

SU(5) Compatible Yukawaon Model With Two Family Symmetries $U(3) \times O(3)$

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Abstract

A yukawaon model which is compatible with an SU(5) GUT model is investigated. In a previous SU(5) compatible yukawaon model with a U(3) family gauge symmetry, we could not build a model with a lower energy scale of the family gauge symmetry breaking scale Λ_{fam} than 10^{13} GeV, so the family gauge boson effects in the previous model were invisible. In the present model, we consider two family symmetries $U(3) \times O(3)$, and we assume that the conventional quarks and leptons ($\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$) of SU(5) are described as ($\bar{\mathbf{5}}_i + \mathbf{10}_\alpha + \mathbf{1}_\alpha$) ($i = 1, 2, 3$ and $\alpha = 1, 2, 3$ are indices of U(3) and O(3), respectively). As a result, we build a model with $\Lambda_{O3} \sim 10^{16}$ GeV and $\Lambda_{U3} \sim 10^3$ GeV. The lightest U(3) family gauge boson A_1^1 will be observed with a mass of the order of 1 TeV.

1. Introduction

In the standard model (SM) of quarks and leptons, their mass spectra and mixings originate in the structures of the Yukawa coupling constants, although the masses themselves originate in the Higgs scalar. The Yukawa coupling constants are fundamental constants in the theory, so that they are not quantities which we can evaluate dynamically. If we intend to understand the observed mass spectra and mixings by a “family symmetry”, we cannot adopt a non-Abelian gauge symmetry, because the Yukawa coupling constants play a role in breaking the symmetry. Of course, instead of such a non-Abelian symmetry, we may assume U(1) symmetries, discrete symmetries, and so on. Then, by requiring that the model is invariant under such a symmetry, we can obtain some constraints on the Yukawa coupling constants. However, even if we consider such symmetries, we still have a trouble [1]: We know that any model with a family symmetry cannot derive a realistic flavor mixing matrix (Cabibbo-Kobayasi-Maskawa [2] (CKM) quark mixing matrix and/or Pontecorvo-Maki-Nakagawa-Sakata [3] (PMNS) lepton mixing matrix) unless we do not consider a multi-Higgs model. However, such the multi-Higgs model usually lead to a flavor changing neutral current (FCNC) problem.

An easy way to escape from these problems is to consider that the mass spectra and mixings originate in vacuum expectation values (VEVs) of new scalars. As one of such models, the so-called “yukawaon” model [4] is known. In the yukawaon model, which is a kind of “flavon” model [5], all effective Yukawa coupling constants Y_f^{eff} ($f = u, d, e, \dots$) are given by VEVs of “yukawaons” Y_f as

$$(Y_f^{eff})_{ij} = \frac{y_f}{\Lambda} \langle (Y_f)_{ij} \rangle, \quad (1.1)$$

that is, would-be Yukawa interactions are given by the following superpotential:

$$W_Y = \frac{y_e}{\Lambda} \ell_i Y_e^{ij} e_j^c H_d + \frac{y_\nu}{\Lambda} \ell_i Y_\nu^{ij} \nu_j^c H_u + \lambda_R \nu_i^c Y_R^{ij} \nu_j^c + \frac{y_u}{\Lambda} u_i^c Y_u^{ij} q_j H_u + \frac{y_d}{\Lambda} d_i^c Y_d^{ij} q_j H_d, \quad (1.2)$$

where ℓ and q are $SU(2)_L$ doublets $\ell = (\nu_L, e_L)$ and $q = (u_L, d_L)$. In order to distinguish each yukawaon from others, Y_f have R charges different from each other, and we assume R charge conservation. (Of course, the R charge conservation is broken at a high energy scale Λ_{fam} at which the family symmetry is broken.)

The most notable characteristic of the yukawaon model is that structures of VEV matrices $\langle Y_f \rangle$ are described in terms of only one fundamental VEV matrix

$$\langle \Phi_e \rangle = k_0 \text{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau}). \quad (1.3)$$

For examples, we describe $\langle Y_f \rangle$ as follows in terms of $\langle \Phi_e \rangle$ [6]:

$$\langle Y_e \rangle = k_e \langle \Phi_e \rangle \langle \Phi_e \rangle, \quad (1.4)$$

$$\langle Y_u \rangle = k_u \langle \Phi_u \rangle \langle \Phi_u \rangle, \quad \langle \Phi_u \rangle = k'_u \langle \Phi_e \rangle (\mathbf{1} + a_u X) \langle \Phi_e \rangle, \quad (1.5)$$

$$\langle Y_d \rangle = k'_d \langle \Phi_e \rangle (\mathbf{1} + a_d X) \langle \Phi_e \rangle, \quad (1.6)$$

where

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \quad (1.7)$$

We can also describe the neutrino mass matrix M_ν in terms of $\langle \Phi_e \rangle$ (see Eqs.(2.19) and (3.20) later). In this scenario, we do not ask why the VEV matrix $\langle \Phi_e \rangle$ takes such a value given in Eq.(1.3). As a result, the model has considerably few adjustable parameters (the charged lepton masses are input values, and the eigenvalues of $\langle \Phi_e \rangle$ are not adjustable parameters). The observed hierarchical structures in quarks and leptons are attributed to the hierarchical structure of $\langle \Phi_e \rangle$.

Here, note that the yukawaons Y_f are singlets under the conventional gauge symmetries $SU(3)_c \times SU(2)_L \times U(1)_Y$, and they have only family indices. This suggests that the yukawaon model may be compatible with a grand unification (GUT) model, for example, $SU(5)$ GUT model [7]. Recently, the author [8] has proposed an $SU(5)$ compatible yukawaon model. The main purpose of the $SU(5)$ compatible model was to build a yukawaon model without a cutoff scale Λ . The purpose also was to develop the yukawaon model and not to discuss problems in a GUT model. That is, possible structures of yukawaons were investigated for the case when we regarded quarks and leptons as $\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$ of $SU(5)$. In the present paper, too, a compatible $SU(5)$ yukawaon model is investigated, but we do not intend to develop a GUT scenario or to resolve problems in the current GUT scenarios.

Let us give a brief review of the previous $SU(5)$ compatible yukawaon model [8] in order to make the purpose of the present paper clear. In the previous model, superpotential terms for up-quark and charged lepton yukawaon sectors have been taken as:

$$W_{Y_u} = y_u \mathbf{10}_i Y_u^{ij} \overline{\mathbf{10}}'_j + M_{10} \overline{\mathbf{10}}'_i \mathbf{10}'^i + y_{10} \mathbf{10}'^i \mathbf{10}_i \mathbf{5}_H, \quad (1.8)$$

$$W_{Y_e} = y_e \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{5}'_j + M_5 \mathbf{5}'_i \bar{\mathbf{5}}'^i + y_5 \bar{\mathbf{5}}'^i \mathbf{10}_i \bar{\mathbf{5}}_H, \quad (1.9)$$

which lead to effective Yukawa interactions

$$W_{Y_u}^{eff} = \frac{y_u y_{10}}{M_{10}} \mathbf{10}_i Y_u^{ij} \mathbf{10}_j \mathbf{5}_H, \quad (1.10)$$

$$W_{Y_e}^{eff} = \frac{y_e y_5}{M_5} \bar{\mathbf{5}}_i Y_e^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H, \quad (1.11)$$

respectively. Here, although \bar{M}_{10} and \bar{M}_5 in Eqs.(1.10) and (1.11) have family-number dependence as we discuss later, for the time being those may be regarded as $\bar{M}_{10} \simeq M_{10}$ and $\bar{M}_5 \simeq M_5$. Anyhow, as seen in Eqs.(1.10) and (1.11), we can introduce two different cutoff scales M_{10} and M_5 for the up-quark and charged lepton sectors, respectively. However, since the model gives

$$M_u = \frac{y_u y_{10}}{M_{10}} \langle Y_u^{ij} \rangle v_{Hu}, \quad M_e = \frac{y_e y_5}{M_5} \langle Y_e^{ij} \rangle v_{Hd}, \quad (1.12)$$

where $v_{Hu} = \langle H_u^0 \rangle = \langle \mathbf{5}_H \rangle$ and $v_{Hd} = \langle H_d^0 \rangle = \langle \bar{\mathbf{5}}_H \rangle$, we are obliged to accept phenomenological constraints

$$\frac{\langle Y_u \rangle}{M_{10}} \sim 1, \quad \frac{\langle Y_e \rangle}{M_5} \sim 10^{-1}, \quad (1.13)$$

from the observed quark and lepton masses (we suppose $\tan \beta \sim 10$). (Here, the order of a VEV matrix $\langle Y_f \rangle$ means the largest value of the eigenvalues of $\langle Y_f \rangle$.) We consider that the family symmetry U(3) is broken at an energy scale Λ_{U3} . The scale is given by the largest one of the VEV values of the whole U(3) non-singlet scalars, i.e. $\Lambda_{U3} \geq \langle Y_u \rangle$. If we want that the family symmetry effects are visible, we must take the value of Λ_{U3} considerably low. However, on the other hand, if we take $M_{10} < 10^{12}$ GeV, such a model with a low value of M_{10} will cause blowing up of the SU(3)_c gauge coupling constant because of the additional fields ($\mathbf{10}' + \bar{\mathbf{10}}'$). In order to avoid such the blowing up, we must take $M_{10} \geq 10^{12}$ GeV. Thus, the scale Λ_{U3} is constrained as

$$\Lambda_{U3} \geq \langle Y_u \rangle \sim M_{10} \geq 10^{12} \text{ GeV}. \quad (1.14)$$

We could not take a lower value of Λ_{U3} in the previous SU(5) compatible model [8].

The main purpose of the present paper is to propose an SU(5)-compatible yukawaon model in which the family symmetry U(3) is broken at a suitably low energy scale $\Lambda_{U3} \sim 10^3$ GeV. The basic idea is quite simple: in the conventional quarks and leptons $\bar{\mathbf{5}} + \mathbf{10}$ of SU(5), the field $\bar{\mathbf{5}}$ is $\mathbf{3}$ of the family symmetry U(3) [we denote it as $\bar{\mathbf{5}}_i$ ($i = 1, 2, 3$)], while the field $\mathbf{10}$ is $\mathbf{3}$ of another family symmetry O(3) [we denote it as $\mathbf{10}_\alpha$ ($\alpha = 1, 2, 3$)]. Thereby, VEVs of the yukawaons Y_e and Y_u are given by $\langle Y_e^{i\alpha} \rangle$ and $\langle Y_u^{\alpha\beta} \rangle$, so that we can assume that those VEV values take different scales $\langle Y_e^{i\alpha} \rangle \sim \Lambda_{U3}$ and $\langle Y_u^{\alpha\beta} \rangle \sim \Lambda_{O3}$, where Λ_{U3} and Λ_{O3} are energy scale at which U(3) and O(3) are broken, respectively. We consider $\Lambda_{O3} \gg \Lambda_{U3}$. [A model with two family symmetries U(3)×O(3) has been proposed by Sumino [9]. A yukawaon model with two family symmetries U(3)×O(3) has been discussed in Ref.[10], although the model was not compatible with SU(5).]

In addition to the above idea, we will propose the following new ideas in the present yukawaon model:

(i) *Economizing of yukawaons*: In the previous SU(5) compatible yukawaon model [8], we have demonstrated that the yukawaon Y_ν in Eq.(1.2) can be substituted with the charged lepton yukawaon Y_e . In the present model, the up-quark yukawaon Y_u will also be removed from the model by modifying the superpotential W_u , (1.8). By considering a double seesaw mechanism, a bilinear form $\langle\Phi_u\rangle\langle\Phi_u\rangle$ can directly couple to the up-quark sector qu^c . As seen in Eq.(2.11) and Fig.2 in the next section, we would like to emphasize that such the double seesaw mechanism becomes possible only when we consider that $\mathbf{10}'_\alpha$ is a triplet of the O(3) family symmetry. Hereafter, we will denote Φ_u as \hat{Y}_u . Thereby, the $\langle\hat{Y}_u\rangle$ - $\langle Y_d\rangle$ correspondence becomes more natural, i.e.

$$\langle\hat{Y}_u\rangle = k'_u\langle\Phi_e\rangle(\mathbf{1} + a_u X)\langle\Phi_e\rangle \leftrightarrow \langle Y_d\rangle = k'_d\langle\Phi_e\rangle(\mathbf{1} + a_d X)\langle\Phi_e\rangle, \quad (1.15)$$

compared with Eqs.(1.5) and (1.6).

(ii) *New model for the factor $(\mathbf{1} + a_f X)$* : So far, it has been considered that the factors $(\mathbf{1} + a_f X)$ in the VEV relations (1.5) and (1.6) originate in VEVs $\langle E\rangle = v_E \mathbf{1}$ and $\langle S\rangle = v_X X$ of new scalars E and S . In the present model, we consider that the factors originate in an S_3 invariant coefficients for $\Phi_e \Phi_e$ as we discuss in Sec.3. In the previous SU(5) compatible yukawaon model, since we considered that $\Phi_e(E + a_f S)\Phi_e$ are cubic forms of fields, a complicated mechanism was required to obtain the VEV relations (1.5) and (1.6). In the present model, since the factors $(\mathbf{1} + a_f X)$ are merely numerical coefficients, we can present the relations (1.5) and (1.6) with a simple mechanism. This change is practically important to build a model without a cut off scale Λ . In Sec.3, we will assume that the fundamental yukawaon Φ_e obeys a transformation of a permutation symmetry S_3 .

Such a modification in a yukawaon model causes considerable change from previous yukawaon models. Especially, in contrast to past yukawaon models which are based on an effective theory with a cut off Λ and with a single family symmetry, the present yukawaon model somewhat becomes complicated. However, we consider that it is important to investigate a possibility that family symmetry effects are visible, even we pay the cost of complicated forms of the superpotential.

2. Would-be Yukawa interactions

Let us consider a superpotential form for would-be Yukawa interactions straightforwardly as

$$W_Y = \frac{y_u}{\Lambda} \mathbf{10}_i Y_{(10,10)}^{ij} \mathbf{10}_j \mathbf{5}_H + \frac{y_{d,e}}{\Lambda} \bar{\mathbf{5}}_i Y_{(5,10)}^{ij} \mathbf{10}_j \bar{\mathbf{5}}_H + \frac{y_\nu}{\Lambda} \bar{\mathbf{5}}_i Y_{(5,1)}^{ij} \mathbf{1}_j \mathbf{5}_H + \lambda_R \mathbf{1}_i Y_{(1,1)}^{ij} \mathbf{1}_j, \quad (2.1)$$

where $\bar{\mathbf{5}} + \mathbf{10} + \mathbf{1}$ are quark and lepton fields and $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ correspond to the conventional two Higgs doublets H_u and H_d , respectively. In the would-be Yukawa interactions (2.1), the charged lepton yukawaon Y_e is identical with the down-quark yukawaon Y_d , i.e. $Y_e = Y_d = Y_{(5,10)}$. In the yukawaon model, the yukawaon Y_e has to be different from Y_d . A splitting mechanism between Y_e and Y_d is needed. Therefore, first, let us give a brief review a Y_e - Y_d splitting mechanism which has been proposed in the previous SU(5)-compatible yukawaon model [8] with one family

symmetry $U(3)$. We introduce vector-like $\mathbf{5}'^i$ and $\bar{\mathbf{5}}'_i$ fields in addition to the fields given in Eq.(1.2). For convenience, we denote one $\mathbf{5}$ and two $\bar{\mathbf{5}}$ as

$$\bar{\mathbf{5}}_i = (D_i^c, \ell_i), \quad \bar{\mathbf{5}}''_i = (d_i^c, L_i), \quad \mathbf{5}''^i = (\bar{D}^{c i}, \bar{L}^i), \quad (2.2)$$

where d^c , D^c and \bar{D}^c are $SU(2)_L$ singlet down-quarks with electric charges $+1/3$, $+1/3$ and $-1/3$, respectively, and ℓ , L and \bar{L} are $SU(2)_L$ lepton doublets. In order to realize that the fields (D^c, \bar{D}^c) , and (L, \bar{L}) become massive and decouple from the present model, we assume the following interactions

$$\lambda_D \bar{\mathbf{5}}_i^A (\Sigma_3)_A^B \mathbf{5}''^i + \lambda_L \bar{\mathbf{5}}_i^A (\Sigma_2)_B^C \mathbf{5}''^i, \quad (2.3)$$

where A, B are indices of $SU(5)$, and $SU(5)$ $\mathbf{24} + \mathbf{1}$ fields Σ_2 and Σ_3 take VEV forms

$$\begin{aligned} \langle \Sigma_2 \rangle &= v_2 \text{diag}(0, 0, 0, 1, 1), \\ \langle \Sigma_3 \rangle &= v_3 \text{diag}(1, 1, 1, 0, 0). \end{aligned} \quad (2.4)$$

Therefore, Eq.(2.3) leads to mass terms

$$\lambda_D v_3 \bar{D}^{c i} D_i^c + \lambda_L v_2 \bar{L}^i L_i. \quad (2.5)$$

We consider that the VEVs of Σ_2 and Σ_3 are of the order of Λ_{GUT} . As we seen in Eq.(2.3), R charges of Σ_2 and Σ_3 (and also $\mathbf{5}$ and $\mathbf{5}'$) have to be different from each other:

$$R(\Sigma_3) - R(\Sigma_2) = R(\bar{\mathbf{5}}''_i) - R(\bar{\mathbf{5}}_i) = R(Y_e) - R(Y_d). \quad (2.6)$$

If we accept such fields with the VEV forms (2.4), we may understand the doublet-triplet splitting of the Higgs fields $\mathbf{5}_H$ and $\bar{\mathbf{5}}_H$ by a similar mechanism $\lambda_H \bar{\mathbf{5}}_H \Sigma_3 \mathbf{5}$. The doublet-triplet splitting mechanism has been already proposed in the framework of an $SO(10)$ GUT scenario [11]. Therefore, the VEV forms (2.4) will also be understood from a GUT scenario based on a higher gauge group and/or on extra-dimensions. For the time being, we do not ask the origin of the VEV forms (2.4). This is still an open question.

For such the fields $\bar{\mathbf{5}}_i$ and $\bar{\mathbf{5}}''_i$, the would-be Yukawa interactions are given by the following superpotential:

$$W_{Y_{e,d}} = y_e \bar{\mathbf{5}}_i Y_e^{i\alpha} \mathbf{5}'_\alpha + y_d \bar{\mathbf{5}}''_i Y_d^{i\alpha} \mathbf{5}'_\alpha + M_5 \mathbf{5}'_\alpha \bar{\mathbf{5}}'_\alpha + y_5 \bar{\mathbf{5}}'_\alpha \mathbf{10}_\alpha \bar{\mathbf{5}}_H. \quad (2.7)$$

[Here and hereafter, for convenience, we sometime denote a field A_α as A^α although those are identical because α is an index of $O(3)$.] Then, we can obtain the effective Yukawa interaction

$$W_{Y_{e,d}}^{eff} = \frac{y_e y_5}{\bar{M}_5} \bar{\mathbf{5}}_i Y_e^{i\alpha} \mathbf{10}_\alpha \bar{\mathbf{5}}_H + \frac{y_d y_5}{\bar{M}_5} \bar{\mathbf{5}}''_i Y_d^{i\alpha} \mathbf{10}_\alpha \bar{\mathbf{5}}_H, \quad (2.8)$$

where \bar{M}_5 is given by

$$\bar{M}_5 \simeq \sqrt{(M_5)^2 + y_{e,d}^2 \langle Y_{e,d} \rangle^2}, \quad (2.9)$$

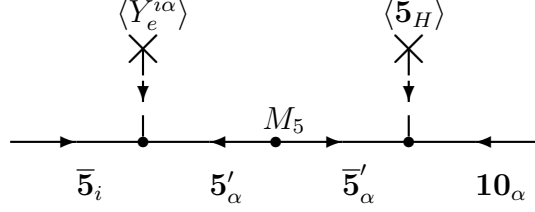


Figure 1: Mass generation mechanism for the charged leptons.

under the approximation $y_5^2 \langle H_d \rangle^2 \ll (M_5)^2$ and in the diagonal basis of $\langle Y_{e,d} \rangle$. The relation (2.9) has been obtain from the diagonalization of mass matrix for $(\bar{\mathbf{5}} \ \bar{\mathbf{5}}', \mathbf{10}, \bar{\mathbf{5}}'', \mathbf{5}'')$

$$\begin{pmatrix} 0 & 0 & 0 & y_{e,d} \langle Y_{e,d} \rangle \\ 0 & 0 & y_5 v_{Hd} & 0 \\ 0 & v_{Hd} & 0 & M_5 \\ y_{e,d} \langle Y_{e,d} \rangle & 0 & M_5 & 0 \end{pmatrix}. \quad (2.10)$$

Note that we can use the relation (2.9) even for the case $y_{e,d} \langle Y_{e,d} \rangle \sim M_5$. (For the mass generation mechanism of the charged leptons, see Fig.1.)

On the other hand, for the up-quark sector, we somewhat change our model from the previous model (1.5). As we discuss in the next section, in the yukawaon model, the VEV matrix of Y_u is given by a bilinear form $\langle Y_u \rangle = k_u \langle \Phi_u \rangle \langle \Phi_u \rangle$, and $\langle Y_d \rangle$ takes the same structure as $\langle \Phi_u \rangle$ except for values of the parameters a_u and a_u . Therefore, in this paper, we denote Φ_u in the previous paper as \hat{Y}_u , and we propose a model without Y_u in the old model:

$$W_{Y_u} = y_u \mathbf{10}_\alpha \hat{Y}_u^{\alpha\beta} \bar{\mathbf{10}}'_\beta + M_{10} \bar{\mathbf{10}}'_\alpha \mathbf{10}'_\alpha + y_{10} \mathbf{10}'_\alpha \mathbf{10}'_\alpha \mathbf{5}_H. \quad (2.11)$$

Note that the Higgs field $\mathbf{5}_H$ couples not to $\mathbf{10}'\mathbf{10}$, but to $\mathbf{10}'\mathbf{10}'$, differently from Eq.(1.8). Therefore, the effective interaction is given by a double seesaw form

$$W_{Y_u}^{eff} = \frac{y_u^2 y_{10}}{(\bar{M}_{10})^2} \mathbf{10}_\alpha \hat{Y}_u^{\alpha\gamma} \hat{Y}_u^{\gamma\beta} \mathbf{10}_\beta \mathbf{5}_H, \quad (2.12)$$

under the approximation $\langle H_u \rangle \ll M_{10}$. We again would like to emphasize that the double seesaw form (2.12) is possible only when we consider the third term in Eq.(2.11), i.e. only when $\mathbf{10}'_\alpha$ is a triplet of $O(3)$ family symmetry. In Eq.(2.10), $\bar{M}_{10}^{\alpha\beta}$ is given by

$$\bar{M}_{10} \simeq M_{10} \quad (2.13)$$

from the diagonalization of the mass matrix for the fields $(\mathbf{10}, \mathbf{10}', \bar{\mathbf{10}}')$

$$\begin{pmatrix} 0 & 0 & y_u \langle Y_u \rangle \\ 0 & y_{10} v_{Hu} & M_{10} \\ y_u \langle Y_u \rangle & M_{10} & 0 \end{pmatrix}. \quad (2.14)$$

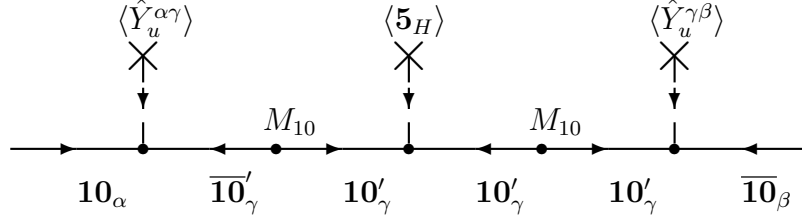


Figure 2: Mass generation mechanism for the up-quarks.

We can use the relation (2.13) even for a case of $y_u \langle Y_u \rangle \sim M_{10}$, although \bar{M}_{10} in the previous SU(5) compatible model has been highly dependent of the value of $\langle \hat{Y}_u \rangle$ [8]. This is because the Higgs field $\mathbf{5}_H$ couples to $\mathbf{10}'_\alpha \mathbf{10}'_\alpha$ in the present model, not to $\mathbf{10}'^i \mathbf{10}_i$ as in the previous model.

Thus, SU(5) non-singlet fields which can contribute to the evolutions of the gauge coupling constants of $SU(3)_c \times SU(2)_L \times U(1)_Y$ below $\mu < \Lambda_{GUT}$ are only

$$(\mathbf{5}'_\alpha + \bar{\mathbf{5}}'_\alpha) + (\bar{\mathbf{10}}'_\alpha + \mathbf{10}'_\alpha). \quad (2.15)$$

in addition to the standard $\bar{\mathbf{5}}_i + \mathbf{10}_\alpha$. The mass parameters M_5 and M_{10} are free parameters in the superpotential. We will consider $M_5 \ll M_{10}$ in the next section. On the other hand, the fields (D_i^c, L_i) and $(\bar{D}^{ci}, \bar{L}^i)$ given in Eq.(2.1) cannot contribute to the evolutions of gauge coupling constants of $SU(3)_c \times SU(2)_L \times U(1)_Y$, because those particles have masses of the order of Λ_{GUT} .

Next, we discuss a seesaw-type mass matrix for neutrinos. Differently from the previous SU(5) compatible yukawaon model [8], we introduce an SU(5) singlet field $\mathbf{1}_\alpha$ instead of $\mathbf{1}_i$ in the previous model. The neutrino Dirac mass term m_D is obtained from the following superpotential

$$W_{Y\nu} = y_e \bar{\mathbf{5}}_i Y_e^{i\alpha} \mathbf{5}'_\alpha + M_5 \mathbf{5}'_\alpha \bar{\mathbf{5}}'_\alpha + y_1 \bar{\mathbf{5}}'_\alpha \mathbf{1}_\alpha \mathbf{5}_H, \quad (2.16)$$

where only the third term is a new term and the first and second terms have already given in Eq.(2.7). The superpotential (2.16) leads to the effective interaction

$$W_{Y\nu}^{eff} = \frac{y_e y_1}{M_5} \bar{\mathbf{5}}_i Y_e^{i\alpha} \mathbf{1}_\alpha \mathbf{5}_H. \quad (2.17)$$

Note that the neutrino Dirac mass matrix has the same structure as the charged lepton mass matrix. On the other hand, the right-handed Majorana neutrino mass matrix M_R is obtained from the superpotential term

$$W_R = \lambda_R \mathbf{1}_\alpha Y_R^{\alpha\beta} \mathbf{1}_\beta. \quad (2.18)$$

Therefore, we can obtain a seesaw-type neutrino mass matrix

$$M_\nu = \frac{y_e^2 y_1^2}{\lambda_R} \left(\frac{v_{Hu}}{M_5} \right)^2 \langle Y_e \rangle \langle Y_R \rangle^{-1} \langle Y_e \rangle. \quad (2.19)$$

From Eq.(2.8), we can rewritten Eq.(2.19) as

$$M_\nu = \frac{y_1^2 \tan^2 \beta}{\lambda_R y_5^2} M_e \langle Y_R \rangle^{-1} M_e, \quad (2.20)$$

where $\tan \beta = \langle H_u^0 \rangle / \langle H_d^0 \rangle$. By taking $m_\tau \simeq 1.777$ GeV, $(M_\nu)_{33} \sim m_{\nu 3} \simeq \sqrt{\Delta m_{atm}^2} \simeq 0.049$ eV [13]) and $\tan \beta \simeq 10$, we can estimate the value of $\langle Y_R \rangle$ as

$$\langle Y_R \rangle \simeq \lambda_R (y_5/y_1)^2 \times 6.4 \times 10^{12} \text{ GeV}. \quad (2.21)$$

However, this does not mean that the value of Λ_{O3} is of the order of $\langle Y_R^{\alpha\beta} \rangle \sim 10^{12-13}$ GeV. The value of Λ_{O3} is determined by the largest one of all O(3)-non-singlet scalars. We can assert only $\Lambda_{O3} \geq \langle Y_R^{\alpha\beta} \rangle$.

3. Yukawaon sector

Priori to discussing VEV relations among yukawaons, we discuss a new idea about the factors $(\mathbf{1} + a_f X)$ in Eqs.(1.5) and (1.6). The factors play an essential role in giving successful results in the phenomenological yukawaon model. In the past yukawaon model, it has been considered that the factors $(\mathbf{1} + a_f X)$ are originated in VEVs of scalars E and S , so that $\Phi_e(E + a_f S)\Phi_e$ were cubic forms of fields. As a result, in order to build a model without Λ and in order to obtain the VEV forms given in Eq.(1.7), very complicated mechanism was required. In the present model, since the factors $(\mathbf{1} + a_f X)$ are merely numerical coefficients, $\Phi_e(\mathbf{1} + a_f X)\Phi_e$ is a bilinear form of the fields (not a cubic form of fields).

When we denote a doublet (ψ_π, ψ_η) and a singlet ψ_σ in a permutation symmetry [12] S_3 as

$$\begin{pmatrix} \psi_\pi \\ \psi_\eta \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\psi_1 - \psi_2) \\ \frac{1}{\sqrt{6}}(\psi_1 + \psi_2 - 2\psi_3) \end{pmatrix}, \quad (3.1)$$

$$\psi_\sigma = \frac{1}{\sqrt{3}}(\psi_1 + \psi_2 + \psi_3), \quad (3.2)$$

the field $\psi = (\psi_1, \psi_2, \psi_3)$ is represented as

$$\psi \equiv \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ 0 & -\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} \psi_\pi \\ \psi_\eta \\ \psi_\sigma \end{pmatrix} \equiv U_0 \begin{pmatrix} \psi_\pi \\ \psi_\eta \\ \psi_\sigma \end{pmatrix}. \quad (3.3)$$

A bilinear form $\psi\psi$ is invariant under the S_3 symmetry only when $\psi_a \xi_{ab} \psi_b$ is given by the form

$$\psi_a \xi_{ab} \psi_b = \psi_a (\mathbf{1} + aX)_{ab} \psi_b, \quad (3.4)$$

where a is a free parameter, and $\mathbf{1}$ and X are defined by Eq.(1.7). The matrix X is diagonalized by U_0 as

$$U_0^T X U_0 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3.5)$$

Therefore, in the present model, we assume that the fundamental yukawaon Φ_e transforms as ψ defined by Eq.(3.3) under the S_3 symmetry, i.e. as $\Phi_e^{a\alpha}$. We assume the following S_3 invariant superpotential terms

$$W_e = \left(\mu_e Y_e^{i\alpha} + \lambda_e \bar{E}^{i\gamma} \hat{Y}_e^{\gamma\alpha} \right) \Theta_{\alpha i}^e + \left(\mu_e' \hat{Y}_e^{\alpha\beta} + \lambda_e' \Phi_e^{T\alpha a} \xi_{ab}^e \Phi_e^{b\beta} \right) \Theta_{\beta\alpha}^{e'}, \quad (3.6)$$

$$W_d = \left(\mu_d Y_d^{i\alpha} + \lambda_d \bar{E}^{i\gamma} \hat{Y}_d^{\gamma\alpha} \right) \Theta_{\alpha i}^d + \left[\lambda_d' (\hat{E}^{\alpha\gamma} \hat{Y}_d^{\gamma\beta} + \hat{Y}_d^{\alpha\gamma} \hat{E}^{\gamma\beta}) + \lambda_d'' \Phi_e^{T\alpha a} \xi_{ab}^d \Phi_e^{b\beta} \right] \Theta_{\beta\alpha}^{d'}, \quad (3.7)$$

$$W_u = \left[\lambda_u \left(P_u^{\alpha\gamma} \hat{Y}_u^{\gamma\beta} + \hat{Y}_u^{\alpha\gamma} P_u^{\gamma\beta} \right) + \lambda_u' \Phi_e^{T\alpha a} \xi_{ab}^u \Phi_e^{b\beta} \right] \Theta_{\beta\alpha}^u, \quad (3.8)$$

where

$$\xi_{ab}^f = (\mathbf{1} + a_f X)_{ab}. \quad (3.9)$$

In Eqs.(3.6) and (3.7), Y_e and Y_d are connected to $(\Phi_e^T \xi \Phi_e)$ via two steps. The introducing \hat{Y}_e is to connect $Y_R^{\alpha\beta}$ to $\hat{Y}_e^{\alpha\gamma} \hat{Y}_u^{\gamma\beta}$ as seen later. Then, it is required that \hat{Y}_e is distinguished from \hat{Y}_d by R charges. Therefore, we have assumed different structures for \hat{Y}_e and \hat{Y}_d as given in Eqs.(3.6) and (3.7). Also, the field P_u has been inserted in Eq.(3.8) in order to distinguish \hat{Y}_u from \hat{Y}_e and \hat{Y}_d under the R charge conservation. Since

$$R(\hat{E}) + R(\hat{Y}_d) = 2R(\Phi_e) = R(\hat{Y}_e), \quad (3.10)$$

$$R(\hat{Y}_u) = R(\hat{Y}_e) - R(P_u) = R(\hat{Y}_d) + R(\hat{E}) - R(P_u), \quad (3.11)$$

we can distinguish \hat{Y}_d from \hat{Y}_e and \hat{Y}_u when $R(\hat{E}) \neq 0$ and $R(\hat{E}) \neq R(P_u)$, respectively.

The values of a_f in Eq.(3.9) are purely phenomenological parameters. At present, there is no reason that we take $a_e = 0$. However, we think that the VEV matrix $\langle \Phi_e \rangle$ is a fundamental VEV matrix in the model, so that it is likely that the value a_e in $\langle \hat{Y}_e \rangle$ takes a specific value $a_e = 0$. However, the true reason is a future task to us.

By using SUSY vacuum conditions $\partial W / \partial \Theta_A = 0$ ($\Theta_A = \Theta^e, \Theta^{e'}, \Theta^d, \Theta^{d'}, \Theta^u$) for the superpotential terms (3.6)-(3.8), and by assuming that our vacuum takes $\langle \Theta_A \rangle = 0$, we obtain the following VEV relations:

$$\langle Y_e^{i\alpha} \rangle = -\frac{\lambda_e}{\mu_e} \langle \bar{E}^{i\gamma} \rangle \langle \hat{Y}_e^{\gamma\alpha} \rangle = \frac{\lambda_e \lambda_e'}{\mu_e \mu_e'} \langle \bar{E}^{i\gamma} \rangle \langle \Phi_e^{T\alpha a} \rangle \xi_{ab}^e \langle \Phi_e^{b\beta} \rangle, \quad (3.12)$$

$$\langle Y_d^{i\alpha} \rangle = -\frac{\lambda_d}{\mu_d} \langle \bar{E}^{i\gamma} \rangle \langle \hat{Y}_d^{\gamma\alpha} \rangle = \frac{\lambda_d \lambda_d''}{2\mu_d \lambda_d'} \langle \bar{E}^{i\gamma} \rangle \langle (\hat{E}^{-1})^{\gamma\delta} \rangle \langle \Phi_e^{T\delta a} \rangle \xi_{ab}^d \langle \Phi_e^{b\alpha} \rangle, \quad (3.13)$$

$$\langle P_u^{\alpha\gamma} \rangle \langle \hat{Y}_u^{\gamma\beta} \rangle + \langle \hat{Y}_u^{\alpha\gamma} \rangle \langle P_u^{\gamma\beta} \rangle = -\frac{\lambda_u'}{\lambda_d} \langle \Phi_e^{T\gamma a} \rangle \xi_{ab}^u \langle \Phi_e^{b\beta} \rangle, \quad (3.14)$$

where we assume that the VEV forms of $\langle \bar{E} \rangle$ and $\langle P_u^{\alpha\gamma} \rangle$ are given by

$$\frac{1}{\bar{v}_E} \langle \bar{E} \rangle_e = \frac{1}{\bar{v}_E} \langle \hat{E} \rangle_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \frac{1}{v_{P_u}} \langle P_u \rangle_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (3.15)$$

Here, the expression $\langle A \rangle_f$ ($f = e, u$) denotes a form of the VEV matrix $\langle A \rangle$ in a basis (we call it f -basis) in which the mass matrix M_f is diagonal. Note that almost VEV forms are represented with simple forms in the e -basis, while only $\langle P_u \rangle$ takes a simple form (3.15) in the u -basis. Therefore, the assumption for the form (3.15) is somewhat strange. Such the form (3.15) was introduced [6] in order to change the sign $(+, -, +)$ of the eigenvalues of $\langle \Phi_u \rangle$ (i.e. $\langle \hat{Y}_u \rangle$ in the present model) to the positive values $(+, +, +)$. We need the field P_u in order to obtain the successful fitting for the observed neutrino mixing and up-quark mass ratios as we discuss in the next section.

Here and hereafter, we denote fields whose VEV values are zeros as Θ_A ($A = e, u, \dots$). Therefore, we can obtain meaningful VEV relations from SUSY vacuum conditions $\partial W / \partial \Theta_A = 0$, while we cannot obtain any relations from other conditions (e.g. $\partial W / \partial Y_f = 0$) because the relations always include $\langle \Theta_A \rangle$. For the time being, we assume that the supersymmetry breaking is induced by a gauge mediation mechanism (not including family gauge symmetries), so that our VEV relations among yukawaons are still valid even after the SUSY was broken in the quark and lepton sectors.

Finally, we comment on the VEV forms of \bar{E} , \hat{E} and P_u which were assumed as in Eq.(3.15). We cannot directly give the forms (3.15), but we can give the relations

$$\langle \hat{E} \rangle = \hat{v}_E \mathbf{1}, \quad \langle E \rangle \langle \bar{E} \rangle = v_E \bar{v}_E \mathbf{1}, \quad \langle P_u \rangle^2 = v_P^2 \mathbf{1}, \quad (3.16)$$

by introducing a new field $E_{i\alpha}$ and by assuming the following superpotential

$$W_{E,P} = \lambda_1 \text{Tr}[E \bar{E} \hat{E}] + \lambda_2 \text{Tr}[E \bar{E}] \text{Tr}[\hat{E}] + \lambda_3 \text{Tr}[P_u P_u P_u] + \lambda_4 \text{Tr}[E \bar{E}] \text{Tr}[P_u], \quad (3.17)$$

where we have assumed

$$R(\hat{E}) = R(P_u) = \frac{2}{3}, \quad R(E) + R(\bar{E}) = \frac{4}{3}. \quad (3.18)$$

The SUSY vacuum conditions $\partial W / \partial \hat{E} = 0$ and $\partial W / \partial P_u = 0$ can give

$$\frac{\partial W}{\partial \hat{E}} = \lambda_1 E \bar{E} + \lambda_2 \text{Tr}[E \bar{E}] \mathbf{1} = 0, \quad (3.19)$$

$$\frac{\partial W}{\partial P_u} = 3\lambda_3 P_u P_u + \lambda_4 \text{Tr}[E \bar{E}] \mathbf{1} = 0, \quad (3.20)$$

which lead to the relations $\langle E \rangle \langle \bar{E} \rangle \propto \mathbf{1}$ and $\langle P_u \rangle^2 \propto \mathbf{1}$, respectively. The remaining conditions $\partial W / \partial E_f = 0$ and $\partial W / \partial \bar{E}^f = 0$ can be satisfied for the case $\langle \hat{E}_f \rangle = v_E \mathbf{1}$. We consider that the forms (3.15) are specific solutions of (3.16).

Next, we discuss a possible form of $\langle Y_R \rangle$. In the previous O_3 yukawaon model [6], the form $\langle Y_R \rangle$ has been given by

$$\langle Y_R \rangle_e = k_R [\langle \Phi_u \rangle_e \langle P_u \rangle_e \langle Y_e \rangle_e + \langle Y_e \rangle_e \langle P_u \rangle_e \langle \Phi_u \rangle_e + \xi_\nu (\langle \Phi_u \rangle_e \langle Y_e \rangle_e \langle P_u \rangle_e + \langle P_u \rangle_e \langle Y_e \rangle_e \langle \Phi_u \rangle_e)], \quad (3.21)$$

ξ_ν	$\tan^2 \theta_{solar}$	$\sin^2 2\theta_{atm}$	$\sin^2 2\theta_{13}$
0	0.6995	0.9872	0.00068
0.0004	0.4881	0.9880	0.00072
0.0005	0.4477	0.9882	0.00073
0.0006	0.4112	0.9884	0.00074

Table 1: ξ_ν dependence of the neutrino mixing parameters. The value of a_u is taken as $a_u = -1.78$ which can give reasonable up-quark mass ratios.

where $\langle P_u \rangle_u$ is given by Eq.(3.15). In contrast to Eq.(3.21), in the present model, $\langle Y_R \rangle$ is derived from the following superpotential

$$W_R = \left\{ \mu_R Y_R^{\alpha\beta} + \lambda'_R \left[\hat{Y}_u^{\alpha\gamma} \hat{Y}_e^{\gamma\beta} + \hat{Y}_e^{\alpha\gamma} \hat{Y}_u^{\gamma\beta} + \xi_\nu \left(\text{Tr}(\hat{Y}_u) \hat{Y}_e^{\alpha\beta} + \text{Tr}(\hat{Y}_e) \hat{Y}_u^{\alpha\beta} \right) \right] \right\} \Theta_{\beta\alpha}^R, \quad (3.22)$$

without P_u . Instead, P_u has been inserted in W_u as given in Eq.(3.8).

We notice that, in Eq.(3.22), the ξ_ν term has been changed from Eq.(3.21) in the O(3) model. Nevertheless, we can again obtain reasonable value of the neutrino mixing parameters by fitting the parameters a_u and ξ_ν : By using the input value $a_u = -1.78$, we can give reasonable up-quark mass ratios

$$\sqrt{\frac{m_u}{m_c}} = 0.04389, \quad \sqrt{\frac{m_c}{m_t}} = 0.05564, \quad (3.23)$$

which are in good agreement with the observed values at $\mu = m_Z$ [14] $\sqrt{m_u/m_c} = 0.045_{-0.010}^{+0.013}$ and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$. Then, the predicted neutrino oscillation parameters are given in Table 1. The results are in favor of the observed values except for that the value of $\sin^2 2\theta_{13}$ is too small. For this problem in $\sin^2 2\theta_{13}$, we may improve the present model by taking some other small effects into consideration.

In Table 2, we list assignments of $SU(5) \times U(3) \times O(3)$ for all fields in the present model. Obviously, the present model is anomaly free in $SU(5)$. In Table 2, in order to make the model anomaly free in the $U(3)$ family symmetry, we have added new fields $T_A^{i\alpha}$, $T_B^{i\alpha}$ and S_i , because we have a sum of the anomaly coefficients $\sum A = 19 - 14 = 5$ except for $T^{i\alpha}$ and S_i . However, for the time being, we do not specify the roles of those fields T_A , T_B and S in the model. At least, the sterile neutrino S_i is harmless, because the sterile neutrino can couple to the massive field $\mathbf{5}''$ (mass $\sim \Lambda_{GUT}$) as $\mathbf{5}''^i S_i \bar{\mathbf{5}}_H$. The existence of $T_A^{i\alpha}$ and $T_B^{i\alpha}$ will play a role in fitting the Cabibbo-Kobayashi-Maskawa mixing parameters.

In the present model, fields which have the same quantum numbers of $SU(5) \times U(3) \times O(3)$ are distinguished from others by R charges. Since we have still free parameters in the assignments of R charges, we do not give explicit numerical assignments in Table 2.

Finally, we would like to comment on R parity assignments. Since we inherit R parity assignments in the standard SUSY model, R parities of yukawaons Y_f (and also Θ_f , $\Phi_{e,u}$, E , \dots) are the same as those of Higgs particles (i.e. $P_R(\text{fermion}) = -1$ and $P_R(\text{scalar}) = +1$), while

	$\bar{\mathbf{5}}_i$	$\mathbf{10}_\alpha$	$\mathbf{1}_\alpha$	$\bar{\mathbf{5}}''_i$	$\mathbf{5}''^i$	$\bar{\mathbf{5}}'_\alpha$	$\mathbf{5}'_\alpha$	$\mathbf{10}'_\alpha$	$\bar{\mathbf{10}}'_\alpha$	$\bar{\mathbf{5}}_H$	$\mathbf{5}_H$
SU(5)	$\mathbf{5}^*$	$\mathbf{10}$	$\mathbf{1}$	$\mathbf{5}^*$	$\mathbf{5}$	$\mathbf{5}^*$	$\mathbf{5}$	$\mathbf{10}$	$\mathbf{10}^*$	$\mathbf{5}^*$	$\mathbf{5}$
U(3)	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
O(3)	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$

Σ_3	Σ_2	$Y_e^{i\alpha}$	$\hat{Y}_e^{\alpha\beta}$	$\Phi_e^{a\alpha}$	$\Theta_{\alpha i}^e$	$\Theta_{\alpha\beta}^{e'}$	$Y_d^{i\alpha}$	$\Theta_{\alpha i}^d$	$\hat{Y}_d^{\alpha\beta}$	$\Theta_{\alpha\beta}^{d'}$
$\mathbf{24} + \mathbf{1}$	$\mathbf{24} + \mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}^*$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$

$\bar{E}^{i\alpha}$	$E_{\alpha i}$	$\hat{E}^{\alpha\beta}$	$\hat{Y}_u^{\alpha\beta}$	$P_u^{\alpha\beta}$	$\Theta_{\alpha\beta}^u$	$Y_R^{\alpha\beta}$	$\Theta_{\alpha\beta}^R$	$\hat{E}_{\alpha\beta}$	$T_A^{i\alpha}$	$T_B^{i\alpha}$	S_i
$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}^*$	$\mathbf{3}^*$	$\mathbf{3}$
$\mathbf{3}$	$\mathbf{3}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{5} + \mathbf{1}$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}$

Table 2: Fields in the present model and their SU(5) \times U(3) \times O(3) assignments.

$(\bar{\mathbf{5}}'' + \mathbf{5}'')$, $(\bar{\mathbf{5}}' + \mathbf{5}')$ and $(\mathbf{10}' + \bar{\mathbf{10}}')$ are assigned to quark and lepton type, i.e. $P_R(\text{fermion}) = +1$ and $P_R(\text{scalar}) = -1$.

4. Energy scales

In the present model, we have introduced three energy scales Λ_{GUT} , Λ_{O3} and Λ_{U3} , which break SU(5), O(3) and U(3), respectively. As seen in Eqs.(3.12)-(3.14), if we take $\mu_e, \mu'_e, \mu_d, \mu_R \sim \Lambda_{O3}$, we can take VEV values as $\langle Y_e^{i\alpha} \rangle, \langle Y_d^{i\alpha} \rangle, \langle \bar{E}^{i\alpha} \rangle, \langle E_{i\alpha} \rangle \sim \Lambda_{U3}$, and $\langle \hat{Y}_e^{\alpha\beta} \rangle, \langle \hat{Y}_d^{\alpha\beta} \rangle, \langle \hat{Y}_u^{\alpha\beta} \rangle, \langle Y_R^{\alpha\beta} \rangle, \langle \hat{E}^{\alpha\beta} \rangle, \langle P_u^{\alpha\beta} \rangle, \langle \Phi_e^{a\alpha} \rangle \sim \Lambda_{O3}$, so as to be consistent with the relations (3.12)-(3.14) and (3.21). (The expression $\langle A \rangle \sim \Lambda$ for a field A means that the largest component of $\langle A \rangle$ is of the order of Λ .)

However, as seen in Table 2, we have many O(3) non-singlet fields in the present model. If we consider $\Lambda_{O3} < \Lambda_{GUT}$, the gauge coupling constant of O(3) will rapidly blow up before μ reaches Λ_{GUT} . Therefore, we are obliged to consider

$$\Lambda_{O3} \sim \Lambda_{GUT}. \quad (4.1)$$

When we simply take

$$\mu_e, \mu'_e, \mu_d, \sim \Lambda_{O3}, \quad (4.2)$$

we can obtain

$$\begin{aligned} \langle Y_e^{i\alpha} \rangle, \langle Y_d^{i\alpha} \rangle, \langle \bar{E}^{i\alpha} \rangle, \langle E_{i\alpha} \rangle &\sim \Lambda_{U3}, \\ \langle \hat{Y}_e^{\alpha\beta} \rangle, \langle \hat{Y}_d^{\alpha\beta} \rangle, \langle \hat{Y}_u^{\alpha\beta} \rangle, \langle \hat{E}^{\alpha\beta} \rangle, \langle P_u^{\alpha\beta} \rangle, \langle \Phi_e^{a\alpha} \rangle &\sim \Lambda_{O3}. \end{aligned} \quad (4.3)$$

The VEVs $\langle Y_e \rangle, \langle Y_d \rangle$ and $\langle \bar{E} \rangle$ contribute to the family gauge boson masses $m(A_i^j)$. The VEVs $\langle Y_e \rangle, \langle Y_d \rangle$ have hierarchical structures, while $\langle \bar{E} \rangle$ takes a structure proportional to a unit matrix.

Since it is not likely that the lightest family gauge boson mass $m(A_1^1)$ is smaller than 10^3 GeV, we take

$$\Lambda_{U3} \sim 10^3 \text{ GeV}. \quad (4.4)$$

Then, we may suppose

$$10^3 \text{ GeV} \sim m(A_1^1) < m(A_2^2) < m(A_3^3) \sim 10^4 \text{ GeV}, \quad (4.5)$$

because $m(A_3^3)$ is contributed from $\langle Y_e \rangle$, $\langle Y_d \rangle$ and $\langle \bar{E} \rangle$, while $m(A_1^1)$ is dominantly contributed only from $\langle \bar{E} \rangle \propto \mathbf{1}$. Note that, usually, a scale of a family symmetry breaking cannot take a too low value, because such a low value contradicts phenomenology in the kaon physics. In contrast to the conventional models, in the present model, we can take a considerably low value of Λ_{U3} , because the U(3) gauge bosons couple only to $SU(2)_L$ singlet down-quark d_i^c , while they cannot couple to $SU(2)_L$ doublet quark $(u_\alpha, d_\alpha)_L$. The value $m(A_1^1) \sim 10^3$ GeV is a value within our reach: The gauge boson A_1^1 can be observed via the characteristic decay $A_1^1 \rightarrow e^+e^-$ (but no $\mu^+\mu^-$) [15] in Z' search experiments at LHC and ILC.

On the other hand, for μ_R , as a trial, let us assume

$$\mu_R \sim M_{Pl} \sim 10^{19} \text{ GeV}, \quad (4.6)$$

where M_{Pl} is the Planck mass. Then, from Eq.(3.21), we obtain

$$\mu_R \langle Y_R^{\alpha\beta} \rangle \sim \langle \hat{Y}_u^{\alpha\gamma} \rangle \langle \hat{Y}_e^{\gamma\beta} \rangle \sim \Lambda_{O3}^2 \sim \Lambda_{GUT}^2, \quad (4.7)$$

which leads to the value of $\langle Y_R^{\alpha\beta} \rangle$

$$\langle Y_R^{\alpha\beta} \rangle \sim (10^{16} \text{ GeV})^2 / (10^{19} \text{ GeV}) \sim 10^{13} \text{ GeV}. \quad (4.8)$$

Thus, we can obtain reasonable neutrino mass scale which is consistent with Eq.(2.21).

Next, we discuss scales of the mass parameters M_5 and M_{10} . The observed relations $m_\tau / \langle H_d \rangle \sim m_b / \langle H_d \rangle \sim 10^{-1}$ (we consider $\tan \beta \sim 10$) suggest

$$M_5 \sim 10 \Lambda_{U3}, \quad (4.9)$$

from Eq.(2.8), where we have regarded the VEVs of Y_e and Y_d as $\langle Y_e \rangle \sim \langle Y_d \rangle \sim \Lambda_{U3}$. On the other hand, the observed relation $m_t / \langle H_u \rangle \sim 1$ means $\langle \hat{Y}_u^{33} \rangle / M_{10} \sim 1$:

$$M_{10} \sim \langle \hat{Y}_u \rangle \sim \Lambda_{O3} \sim 10^{16} \text{ GeV}. \quad (4.10)$$

The assumption (4.9) is somewhat queer, because M_5 and M_{10} are mass parameters of $(\mathbf{5}'_\alpha + \bar{\mathbf{5}}'_\alpha)$ and $(\bar{\mathbf{10}}'_\alpha + \mathbf{10}'_\alpha)$, respectively, and both fields are triplets of O(3). (Note that the constraints (4.9) and (4.10) are phenomenological ones, and they are not based on theoretical reasons.) In this paper, we regard M_5 and M_{10} as merely parameters in the superpotential differently from the realistic masses of $(\mathbf{5}'_\alpha + \bar{\mathbf{5}}'_\alpha)$ and $(\bar{\mathbf{10}}'_\alpha + \mathbf{10}'_\alpha)$. Therefore, for the time being, the value of Λ_{U3} is free, although we consider $\Lambda_{O3} \sim \Lambda_{GUT} \sim 10^{16}$ GeV.

We investigate what value of M_5 is acceptable without blowing up the gauge coupling constants of the $SU(3)_c \times SU(2)_L \times U(1)_Y$ as seen in Table 3. Results are very sensitive to the

M_5	10^8 GeV	10^6 GeV	10^5 GeV	10^4 GeV	10^3 GeV
$M_{10} = 10^{15}$ GeV	10.4	8.2	7.1	6.0	4.9
$M_{10} = 10^{14}$ GeV	7.1	4.9	3.8	2.7	1.6

Table 3: Value of α_5^{-1} at $\mu = \Lambda_{GUT}$ for typical values of M_5 and M_{10} .

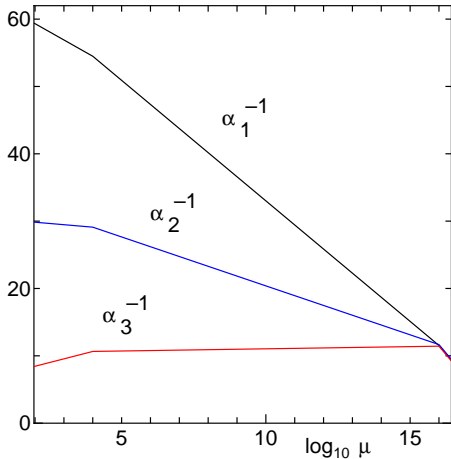


Figure 3: Behavior of gauge coupling constants α_i^{-1} ($i = 1, 2, 3$) in the case of $M_5 = 10^4$ GeV and $M_{10} = 10^{16}$ GeV. For simplicity, we have neglected the SUSY breaking effects at $\mu \sim 10^3$ GeV in this figure.

value of M_{10} . When we take $M_{10} = 10^{14}$ GeV and $M_5 = 10^8$ GeV, 10^6 GeV and 10^3 GeV, we obtain $\alpha_5^{-1} = 7.1, 3.8$ and 1.6 , respectively (α_5 is the SU(5) unification gauge coupling constant) without blowing up. (We show an example of the behavior of the gauge coupling constants in Fig.3.) Therefore, we can choose any low value of Λ_{U3} (but $\Lambda_{U3} \geq 10^2$ GeV) as far as $M_{10} \geq 10^{14}$ GeV and $M_5 \geq 10^3$ GeV are concerned. However, a too low value of Λ_{U3} is still not unlikely. In this paper, we suppose

$$M_5 \sim 10^4 \text{ GeV}, \quad \Lambda_{U3} \sim 10^3 \text{ GeV}. \quad (4.11)$$

As seen in Table 2, we have many U(3) non-singlet fields in the present model, so that the model does not give an asymptotic free theory. The evolution of the U(3) family gauge coupling constant $\alpha_F(\mu)$ is given by

$$\frac{d}{d \log \mu} \alpha_F^{-1}(\mu) = \frac{1}{2\pi} \left(9 - \frac{1}{2} \sum \ell(R) \right), \quad (4.12)$$

where $\ell(R)$ is an index of the representation R of the group U(3). The sum $\sum \ell(R)$ is given by $\sum \ell(R) = 6$ for $\mu < \Lambda_{U3}$ and $\sum \ell(R) = 15 + 15$ for $\Lambda_{U3} < \mu < \Lambda_{GUT}$, where we do not consider contribution from $\bar{\mathbf{5}}'' + \mathbf{5}''$ because they have masses of the order of Λ_{GUT} . We find that $\alpha_F(\mu)$ does not blow up even in the case of $\Lambda_{U3} = 10^3$ GeV unless $\alpha_F(m_Z) > 0.033$. We show behavior of $\alpha_F(\mu)$ in a typical case with $\Lambda_{U3} = 2$ TeV and $\alpha_F(m_Z) = 0.02$ in Fig.4.

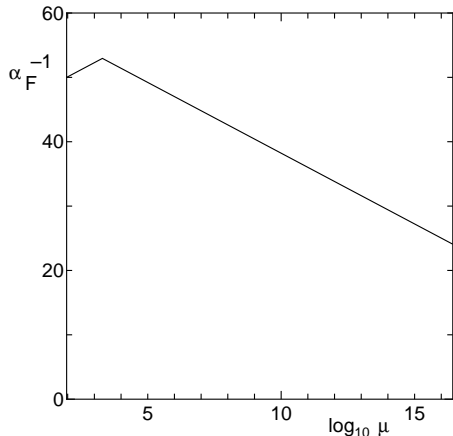


Figure 4: Behavior of the inverse α_F^{-1} of the U(3) family gauge coupling constant in the case with $\Lambda_{U3} = 2$ TeV and $\alpha_F(m_Z) = 0.02$.

We do not discuss the behaviors of gauge coupling constants above $\mu = \Lambda_{GUT}$ because we have no scenario at $\mu > \Lambda_{GUT}$ at present.

5. Concluding Remarks

In conclusion, we have investigated a possibility that a family gauge symmetry U(3) has a comparatively low energy scale by considering an SU(5) compatible yukawaon model with two family symmetries U(3) \times O(3). Since all of yukawaons are SU(5) singlets, the existence of the yukawaons do not affect the SU(5) GUT model, so that we can inherit the successful results in the SU(5) GUT. However, the purpose of the present model is not to discuss problems which are peculiar to the SU(5) GUT scenario. We optimistically consider that those problems will be resolved by considering further higher GUT groups (SO(10) or E₆, and so on) and/or an extra-dimension scenario.

In the present model, we have the following matter fields:

$$(\bar{\mathbf{5}}_i + \mathbf{10}_\alpha + \mathbf{1}_\alpha) + (\bar{\mathbf{5}}''_i + \mathbf{5}''^i) + (\bar{\mathbf{5}}'_\alpha + \mathbf{5}'_\alpha) + (\mathbf{10}'_\alpha + \overline{\mathbf{10}}'_\alpha), \quad (5.1)$$

where i and α are indices of U(3) and O(3), respectively. The particles $(\bar{\mathbf{5}}''_i + \mathbf{5}''^i)$ and $(\overline{\mathbf{10}}' + \mathbf{10}')_\alpha$ have masses of the orders of $\Lambda_{GUT} \sim 10^{16}$ GeV, while $(\mathbf{5}' + \bar{\mathbf{5}}')_\alpha$ have masses of the order of 10^4 GeV. The U(3) family symmetry is broken at $\mu = \Lambda_{U3} \sim 10^3$ GeV.

The most notable result is that we have been able to consider a double seesaw mechanism for up-quark mass generation as shown in Fig.2 by introducing O(3) family symmetry. (If we consider U(3) \times U(3) family symmetries, we cannot obtain the effective Yukawa interaction (2.12).) As a result, the $Y_d - \hat{Y}_u$ corresponding has been improved as seen in Eq.(1.15). Also, by considering that the fundamental yukawaon Φ_e is transformed a triplet (doublet + singlet) of a permutation symmetry as defined in Eq.(3.3), our model without a cutoff Λ can take more simple forms.

In this paper, we did not give numerical results on the basis of the present model, because the phenomenology is almost the same as the previous model [6]. Phenomenology for the family gauge bosons with the scale 10^3 GeV will be given elsewhere.

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