OU-HET-687/2010,

Neutrino Mass Matrix Speculated from U(3)×O(3) Family Symmetries

Yoshio Koide

Department of Physics, Osaka University, Toyonaka, Osaka 560-0043, Japan

E-mail address: koide@het.phys.sci.osaka-u.ac.jp

Abstract

A quark and lepton mass matrix model with family symmetries U(3)×O(3) is investigated on the basis of the so-called yukawaon model. In the present model, quarks and leptons are assigned as $(\ell, e^c, u^c) \sim \mathbf{3}$ of U(3) and $(q, d^c, \nu^c) \sim \mathbf{3}$ of O(3). Then, the neutrino mass matrix is given by $M_\nu = m_D M_R^{-1} m_D^T$ with $m_D \propto M_e^{1/2}$, where the charged lepton mass matrix is given by $M_e = M_e^{1/2} M_e^{1/2}$.

1. Introduction

One of the most challenging problems in contemporary particle physics is to clarify the origin of flavors. For such a purpose, it is interesting to investigate whether the observed flavor physics phenomena can be understood from a concept of a family gauge symmetry or not. The present data [1] seem to suggest that numbers of lepton- and quark-families are both three. Then, from a point of view of a unification model of quarks and leptons, it will be natural to consider that the quarks and leptons obey the same family symmetry. However, at present, this is experimentally not yet confirmed. In this paper, we investigate a possibility that quarks and leptons obey different family symmetries from each other.

In the present paper, by assuming family symmetries U(3)×O(3), we will propose a new version of the so-called "supersymmetric yukawaon" model [2, 3] (a kind of "flavon" model [4]) with the following would-be Yukawa interaction terms:

$$W = \frac{y_e}{\Lambda^2} \ell_i Y_e^{ij} e_j^c H_d + \frac{y_\nu}{\Lambda} \ell_i \Phi_e^{i\alpha} \nu_\alpha^c H_u + \lambda_R \nu_\alpha^c Y_R^{\alpha\beta} \nu_\beta^c + \frac{y_u}{\Lambda} u_i^c Y_u^{i\beta} q_\beta H_u + \frac{y_d}{\Lambda} d_\alpha^c Y_d^{\alpha\beta} q_\beta H_d, \tag{1}$$

with vacuum expectation value (VEV) relations

$$\langle Y_e^{ij} \rangle \propto \langle \Phi_e^{i\alpha} \rangle \langle \Phi_e^{T\alpha j} \rangle, \quad \langle Y_u^{i\alpha} \rangle \propto \langle \Phi_u^{i\beta} \rangle \langle E_{\beta j}^T \rangle \langle \Phi_u^{j\alpha} \rangle, \tag{2}$$

where $\langle E_{i\alpha} \rangle = v_E \delta_{i\alpha}$. We call the fields Y_f "yukawaons". Here, ℓ and q are defined by $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$, and indexes $i, j, \cdots$ and $\alpha, \beta, \cdots$ denote those of U(3) and O(3), respectively. One of the motivations to consider such a model with two family symmetries is as follows: In general, if we consider that the family symmetry is global as in the previous yukawaon model, massless scalars appear in the model. However, if we want to consider that the family symmetry is local, the gauge coupling constants will burst at $\mu = \Lambda$, because, in the yukawaon model, there are many family symmetry non-singlet fields. Therefore, if we consider a model with different family symmetries for quarks and leptons, we can evade from such a trouble.

As seen in Eq.(1), a Majorana mass matrix M_R of the right-handed neutrinos is given by $M_R = \lambda_R \langle Y_R \rangle$. The characteristic of this model (1) is that the Dirac neutrino mass matrix m_D is inevitably given by $\langle \Phi_e \rangle$, i.e. by $M_e^{1/2}$. Then, the seesaw-type neutrino mass matrix M_ν is given by $M_\nu \propto M_e^{1/2} \langle Y_R \rangle^{-1} M_e^{1/2}$. (In the previous yukawaon model [3], m_D was given not by $m_D \propto M_e^{1/2}$, but by $m_D \propto M_e$, and it was ad hoc assumption.)

In order to give the observed tiny neutrino masses, we suppose $\langle Y_R \rangle \sim \Lambda$. A naive estimate of Λ suggests $\Lambda \sim 10^{14}$ GeV. However, as we state later, we have a possibility that we can take a considerably lower value of the scale Λ due to the VEV relation $\langle Y_e^{ij} \rangle \propto \langle \Phi_e^{i\alpha} \rangle \langle \Phi_e^{T\alpha j} \rangle$.

The present model with U(3)×O(3) symmetries has received a hint from a charged lepton mass matrix model with U(3)×O(3) symmetries which has recently been proposed by Sumino [5, 6]. In the Sumino model, the charged lepton mass term is generated by a would-be Yukawa interaction term

$$H_e = \frac{y_e}{\Lambda^2} \bar{\ell}_L^i \Phi_{i\alpha}^e \Phi_{\alpha j}^{eT} e_R^j H. \tag{3}$$

The charged lepton masses m_{e_i} are acquired from the vacuum expectation value (VEV) of the scalar Φ^e [7], i.e. the VEV is given by $\langle \Phi^e \rangle_e \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, where the suffix "$e$" denotes that a VEV matrix $\langle A \rangle$ takes a form $\langle A \rangle_e$ in a flavor basis in which the charged lepton mass matrix M_e is diagonal. Here, the fields ℓ_L and e_R belong to $(3,1)$ and $(3^*,1)$ of U(3)×O(3) symmetries, respectively, and Φ_e belongs to $(3,3)$. However, in his model, O(3) is not a family symmetry. Besides, Sumino has mentioned nothing about quark and neutrino family assignments explicitly. Therefore, in this paper, we regard the O(3) symmetry as another family symmetry which is related to quarks and neutrinos. We note that, in Sumino model (3), the lepton doublet ℓ is assigned to a triplet of U(3), but the charged lepton singlet e_R is done to an anti-triplet of U(3). The reason is as follow: His interest is in the charged lepton mass relation [8] $K \equiv (m_e + m_\mu + m_\tau)/(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2 = 2/3$. The relation $K = 2/3$ is satisfied with the order of 10^{-5} for the pole masses, i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ [1], while it is only valid with the order of 10^{-3} for the running masses, e.g. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$. However, in conventional mass matrix models, "mass" means not "pole mass" but "running mass." Sumino has seriously taken why the mass formula is so remarkably satisfied with the pole masses. The deviation of $K(\mu)$ from K^{pole} is caused by a logarithmic term $m_{ei} \log(\mu/m_{ei})$ in the radiative correction term due to photon. Therefore, he assumed that a family symmetry is gauged, and that the logarithmic term in the radiative correction due to photon is canceled by that due to family gauge bosons. As a result, we can obtain $K(\mu) = K^{pole}$.

However, the purpose of the present model is not to develop Sumino model, but to propose a revised version of the yukawaon model. Although we partially adopt his model with U(3)×O(3) and his assignments for ℓ and e_R as far as the charged lepton sector is concerned, we do not adopt his mechanism of $K(\mu) = K^{pole}$. As seen later, we will use running mass values at $\mu = m_Z$ without using the Sumino mechanism (only with a radiative correction due to photon) as input values of the eigenvalues v_i of $\langle \Phi_e \rangle$. According to the Sumino model, we assume that O(3) is broken at an energy scale $\mu = \Lambda_{O3}$ with $\Lambda_{O3} > \Lambda$ (Λ is an energy scale of the present effective theory), so that, in the present effective theory, only family gauge symmetry is U(3). Nevertheless, we consider that the description based on U(3)×O(3) is still useful below $\mu = \Lambda$.

In Sumino model, in order that the cancellation works correctly, it is essential that the left-handed lepton field e_L and its right-handed partner e_R are assigned to **3** and **3*** of U(3), respectively. (For such an assignment, also see Ref.[9].) However, such an assignment destroys anomaly free of the model. Of course, the model is an effective theory, so that it does not need to require that the theory is anomaly free below a scale Λ of the effective theory. Nevertheless, it is interesting to require that the model should be anomaly free even below the scale Λ. We note that if we suppose a supersymmetric (SUSY) version of the Sumino mechanism, the fermion set $(\ell_i, e^c_i, \Phi_e^{i\alpha})$ becomes anomaly free. [Here, we have taken Φ_e as $(3^*, 3)$ of U(3)×O(3).] Then, we cannot regard the SU(2)$_L$ singlet neutrino ν^c as a triplet of U(3), because the model with $(\ell_i, e^c_i, \nu^c_i, \Phi_e^{i\alpha})$ cannot become anomaly free. We must regard ν^c as a triplet of O(3) if we require that the model should be anomaly free in the lepton sector itself. Therefore, we consider the model given in Eq.(1) with anomaly free for lepton sector. Here, the VEV of Y_e is related to the VEV of Φ_e as follows: We assume superpotential terms

$$W_e = \left(\mu_e Y_e^{ij} + \lambda_e \Phi_e^{i\alpha} \Phi_e^{T\alpha j}\right) \Theta^e_{ji}. \tag{4}$$

From a SUSY vacuum condition $\partial W/\partial \Theta_e = 0$, we obtain a VEV relation

$$\langle Y_e^{ij} \rangle = -\frac{\lambda_e}{\mu_e} \langle \Phi_e^{i\alpha} \rangle \langle \Phi_e^{T\alpha j} \rangle. \tag{5}$$

($\partial W/\partial Y_e = 0$ leads to $\langle \Theta^e \rangle = 0$.) Since the new fields Y_e and Θ_e are **6*** and **6** of U(3), respectively, they do not affect anomaly free condition, so that the lepton sector is still anomaly free.

2. Review of the yukawaon model

The yukawaon model intends to describe all quark and lepton mass matrices based on only one matrix, the charged lepton mass matrix (exactly speaking, based on $\langle \Phi_e \rangle$). In the yukawaon model, in which all effective Yukawa coupling constants Y_f^{eff} $(f = u, d, e, \cdots)$ are given by VEV of yukawaons Y_f as $Y_f^{eff} = \langle Y_f \rangle / \Lambda$. The VEV relations among $\langle Y_f \rangle$ are obtained from SUSY vacuum conditions. We cannot always uniquely determine superpotential form from a flavor symmetry alone. In order to distinguish a yukawaon from other yukawaons and in order to obtain phenomenologically desirable VEV relations, we assign R charge to each yukawaon by hand. Our superpotential is usually given by a form

$$W = \sum_A f_A(Y_f, Y_{f'}, \cdots) \Theta_A. \tag{6}$$

Therefore, a SUSY vacuum condition $\partial W/\partial \Theta_A = 0$ leads to a VEV relation $f_A(\langle Y_f \rangle, \langle Y_{f'} \rangle, \cdots) = 0$ as you see one example in Eq.(5). The other vacuum conditions $\partial W/\partial Y_f = 0$ and so on do not give any VEV relation, because each term in those equations always contains one Θ field.

In the previous yukawaon mode [3], a global family symmetry O(3) has been assumed, and by using SUSY vacuum conditions, we could successfully obtain reasonable quark and lepton mass matrices, especially excellent predictions for up-quark mass ratios and neutrino mixing parameters. In the present paper, since we assume U(3)×O(3) as family symmetries, the theoretical framework is completely changed. However, we want to inherit the phenomenological success from the old yukawaon model. Therefore, for convenience of later arguments, in this section, we give a short review of results in the old yukawaon model in terms of mass matrices (not in terms of VEVs).

Previously, the author [3] has proposed a neutrino mass matrix model based on a yukawaon model with an O(3) family symmetry: the neutrino mass matrix is given by a seesaw-type mass matrix $M_\nu = m_D M_R^{-1} m_D^T$, where the Dirac and Majorana mass matrices are given by

$$m_D \propto M_e, \qquad M_R \propto M_u^{1/2} M_e + M_e M_u^{1/2}, \tag{7}$$

respectively, and M_u is given by $M_u^{1/2} = U_u (M_u^{1/2})^{diag} U_u^T$ and $(M_u^{1/2})^{diag} \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$. By assuming that the quark mass matrix $M_u^{1/2}$ and M_d are given by

$$M_u^{1/2} \propto M_e^{1/2} S_u M_e^{1/2}, \quad M_d \propto M_e^{1/2} S_d M_e^{1/2}, \tag{8}$$

where S_q $(q = u, d)$ have forms

$$S_u = \mathbf{1} + a_u X, \quad S_d = \mathbf{1} + a_d X, \tag{9}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{10}$$

By adjusting parameters a_u, a_d and an additional one parameter ξ_ν in M_R (see Eq.(16) later), we have also obtained [3] not only the nearly tribimaximal neutrino mixing, but also reasonable CKM quark mixing (however, the fitting of CKM mixing is not so excellent compared with that of neutrino mixing). Since we take interest only in the relative ratios among the matrix elements, here and hereafter, for convenience, we drop numerical coefficients even if they have mass dimensions. If we take a value $a_u \simeq -1.8$ in the up-quark mass matrix relation given in Eq.(8), we can give reasonable up-quark mass ratios, but signs of the eigenvalues of $M_u^{1/2}$ show $(+, -, +)$, i.e. $(M_u^{1/2})^{diag} \propto \mathrm{diag}(\sqrt{m_u}, -\sqrt{m_c}, \sqrt{m_t})$ for $a_u \simeq -1.8$. Therefore, in Eq.(8), we replace $M_u^{1/2}$ with $M_u^{1/2} P_u$ (e.g. $M_u^{1/2} P_u \propto M_e^{1/2} S_u M_e^{1/2}$), where P_u is defined as a field with a VEV matrix form

$$\langle P_u \rangle_u = v_P \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{11}$$

and the index "u" denotes that a VEV matrix $\langle A \rangle$ takes a form $\langle A \rangle_u$ at the diagonal basis of the up-quark mass matrix M_u. Then, we can obtain [3] the observed nearly tribimaximal mixing by taking $a_u \simeq -1.8$.

In the present model, the Dirac neutrino mass matrix is given not by $m_D \propto M_e$ but by $m_D \propto M_e^{1/2}$, the structure of M_R given in Eq.(7) must be changed. Besides, in order to give a good fit to the neutrino mixing parameter $\tan^2\theta_{solar}$, we needed an additional term in Ref.[3], while in the present model with U(3)×O(3) it is impossible to give such the term.

3. Model for quark sector

First, we investigate possible superpotential forms in the quark sector. Although, correspondingly to the lepton sector with $(\ell_i, e^c_i, \nu_\alpha)$, we can naively consider a model with (q_i, d^c_i, u^c_α), we do not adopt such the assignment, because the number of quarks with **3** of U(3) is three times compared with that of the leptons due to the degree of color. We wish that the number of **3** (and also $\mathbf{3}^*$) is as few as possible. In this paper, we propose another model for the quark sector, i.e. with $(q_\alpha, d^c_\alpha, u^c_i)$ as given in Eq.(1). Although we consider $\Phi_u^{i\alpha}$ similar to Eq.(5), we do not identify Φ_u as Y_d differently from the case $Y_\nu^{Dirac} = \Phi_e$ in the lepton sector given in Eq.(1). Note that in this model, the U(3) gauge bosons A_i^j which couple to charged lepton sector cannot couple to the down-quark sector. Then, for yukawaons, we assume the following superpotential terms:[1]

$$W_u = \left(\mu_u Y_u^{i\alpha} + \frac{\lambda_u}{\Lambda}\Phi_u^{i\beta}E^T_{\beta j}\Phi_u^{j\alpha}\right)\Theta^u_{\alpha i} + \left(\frac{\lambda'_u}{\Lambda^2}\Phi_u^{i\alpha}P_u^{\alpha j} + \frac{\lambda''_u}{\Lambda^3}\Phi_e^{i\alpha}S^u_{\alpha\beta}\Phi_e^{T\beta j}\right)E_{j\gamma}\Theta^{u\prime}_{\gamma\delta}E_{\delta i}, \qquad (12)$$

$$W_d = \left(\mu_d Y_d^{\alpha\beta} + \frac{\lambda_d}{\Lambda^3}\Phi_e^{T\alpha i}E_{i\gamma}S^d_{\gamma\delta}E^T_{\delta j}\Phi_e^{j\beta}\right)\Theta^d_{\beta\alpha}, \qquad (13)$$

where the VEV forms of $\langle S_u\rangle$ and $\langle S_u\rangle$ are given by Eq.(9) and $\langle E\rangle = v_E\mathbf{1}$. Here, since we have take anomaly free conditions into consideration, we have again be obligated to adopt somewhat factitious forms, e.g. $E^T_{j\gamma}\Theta^{u\prime}_{\gamma\delta}E_{\delta i}$ instead of $\Theta^{u\prime}_{ji}$, and so on. Anyhow, since we have three **3** (u^c, E and Θ_u) and three $\mathbf{3}^*$ (Y_u, Φ_u and P_u), so that quark sector is also anomaly free.

4. Neutrino mass matrix

In this paper, we do not discuss phenomenological results for the CKM mixing, because the model for quark sector is practically the same as that given in Ref.[3] (we refer to it as the model A). As stated in Ref.[3], the model A for down-quark sector may be improved. On the other hand, the present model for neutrino sector is somewhat different from the model A.

In the present model, differently from the model A, since $m_D \propto M_e^{1/2}$ as shown in Eq.(1), we may consider $M_R \propto M_e^{-1/2}M_u^{1/2}M_e^{1/2} + M_e^{1/2}M_u^{1/2}M_e^{-1/2}$, i.e.

$$\Phi_e Y_R \Phi_e = \Phi_u Y_e + Y_e \Phi_u. \qquad (14)$$

Therefore, we assume the following superpotential terms for Y_R sector;

$$W_R = \frac{1}{\Lambda^3}\left[\lambda'_R\Phi_e^{i\alpha}Y_R^{\alpha\beta}\Phi_e^{T\beta j} + \lambda''_R\left(\Phi_u^{i\alpha}E^T_{\alpha k}Y_e^{kj} + Y_e^{ik}E_{k\alpha}\Phi_u^{T\alpha j}\right)\right]E_{j\gamma}\Theta^R_{\gamma\delta}E^T_{\delta i}, \qquad (15)$$

where E is a field which has a VEV $\langle E\rangle = v_E\mathbf{1}$ and $\langle\Phi_u\rangle$ corresponds to $M_u^{1/2}$. Here, we have assumed somewhat factitious form $E_{j\gamma}\Theta^R_{\gamma\delta}E^{T\delta i}$ instead of Θ^R_{ji} in order to keep the correspondence

[1] In order to distinguish $\Phi_u^{i\alpha}$ from $\Phi_e^{i\alpha}$, $S^u_{\alpha\beta}$ from $S^u_{\alpha\beta}$, and so on, we must assume different R charges for those fields.

ξ_ν	$\tan^2\theta_{solar}$	$\sin^2 2\theta_{atm}$	$\lvert U_{13}\rvert^2$
0	0.6995	0.9872	1.72×10^{-4}
0.009	0.4610	0.9902	2.28×10^{-4}
0.010	0.4408	0.9905	2.35×10^{-4}

Table 1: ξ_ν dependence of the neutrino parameters. The value of a_u is taken as $a_u = -1.78$ which can give reasonable up-quark mass ratios.

of Θ_R to Y_R. We will find that the field E plays an essential role in building an anomaly-free model as we state later. Since the fields Φ_u and E are not peculiar fields in the lepton sectors, we do not count as fields in the right-handed neutrino sector. Then, anomalies are free in the right-handed neutrino sector, too.

In the model A with O(3) family symmetry, the Majorana mass matrix $\langle Y_R\rangle$ was given by

$$\langle Y_R\rangle \propto \langle\Phi_u\rangle\langle P_u\rangle\langle Y_e\rangle + \langle Y_e\rangle\langle P_u\rangle\langle\Phi_u\rangle + \xi_\nu\left(\langle\Phi_u\rangle\langle Y_e\rangle\langle P_u\rangle + \langle P_u\rangle\langle Y_e\rangle\langle\Phi_u\rangle\right), \qquad (16)$$

where these fields are $\mathbf{1}+\mathbf{5}$ of O(3), and $\langle\Phi_u\rangle_u \propto \mathrm{diag}(\sqrt{m_u}, -\sqrt{m_c}, \sqrt{m_t})$. [Note that the eigenvalue of $\langle\Phi_u\rangle_u$ in the present model is given by $\mathrm{diag}(\sqrt{m_u}, +\sqrt{m_c}, \sqrt{m_t})$ differently from $\langle\Phi_u\rangle_u$ in the model A, because $\langle\Phi_u\rangle$ in the model A is given by $\langle\Phi_u\rangle \propto \langle\Phi_e\rangle\langle S_u\rangle\langle\Phi_e\rangle$, while $\langle\Phi_u\rangle$ in the present model is given by $\langle\Phi_u\rangle\langle P_u\rangle = \langle\Phi_e\rangle\langle S_u\rangle\langle\Phi_e\rangle$.] In the model A which was a O(3) model, when a term ABC (A, B and C are fields with $\mathbf{5}+\mathbf{1}$ of O(3)) is allowed in a superpotential, then a term ACB is also allowed. Thus, the ξ_ν term in Eq.(16) could appear. However, in the present model, we cannot consider such a term which corresponds to the ξ_ν term in the model A. In the model A, in order to give the observed value [10] $\tan^2\theta_{solar} \simeq 1/2$, it was indispensable that we take a non-vanishing value of ξ_ν, although we could give the observed values [11] $\sin^2 2\theta_{atm} \simeq 1$ and $|U_{13}|^2 \simeq 0$ even if $\xi_\nu = 0$. Therefore, in the present model, we assume an alternative term (ξ_ν-term) as follows:

$$\lambda_R(\Phi_e Y_R \Phi_e^T)^{ij} + \lambda'_R\left\{\left[(\Phi_u E^T)^i_k Y_e^{kj} + Y_e^{ik}(E\Phi_u^T)^j_k\right] + \xi_\nu(\Phi_u E^T)^k_k Y_e^{ij}\right\} = 0. \qquad (17)$$

According to the modified relation (17) for the form $\langle Y_R\rangle$, we obtain the following neutrino mass matrix M_ν:

$$M_\nu \propto M_e^{1/2}\left\{M_e^{-1/2}\left[M_u^{1/2}M_e + M_e M_u^{1/2} + \xi_\nu \mathrm{Tr}[M_u^{1/2}]M_e\right]M_e^{-1/2}\right\}^{-1} M_e^{1/2}, \qquad (18)$$

where $M_e^{1/2} \propto \langle\Phi_e\rangle_e \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ and $M_u^{1/2} \propto \langle\Phi_u\rangle_e \propto \langle\Phi_e\rangle_e\langle S_u\rangle_e\langle\Phi_e\rangle_e\langle P_u\rangle_e^{-1}$. In Table 1, we demonstrate ξ_ν-dependence of the neutrino mixing parameters in a case with $a_u = -1.78$, which gives reasonable up-quark mass ratios $\sqrt{m_u/m_c} = 0.04389$ and $\sqrt{m_c/m_t} = 0.05564$. The predicted values are in good agreement with the observed values at $\mu = m_Z$ [12] $\sqrt{m_u/m_c} = 0.045^{+0.013}_{-0.010}$ and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$. As seen in Table 1, the neutrino mixing parameters $\sin^2 2\theta_{atm}$ and $|U_{13}|^2$ are almost independent of the parameter ξ_ν, while the value of

$\tan^2\theta_{solar}$ is highly dependent on the value of ξ_ν. We note that the model can give excellent fits with the observed values of the neutrino mixing parameters in spite of its phenomenologically different form of the ξ_ν term.

5. Concluding remarks

In conclusion, stimulated by the Sumino's model [5, 6] with U(3)×O(3) symmetries for the charged leptons, we have considered a SUSY version of his model with a U(3) family gauge symmetry by extending it to U(3)×O(3) family symmetries for the quarks and leptons (but O(3) is already broken at $\mu = \Lambda_{O3} > \Lambda$). Although, in order to distinguish each yukawaon from other ones, we assumed an additional U(1) symmetry [say $U(1)_X$] in the old yukawaon model, in the present model with such two family symmetries, we do not need such $U(1)_X$ (but we still assume R charge conservation).

The original Sumino model is a model for the charged leptons, so that he has mentioned nothing as to quark and neutrino sectors. However, if we require that the model should be anomaly free in individual sectors of leptons and quarks, the model leads to a seesaw-type neutrino mass matrix $M_\nu = m_D M_R^{-1} m_D^T$, with $m_D \propto M_e^{1/2}$ (under the Sumino assignment for e_L and e_R). We have investigated a possible form of the Majorana mass matrix $M_R = \lambda_R \langle Y_R \rangle$) of the right-handed neutrinos by referring to a supersymmetric yukawaon model (model A) [3] for the neutrino sector. The present form (17) of M_R is similar to the form (16) in the model A, but the ξ_ν term is completely different from the model A. Nevertheless, in this model, too, we can successfully obtain the observed (nearly) tribimaximal neutrino mixing by adjusting the parameter ξ_ν as seen in Table 1. It is worthwhile noticing this.

Although the present model for the quark sector is also anomaly free, the model is somewhat factitious. The present model phenomenologically leads to the same results as the model A as far as the quark sector is concerned. The model A can roughly give reasonable CKM mixings, but, numerically, the predictions for $|V_{ub}|$ and $|V_{td}|$ are somewhat larger then the observed values, so that the present model, too, needs to be improved for the down-quark sector. Therefore, at present, it may be useless to adhere to the requirement of anomaly free for the quark sector. The anomaly free problem may be solved after the improvement in the down-quark sector is achieved. Then, the expressions given in Eqs.(12) and (15) will be naturally improved.

So far, we did not discuss the energy scale Λ of the effective theory. We obtain $m_{\nu 3} \sim \langle H_u^0 \rangle^2 (\langle \Phi_e \rangle/\Lambda)^2 \langle Y_R \rangle^{-1}$ from $M_\nu = m_D M_R^{-1} m_D^T$ and $m_\tau \sim \langle H_d^0 \rangle \langle Y_e \rangle/\Lambda \sim \langle H_d^0 \rangle (\langle \Phi_e \rangle/\Lambda)^2 (\mu_e/\Lambda)^{-1}$ from Eq.(5), so that we obtain

$$\frac{m_{\nu 3}}{m_\tau} \sim \frac{\langle H_u^0 \rangle^2}{\langle H_d^0 \rangle \langle Y_R \rangle} \frac{\mu_e}{\Lambda}. \tag{21}$$

By taking $m_{\nu 3}/m_\tau \sim 10^{-11}$, $\langle H_u^0 \rangle \sim 10^2$ GeV and $\langle H_d^0 \rangle \sim 10$ GeV, and by assuming $\langle Y_R \rangle \sim \Lambda$ (a maximum value in the present effective theory) , we obtain $\Lambda \sim \sqrt{10^{14}\mu_e}$ GeV. Therefore, we can, in principle, take any small value of Λ by assuming a small value of μ_e, but too small value of μ_e is unlikely. If we take $\mu_e \sim 1$ GeV [$\mu_e \sim 10^2$ GeV], we obtain $\Lambda \sim 10^7$ GeV [$\Lambda \sim 10^8$ GeV]. Even the value $\Lambda \sim 10^7$ GeV is still considerably large to bring visible flavor changing neutral current (FCNC) effects. Besides, the U(3) gauge bosons cannot couple to the down-quark sector and do only to u^c. Nevertheless, the present model with such low energy scale Λ has a possibility that it bring some visible effects [for example, a family gauge boson A_1^1 with the lowest mass mass (maybe $m(A_1^1) \sim 1$ TeV)]. Phenomenological meanings of the present model in TeV region physics will be discussed elsewhere. We again would like to emphasize that such a lower scale of Λ would not be realized without introducing the yukawaon Y_e^{ij} instead of $\Phi_e^{i\alpha}\Phi_e^{\alpha j}$ in the would-be Yukawa interactions as given in Eq.(1) [not as in Eq.(3)].

Acknowledgments

The authors would like to thank Y. Sumino and M. Yamanaka for enjoyable conversations. He also thank T. Yamashita for helpful comments on anomaly-free conditions and effective theories, and especially for pointing out an mistake in an earlier version of this paper, and J. Kubo for useful comments for a FCNC problem. The work is supported by JSPS (No. 21540266).

References

[1] Particle Data Group, K. Nakamura, *et al.*, J. Phys. **G 37**, 075021 (2010).

[2] Y. Koide, Phys. Rev. **D 79**, 033009 (2009).

[3] Y. Koide, Phys. Lett. **B 680**, 76 (2009).

[4] C. D. Froggatt and H. B. Nelsen, Nucl. Phys. **B 147**, 277 (1979).

[5] Y. Sumino, Phys. Lett. **B671**, 477 (2009).

[6] Y. Sumino, JHEP **0905**, 075 (2009).

[7] Y. Koide, Mod. Phys. Lett. **A5**, 2319 (1990).

[8] Y. Koide, Lett. Nuovo Cimento **34** (1982) 201; Phys. Lett. **B120**, 161 (1983); Phys. Rev. **D28**, 252 (1983).

[9] T. Applequist, Y. Bai and M. Piai, Phys. Rev. **D 74** (2006) 076001; Phys. Lett. **B 637** (2006) 245.

[10] D. G. Michael *et al.*, MINOS collaboration, Phys. Rev. Lett. **97**, 191801 (2006); J. Hosaka, *et al.*, Super-Kamiokande collaboration, Phys. Rev. **D 74**, 032002 (2006).

[11] B. Aharmim, et al., SNO collaboration, Phys. Rev. Lett. **101**, 111301 (2008). Also, see S. Abe, *et al.*, KamLAND collaboration, Phys. Rev. Lett. **100**, 221803 (2008).

[12] Z.-z. Xing, H. Zhang and S. Zhou, Phys. Rev. **D 77**, 113016 (2008). And also see, H. Fusaoka and Y. Koide, Phys. Rev. **D 57**, 3986 (1998).