. Note that, differently from the O(3) model, we have the phase matrix $\langle\bar{P}_d\rangle$ as seen in Eq.(3.2). This will improve the fitting

ξ_ν	$\tan^2\theta_{solar}$	$\sin^2 2\theta_{atm}$	$\|U_{13}\|^2$
0	0.6995	0.9872	1.72×10^{-4}
0.009	0.4610	0.9902	2.28×10^{-4}
0.010	0.4408	0.9905	2.35×10^{-4}

Table 1: ξ_ν dependence of the neutrino parameters. The value of a_u is taken as $a_u=-1.78$ which can give reasonable up-quark mass ratios.

for the CKM mixing parameters. (However, such a numerical fitting is not our purpose in the present paper.)

4. Neutrino mass matrix

In the present model, since $m_D \propto \langle\Phi_e\rangle$ differently from the O(3) model, we cannot consider the form $Y_R=\Phi_u P_u Y_e + Y_e P_u \Phi_u$ as shown in Eq.(2.5). Therefore, we consider the following superpotential terms for the Y_R sector:

$$\frac{1}{\Lambda}\left[\lambda_R \Phi_e^{i\alpha} Y_R^{\alpha\beta}\Phi_e^{T\beta j} + \lambda'_R\left(\bar{P}_u^{i\alpha}\Phi_{\alpha k}^{Tu} Y_e^{kj} + Y_e^{ik}\Phi_{k\alpha}^{u}\bar{P}_u^{T\alpha j}\right)\right]\Theta_{ji}^R. \tag{4.1}$$

Note that, in the present model, we cannot consider such a term which corresponds to the ξ_ν term in the O(3) model [Eq.(2.5)]. In the O(3) model, in order to give the observed value [17] $\tan^2\theta_{solar}\simeq 1/2$, it was indispensable that we take a non-vanishing value of ξ_ν, although we could give the observed values [15] $\sin^2 2\theta_{atm}\simeq 1$ and $|U_{13}|^2\simeq 0$ even if $\xi_\nu=0$. Therefore, in the present model, too, we need some additional term to Eq.(4.1). We assume the following superpotential for Y_R with a new ξ_ν term:

$$W_R=\frac{1}{\Lambda}\Big\{\lambda_R(\Phi_e Y_R \Phi_e^T)^{ij} + \lambda'_R\Big[(\bar{P}_u\Phi_u^T)_k^i Y_e^{kj} + Y_e^{ik}(\Phi_u \bar{P}_u^T)_k^j$$

$$+\xi_\nu(\Phi_u\bar{P}_u^T)_k^k Y_e^{ij}\Big] + \lambda''_R Y_e^{ik} E_{kl}^{u6} Y_e^{lj}\Big\}\Theta_{ji}^R. \tag{4.2}$$

The last term (λ''_R term) has been added in order to adjust the neutrino mass ration $R_\nu=\Delta m^2_{solar}/\Delta m^2_{atm}$ similar to the ξ_0 term in Eq.(2.5) in the O(3) model. From the superpotential (4.2), we obtain the following neutrino mass matrix M_ν:

$$M_\nu \propto \langle\Phi_e\rangle\left\{\langle\Phi_e\rangle^{-1}\left[\langle P_u\rangle\langle\Phi_u\rangle\langle Y_e\rangle + \langle Y_e\rangle\langle\Phi_u\rangle\langle P_u\rangle + \xi_\nu \mathrm{Tr}[\langle\Phi_u\rangle\langle P_u\rangle\right.\right.$$

$$\left.\left.+\xi_0\langle Y_e\rangle\langle E^{u6}\rangle\langle Y_e\rangle]\langle Y_e\rangle\right]\langle\Phi_e\rangle^{-1}\right\}^{-1}\langle\Phi_e\rangle, \tag{4.3}$$

where $\langle\Phi_e\rangle_e\propto \mathrm{diag}(\sqrt{m_e},\sqrt{m_\mu},\sqrt{m_\tau})$ and $\langle\Phi_u\rangle_e = k_u\langle\Phi_e\rangle_e(\langle E\rangle + a_u\langle S_u\rangle_e)\langle\Phi_e\rangle_e$.

In Table 1, we demonstrate ξ_ν-dependence of the neutrino mixing parameters in a case with $a_u=-1.78$, which gives reasonable up-quark mass ratios

$$\sqrt{\frac{m_u}{m_c}}=0.04389,\qquad \sqrt{\frac{m_c}{m_t}}=0.05564. \tag{4.4}$$

The predicted values are in good agreement with the observed values at $\mu = m_Z$ [19] $\sqrt{m_u/m_c} = 0.045^{+0.013}_{-0.010}$ and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$. As seen in Table 1, the magnitudes of $\sin^2 2\theta_{atm}$ and $|U_{13}|^2$ are almost independent of the parameter ξ_ν (e.g. $\sin^2 2\theta_{atm} \simeq 1$ and $|U_{13}|^2 \simeq O(10^{-4})$, while the value of $\tan^2\theta_{solar}$ is highly dependent on the value of ξ_ν (e.g. $\tan^2\theta_{solar} = 0.70$ for $\xi_\nu = 0$ and $\tan^2\theta_{solar} = 0.44$ for $\xi_\nu = 0.01$). We note that the model can give excellent fits with the observed values of the neutrino mixing parameters in spite of its phenomenologically different form of the ξ_ν term.

5. Energy scale of the cutoff Λ

So far, we did not discuss the energy scale Λ of the effective theory. In this section, we discuss that we can lower the value of Λ in the present model.

In the O(3) model (also in the present model, too), the charged lepton mass matrix M_e is given by

$$(M_e)_{33} = y_e \frac{\langle (Y_e)_{33}\rangle}{\Lambda} \langle H_d^0 \rangle, \tag{5.1}$$

so that, in order to give $m_\tau \sim 1$ GeV with $\langle H_d^0 \rangle \sim 10$ GeV ($\tan\beta \sim 10$), we have to take $\langle Y_e \rangle/\Lambda \sim 10^{-1}$. Since the neutrino mass matrix in the O(3) model is given by

$$(M_\nu)_{33} = (m_D)_{3k}(M_R^{-1})_{kl}(m_D)_{l3} = \frac{(y_\nu)^2}{\lambda_R}\langle H_u^0\rangle^2 \frac{1}{\Lambda^2}\langle (Y_e)_{33}\rangle(\langle Y_R\rangle^{-1})_{33}(\langle Y_e\rangle)_{33}, \tag{5.2}$$

we have to take $\langle Y_R \rangle \sim 10^{12}$ GeV in order to give $m_{\nu 3}/m_\tau \sim 10^{-10}$.

On the other hand, in contrast to the O(3) model, the neutrino mass matrix in the present model is given by

$$(M_\nu)_{33} = (m_D M_R^{-1} m_D^T)_{33} = \frac{(y_\nu)^2}{\lambda_R}\frac{1}{\Lambda^2}\langle H_u^0\rangle^2 \langle (\Phi_e)^{33}\rangle(\langle Y_R\rangle^{-1})_{33}\langle (\Phi_e)^{33}\rangle, \tag{5.3}$$

and the charged lepton mass matrix is given by

$$(M_e)_{33} = y_e \frac{\langle (Y_e)^{33}\rangle}{\Lambda}\langle H_d^0\rangle = -y_e\lambda_e \frac{\Lambda}{\mu_e}\frac{\langle (\Phi_e)^{33}\rangle}{\Lambda}\frac{\langle (\Phi_e)^{33}\rangle}{\Lambda}\langle H_d^0\rangle, \tag{5.4}$$

from Eq.(2.3). Therefore, we can estimate

$$\frac{(M_\nu)_{33}}{m_\tau} = \frac{(y_\nu)^2}{y_e\lambda_e\lambda_R}\frac{\langle H_u^0\rangle^2}{\langle H_d^0\rangle}\left(\langle Y_R^{-1}\rangle\right)_{33}\frac{\mu_e}{\Lambda}. \tag{5.5}$$

By taking $(M_\nu)_{33}/m_\tau \sim 10^{-10}$ (by regarding $(M_\nu)_{33}$ as $(M_\nu)_{33} \sim m_{\nu 3} \simeq \sqrt{\Delta m^2_{atm}} \simeq 0.047$ eV [15]), $\langle H_u^0\rangle \sim 10^2$ GeV and $\langle H_d^0\rangle \sim 10$ GeV, and by assuming $\langle Y_R\rangle \sim \Lambda$ (a maximum value of $\langle Y_R\rangle$ in the present effective theory) , we obtain

$$\Lambda \sim \sqrt{10^{13}[\mathrm{GeV}]\,\mu_e[\mathrm{GeV}]}. \tag{5.6}$$

Therefore, we can, in principle, take any small value of Λ by assuming a small value of μ_e. (However, a too small value of μ_e is unlikely.) For example, if we take $\mu_e \sim 1$ TeV, we obtain $\Lambda \sim 10^8$ GeV.

On the other hand, the U(3) gauge bosons A_i^j acquire their masses from VEVs $\langle Y_e^{ij} \rangle$, $\langle \Phi_e^{i\alpha} \rangle$, $\langle Y^u_{i\alpha} \rangle$, and so on. In general, the gauge boson masses $m(A_i^j)$ are given by

$$m^2(A_i^j) = \frac{1}{2}\left[\sum_{\mathbf{6}} |\langle \mathbf{6}_i \rangle + \langle \mathbf{6}_j \rangle|^2 + \sum_{\mathbf{3}} \left(|\langle \mathbf{3}_i \rangle|^2 + |\langle \mathbf{3}_j \rangle|^2 \right) \right], \tag{5.7}$$

where $\langle \mathbf{6}_i \rangle$ and $\langle \mathbf{3}_i \rangle$ are eigenvalues of VEV matrices of fields $\mathbf{6}$ ($\mathbf{6}^*$) and $\mathbf{3}$ ($\mathbf{3}^*$), respectively, and the observed CKM mixing has been neglected. At present, magnitudes of $\langle \bar{E}_{u6} \rangle$, $\langle \bar{E}_{u3} \rangle$, and so on, are free parameters. If we consider that the dominant contributions to $m(A_i^j)$ are, for example, $\langle \bar{E}_{u6}^{ij} \rangle = \delta_{ij}\Lambda$, we obtain $m(A_i^j) \simeq \sqrt{2} g \Lambda$, so that we cannot observe the gauge boson effects. Here, let us optimistically suppose that dominant contributions are $\langle Y_u^{ii} \rangle \sim (m_{ui}/\langle H_u^0 \rangle)\Lambda$. (We can easily demonstrate that $\langle Y_u \rangle$ is dominant, at least, compared with $\langle Y_e \rangle$ and $\langle \Phi_e \rangle$.) Then, we can estimate the gauge boson masses as follows:

$$m(A_i^j) \simeq \frac{g}{\sqrt{2}} \sqrt{(v_i^u)^2 + (v_j^u)^2} = \frac{g}{\sqrt{2}} \sqrt{m_{ui}^2 + m_{uj}^2} \frac{\Lambda}{y_u \langle H_u^0 \rangle}, \tag{5.8}$$

where we have taken a flavor basis which is defined by $\langle Y^u_{i\alpha} \rangle = \delta_{i\alpha} v_i^u$, so that we can estimate gauge boson mass ratios as

$$\frac{m(A_1^1)}{m(A_3^3)} \simeq \frac{m_u}{m_t} \simeq 0.74 \times 10^{-5}, \qquad \frac{m(A_2^1)}{m(A_3^3)} \simeq \frac{m_c}{\sqrt{2} m_t} \simeq 2.6 \times 10^{-3}, \tag{5.9}$$

where $m(A_3^3) \sim \Lambda$. If we suppose $\Lambda \sim 10^8$ GeV, we obtain $m(A_1^1) \sim 1$ TeV and $m(A_2^1) \sim 10^2$ TeV. The gauge boson A_1^1 with $m(A_1^1) \sim 1$ TeV will be observed in Z' searches at the LHC at which Z' can decay into $e^+ + e^-$ but not into $\mu^+ + \mu^-$. (More details of the A_1^1 search at LHC have substantially been discussed in Ref.[20].) Since the U(3) gauge bosons cannot couple to the down-quark sector and do only to u^c, the gauge boson A_2^1 with the next lower mass can contribute to the D^0-$\bar{D}^0$ mass difference. In this paper, we do not give further numerical predictions. Phenomenological meanings of the present model in TeV region physics will be discussed elsewhere. We again would like to emphasize that such a lower scale of Λ would not be realized without introducing the yukawaon Y_e^{ij} which is related to $\Phi_e^{i\alpha} \Phi_e^{T\alpha j}$ by Eq.(2.3).

6. Concluding remarks

In conclusion, stimulated by the Sumino's model [8, 9] with U(3)×O(3) symmetries for the charged leptons, we have considered a SUSY version of his model with a U(3) family gauge symmetry by extending it to U(3)×O(3) family symmetries for the quarks and leptons (but O(3) is already broken at $\mu = \Lambda$). Although, in order to distinguish each yukawaon from other ones, an additional U(1) symmetry [i.e. U(1)$_X$] was assumed in the previous O(3) yukawaon

	ℓ	e^c	ν^c	q	u^c	d^c	H_u	H_d	Y_e	$\bar{E}_{u6}$
U(3)	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{6}^*$
O(3)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
R	1	$-2r_e$	$-r_e$	1	$-(2r_u+\bar{r}_{u3})$	$-r_d$	1	1	$2r_e$	1

	Θ^e	Θ^R	$\Theta^{u\prime}$	Θ^d	E^{u6}	Φ_e	$\bar{P}_u$	$\bar{P}_d$	Θ_u	$\bar{E}_{u3}$
U(3)	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$
O(3)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$
R	$2-2r_e$	$R(\Theta^R)$	$R(\Theta^{u\prime})$	$R(\Theta^d)$	0	r_e	$\bar{r}_{Pu}$	$\bar{r}_{Pd}$	$R(\Theta_u)$	$\bar{r}_{u3}$

	Y^u	Φ^u	P^d	E^{u3}	Y_R	Y_d	S & E
U(3)	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
O(3)	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5}+\mathbf{1}$	$\mathbf{5}+\mathbf{1}$	$\mathbf{5}+\mathbf{1}$
R	$2r_u+\bar{r}_{u3}$	r_u	$1-\bar{r}_{Pd}$	$1-\bar{r}_{u3}$	$2r_e$	r_d	$\frac{2}{3}$

Table 2: Quantum numbers of U(3)×O(3) family symmetries and R charges, where $R(\Theta^R)=2-4r_e$, $R(\Theta^{u\prime})=1-(r_u+\bar{r}_{u3})$, $R(\Theta^d)=2-(r_d+2\bar{r}_{Pd})$ and $R(\Theta_u)=2-(2r_u+\bar{r}_{u3})$. For simplicity, we have taken as $R(H_u)=R(H_d)=1$ and $R(\ell)=R(q)=1$. Here, relations $\bar{r}_{Pu}=2r_e-r_u$ and $2r_e+\frac{2}{3}=r_u+\bar{r}_{E3}+1=r_d+2\bar{r}_{Pd}$ are required. The value of $R(E^{u6})$ is fixed as $R(E^{u6})=0$ due to the relation $R(Y_R)+2R(\Phi_e)=2R(Y_e)+R(E^{u6})$ term in Eq.(4.2).

model, we do not need such $U(1)_X$ charges in the present model with the two family symmetries (but we still assume the R charge conservation). Quantum number assignments of the fields are summarized in Table 2. Since we consider a superpotential term $\mu_H H_u H_d$, R charges for Higgs fields H_u and H_d satisfy $R(H_u)+R(H_d)=2$. For simplicity, in Table 2, we have taken $R(H_u)=R(H_d)=1$. Furthermore, we have taken $R(\ell)=R(q)=1$. In spite of such simplified assignments, we still have free parameters in the R charge assignments as seen in Table 2. In an explicit R charge assignments, we must take care that yukawaon fields with the same U(3)×O(3) assignments cannot have the same R charges, because those are singlets under the conventional gauge symmetry, and they should distinguished from each other only by R charges.

The original Sumino model is a model for the charged leptons, so that he has explicitly mentioned nothing as to quark and neutrino sectors. If we adopt Sumino's assignments $\ell\sim\mathbf{3}$ and $e^c\sim\mathbf{3}$ of U(3), while we assume $\nu^c\sim\mathbf{3}$ of O(3), the model leads to a seesaw-type neutrino mass matrix $M_\nu=\langle\Phi_e\rangle M_R^{-1}\langle\Phi_e\rangle^T$. We have investigated a possible form of the Majorana mass matrix $M_R=\lambda_R\langle Y_R\rangle$ of the right-handed neutrinos by referring to a supersymmetric yukawaon model (O(3) model) [3] for the neutrino sector. The present form (4.2) of M_R is similar to the form (2.5) in the O(3) model, but the ξ_ν term is completely different from the O(3) model. Nevertheless, in this model, too, we can successfully obtain the nearly tribimaximal neutrino mixing by adjusting the parameter ξ_ν as seen in Table 1. It is worthwhile noticing this.

As seen in Table 2, we have two $\mathbf{6}^*$, five $\mathbf{6}$, five $\mathbf{3}^*$ and four $\mathbf{3}$ of U(3) except for the quarks and leptons. Therefore, the present model is not anomaly free. At present, the present model for M_d cannot give precise numerical fits for the observed CKM mixing. The improvement of Y_d structure is an open question at present. For such improvement, we will need further fields which are singlets under SU(3)$_c\times$SU(2)$_L\times$U(1)$_Y$, but non-singlets under family symmetries. Inversely, since we have too many yukawaons, some of them will be economized in future. Although we consider that the theory should be anomaly free, at this stage of the yukawaon model, it will not be fruitful to adhere to the anomaly freedom problem.

The greatest merit in considering U(3)$\times$O(3) family gauge symmetries lies in that we can lower the cutoff scale Λ in the present yukawaon model. In the previous yukawaon model, in order to give the observed tiny neutrino masses, we had been obliged to consider $\Lambda \sim 10^{12}$ GeV. In the O(3) model, the relation (2.4) was ad hoc assumed (i.e. the yukawaon Y_ν was regarded as Y_e). In the present model, the field Φ_e can couple to the neutrino Dirac term because of the same quantum numbers of U(3)$\times$O(3), so that we obtain Eq.(5.3) instead of the relation (5.2). This has enabled us to lower the scale Λ as we have seen in Sec.5.

If we suppose a value $\mu_e \sim 1$ TeV which gives $\Lambda \sim 10^8$ GeV, we obtain $m(A_1^1) \sim 1$ TeV and $m(A_2^1) \sim 10^2$ TeV in an optimistic case. The gauge boson A_1^1 with $m(A_1^1) \sim 1$ TeV will be observed in Z' searches at the LHC at which Z' can decay into $e^+ + e^-$ but not into $\mu^+ + \mu^-$. The gauge boson A_2^1 with the next lower mass can contribute to D^0-$\bar{D}^0$ mass difference. Phenomenological studies of the present model in TeV region physics will be discussed elsewhere. We would like to emphasize that, in order to make yukawaon effects visible in the terrestrial experiments, it has been inevitable to adopt the present model with two family symmetries.

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