Neutrino Mass Matrix in a U(3) Yukawaon Model with a New Fundamental VEV Matrix

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In the so-called "Yukawaon" model, it is assumed that Yukawa coupling constants originate in vacuum expectation values (VEVs) $\langle Y_f \rangle$ of scalars Y_f. In the previous Yukawaon model based on an O(3) family symmetry, the VEV matrices $\langle Y_f \rangle$ have been given in terms of a fundamental VEV matrix $\langle \Phi_e \rangle \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$. We propose a new model based on a U(3) family symmetry, in which all quark and lepton mass matrices are described in terms of a new fundamental VEV matrix $\langle \Phi_0 \rangle$ (not $\langle \Phi_e \rangle$ in the previous model). The new model has a simple form of the neutrino mass matrix and, as a result, it predicts reasonably good values not only for neutrino mixing parameters and up-quark mass ratios but also for the ratio of the neutrino mass squared differences $\Delta m^2_{solar}/\Delta m^2_{atm}$.

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I. INTRODUCTION

The observed masses and mixings of the quarks and leptons will provide a promising clue to a unified understanding of those fundamental particles. As a phenomenological mass matrix model of the quarks and leptons, the so-called "Yukawaon" model [1] has been proposed, in which quark and lepton mass matrices are described in terms of a fundamental matrix of vacuum expectation values (VEVs) $\langle \Phi_e \rangle \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, as we give a brief review in Subsection 2.2. Here m_e, m_μ, and m_τ are the masses of the charged leptons. It is shown that the Yukawaon model can give a successful description of the observed neutrino mixing parameters, such as a nearly tribimaximal mixing [2], together with reasonable up-quark mass ratios by using only two adjustable parameters [3]. The model can also give a successful description of the observed Cabibbo-Kobayashi-Maskawa (CKM) mixing [4] and the reasonable down-quark mass ratios, by using some additional parameters [5].

In the present paper, we propose a new and predictable model which is a revised version of the previous Yukawaon model by using U(3) family symmetry. In this model, all of the quark and lepton mass matrices are given in terms of a new fundamental matrix $\langle \Phi_0 \rangle$ (not $\langle \Phi_e \rangle$) as we state in Subsection 2.3. The model has the following features: The charged lepton mass matrix M_e is not given by a bilinear form as $M_e \propto \langle \Phi_0 \rangle \langle \Phi_0 \rangle$, although M_e was given by a bilinear form as $M_e \propto \langle \Phi_e \rangle \langle \Phi_e \rangle$ in the previous model. The neutrino mass matrix, which is related to the up-quark and charged lepton mass matrices, can have a simple form unlike the previous model. As a result, the model predicts a good value for the atmospheric neutrino mixing parameter $\sin^2 2\theta_{atm}$ and a detectable 1-3 neutrino mixing parameter $|U^2_{13}|$ as well as a reasonable value for the ratio of the neutrino mass squared differences $\Delta m^2_{solar}/\Delta m^2_{atm}$ by fitting free parameters of the model to observed values of the quark mass ratio $\sqrt{m_c/m_t}$ and the solar neutrino mixing parametertan$^2\,\theta_{solar}$ as inputs. These merits of new model will be stated in details in Subsection 2.4.

In Sec. II, we give a brief review of basic idea of the Yukawaon model, previous O(3) and the present U(3) Yukawaon model. In Sec. III, superpotentails for Yukawaons and the assignments of the fields in the present U(3) Yukawaon model are presented. In Sec. IV, numerical results of the model are discussed. Sec. V devoted to the concluding remarks.

II. YUKAWAON MODEL

2.1 Basic idea of the Yukawaon model

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In the Yukawaon model, all effective Yukawa coupling constants Y_f^{eff} ($f = u, d, e, \cdots$) are given by VEVs of Yukawaons Y_f as

$$(Y_f^{eff})_{ij} = \frac{y_f}{\Lambda}\langle (Y_f)_{ij}\rangle \qquad (i,j = 1,2,3). \tag{2.1}$$

Here y_f is a coupling constant and Λ is an energy scale of the effective theory. Although the Yukawaon model is a kind of "flavon" models [6], we assume that quarks and leptons are assigned to triplets (and/or anti-triplets) of a non-Abelian symmetry G. (For example, we do not consider that the quark and leptons are $\mathbf{2}+\mathbf{1}$, $\mathbf{1}+\mathbf{1}'+\mathbf{1}''$, and so on, of G.) The Yukawaons Y_f are assumed to be singlets under the conventional gauge symmetries SU(3)$_c\times$SU(2)$_L\times$U(1)$_Y$, and have only family indices. We also do not consider $c_1 Y_f(R_1) + c_2 Y_f(R_2) + \cdots$ instead of a single Y_f, where R_a ($a = 1, 2, \cdots$) are representations of G. It should also be noted that at the energy scale μ such that $\mu < \Lambda$ the effective Yukawa coupling constants Y_f^{eff} evolve as those in the standard model.

When we consider a supersymmetric Yukawaon model, VEV relations among $\langle Y_f\rangle$ are obtained by using SUSY vacuum conditions. For example, the VEV relation $\langle Y_e\rangle \propto \langle \Phi_e\rangle\langle \Phi_e^T\rangle$ was derived from a superpotential among superfields Y_e, Φ_e, and Θ_e given by

$$W_e = \mu_e \mathrm{Tr}[Y_e\Theta_e] + \lambda_e \mathrm{Tr}[\Phi_e\Phi_e\Theta_e], \tag{2.2}$$

in a model with an O(3) family symmetry. Therefore, a SUSY vacuum condition $\partial W/\partial\Theta^e = 0$ leads to a charged lepton mass matrix with a bilinear form

$$\langle Y_e\rangle = -\frac{\lambda_e}{\mu_e}\langle \Phi_e\rangle\langle \Phi_e^T\rangle. \tag{2.3}$$

The purpose in the early stage of the Yukawaon model was to predict the charged lepton mass relation [7]

$$\frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e}+\sqrt{m_\mu}+\sqrt{m_\tau})^2} = \frac{2}{3}. \tag{2.4}$$

Thus the bilinear form (2.3) for the charged lepton mass matrix is indispensable to predict [8] the relation (2.4). However, in this paper, we do not use the relation (2.4), but only use the observed charged lepton mass values as input values. Therefore the bilinear form such as (2.3) is not necessarily required in this paper.

In Eq.(2.2), we have assumed a vacuum with $\langle\Theta^e\rangle = 0$, so that the conditions $\partial W/\partial Y_e = 0$ and $\partial W/\partial \Phi_e = 0$ do not affect other VEV relations obtained from SUSY vacuum conditions $\partial W/\partial \Theta_A = 0$ ($A \neq e$). We assume that the observed SUSY symmetry breaking is induced by a gauge mediation mechanism (not including family symmetry), so that our VEV relations among Yukawaons are still valid in the quark and lepton sectors after the SUSY is broken .

2.2 Review of results in the previous Yukawaon model

For comparison with a new model, we give a brief review of results in the previous Yukawaon model [3] which is described by using an O(3) family symmetry (hereafter, we refer to it as the O(3) model). In the O(3) model, in order to distinguish a Yukawaon Y_f from other Yukawaons, we assign sector charges (U(1)$_X$ charges) as $Q_X(Y_f) = x_f$ for Y_f, $Q_X(f^c) = -x_f$ for f^c, and $Q_X = 0$ for the SU(2)$_L$ doublet fields. The mass matrices M_e, M_u, and M_d for the charged leptons, up- and down-quarks are, respectively, described in terms of Yukawaon VEV matrices $\langle\Phi_e\rangle$ as follows:

$$M_e \propto \langle Y_e\rangle_e \propto \langle\Phi_e\rangle_e\langle\Phi_e\rangle_e, \tag{2.5}$$

$$M_u \propto \langle Y_u\rangle_e \propto \langle\Phi_u\rangle_e\langle\Phi_u\rangle_e, \tag{2.6}$$

$$\langle\Phi_u\rangle_e \propto \langle\Phi_e\rangle_e(\langle E\rangle_e + a_u\langle X\rangle_e)\langle\Phi_e\rangle_e, \tag{2.7}$$

$$M_d \propto \langle Y_d\rangle_e \propto \langle\Phi_e\rangle_e(\langle E\rangle_e + a_d\langle X\rangle_e)\langle\Phi_e\rangle_e, \tag{2.8}$$

where $\langle\Phi_e\rangle_e \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$. Here m_e, m_μ, and m_τ are the masses of the charged leptons and the VEV matrices $\langle E\rangle_e$ and $\langle X\rangle_e$ of the field E and X are assumed to take the forms as

$$\langle E\rangle_e = v_E \mathbf{1} = v_E\begin{pmatrix} 1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}, \quad \langle X\rangle_e = v_X S_3 \equiv \frac{1}{3}v_X\begin{pmatrix} 1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{pmatrix}. \tag{2.9}$$

Here, the index "e" denotes that a VEV matrix $\langle A\rangle$ takes a form $\langle A\rangle_e$ in the diagonal basis of the charged lepton mass matrix M_e. (Hereafter, $\langle A\rangle_f$ denotes a VEV matrix form $\langle A\rangle$ in a diagonal basis of a mass matrix M_f.) The Yukawaon model intends to describe all quark and lepton mass matrices in terms of only a fundamental VEV matrix $\langle\Phi_e\rangle$ of the field Φ_e.

For a neutrino mass matrix M_ν, we assume a seesaw-type mass matrix $M_\nu = m_D M_R^{-1} m_D^T$. Here, the Dirac and the Majorana mass matrices m_D and M_R are respectively obtained as follows:

$$m_D \propto \langle Y_e\rangle_e, \qquad M_R \propto \langle Y_R\rangle_e + m_{0\nu}^{-1}\langle Y_e\rangle_e\langle Y_e\rangle_e, \tag{2.10}$$

$$\langle Y_R\rangle_e \propto \langle\Phi_u\rangle_e\langle P_u\rangle_e\langle Y_e\rangle_e + \langle Y_e\rangle_e\langle P_u\rangle_e\langle\Phi_u\rangle_e + \xi_\nu\left(\langle\Phi_u\rangle_e\langle P_u\rangle_e\langle Y_e\rangle_e + \langle Y_e\rangle_e\langle P_u\rangle_e\langle\Phi_u\rangle_e\right). \tag{2.11}$$

The $m_{0\nu}^{-1}$ term in Eq.(2.10), which has been added because of $Q_X(Y_R) = 2Q_X(Y_e)$, can affect the neutrino masses, but does not the neutrino mixing matrix. Therefore, in the O(3) model, we have not discussed the ratio of neutrino mass squared differences $R_\nu \equiv \Delta m^2_{solar}/\Delta m^2_{atm}$, but confined ourselves only to the lepton mixing matrix. In Eq.(2.11), P_u is introduced as a field with a VEV matrix form

$$\langle P_u\rangle_u \propto \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{2.12}$$

in the basis in which the up-quark mass matrix M_u (also Φ_u) is diagonal. The reason of the existence of the matrix $\langle P_u\rangle_u$ is as follows: If we take a value $a_u \simeq -1.8$ in Eq.(2.7), with taking $v_E = v_X$ for simplicity, we can obtain reasonable up-quark mass ratios $\sqrt{m_c/m_t}$ and $\sqrt{m_u/m_c}$, but signs of the eigenvalues of $\langle\Phi_u\rangle$ take $(+,-,+)$ for $a_u \simeq -1.8$, i.e. $\langle\Phi_u\rangle_u \propto \mathrm{diag}(\sqrt{m_u}, -\sqrt{m_c}, \sqrt{m_t})$. Thus, the introduction of the phase matrix $\langle P_u\rangle$ has been indispensable to give the observed value [9] $\sin^2 2\theta_{atm} \simeq 1$, although it is irrelevant to describe the up-quark mass matrix $\langle Y_u\rangle$ given by Eq.(2.6). The ξ_ν term in Eq.(2.11) has been added from a reason that since Yukawaons are $\mathbf{5}+\mathbf{1}$ of O(3), terms $ACB + BCA$ are also possible in addition to terms $ABC + CBA$.

In summary, we have found that the O(3) model, by adjusting parameters a_u, a_d and ξ_ν, has predicted [3] not only the nearly tribimaximal neutrino mixing [2], but also the reasonable CKM quark mixing. (However, the fitting of the CKM mixings is not so excellent compared with that of the neutrino mixing. This can be overcome by using some additional parameters in the down-quark mass matrix [5].)

2.3 New idea in the present model

In the O(3) model, a fundamental VEV matrix was regarded as $\langle\Phi_e\rangle$. The down-quark mass matrix $\langle Y_d\rangle$ was given by Eq.(2.8), i.e. $\langle Y_d\rangle \propto \langle\Phi_e\rangle(\langle E\rangle + a_d\langle X\rangle)\langle\Phi_e\rangle$. We imagine that a structure of the down-quark mass matrix is similar to that of the charged lepton mass matrix. Then, it is interesting to introduce a fundamental VEV matrix other than $\langle\Phi_e\rangle$ (we denote it by $\langle\Phi_0\rangle$) and to assume that the mass matrices $\langle Y_e\rangle$ and $\langle Y_d\rangle$ take the same form as given by $\langle Y_e\rangle \propto \langle\Phi_0\rangle(\langle E\rangle + a_e\langle X\rangle)\langle\Phi_0\rangle$ and $\langle Y_d\rangle \propto \langle\Phi_0\rangle(\langle E\rangle + a_d\langle X\rangle)\langle\Phi_0\rangle$, respectively. However, this idea, as it is, fails phenomenologically because we cannot obtain reasonable predictions.

Instead, we investigate another possibility in this paper by assuming the following forms:

$$\langle\bar{Y}_e\rangle_0 \propto \langle\bar{\Phi}_0\rangle_0(\langle E'\rangle_0 + a_e\langle X'\rangle_0)\langle\bar{\Phi}_0\rangle_0, \tag{2.13}$$

$$\langle\bar{Y}_d\rangle_0 \propto \langle\bar{\Phi}_0\rangle_0(\langle E\rangle_0 + a_d\langle X\rangle_0)\langle\bar{\Phi}_0\rangle_0, \tag{2.14}$$

$$\langle\bar{\Phi}_u\rangle_0 \propto \langle\bar{\Phi}_0\rangle_0(\langle E\rangle_0 + a_u\langle X\rangle_0)\langle\bar{\Phi}_0\rangle_0, \tag{2.15}$$

where $\langle E'\rangle \propto \langle E\rangle \propto \mathbf{1}$. We can take a diagonal basis of $\langle\bar{\Phi}_0\rangle$ without loosing the generality:

$$\langle\bar{\Phi}_0\rangle_0 = \mathrm{diag}(v_1, v_2, v_3) = v_0\,\mathrm{diag}(x_1, x_2, x_3), \tag{2.16}$$

where we have normalized x_i as $x_1^2 + x_2^2 + x_3^2 = 1$. Here and hereafter, we denote a VEV matrix $\langle A\rangle$ in this base by the index "0" as $\langle A\rangle_0$. Then, we assume that the VEV matrix $\langle X\rangle_0$ takes a democratic form $\langle X\rangle = v_X S_3$, defined by Eq.(2.9).

Note that the VEV matrix $\langle\bar{Y}_e\rangle_0$ in Eq.(2.13) is no more diagonal in this basis, which is different from the case in the basis given in Eq.(2.5). It is an essential assumption that the VEV matrix $(\langle E\rangle_0 + a_u\langle X\rangle_0)$ takes a invariant form of the permutation symmetry S_3 in the diagonal basis of $\langle\bar{\Phi}_0\rangle$. We also assume that the VEV matrix $\langle X'\rangle$ takes a form S', i.e. $\langle X'\rangle = v_{X'}S'$, $(S')^2 = S'$ similarly to S_3, but we consider $S' \neq S_3$ in this paper.

In the expressions given by (2.13) - (2.15), the order of the fields is important. Therefore, in this paper, we have assumed a U(3) family symmetry instead of O(3) and we have denoted fields $\mathbf{6}^*$ and $\mathbf{6}$ of U(3) as $\bar{A}$ and A, respectively. (Therefore, it should be noted that a term $\bar{A}B\bar{C}$ is allowed, but $\bar{A}\bar{C}B$ and $B\bar{A}\bar{C}$ are forbidden.) In the U(3) model, the relation (2.6) is re-expressed as

$$M_u \propto \langle \bar{Y}_u \rangle_0 \propto \langle \bar{\Phi}_u \rangle_0 \langle E^u \rangle_0 \langle \bar{\Phi}_u \rangle_0, \tag{2.17}$$

with $\langle E^u \rangle = v_E \mathbf{1}$.

In this U(3) model, we assume only R charge conservation without introducing $\mathrm{U}(1)_X$ charge, although we assumed $\mathrm{U}(1)_X$ charge in the O(3) model in order to distinguish each Yukawaon from other Yukawaons.

For the right handed neutrino sector ($\bar{Y}_R$), it should be noted that we cannot add $\bar{Y}_e \bar{Y}_e$ term to $\bar{Y}_R$, because the R charge of $\bar{Y}_R$ is not the same as $\bar{Y}_e E_e \bar{Y}_e$, which is different from the case of the O(3) model given by Eq.(2.10). Besides, in this U(3) model, we cannot introduce a ξ_ν term such as in Eq.(2.11). As a result, the Majorana mass matrix M_R is simply given by

$$M_R \propto \langle \bar{Y}_R \rangle_0 \propto \langle \bar{\Phi}_u \rangle_0 \langle E^u \rangle_0 \langle \bar{Y}_e \rangle_0 + \langle \bar{Y}_e \rangle_0 \langle E^u \rangle_0 \langle \bar{\Phi}_u \rangle_0, \tag{2.18}$$

which is different from the case of Eq.(2.11) in the O(3) model.

2.4 What are new results?

Since a case $\langle X' \rangle_0 \propto \langle X \rangle_0$ is ruled out phenomenologically as we already stated, we investigate a case that the VEV matrix of $\langle X' \rangle_0$ in Eq.(2.13) is given by

$$\langle X' \rangle_0 = v_{X'} S_2 \equiv \frac{1}{2} v_{X'} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2.19}$$

Of course, there are no theoretical reasons why $\langle (E + aX) \rangle_0$ takes a invariant form under permutation symmetry S_3 in the quark sector, and why $\langle (E' + aX') \rangle_0$ takes an S_2 invariant form in the lepton sector. This is only a trial hypothesis at present. Nevertheless, as we see in Sec.4, this choice will bring the following fruitful results to the present model: (i) We can predict the neturino mixing $\sin^2 2\theta_{atm} \simeq 1$ consistent with the observed one, without introducing the phase matrix given in the form (2.12). That is, we can use the simple form (2.18) for the right-handed neutrino Majorana mass matrix M_R. (ii) We can fit the observed neutrino mixing [10] $\tan^2 \theta_{solar} \simeq 0.5$ without introducing the ξ_ν term such that appears in Eq.(2.11). (iii) We can give the observed value [11] of

$$R_\nu \equiv \frac{\Delta m^2_{solar}}{\Delta m^2_{atm}} = (3.12^{+0.27}_{-0.23}) \times 10^{-2}, \tag{2.20}$$

because we do not have an additional term $\langle \bar{Y}_e \rangle \langle E \rangle \langle \bar{Y}_e \rangle$ to $\langle \bar{Y}_R \rangle$. (In the O(3) model, we have a term $Y_e Y_e / \Lambda$ in addition to Y_R.) (iv) We predict $|U_{13}|^2 \sim 0.004$ which will be within the reach of future experimental observations. (Note that in the O(3) model we predicted an invisible value of $|U_{13}|^2$, $|U_{13}|^2 \sim 10^{-4}$.)

III. SUPERPOTENTIAL AND R CHARGE ASSIGNMENTS

In this section, we give superpotentails for the Yukawaons and the assignments of the fields in the present $U(3)$ Yukawaon model.

3.1 Superpotential

We assume the following superpotential $W = W_Y + W_e + W_d + W'_u + W_u + W_R + W_E$:

$$W_Y = \frac{y_e}{\Lambda} \ell_i \bar{Y}_e^{ij} e^c_j H_d + \frac{y_\nu}{\Lambda} \ell_i \bar{Y}_e^{ij} \nu^c_j H_u + \lambda_R \nu^c_i \bar{Y}_R^{ij} \nu^c_j$$

$$+ \frac{y_u}{\Lambda} u^c_i \bar{Y}_u^{ij} q_j H_u + \frac{y_d}{\Lambda} d^c_i \bar{Y}_d^{ij} q_j H_d + \mu_H H_u H_d, \tag{3.1}$$

$$W_e = \mu_e \mathrm{Tr}[\bar{Y}_e \Theta^e] + \frac{\lambda_e}{\Lambda} \mathrm{Tr}[\bar{\Phi}_0 (E' + a_e X') \bar{\Phi}_0 \Theta^e], \tag{3.2}$$

$$W_d = \mu_d \mathrm{Tr}[\bar{Y}_d \Theta^d] + \frac{\lambda_d}{\Lambda} \mathrm{Tr}[\bar{\Phi}_0 (E + a_d e^{i\alpha_d} X) \bar{\Phi}_0 \Theta^d], \tag{3.3}$$

$$W'_u = \mu'_u \mathrm{Tr}[\bar{\Phi}_u \Theta^{u\prime}] + \frac{\lambda'_u}{\Lambda} \mathrm{Tr}[\bar{\Phi}_0 (E + a_u e^{i\alpha_u} X) \bar{\Phi}_0 \Theta^{u\prime}], \tag{3.4}$$

$$W_u = \mu_u \mathrm{Tr}[\bar{Y}_u \Theta^u] + \frac{\lambda_u}{\Lambda} \mathrm{Tr}[\bar{\Phi}_u E^u \bar{\Phi}_u \Theta_u], \tag{3.5}$$

$$W_R = \mu_R \mathrm{Tr}[\bar{Y}_R \Theta^R] + \frac{\lambda_R}{\Lambda} \mathrm{Tr}[\left(\bar{\Phi}_u E^u \bar{Y}_e + \bar{Y}_e E^u \bar{\Phi}_u\right) \Theta^R], \tag{3.6}$$

where, in Eq.(3.1), q and ℓ are $\mathrm{SU}(2)_L$ doublet fields, and f^c $(f = u, d, e, \nu)$ are $\mathrm{SU}(2)_L$ singlet fields. The other fields in Eqs.(3.2)-(3.7) have quantum numbers defined in Table I.

For the field E^u, we assume an additional field $\bar{E}_u$, and consider a superpotential with a form

$$W_E = \lambda_E \mathrm{Tr}[E^u \bar{E}_u \Theta_{8+1}] + \lambda'_E \mathrm{Tr}[E^u \bar{E}_u]\, \mathrm{Tr}[\Theta_{8+1}], \tag{3.7}$$

where Θ_{8+1} is a field $\mathbf{8}+\mathbf{1}$ of U(3) with $\langle \Theta_{8+1} \rangle = 0$. The superpotential W_E leads to $\langle E^u \rangle \langle \bar{E}_u \rangle \propto \mathbf{1}$. We assume that the form

$$\langle \bar{E}_u \rangle \propto \langle E^u \rangle = v_E \,\mathrm{diag}(1,1,1), \tag{3.8}$$

is given by a specific form of the solutions $\langle E^u \rangle \langle \bar{E}_u \rangle \propto \mathbf{1}$.

In this paper, we do not discuss a superpotential which gives the observed charged lepton mass spectrum. We only use the observed charged lepton mass values as the input values in $\langle \bar{Y}_e \rangle_e$.

3.2 R charge assignments

In the present model, as well as in the O(3) model, we construct a model without introducing a Yukawaon Y_ν by replacing Y_ν by Y_e. The simple way to guarantee that the Yukawaon Y_e couples not only to the charged lepton sector but also to the Dirac neutrino sector is to introduce the following R charge assignment,

$$R(\nu^c) = R(e^c) \equiv r_e, \tag{3.9}$$

$$R(H_u) = R(H_d) = 1. \tag{3.10}$$

The R charge of $(\bar{E}_u E^u)$ is free parameter in the form (3.7). For simplicity, we take

$$R(\bar{E}_u E^u) = R(\Theta_{8+1}) = 1. \tag{3.11}$$

Hereafter, we will denote $R(\bar{E}_u)$ and $R(E^u)$ as $\bar{r}_E$ and $1 - \bar{r}_E$, respectively. Each Yukawaon is distinguished from other Yukawaons by the R charges. If we define a parameter n as

$$n \equiv 2[R(\bar{Y}_R) - R(\bar{Y}_e)], \tag{3.12}$$

then, we can express the R charges of the other fields from Eq.(3.1) as follows:

$$R(\ell) = r_e + \frac{1}{2}(n-2), \tag{3.13}$$

$$R(\bar{Y}_e) = \frac{1}{2}(4-n) - 2r_e, \tag{3.14}$$

$$R(\bar{Y}_R) = 2 - 2r_e, \tag{3.15}$$

$$R(\bar{Y}_u) = n - 1 + \bar{r}_E, \tag{3.16}$$

$$R(\bar{Y}_d) = \frac{1}{2}(n-2) + \bar{r}_E, \tag{3.17}$$

$$R(u^c) + R(q) = 2 - n - \bar{r}_E, \tag{3.18}$$

$$R(d^c) + R(q) = 2 - \frac{1}{2}n - \bar{r}_E. \tag{3.19}$$

From Eqs.(3.5) and (3.6), we obtain

$$R(\bar{Y}_u) = 2R(\bar{\Phi}_u) + R(E^u), \tag{3.20}$$

$$R(\bar{Y}_R) = R(\bar{\Phi}_u) + R(\bar{Y}_e) + R(E^u), \tag{3.21}$$

respectively. From Eqs.(3.20) and (3.21), we obtain a relation

$$R(\bar{Y}_u) = n - R(E^u) = n - 1 + \bar{r}_E. \tag{3.22}$$

The relation (3.22) leads to

$$R(\bar{Y}_u \Theta^u) = (n-1)R(\bar{E}_u E^u) + R(\bar{E}_u \Theta^u). \tag{3.23}$$

Only when the value n is a positive integer, Eq.(3.23) means that an additional term

$$\frac{\lambda_{0u}}{\Lambda^{2n-1}} \mathrm{Tr}[(\bar{E}_u E^u)^{n-1} \bar{E} \Theta^u], \tag{3.24}$$

can appear in the expression (3.5). Note that if n is not a positive integer, the factor $(\bar{E}_u E^u)^{n-1}$ does not have a physical meaning, because a term with $(E^u)^{-1}$ cannot appear in the superpotential terms. Therefore, the n defined in Eq.(3.12) is allowed only for $n = 1, 2, \cdots$.

As we see in Eqs.(3.13) - (3.19), these R charges are described by four parameters r_e, $R(q)$, $\bar{r}_E$ and n. Therefore, in order to fix these R charge values, we have to assume four constraints for these R charges. On the other hand, the fields $\bar{Y}_e$, $\bar{Y}_R$, $\bar{Y}_u$, $\bar{Y}_d$, $\bar{\Phi}_u$, and $\bar{E}_u$ are gauge singlets, so that they must be distinguished only by R charges. We can choose a suitable parameter set $(n, r_e, r_q, \bar{r}_E)$. Here, let us demonstrate an example of R charge assignments, although it is not the purpose of the present paper to give such an explicit R charge assignment.

For example, we put the following working hypothesis:

$$R(\bar{Y}_\nu) + R(\bar{Y}_e) = 0, \quad R(\bar{Y}_u) + R(\bar{Y}_d) = 0, \tag{3.25}$$

$$R(u^c) + R(d^c) = 0, \quad R(\nu^c) + R(e^c) = 0. \tag{3.26}$$

The constraint (3.25) is an analogy that the Yukawa coupling constants in the standard model do not have R charges. The constraints (3.25) and (3.26) leads to the relation $R(\ell) = R(q) = 1$. Of course, since the Yukawaon $\bar{Y}_\nu$ has been replaced by $\bar{Y}_e$ in the present model, the first constraint in Eq.(3.25) reads as $R(\bar{Y}_e) = 0$, and since $R(\nu^c) = R(e^c)$ in the model, the second constraint in Eq.(3.26) reads as $R(e^c) = 0$. Since $R(\bar{Y}_e)$ is given by Eq.(3.15), the requirement $R(\bar{Y}_e) = 0$ together with $R(e^c) = 0$ requires $n = 4$. Thus, the constraints (3.25) and (3.26) fix the parameters $(n, r_e, r_q, \bar{r}_E)$ as

$$n = 4, \quad R(e^c) = 0, \quad R(q) = 1, \quad R(\bar{E}_u) = -2. \tag{3.27}$$

The explicit values of these R values are listed in Table I. Since the R charges of $\bar{\Phi}_0$, X' and X are still free parameters, we take $R(\bar{\Phi}_0) = \frac{1}{2}$ for simplicity. As we see in Table I, the fields $\bar{Y}_e$, $\bar{Y}_R$, $\bar{Y}_u$, $\bar{Y}_d$ and $\bar{E}_u$ can safely have different R charges from each other.

Thus, the assumption can lead to plausible R charge values (3.27), so that we consider that the assumption is reasonable. Now we have an additional term,

$$\frac{\lambda_{0u}}{\Lambda^5} \mathrm{Tr}[(\bar{E}_u E^u)^3 \bar{E}_u \Theta^u], \tag{3.28}$$

which should be included in M_u given in Eq.(3.5).

	H_u	H_d	E^u	$\bar{E}_u$	Θ_{8+1}
U(3)	**1**	**1**	**6**	$\mathbf{6}^*$	$\mathbf{8+1}$
R	1	1	$1-\bar{r}_E$	$\bar{r}_E$	1
Model	1	1	3	-2	1

ℓ	e^c	ν^c	$\bar{Y}_e$	$\bar{\Phi}_0$	E'	X'	Θ^e	$\bar{Y}_R$	Θ^R
3	**3**	**3**	$\mathbf{6}^*$	$\mathbf{6}^*$	**6**	**6**	**6**	$\mathbf{6}^*$	**6**
r_ℓ	r_e	r_e	r_{Ye}	r_0	$r_{X'}$	$r_{X'}$	$2-r_{Ye}$	r_R	$2-r_R$
1	0	0	0	$\frac{1}{2}$	-1	-1	2	2	0

q	u^c	d^c	$\bar{Y}_u$	$\bar{\Phi}_u$	Θ^u	$\Theta^{u\prime}$	$\bar{Y}_d$	Θ^d	E	X
3	**3**	**3**	$\mathbf{6}^*$	$\mathbf{6}^*$	**6**	**6**	$\mathbf{6}^*$	**6**	**6**	**6**
r_q	r_u	r_d	r_{Yu}	r_{Yd}	$2-r_{Yu}$	$2-r_{Yd}$	r_{Yd}	$2-r_{Yd}$	r_X	r_X
1	-1	$+1$	$+1$	-1	1	3	-1	3	-2	-2

TABLE I: Assignments of R charges, where $r_{X'} = r_{Ye} - 2r_0$, $r_X = r_{Yd} - 2r_0$ and $R(\bar{\Phi}u) = R(\bar{Y}_d) \equiv r_{Yd}$. The values in the third raw denote R charge values in a special case under the assumptions (3.11) and (3.25). For more details, see Eqs.(3.9) - (3.28) in the text.

3.3 VEV relations

Under the assumption that all Θ fields take $\langle\Theta\rangle = 0$, SUSY vacuum conditions lead to the following VEV relations from Eqs.(3.2)-(3.6) with (3.28):

$$M_e \propto \langle \bar{Y}_e \rangle_0 \propto \langle \bar{\Phi}_0 \rangle_0 (\mathbf{1} + a_e S_2) \langle \bar{\Phi}_0 \rangle_0, \tag{3.29}$$

$$M_d \propto \langle \bar{Y}_d \rangle_0 \propto \langle \bar{\Phi}_0 \rangle_0 (\mathbf{1} + a_d e^{i\alpha_d} S_3) \langle \bar{\Phi}_0 \rangle_0, \tag{3.30}$$

$$M_u^{1/2} \propto \langle \bar{\Phi}_u \rangle_0 \propto \langle \bar{\Phi}_0 \rangle_0 (\mathbf{1} + a_u e^{i\alpha_u} S_3) \langle \bar{\Phi}_0 \rangle_0, \tag{3.31}$$

$$M_u \propto \langle \bar{Y}_u \rangle_0 \propto \langle \bar{\Phi}_u \rangle_0 \cdot \mathbf{1} \cdot \langle \bar{\Phi}_u \rangle_0 + (v_{\Phi u})^2 \xi_u \mathbf{1}, \tag{3.32}$$

$$M_R \propto \langle \bar{Y}_R \rangle_0 \propto \langle \bar{\Phi}_u \rangle_0 \cdot \mathbf{1} \cdot \langle \bar{Y}_e \rangle_0 + \langle \bar{Y}_e \rangle_0 \cdot \mathbf{1} \cdot \langle \bar{\Phi}_u \rangle_0. \tag{3.33}$$

Here the numerical matrices S_3 and S_2 are defined by

$$S_3 = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad S_2 = \frac{1}{2}\begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{3.34}$$

In obtaining the mixing matrices, the common coefficients are not important. Here we have taken $v_{E'} = v_{X'}$ and $v_E = v_X$ for simplicity. The ξ_u term in Eq.(3.32) comes from the new term given in Eq.(3.28). This term contributes to the up-quark mass ratios, while not to the up-quark mixing matrix, so that it does not change the predictions for the neutrino mixing parameters. We suppose that the contribution from such the higher dimensional term (3.28) is considerably small, so that it also does not visibly affect the up-quark mass ratio m_c/m_t, although it can slightly affect m_u/m_c.

IV. NUMERICAL RESULTS IN THE UP-QUARK AND NEUTRINO MASS MATRICES

In this section, we investigate whether the new VEV matrix relations (3.29) - (3.33) can well describe the observed neutrino mixing parameters together with the observed up-quark mass ratios or not.

Since the charged lepton mass matrix given by Eq.(3.29) is not diagonal, the lepton mixing matrix U in the present conventions is defined by

$$U = U_{eL}^\dagger U_{\nu L}, \tag{4.1}$$

where U_{eL} and $U_{\nu L}$ are defined by

$$U_{eL}^\dagger \langle \bar{Y}_e \rangle_0 U_{eL} = \langle \bar{Y}_e \rangle^{diag} \equiv \langle \bar{Y}_e \rangle_e, \tag{4.2}$$

$$U_{\nu L}^\dagger (M_\nu^\dagger M_\nu) U_{\nu L} = (M_\nu^\dagger M_\nu)^{diag}, \tag{4.3}$$

and M_ν is given by

$$M_\nu = \frac{y_\nu^2}{\lambda_R} \left(\frac{\langle H_u^0 \rangle}{\Lambda} \right)^2 \langle \bar{Y}_e \rangle_0 \langle \bar{Y}_R \rangle_0^{-1} \langle \bar{Y}_e \rangle_0. \tag{4.4}$$

Neutrino mixing parameters we discuss are $\tan^2\theta_{solar} = |U_{12}|^2/|U_{11}|^2$, $\sin^2 2\theta_{atm} = 4|U_{23}|^2|U_{33}|^2$, and $|U_{13}|^2$. Here U_{ij} are the matrix elements of the lepton mixing matrix defined by (4.1)

The M_u in (3.32) is diagonalized as

$$U_{uL}^\dagger (M_u^\dagger M_u) U_{uL} = (M_u^\dagger M_u)^{diag}. \tag{4.5}$$

Here U_{uL} is a mixing matrix among left-handed up-quarks u_{Li} . (In the present paper, the mass matrices (i.e. $\langle Y_f \rangle$) are defined by Eq.(3.1). Therefore, the conventions of the mixing matrices are somewhat changed from the conventional ones.) Note that since the VEV matrix $\langle \bar{\Phi}_u \rangle_0$ is complex and $\langle \bar{Y}_u \rangle_0$ is given by Eq.(3.32), the diagonalization of the up-quark mass matrix must be done by Eq.(4.5).

4.1 Parameters in the model

The mass matrices for quarks and neutrinos in the O(3) model have been described in terms of the fundamental VEV matrix $\langle \Phi_e \rangle$. On the other hand, the fundamental VEV matrix in the present model is $\langle \bar{\Phi}_0 \rangle$ defined by Eq.(3.29) in which we have new parameter a_e. Thus the number of parameters are increased by one in addition to the thee charged lepton masses. Note that we cannot bring neither the ξ_ν term given in Eq.(2.11) nor $\langle P_u \rangle_u$ defined in Eq.(2.12) into the present model.

The VEV of $\langle \Phi_0 \rangle_0 = \mathrm{diag}(v_1, v_2, v_3)$ is related to the charged lepton mass matrix M_e as follows:

$$M_e = k \begin{pmatrix} \left(1+\frac{1}{2}a_e\right) v_1^2 & \frac{1}{2}a_e v_1 v_2 & 0 \\ \frac{1}{2}a_e v_1 v_2 & \left(1+\frac{1}{2}a_e\right) v_2^2 & 0 \\ 0 & 0 & v_3^2 \end{pmatrix}, \tag{4.6}$$

where $k = -(\lambda_e/\mu_e\Lambda)(y_e\langle H_d^0\rangle/\Lambda)v_{X'}$, so that we obtain

$$m_e + m_\mu = k \left(1 + \frac{1}{2}a_e\right)(v_1^2 + v_2^2), \tag{4.7}$$

$$m_e m_\mu = k^2(1 + a_e)v_1^2 v_2^2, \tag{4.8}$$

and $m_\tau = k v_3^2$. Therefore, when we give a value of the parameter a_e, the values of v_i are completely determined by the input values of the charged lepton masses.

Thus, in the present model, we have 4 parameters a_e, a_u, α_u and ξ_u (except for the input values m_e, m_μ, and m_τ) for the up-quark and neutrino mass matrices. On the other hand, the number of the predictable values are six, i.e., $\sqrt{m_u/m_c}$, $\sqrt{m_c/m_t}$, $\tan^2\theta_{solar}$, $\sin^2 2\theta_{atm}$, $|U_{13}|^2$, and R_ν. (Note that we cannot add a term $\nu^c \bar{Y}_e E \bar{Y}_e \nu^c$ to $\nu^c \bar{Y}_R \nu^c$ in Eq.(3.1) because of R charge conservation in the present U(3) model, although we could add $\nu^c Y_e Y_e \nu^c$ to $\nu^c Y_R \nu^c$ in the O(3) model. Therefore, in the O(3) model, the value R_ν was always adjustable by an additional parameter which is a coefficient of the $Y_e Y_e$ term.)

The term $\xi_u \mathbf{1}$ in M_u given in Eq.(3.32) does not affect the up-quark mixing matrix, so that it also affects neither quark or lepton mixing matrices. Since we suppose $|\xi_u|^2 \ll 1$, the term almost does not affect $\sqrt{m_c/m_t}$, although it can slightly affect $\sqrt{m_u/m_c}$. As a result, the present model predicts five observables $\sqrt{m_c/m_t}$, $\tan^2\theta_{solar}$, $\sin^2 2\theta_{atm}$, $|U_{13}|^2$, and R_ν by using the three parameters (a_e, a_u, α_u). In other words, the value of $\sqrt{m_u/m_c}$ is not " prediction", and it is a quantity which can be adjustable by the additional parameter ξ_u freely. More precisely speaking, as seen in the next section, our three parameters (a_e, a_u, α_u) are determined only by fitting two observed values $\sqrt{m_c/m_t}$ and $\tan^2\theta_{solar}$, and thereby, the values of the other three quantities $\sin^2 2\theta_{atm}$, $|U_{13}|^2$, and R_ν will be predicted.

4.2 Numerical results

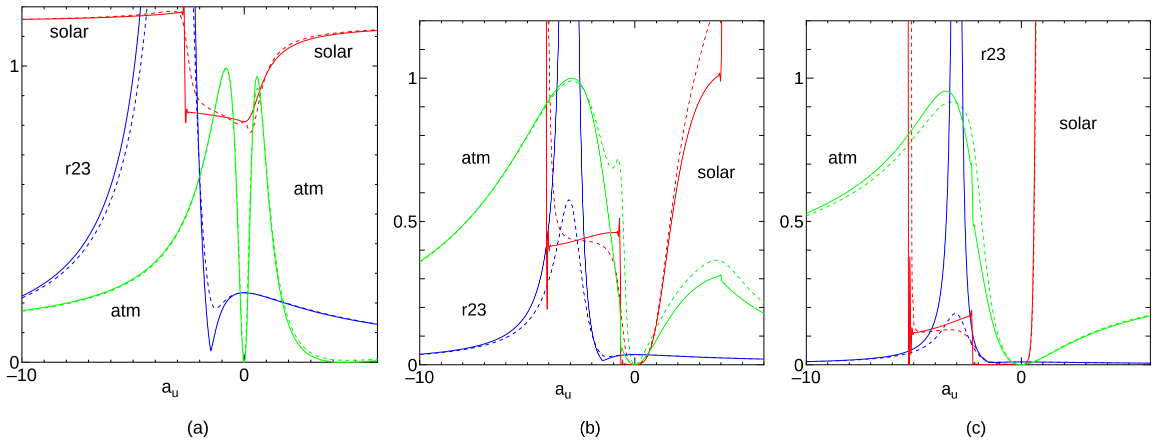

FIG. 1: $\sqrt{m_c/m_t}$, $\tan^2\theta_{solar}$, and $\sin^2 2\theta_{atm}$ versus a parameter a_u for typical parameter values $a_e = 3$ (Fig.1 (a)), $a_e = 30$ (Fig.1 (b)), and $a_e = 100$ (Fig.1 (c)) with $\alpha_u = 0°$ (solid curves) and $\alpha_u = 15°$ (dashed curves). Curves "r23", "solar", and "atm" denote "r23"$= \sqrt{m_c/m_t} \times 10$, "solar"$= \tan^2\theta_{solar}$, and "atm"$= \sin^2 2\theta_{atm}$, respectively.

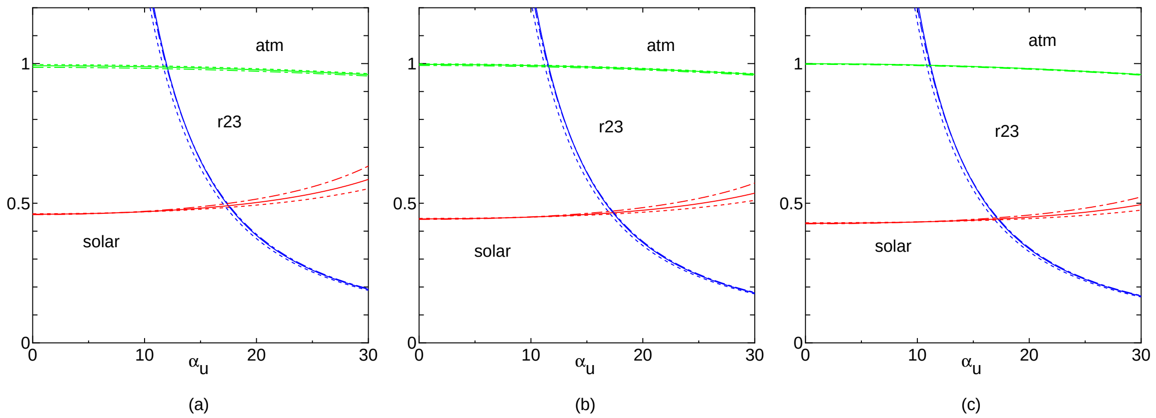

FIG. 2: $\sqrt{m_c/m_t}$, $\tan^2\theta_{solar}$, and $\sin^2 2\theta_{atm}$ versus a parameter α_u for typical parameter values $a_e = 26$ (Fig.2 (a)), $a_e = 28$ (Fig.2 (b)), and $a_e = 30$ (Fig.2 (c)) with $a_u = -2.9$ (dashed curves), $a_u = -3.0$ (solid curves), and $a_u = -3.1$ (dot-dashed curves). Curves "r23", "solar", and "atm" denote "r23"$= \sqrt{m_c/m_t} \times 10$, "solar"$= \tan^2\theta_{solar}$, and "atm"$= \sin^2 2\theta_{atm}$, respectively.

Now let us show the results of numerical analysis of the model. First, we show, in Fig. 1, the a_u dependences of the quantities $\sqrt{m_c/m_t}$, $\tan^2\theta_{solar}$, and $\sin^2 2\theta_{atm}$ with taking typical values of $a_e = 3,\ 30,\ 100$ and $\alpha_u = 0°,\ 15°$ in order to see rough parameter behaviors.

As seen in Fig. 1, we can find that (i) the value of $\sqrt{m_c/m_t}$ takes a maximum value at $a_u \sim -3$ insensitively to the values of a_e and α_u; (ii) since the maximum value of $\sin^2 2\theta_{atm}$ shows $\sin^2 2\theta_{atm} \simeq 1$ which is in favor of the observed value, we must search for a parameter set (a_e, a_u, α_u) which gives a maximum value of $\sin^2 2\theta_{atm}$; (iii) a case with a small value of a_e gives a large value of $\tan^2\theta_{solar}$ compared with the observed value $\tan^2\theta_{solar} \sim 0.5$ (see Fig.1 (a)), so that such a case is ruled out; on the other hand, a case with a large value of a_e gives a small $\tan^2\theta_{solar}$ (see Fig.1 (c)), so that such a case is also ruled out; (iv) as a result, a region of (a_e, a_u) which can give $\sin^2 2\theta_{atm} \simeq 1$ and $\tan^2\theta_{solar} \sim 0.5$ is $(a_e, a_u) \sim (30, -3)$.

Next, in order to determine parameter values (a_e, a_u, α_u), let us illustrate, in Fig. 2, the α_u behaviors of $\sqrt{m_c/m_t}$, $\tan^2\theta_{solar}$ and $\sin^2 2\theta_{atm}$ at $a_e \sim 28$ and $a_u \sim -3$. From Fig.2, we search for the value α_u which gives the observed value [12] $\sqrt{m_c/m_t} = 0.0600^{+0.0045}_{-0.0047}$ at $\mu = m_Z$. We find that the value $\alpha_u \simeq 15°$ can give a reasonable fit $\sqrt{m_c/m_t} = 0.0600$ insensitively to the other parameters.

Therefore, by fixing the value $\alpha_u = 15°$, we illustrate the contour lines of $\sqrt{m_c/m_t}$ and $\tan^2\theta_{solar}$ in the (a_e, a_u) plane in Fig.3. The curves denote (a_e, a_u) which gives the observed values $\sqrt{m_c/m_t} = 0.0600^{+0.0045}_{-0.0047}$ [12] and $\tan^2\theta^{obs}_{solar} = 0.47^{+0.05}_{-0.03}$ [11]. As seen in Fig.3, we have two intersection points of the curves of $\sqrt{m_c/m_t}$ and $\tan^2\theta_{solar}$.

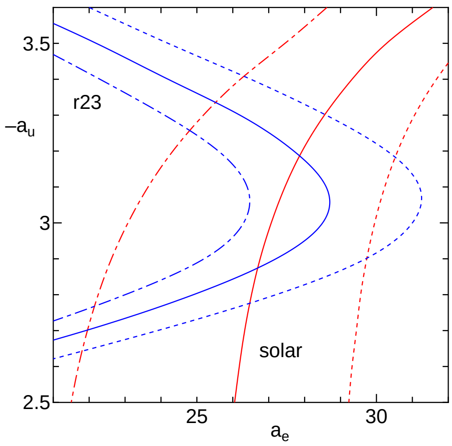

FIG. 3: Contour lines of $\sqrt{m_c/m_t}$ and $\tan^2\theta_{solar}$ in the (a_e, a_u) plane in the case of $\alpha_u = 15°$. As input values, $\sqrt{m_c/m_t} = 0.0600^{+0.0045}_{-0.0047}$ and $\tan^2\theta^{obs}_{solar} = 0.47^{+0.05}_{-0.03}$ have been used. Curves with dot-dash, solid, and dash denote the upper, center, and lower observed values, respectively. Curves "r23" and "solar" denote "r23"$= \sqrt{m_c/m_t} \times 10$ and "solar"$= \tan^2\theta_{solar}$, respectively.

Input (a_e, a_u, α_u)	$\sqrt{m_c/m_t}$	$\tan^2\theta_{solar}$	$\sin^2 2\theta_{atm}$	$\lvert U_{13}\rvert^2$	R_ν
$(26.7, -2.88, 15°)$	0.0603	0.470	0.990	0.00377	0.0303
$(27.9, -3.18, 15°)$	0.0601	0.469	0.980	0.00264	0.0204
upper	+0.0045	+0.05			+0.0027
Observed value	0.0600	0.47	> 0.92 [11]	< 0.039 [11]	0.0312
lower	−0.0047	−0.03			−0.0023

TABLE II: Predicted values for the parameter values (a_e, a_u, α_u).

For the center values $\sqrt{m_c/m_t} = 0.060$ and $\tan^2\theta_{solar} = 0.47$, the solutions (a_e, a_u, α_u) are

$$(26.7, -2.88, 15°), \qquad (27.9, -3.18, 15°). \tag{4.9}$$

We list our prediction values for these parameter solutions in Table I. Of the two solutions obtained from the input data $\sqrt{m_c/m_t}$ and $\tan^2\theta_{solar}$, Table II suggests that we should take the former one considering the observed value of R_ν, Eq.(2.20). For reference, we also illustrate the behavior of predicted values for input values (a_e, a_u, α_u) around the parameter solutions (4.9) in Fig.4. As seen in Fig. 4, the predicted values $\sin^2\theta_{atm}$ and $\tan^2\theta_{solar}$ are insensitive to the parameter values a_e, a_u, and α_u around the values $(a_e, a_u, \alpha_u) = (26.7, -2.88, 15°)$. However, $\sqrt{m_c/m_t}$ and $|U_{13}|^2$ (and also R_ν) are somewhat dependent on these parameters. Since these parameter values are mainly obtained by taking the input value $\sqrt{m_c/m_t} = 0.0600$, if the input value change, then the predicted values will also change.

So far, we have not discussed the value of m_u/m_c. In the present model, the value of m_u/m_c is always adjustable by the parameter ξ_u given in Eq.(3.32) without affecting other predicted values. In order to fit the predicted value of $\sqrt{m_u/m_c}$ to the observed value [12] $\sqrt{m_u/m_c} = 0.0453^{+0.012}_{-0.010}$, we choose ξ_u as $\xi_u = 3.8 \times 10^{-7}$. As seen in Table III, the value of ξ_u almost does not change the numerical predictions given in Table II.

V. CONCLUDING REMARKS

In this paper we have constructed a new Yukawaon model based on U(3) family symmetry, in which the Yukawaon VEV matrices are described in terms of a new fundamental VEV matrix $\langle\Phi_0\rangle$. For example, the Yukawaon VEV matrix $\langle Y_e\rangle$ for the charged leptons is given by (3.29) which has the structure of $(\mathbf{1} + a_e S_2)$ in it with a new parameter a_e. This structure in $\langle Y_e\rangle$ has been chosen from a phenomenological point of view and there is no reason why $\langle Y_e\rangle$ takes such a form. Nevertheless if we accept the form (3.29), then we can obtain a simple form of VEV matrix $\langle Y_R\rangle$ for

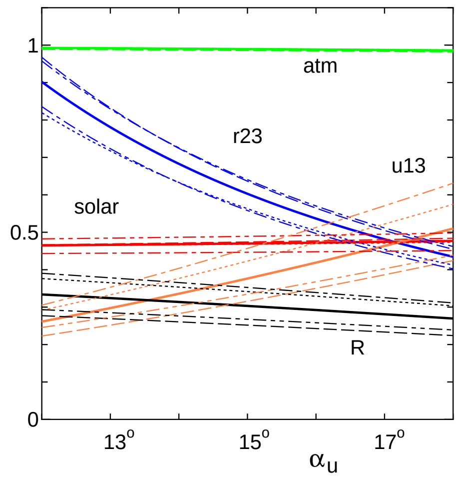

FIG. 4: Predicted values versus a parameter α_u for $a_e = 26.7$ and $a_u = -2.88$. Curves "r23", "solar", "atm", "u13", and "R" denote "r23"$= \sqrt{m_c/m_t} \times 10$, "solar"$= \tan^2\theta_{solar}$, "atm"$= \sin^2 2\theta_{atm}$, "u13"$= |U_{13}|^2 \times 100$, and "R"$= R_\nu \times 10$, respectively. For reference, curves for $(a_e, a_u) = (26.7, -3.00)$ (dash curve), $(26.7, -2.80)$ (dot curve), $(29, -2.88)$ (dot dash curve), and $(25, -2.88)$ (2-dot dash curve) are illustrated in addition to the curve (solid) for $(26.7, -2.88)$.

Input ξ_u	$\sqrt{m_u/m_c}$	$\sqrt{m_c/m_t}$	$\tan^2\theta_{solar}$	$\sin^2 2\theta_{atm}$	$\|U_{13}\|^2$	R_ν
0	0.0180	0.0603	0.470	0.990	0.00377	0.0303
3.8×10^{-7}	0.0454	0.0602	0.470	0.990	0.00376	0.0303

TABLE III: Predicted values versus ξ_u parameter. Other parameters (a_e, a_u, α_u) have taken the same values $(26.7, -2.88, 15^\circ)$ as those in Table II.

right-handed neutrinos without introducing the somewhat strange VEV matrix P_u and ξ_u term that were introduced in the O(3) model [see (2.11)] to get the observed nearly tribimaximal neutrino mixing. As a result, it turns out that our U(3) model have the following interesting features: (i) It predicts $|U_{13}|^2 \sim 0.004$, which is within our experimental reach. In the previous O(3) model, the predicted value of $|U_{13}|^2$ was invisibly small, i.e. $|U_{13}|^2 \sim 10^{-4}$. (ii) It also predicts a reasonable value of $R_\nu \equiv \Delta m^2_{solar}/\Delta m^2_{atm} \sim 0.03$ in contrast to the case of the O(3) model in which we cannot predict R_ν because of the possible presence of an additional free parameter.

Finally let us comment on the structure of $\langle Y_e \rangle$ and on the CKM mixing parameters in our U(3) model.

The purpose of the earlier Yukawaon model was to understand the charged lepton mass relation (2.4), so that it was essential that $\langle Y_e \rangle$ was given by a bilinear form $\langle Y_e \rangle = k_e \langle \Phi_e \rangle \langle \Phi_e \rangle$, Eq.(2.5). In contrast to such a previous model, in the present model, $\langle Y_e \rangle$ is given by the form (3.29). However, this is not so vital alteration. In the Sumino model [13], the effective Yukawa coupling constant of the charged leptons Y_e^{eff} has been given by a form $(Y_e^{eff})^{ij} \propto \langle \Phi_e^{i\alpha} \rangle \langle \Phi_e^{T\alpha j} \rangle$, [$\alpha$ is an index of O(3)] with a non-diagonal form of $\langle \Phi_e \rangle$, and thereby, the charged lepton mass relation (2.4) has been derived. Therefore, according to the Sumino model, if we define

$$\langle \bar{\Phi}_e^{i\alpha} \rangle = \frac{1}{\Lambda} \langle \bar{\Phi}_0^{i\beta} \rangle (\langle (E')^{\beta\alpha} \rangle + a'_e \langle (X')^{\beta\alpha} \rangle), \tag{5.1}$$

then we can again express the VEV matrix $\langle \bar{Y}_e \rangle$ by a bilinear form

$$\langle \bar{Y}_e^{i\alpha} \rangle = k_e \langle \bar{\Phi}_e^{i\alpha} \rangle \langle \bar{\Phi}_e^{T\alpha j} \rangle = k_e \frac{v_E^2}{\Lambda^2} \langle \bar{\Phi}_0^{i\alpha} \rangle \left(\mathbf{1} + a_e S_2 \right)^{\alpha\beta} \langle \bar{\Phi}_0^{T\beta j} \rangle, \tag{5.2}$$

where, for simplicity, we have taken $v_{E'} = v_{X'}$ and $a_e = a'_e(a'_e + 2)$ because of $(S_2)^2 = S_2$. Although we have obtained the parameter value $a_e = 26.7$ from the phenomenological analysis in the previous section, the value is considerably large compared with the value $a_u = -2.88$ in the quark sector. If we accept the mechanism (5.2), then we obtain a mild value $a'_e = 4.26$.

We have discussed only mixings and masses in the lepton sector (and up-quark masses), and we have not discussed the CKM mixing parameters. Since a naive extension of the O(3) Yukawaon model cannot exceed the previous O(3) model in the numerical fits of the CKM mixing parameters, we need a further improvement of the VEV structure in Y_d. In other words, whether we can build a model of Y_d which yields reasonable CKM mixing parameters with keeping the present successful results on the lepton mixing parameters and up-quark mass ratios or not is a touchstone (diverging point) as to whether the ad hoc structure (3.29) and the numerical success in the present model are true or accidental. This is our next task.

Note added. When this manuscript was almost completed, Ref.[14] appeared. It reports an experimental lower bound for $|U_{13}|^2$, $|U_{13}|^2 > 0.0076$ (90% CL), which is two times larger than our predicted value $|U_{13}|^2 = 0.0038$. If we take this bound seriously, we may need some mechanism to lead a further enhancement of $|U_{13}|^2$ in the present model.

Acknowledgment

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