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SU(5)-Compatible Yukawaon Model

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Abstract

A yukawaon model which is compatible with an SU(5) GUT model is investigated. In previous yukawaon models, the effective Yukawa coupling constants Y_f^{eff} are given by vacuum expectation values of fields Y_f ("yukawaons") as $(Y_f^{eff})_{ij} = (y_f/\Lambda)\langle (Y_f)_{ij}\rangle$. In order to build a model without a cutoff scale Λ, vector-like fields $(\bar{\mathbf{5}} + \mathbf{10} + \mathbf{5} + \bar{\mathbf{10}})$ are introduced in addition to the conventional quarks and leptons $(\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1})$. The model naturally leads to a model in which a yukawaon Y_e in the charged lepton sector plays a role of a substitute for a yukawaon Y_ν in the neutrino Dirac mass sector. Stimulated by Sumino's model, U(3)×O(3) are assumed as family symmetries. The U(3) symmetry is broken at $\langle Y_f\rangle \sim 10^{13}$ GeV.

1. Introduction

In the standard model of the quarks and leptons, the mass spectra and mixings are due to the Yukawa coupling constants. Then, one may investigate relations among those fundamental constants by assuming a family symmetry (e.g. a U(1) symmetry, a discrete symmetry, and so on). In contrast to such conventional models, there is another idea: the mass spectra and mixings are due to vacuum expectation values (VEVs) of new scalars. As one of such models, the so-called "yukawaon" model [1] is known.

In the yukawaon model, which is a kind of "flavon" model [2], all effective Yukawa coupling constants Y_f^{eff} $(f = u, d, e, \cdots)$ are given by VEVs of "yukawaons" Y_f as

$$(Y_f^{eff})_{ij} = \frac{y_f}{\Lambda}\langle (Y_f)_{ij}\rangle. \tag{1.1}$$

For example, would-be Yukawa interactions are given by the following superpotential:

$$W_Y = \frac{y_e}{\Lambda}\ell_i Y_e^{ij} e_j^c H_d + \frac{y_\nu}{\Lambda}\ell_i Y_\nu^{ij}\nu_j^c H_u + \lambda_R \nu_i^c Y_R^{ij}\nu_j^c + \frac{y_u}{\Lambda}u_i^c Y_u^{ij} q_j H_u + \frac{y_d}{\Lambda} d_i^c Y_d^{ij} q_j H_d, \tag{1.2}$$

where ℓ and q are SU(2)$_L$ doublets $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$. By using supersymmetric vacuum conditions, those VEVs $\langle Y_f\rangle$ are related to a fundamental VEV matrix $\langle \Phi_e\rangle$, for example, as $\langle Y_e\rangle = k_e\langle\Phi_e\rangle\langle\Phi_e\rangle$. For neutrino mass matrix, we assume a seesaw-type mass matrix

$$M_\nu = m_D M_R^{-1} m_D^T = \frac{y_\nu^2}{\lambda_R}\left(\frac{\langle H_u^0\rangle}{\Lambda}\right)^2 \langle Y_\nu\rangle\langle Y_R\rangle^{-1}\langle Y_\nu\rangle. \tag{1.3}$$

As we see later, the VEV matrix $\langle Y_R\rangle$ is also described in terms of $\langle\Phi_e\rangle$.

Note that the yukawaons Y_f are singlets under the conventional gauge symmetries SU(3)$_c\times$ SU(2)$_L\times$U(1)$_Y$, and they have only family indices. This suggests that the yukawaon model may be compatible with an SU(5) grand unification (GUT) model [3]. For example, we may consider:

$$W_Y = \frac{y_u}{\Lambda}\mathbf{10}_i Y^{ij}_{(10,10)}\mathbf{10}_j\mathbf{5}_H + \frac{y_d}{\Lambda}\bar{\mathbf{5}}_i Y^{ij}_{(5,10)}\mathbf{10}_j\bar{\mathbf{5}}_H + \frac{y_\nu}{\Lambda}\bar{\mathbf{5}}_i Y^{ij}_{(5,1)}\mathbf{1}_j\mathbf{5}_H + \lambda_R\mathbf{1}_i Y^{ij}_{(1,1)}\mathbf{1}_j, \tag{1.4}$$

where $\bar{\mathbf{5}}+\mathbf{10}+\mathbf{1}$ are quark and lepton fields and $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ correspond to the conventional two Higgs doublets H_u and H_d, respectively. However, in the would-be Yukawa interactions (1.4), the charged lepton yukawaon Y_e is identical with the down-quark yukawaon Y_d, i.e. $Y_e = Y_d = Y_{(5,10)}$, although Y_u and Y_ν are reasonably given by $Y_u = Y_{(10,10)}$ and $Y_\nu = Y_{(5,1)}$. In the yukawaon model, it is essential that Y_e has a different family structure from Y_d. We must build a model where yukawaon Y_e is an independent field from Y_d. For this problem, we will introduce matter fields $f' = (\bar{\mathbf{5}}'_i + \mathbf{5}'^i)$ in addition to the quarks and leptons $f_i = (\bar{\mathbf{5}}+\mathbf{10}+\mathbf{1})_i$ [the index i denotes $i = 1, 2, 3$ in a family symmetry U(3)].

So far, as seen in Eq.(1.2), the yukawaon model has been based on an effective theory with the cutoff Λ. In the present SU(5) compatible model, we must make clear whether the cutoff scale Λ is lower or higher than the grand unification scale Λ_{GUT}. In order to build a model without a cutoff scale Λ, we will introduce matter fields $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i$ in addition to the fields f_i and f'.

The purpose of the present paper is to propose a yukawaon model which is compatible with an SU(5) GUT scenario. In this paper, we do not try to unify a family symmetry into the SU(5) gauge symmetry. We do not intend to resolve current problems in the minimum SU(5) GUT by considering the yukawaon model. Since yukawaons are SU(5) singlets, the existence of the yukawaons do not affect an SU(5) GUT model, so that we can inherit the successful results in the SU(5) GUT and we also inherit the current problems in the minimum SU(5) GUT scenario. On the other hand, we may inherit successful results of the yukawaon model without disturbing the SU(5) GUT model. However, as we discuss later, the existence of the new matter fields f' and f'' will put some constraints on the yukawaon structures.

2. Would-be Yukawa interactions

First, le us discuss the Y_e-Y_d splitting. We introduce vector-like $\mathbf{5}'^i$ and $\bar{\mathbf{5}}'_i$ fields in addition to the fields given in Eq.(1.4). For convenience, we denote one $\mathbf{5}$ and two $\bar{\mathbf{5}}$ as

$$\mathbf{5}'^i = (D^i, L^{ci}), \quad \bar{\mathbf{5}}_i = (D^c_i, \ell_i), \quad \bar{\mathbf{5}}'_i = (d^c_i, L_i), \tag{2.1}$$

where d^c, D^c and D are SU(2)$_L$ singlet down-quarks with electric charges $+1/3$, $+1/3$ and $-1/3$, respectively, and ℓ and L are SU(2)$_L$ lepton doublets. In order to realize that the fields (D, D^c), and (L, L^c) become massive and decouple from the present model, we assume the following interactions

$$\lambda_D \bar{\mathbf{5}}^A_i (\Sigma_3)^B_A \mathbf{5}'^i_B + \lambda_L (\bar{\mathbf{5}}')^A_i (\Sigma_2)^B_A \mathbf{5}'^i_B, \tag{2.2}$$

where indices $A,\ B = 1, 2, \cdots, 5$ are those of SU(5), and SU(5) $\mathbf{24}+\mathbf{1}$ fields Σ_2 and Σ_3 take VEV forms

$$\langle\Sigma_2\rangle = v_2 \begin{pmatrix} 0 & 0 \\ 0 & \mathbf{1}_2 \end{pmatrix}, \quad \langle\Sigma_3\rangle = v_3 \begin{pmatrix} \mathbf{1}_3 & 0 \\ 0 & 0 \end{pmatrix}, \tag{2.3}$$

$$\mathbf{1}_2 = \mathrm{diag}(1,1), \quad \mathbf{1}_3 = \mathrm{diag}(1,1,1). \tag{2.4}$$

Therefore, Eq.(2.2) leads to mass terms

$$\lambda_D v_3 D^c_i D^i + \lambda_L v_2 L_i L^{ci}. \tag{2.5}$$

As we discuss later, we will assign different R charges to Σ_2 and Σ_3 (and also to $\mathbf{5}$ and $\mathbf{5}'$). If we once admit the existence of Σ_3 with VEV given in Eq.(2.3) phenomenologically, we can use Σ_3 for the triplet-doublet splitting of the Higgs fields $\bar{\mathbf{5}}_H$ and $\mathbf{5}_H$ by considering an interaction $\bar{\mathbf{5}}_H \Sigma_3 \mathbf{5}_H$. For the triplet-doublet splitting in Higgs fields, already a reasonable mechanism has been proposed based on SO(10) GUT scenario [4]. Since the purpose of the present paper to investigate the GUT scenario, we ad hoc assume the VEV forms (2.3) in this paper. The reason of the forms (2.3) may be understood by an SO(10) GUT scenario.

Next, we discuss a seesaw-type mass matrix mechanism in order to realize the effective interactions given by Eq.(1.1). We assume further vector-like matter fields $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i$. The up-quark, charged lepton, and down-quark mass matrices, M_u, M_e and M_d, are generated by the following superpotentials:

$$W_u = y_u \mathbf{10}_i Y_u^{ij} \bar{\mathbf{10}}''_j + M_{10} \bar{\mathbf{10}}''_i \mathbf{10}''^i + y_{10} \mathbf{10}''^i \mathbf{10}_i \mathbf{5}_H, \tag{2.6}$$

$$W_e = y_e \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{5}''_j + M_5 \mathbf{5}''_i \bar{\mathbf{5}}''^i + y_5 \bar{\mathbf{5}}''^i \mathbf{10}_i \bar{\mathbf{5}}_H, \tag{2.7}$$

$$W_d = y_d \bar{\mathbf{5}}'_i Y_d^{ij} \mathbf{5}''_j + M_5 \mathbf{5}''_i \bar{\mathbf{5}}''^i + y_5 \bar{\mathbf{5}}''^i \mathbf{10}_i \bar{\mathbf{5}}_H, \tag{2.8}$$

which lead to effective Yukawa interactions

$$\frac{y_u y_{10}}{M_{10}} \mathbf{10}_i Y_u^{ij} \mathbf{10}_j \mathbf{5}_H, \tag{2.9}$$

$$\frac{y_e y_5}{M_5} \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H, \tag{2.10}$$

$$\frac{y_d y_5}{M_5} \bar{\mathbf{5}}'_i Y_d^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H, \tag{2.11}$$

respectively.

For a neutrino Dirac mass term, we also assume

$$W_\nu = y_e \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{5}''_j + M_5 \mathbf{5}''_i \bar{\mathbf{5}}''^i + y_1 \bar{\mathbf{5}}''^i \mathbf{1}_i \mathbf{5}_H, \tag{2.12}$$

which leads to the effective interaction

$$\frac{y_e y_1}{M_5} \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{1}_j \mathbf{5}_H. \tag{2.13}$$

(In the present model, we do not assume $\mathbf{1}''_i + \mathbf{1}''^i$.) Note that the neutrino Dirac mass matrix m_D is proportional to $\langle Y_e \rangle$ as well as the charged lepton mass matrix M_e. When we assume that the SU(5) singlet matter field $\mathbf{1}_i$ can have a Majorana mass term

$$W_R = \lambda_R \mathbf{1}_i Y_R^{ij} \mathbf{1}_j, \tag{2.14}$$

we can obtain the seesaw-type neutrino mass matrix

$$M_\nu = \frac{y_e^2 y_1^2}{\lambda_R} \left(\frac{v_{Hu}}{M_5}\right)^2 \langle Y_e\rangle \langle Y_R\rangle^{-1} \langle Y_e\rangle, \tag{2.15}$$

where $v_{Hu} = \langle H_u^0\rangle = \langle \mathbf{5}_H\rangle$. From Eq.(2.10), we can rewritten Eq.(2.15) as

$$M_\nu = \frac{y_1^2 \tan^2\beta}{\lambda_R y_5^2} M_e \langle Y_R\rangle^{-1} M_e, \tag{2.16}$$

where $\tan\beta = \langle H_u^0\rangle / \langle H_d^0\rangle$. By taking $m_\tau \simeq 1.777$ GeV, $(M_\nu)_{33} \sim m_{\nu 3} \simeq \sqrt{\Delta m^2_{atm}} \simeq 0.049$ eV [5]) and $\tan\beta \simeq 10$, we can estimate the value of $\langle Y_R\rangle$ as

$$\langle Y_R\rangle \simeq \lambda_R (y_5/y_1)^2 \times 6.4 \times 10^{12} \text{ GeV}. \tag{2.17}$$

R charges of these matter fields must satisfy the following relations:

$$R(\mathbf{10}_i) - R(\mathbf{1}_i) = R(\bar{\mathbf{5}}''_i) - R(\mathbf{10}''_i) = R(\bar{\mathbf{10}}''^i) - R(\mathbf{5}''^i) = R(\mathbf{5}_H) - R(\bar{\mathbf{5}}_H) \equiv a, \tag{2.18}$$

$$R(\bar{\mathbf{5}}'_i) - R(\bar{\mathbf{5}}_i) = R(\Sigma_3) - R(\Sigma_2) \equiv b, \tag{2.19}$$

$$R(\bar{\mathbf{5}}''^i) + R(\mathbf{5}''_i) = 2, \tag{2.20}$$

$$R(\bar{\mathbf{5}}_i) + R(\mathbf{5}'^i) + R(\Sigma_3) = 2, \tag{2.21}$$

$$R(Y_R) + 2R(\mathbf{1}_i) = 2. \tag{2.22}$$

Since there are still free parameters in these assignments, we do not show an explicit assignment of R charges in this paper.

3. Superpotential for yukawaons

The successful results in the previous yukawaon model with an O(3) family symmetry [6] have been derived on the basis of the following VEV relations:

$$\langle Y_e\rangle = k_e \langle \Phi_e\rangle \langle \Phi_e\rangle, \tag{3.1}$$

$$\langle Y_u\rangle = k_u \langle \Phi_u\rangle \langle \Phi_u\rangle, \tag{3.2}$$

$$\langle \Phi_u\rangle = k'_u \langle \Phi_e\rangle \langle (E + a_u X)\rangle \langle \Phi_e\rangle, \tag{3.3}$$

$$\langle Y_d\rangle = k_d \langle \Phi_e\rangle (E + a_d X) \langle \Phi_e\rangle, \tag{3.4}$$

$$\langle Y_R\rangle = k_R \left[\langle \Phi_u\rangle \langle P_u\rangle \langle Y_e\rangle + \langle Y_e\rangle \langle P_u\rangle \langle \Phi_u\rangle + \xi_\nu (\langle \Phi_u\rangle \langle Y_e\rangle \langle P_u\rangle + \langle P_u\rangle \langle Y_e\rangle \langle \Phi_u\rangle)\right], \tag{3.5}$$

where

$$\langle \Phi_e\rangle_e \propto \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}), \tag{3.6}$$

$$\langle E\rangle_e = v_E \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \langle X\rangle_e = \frac{1}{3} v_X \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \langle P_u\rangle_u = v_{Pu} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{3.7}$$

Note that the VEV matrix forms (3.6) and (3.7) are dependent on the flavor basis. The expression $\langle A \rangle_f$ $(f = e, u)$ denotes a form of the VEV matrix $\langle A \rangle$ in a basis in which the mass matrix M_f is diagonal. The bilinear form of Y_e in Eq.(3.1) is required to give a charged lepton mass relation (see Eq.(3.9) later). The bilinear form of Y_u in Eq.(3.2) plays an inevitable role in obtaining a successful neutrino mass matrix via Eq.(3.5).

Since Y_e was substituted for Y_ν in the previous model [6], the neutrino mass matrix M_ν was given by the form (1.3). Therefore, the neutrino mixing comes only from the structure of $\langle Y_R \rangle$. When a reasonable structure of $\langle Y_R \rangle$ is assumed, the model can give excellent agreement with the nearly tribimaximal mixing [7] together with the up-quark mass ratios only by adjusting two parameters [a_u in Eq.(3.3) and ξ_ν in Eq.(3.5)]. Besides, the model can give rough agreement with the CKM mixing parameters and down-quark mass ratios by adjusting remaining two parameters [the magnitude and phase of a_d in Eq.(3.4)].

In the previous model [6], these VEV relations (3,1) – (3,5) have been derived from the SUSY vacuum conditions by assuming an O(3) family symmetry. However, the superpotential terms have (1.2) included a cutoff parameter Λ. As seen in the previous section, in the present model, we want to build a model without such a cutoff parameter Λ. Moreover, in the present paper, we will investigate a model with a U(3) family symmetry instead of the O(3) family symmetry, because, in the yukawaon model, the order of the VEV matrices is important [e.g. absence of $(E + aX)\Phi_e\Phi_e + \Phi_e\Phi_e(E + aX)$ in contrast to the existence of $\Phi_e(E + aX)\Phi$].

Recently, Sumino has proposed a charged lepton mass matrix model based on a U(3) gauge family symmetry [9]. In the Sumino model, the charged lepton mass term is generated by a would-be Yukawa interaction

$$H_e = \frac{y_e}{\Lambda^2} \bar{\ell}_L^i \Phi^e_{i\alpha} \Phi^{eT}_{\alpha j} e^j_R H, \tag{3.8}$$

where i and α are indices of U(3) and O(3), respectively, and H is the Higgs scalar in the standard non-SUSY model. (Sumino' model has not been based on a SUSY scenario.) The charged lepton masses m_{e_i} are acquired from a VEV of the scalar Φ^e [10], i.e. the masses m_{ei} are given by $m_{ei} = (y_e/\Lambda^2)\langle \Phi^e_{i\alpha} \rangle \langle \Phi^{eT}_{\alpha i} \rangle \langle H^0 \rangle$. Sumino's interest was in the charged lepton mass relation [12]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}. \tag{3.9}$$

The relation $K = 2/3$ is satisfied with the order of 10^{-5} for the pole masses, i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ [5], while it is only valid with the order of 10^{-3} for the running masses, e.g. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$. However, in conventional mass matrix models, "mass" means not "pole mass" but "running mass". Sumino has seriously taken why the mass formula $K = 2/3$ is so remarkably satisfied with the pole masses. The deviation of $K(\mu)$ from K^{pole} is caused by a logarithmic term $m_{ei} \log(\mu/m_{ei})$ in the radiative correction term [13] due to photon

$$m_{ei}(\mu) = m^{pole}_{ei} \left[1 - \frac{\alpha(\mu)}{\pi} \left(1 + \frac{3}{4} \log \frac{\mu}{m_{ei}(\mu)} \right) \right]. \tag{3.10}$$

Therefore, he have assumed that a family symmetry is local, and that the logarithmic term in the radiative correction due to photon is canceled by that due to family gauge bosons. In the Sumino model, it is essential that the left- and right-handed charged leptons e_{Li} and e_{Ri} are assigned to **3** and **3*** of U(3) family symmetry, respectively. (A similar fermion assignment has been proposed by Applequist, Bai and Piai [11].) As a result, we can obtain $K(\mu) = K^{pole}$. [However, it does not mean $m_{ei}(\mu) = m_{ei}^{pole}$. The cancellation takes place only for the term with $\log m_{ei}$ in Eq.(3.10).]

Stimulated by the Sumino model, in this paper, we assume U(3)×O(3) family symmetries. The VEV relations (3.1) and (3.2) are derived from the following superpotential terms:

$$W_e = \mu_e (Y_e^{ij} \Theta^e_{ji}) + \lambda_e (\Phi_e^{i\alpha} \Phi_e^{T\alpha j} \Theta^e_{ji}), \tag{3.11}$$

$$W_u = \lambda_u [Y_u^{ik} E^u_{kj} (\Theta_u)^j_i] + \lambda'_u [(\Phi_u)^i_k (\Phi_u)^k_j (\Theta_u)^j_i], \tag{3.12}$$

where E^u takes a VEV form $\langle E^u \rangle = v_E \mathrm{diag}(1,1,1)$. Here and hereafter, we denote fields whose VEV values are zeros as Θ_A ($A = e, u, \cdots$). (Therefore, we can obtain meaningful VEV relations from SUSY vacuum conditions $\partial W/\partial \Theta_A = 0$, while we cannot obtain any relations from other conditions (e.g. $\partial W/\partial Y_f = 0$) because the relations always include $\langle \Theta_A \rangle$. For the time being, we assume that the supersymmetry breaking is induced by a gauge mediation mechanism (not including family gauge symmetries), so that our VEV relations among yukawaons are still valid even after the SUSY was broken in the quark and lepton sectors. In Eqs.(3.11) and (3.12), according to Sumino, we assume that the gauge symmetry O(3) is already broken at $\mu = \Lambda_{GUT}$.

Note that, for the VEV relations (3.3) – (3.5), we cannot use the similar prescriptions which were used for Eqs.(3.11) and (3.12), because we want a model without a cutoff Λ. We assume the following superpotential forms without Λ for the VEV relations (3.3) and (3.4):

$$W'_u = \lambda''_u (\Phi_u)^i_k (P_u)^{kj} (\Theta'_u)_{ji} + \lambda'''_u [\Phi_E^{i\alpha} \Phi_E^{T\alpha j} + a_u \Phi_X^{i\alpha} \Phi_X^{T\alpha j}] (\Theta'_u)_{ji}, \tag{3.13}$$

$$W_d = \mu_d Y_d^{ij} \Theta^d_{ji} + \lambda_d [\Phi_E^{i\alpha} \Phi_E^{T\alpha j} + a_d \Phi_X^{i\alpha} \Phi_X^{T\alpha j}] \Theta^d_{ji}, \tag{3.14}$$

together with

$$W_{E,X} = \mu_E \Phi_E^{i\alpha} \Theta^E_{\alpha i} + \lambda_E \Phi_e^{i\beta} E^\alpha_\beta \Theta^E_{\alpha i} + \mu_X \Phi_X^{i\alpha} \Theta^X_{\alpha i} + \lambda_X \Phi_e^{i\beta} X^\alpha_\beta \Theta^X_{\alpha i}. \tag{3.15}$$

where E, X and P_u have the VEV forms defined in Eq.(3.7). For such VEV forms of X and E, for example, we may consider the following superpotential forms [14]:

$$W_X = \lambda_X \, \mathrm{det} X \equiv \lambda_X \left(\frac{1}{3} \mathrm{Tr}[XXX] - \frac{1}{2} \mathrm{Tr}[XX] \mathrm{Tr}[X] + \frac{1}{6} (\mathrm{Tr}[X])^3 \right), \tag{3.16}$$

$$W_E = \lambda_E \mathrm{Tr}[EE] \mathrm{Tr}[E] + \lambda'_E (\mathrm{Tr}[E])^3. \tag{3.17}$$

For the VEV relation (3.5), we assume the superpotential

$$W_R = \mu_R Y_R^{ij} \Theta^R_{ji} + \lambda'_R [(\Phi_u)^i_k Y_e^{kj} + Y_e^{ik} (\Phi_u)^j_k + \xi_\nu (\Phi_u)^k_k Y_e^{ij}] \Theta^R_{ji}. \tag{3.18}$$

ξ_ν	$\tan^2\theta_{solar}$	$\sin^2 2\theta_{atm}$	$\lvert U_{13}\rvert^2$
0	0.6995	0.9872	1.72×10^{-4}
0.009	0.4610	0.9902	2.28×10^{-4}
0.010	0.4408	0.9905	2.35×10^{-4}

Table 1: ξ_ν dependence of the neutrino parameters. The value of a_u is taken as $a_u = -1.78$ which can give reasonable up-quark mass ratios.

Here, the final term is somewhat different from the form (3.5). In the O(3) model based on U(3), a term $(\Phi_u Y_e P_u + P_u Y_e \Phi_u)$ is allowed in addition to $(\Phi_u P_u Y_e + Y_e P_u \Phi_u)$, because these fields are $(\mathbf{5}+\mathbf{1})$ of O(3). However, in the present model, such a term is forbidden. Instead of such a term, we have added $\mathrm{Tr}[\Phi]\, Y_e^{ij}$ to the term $(\Phi_u Y_e + Y_e \Phi_u)^{ij}$ as a new ξ_ν term in Eq.(3.18). This term is required [14] in order to fit the observed value of $\tan^2\theta_{solar}$.

Thus, we can obtain the neutrino mass matrix Eq.(2.15) with

$$\langle Y_R\rangle = -\frac{\lambda_R}{\mu_R}\left(\langle\Phi_u\rangle\langle Y_e\rangle + \langle Y_e\rangle\langle\Phi_u\rangle + \xi_\nu \mathrm{Tr}[\langle\Phi_u\rangle]\langle Y_e\rangle\right), \tag{3.19}$$

$$\langle\Phi_u\rangle\langle P_u\rangle = -\frac{\lambda_u'''}{\lambda_u''}\left(\langle\Phi_E\rangle\langle\Phi_E^T\rangle + a_u\langle\Phi_X\rangle\langle\Phi_X^T\rangle\right) = -\frac{\lambda_u'''\lambda_X^2}{\lambda_u''\mu_X^2} v_X \langle\Phi_e\rangle(\langle E\rangle + a_u\langle X\rangle)\langle\Phi_e\rangle. \tag{3.20}$$

where, for simplicity, we have put $\lambda_E/\mu_E = \lambda_X/\mu_X$ and $v_E = v_X$. Numerical results for neutrino mixing parameters and up-quark mass rations are identical with those given in Ref.[14]. We quote the numerical results from Ref.[14] in Table 1. Here, we have taken a value $a_u = -1.78$ which can give reasonable up-quark mass ratios:

$$\sqrt{\frac{m_u}{m_c}} = 0.04389, \qquad \sqrt{\frac{m_c}{m_t}} = 0.05564. \tag{3.21}$$

The predicted values (3.21) are in good agreement with the observed values at $\mu = m_Z$ [15] $\sqrt{m_u/m_c} = 0.045^{+0.013}_{-0.010}$ and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$.

In Eqs.(3.13) and (3.14), we have considered somewhat unbalanced assignments between Φ_e and Φ_u, i.e. $\Phi_e^{i\alpha} = (3,3)$ and $(\Phi_u)^i_j = (8+1,1)$ of U(3)×O(3). We may consider an alternative model with the same assignments $(\Phi_e)^i_j$ and $(\Phi_u)^i_j$. However, since we want to inherit the Sumino mechanism [9] for the charged lepton mass relation, we adopt the assignment $\Phi_e^{i\alpha}$. On the other hand, we do not adopt the assignment $\Phi_u^{i\alpha}$, because if we adopt such the assignment, we are forced to modify the structure of W_u' given in Eq.(3.13) and W_R given in Eq.(3.18) into more complicated forms in order to express these terms without Λ.

In Table 2, we list assignments of SU(5)×U(3)×O(3) for all fields in the present model. The model is anomaly free in SU(5), while it is not so in the U(3) gauge symmetry. Since the

	$\bar{\mathbf{5}}_i$	$\mathbf{10}_i$	$\mathbf{1}_i$	$\bar{\mathbf{5}}'_i$	$\mathbf{5}'^i$	$\bar{\mathbf{5}}''^i$	$\mathbf{10}''^i$	$\mathbf{5}''_i$	$\overline{\mathbf{10}}''_i$	$\bar{\mathbf{5}}_H$	$\mathbf{5}_H$
SU(5)	$\mathbf{5}^*$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{5}^*$	$\mathbf{5}$	$\mathbf{5}^*$	$\mathbf{10}$	$\mathbf{5}$	$\mathbf{10}^*$	$\mathbf{5}^*$	$\mathbf{5}$
SU(3)	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
O(3)	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

Σ_3	Σ_2	Y_e	Y_d	Y_u	Y_R	Φ_e	Φ_u	Φ_E	Φ_X
$\mathbf{24+1}$	$\mathbf{24+1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{6}^*$	$\mathbf{3}$	$\mathbf{8+1}$	$\mathbf{3}^*$	$\mathbf{3}^*$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$

E^u	P_u	E	X	Θ^e	Θ_u	Θ'_u	Θ_d	Θ^E	Θ^X	Θ^R
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{6}$	$\mathbf{6}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{6}^*$	$\mathbf{8+1}$	$\mathbf{6}$	$\mathbf{6}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{6}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{5+1}$	$\mathbf{5+1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$

Table 2: Fields in the present model and their SU(5)×U(3)×O(3) assignments.

anomaly coefficients of $\mathbf{3}$, $\mathbf{6}$ and $\mathbf{8}$ of SU(3) are $A(\mathbf{3}) = 1$, $A(\mathbf{6}) = 7$ and $A(\mathbf{8}) = 0$, the sum of the anomaly coefficients A is $A = 17$. In order that the model is anomaly free for U(3) family symmetry, we need further fields which give $A = -17$, so that we may consider, for example, A^{ij}, B^{ij} and $C^{i\alpha}$. However, for the time being, we do not specify the roles of those fields A, B and C in the model.

R charges of these fields in the yukawaon sector must satisfy the following relations:

$$R(Y_e) = 2R(\Phi_e) \equiv r_e, \tag{3.22}$$

$$R(Y_u) = 2R(\Phi_u) - R(E^u) \equiv r_u, \tag{3.23}$$

$$R(Y_d) = R(\Phi_u) + R(P_u) = 2R(\Phi_X) = 2R(\Phi_E) = 2\left(r_e + \frac{2}{3}\right), \tag{3.24}$$

$$R(Y_R) = R(\Phi_u) + R(Y_e) \equiv r_R. \tag{3.25}$$

In these assignments, since we have still free parameters, we do not give numerical assignments of R charges for these fields.

4. Energy scales

Masses of the additional particles $f' = (\bar{\mathbf{5}}' + \mathbf{5}')$ and $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'') + (\mathbf{5}'' + \overline{\mathbf{10}}'')$ are given by

$$m(f') \sim \langle \Sigma_{2,3} \rangle, \quad m(f'') \sim M_{5,10}. \tag{4.1}$$

The existence of these additional particles does not affect the value of Λ_{GUT} in the minimal supersymmetric standard model (MSSM), so that the scale of Λ_{GUT} is still given by

$$\Lambda_{GUT} = 2 \times 10^{16}\ \text{GeV}. \tag{4.2}$$

Since the VEV forms of $\Sigma_{2,3}$ break SU(5), it seems to be natural to consider

$$\langle\Sigma_2\rangle \sim \langle\Sigma_3\rangle \sim \Lambda_{GUT} \tag{4.3}$$

On the other hand, if we consider a lower value of $M_{5,10}$, the gauge coupling constants $\alpha_3(\mu)$ will blow up before μ reaches to the GUT scale Λ_{GUT}. We have a constraint

$$M_{5,10} \geq 10^{12}\ \text{GeV}. \tag{4.4}$$

Masses of the quarks and leptons are given as

$$M_u = y_u\, y_{10} \frac{\langle Y_u\rangle}{M_{10}} \langle H_u^0\rangle, \qquad M_{e,d} = y_{e,d}\, y_5 \frac{\langle Y_{e,d}\rangle}{M_5} \langle H_d^0\rangle, \tag{4.5}$$

from Eqs.(2.9) – (2.11), so that we have constraints

$$\langle Y_u\rangle \sim M_{10}, \qquad \langle Y_e\rangle \sim \langle Y_d\rangle \sim 10^{-1} M_5. \tag{4.6}$$

If we suppose $M_{5,10} \sim \Lambda_{GUT}$, then the scale Λ_{fam} at which the U(3) family symmetry is completely broken must be $\langle Y_f\rangle \sim \Lambda_{fam} \sim 10^{15}$ GeV. On the other hand, we have estimated the value of $\langle Y_R\rangle$ as $\langle Y_R\rangle \sim 10^{13}$ GeV in Eq.(2.17). Therefore, we must assume that the mass term $\mathbf{1}_i \langle Y_R^{ij}\rangle \mathbf{1}_j$ has a different energy scale from those of the mass terms $f_i\langle Y_f^{ij}\rangle f''_j$. However, in this paper, we consider an alternative scenario: We assume that particles f'' have masses of the order of

$$M_5 \sim M_{10} \sim 10^{14}\ \text{GeV}, \tag{4.7}$$

which satisfies the constraint (4.4), so that an energy scale at which U(3) is broken is

$$\langle Y_f\rangle \sim \langle Y_R\rangle \sim \Lambda_{fam} \sim 10^{13}\ \text{GeV}. \tag{4.8}$$

For reference, we illustrate the behaviors of the gauge coupling constants $\alpha_i(\mu)$ $(i = 1, 2, 3)$ for the case $M_{5,10} = 10^{14}$ GeV in Fig.1.

As seen in Table 2, we have many U(3) non-singlet fields in the present model, so that the model does not give an asymptotic free theory. The gauge coupling constants of U(3) will blow up before μ reaches to $\mu = \Lambda_{GUT}$ if we take a lower value of Λ_{fam}. In this paper, we adopt a scenario based on Eq.(4.8). Then, since the scale $M_{5,10}$ is very high, the family gauge coupling constants α_f does not blow up from $\mu = M_{5,10}$ up to $\mu = \Lambda_{GUT}$.

We do not discuss the behaviors of gauge coupling constants above $\mu = \Lambda_{GUT}$ because we have no scenario at $\mu > \Lambda_{GUT}$ at present.

5. Concluding Remarks

In conclusion, we have investigated a yukawaon model which is compatible with an SU(5) GUT scenario. Since yukawaons are SU(5) singlets, the existence of the yukawaons do not affect the SU(5) GUT model, so that we can inherit the successful results in the SU(5) GUT and we also inherit the current problems in the minimum SU(5) GUT scenario.

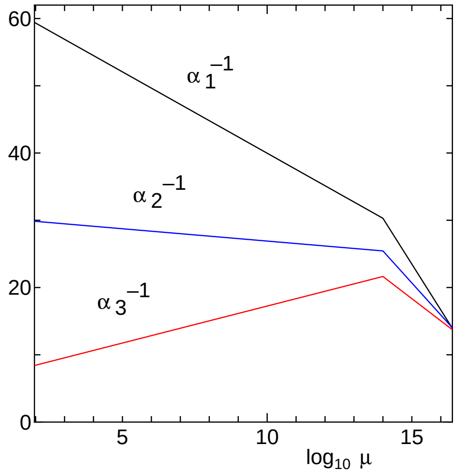

Figure 1: Behaviors of gauge coupling constants α_i^{-1} $(i = 1, 2, 3)$ in the case of $M_{5,6} = 10^{14}$ GeV. For simplicity, we have neglected the SUSY breaking effects at $\mu \sim 10^3$ GeV in this figure.

In the present model, we have the following matter fields:

$$(\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1})_i + (\bar{\mathbf{5}}'_i + \mathbf{5}'^i) + (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i. \tag{5.1}$$

We do not consider $\mathbf{1}''_i + \mathbf{1}''^i$ The particles $f' = (\bar{\mathbf{5}}'_i + \mathbf{5}'^i)$ and $f'' = (\bar{\mathbf{5}}'' + \mathbf{10}'')^i + (\mathbf{5}'' + \bar{\mathbf{10}}'')_i$ have masses of the orders of $\Lambda_{GUT} \sim 10^{16}$ GeV and $M_{5,10} \sim 10^{14}$ GeV, respectively. The U(3) family symmetry is broken at $\mu = \Lambda_{fam} \sim 10^{13}$ GeV, whose value has been settled by a neutrino seesaw mass.

The most notable result in the present SU(5) compatible model is that the model naturally lead to a model without Y_ν in which Y_e plays a role of a substitute for Y_ν, although it was an ac hoc assumption in the previous yukawaon model [6].

However, regrettably, we have failed to build a model in which U(3) family gauge boson effects are visible (for instance, Ref.[16]). It seems to be hard to embed a lower scale of Λ_{fam} (e.g. $\Lambda_{fam} \sim 10^{7-8}$ GeV [14]) into the present SU(5) compatible yukawaon model. This is still our future task.

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References

[1] Y. Koide, Phys. Rev. **D 79**, 033009 (2009).

[2] C. D. Froggatt and H. B. Nelsen, Nucl. Phys. **B 147**, 277 (1979).

[3] H. Georgi and S. L. Glashow, Phys.Rev.Lett. **32**, 438 (1974).

[4] S. Dimopoulos and F. Wilczek, in *The Unity of the Fundamental Interactions*, Proceedings of the 19th Course of the International School of Subnuclear Physics, Erice, Italy, 1981, edited by A. Zichichi (Plenum Press, New York, 1983); M. Srednicki, Nucl. Phys. B202, 327 (1982).

[5] Particle Data Group, K. Nakamura, *et al.*, J. Phys. **G 37**, 075021 (2010).

[6] Y. Koide, Phys. Lett. **B 680**, 76 (2009).

[7] P. F. Harrison, D. H. Perkins and W. G. Scott, Phys. Lett. **B 458**, 79 (1999); Phys. Lett. **B 530**, 167 (2002); Z.-z. Xing, Phys. Lett. **B 533**, 85 (2002); P. F. Harrison and W. G. Scott, Phys. Lett. **B 535**, 163 (2002); Phys. Lett. **B 557**, 76 (2003); E. Ma, Phys. Rev. Lett. **90**, 221802 (2003); C. I. Low and R. R. Volkas, Phys. Rev. **D 68**, 033007 (2003).

[8] H. Nishiura and Y. Koide, Phys. Rev. **D 83**, 035010 (2011).

[9] Y. Sumino, Phys. Lett. **B671**, 477 (2009); JHEP **0905**, 075 (2009).

[10] Y. Koide, Mod. Phys. Lett. **A5**, 2319 (1990).

[11] T. Applequist, Y. Bai and M. Piai, Phys. Rev. **D 74**, 076001 (2006); Phys. Lett. **B 637**, 245 (2006).

[12] Y. Koide, Lett. Nuovo Cimento **34**, 201 (1982); Phys. Lett. **B120**, 161 (1983); Phys. Rev. **D28**, 252 (1983).

[13] H. Arason, *et al.*, Phys. Rev. **D 46**, 3945 (1992).

[14] Y. Koide, arXiv: 1011.1064 [hep-ph].

[15] Z.-z. Xing, H. Zhang and S. Zhou, Phys. Rev. **D 77**, 113016 (2008). And also see, H. Fusaoka and Y. Koide, Phys. Rev. **D 57**, 3986 (1998).

[16] Y. Koide, Y. Sumino and M. Yamanaka, Phys. Lett. **B695**, 279 (2011).