Quark Mass Matrix Model for Neutrino Mixing

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Abstract. For the purpose of deriving the observed nearly tribimaximal neutrino mixing, a possible quark mass matrix model is investigated based on a supersymmetric yukawaon model, where a neutrino mass matrix has been related to the up-quark mass. Therefore, a quark mass matrix model is proposed in this paper. As a result, quark and lepton mixing matrices and quark mass ratios are described by quite few parameters, e.g. five observable quantities (two up-quark mass ratios and three neutrino mixing parameters) are excellently fitted by two parameters and the CKM mixing parameters and down-quark mass ratios are given under the other two parameters.

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WHAT IS A YUKAWAON MODEL?

Usually, the tribimaximal neutrino mixing [1] has been derived from scenarios based on discrete symmetries. In contrast to the conventional approach, in the present work, we will try another approach without such discrete symmetries. For the purpose of deriving the observed nearly tribimaximal neutrino mixing, a quark mass matrix model is investigated based on the so-called “supersymmetric yukawaon" model. In this model, Yukawa coupling constants Y_f $(f = u, d, e, \nu)$ can be related among them. [Although, in grand unification theory (GUT), the mass matrices can also be related among them, e.g. as $M_d = M_e$, the relations are linear. In contrast to GUT models, in the yukawaon model, relations among Y_f are not always linear. See Eqs.(6), (15) and (16) later.] For example, a neutrino mass matrix has been related to the up-quark mass matrix M_u. Therefore, in the present paper, a quark mass matrix model is proposed in order to give the observed neutrino mixing. As we see later, quark and lepton mixing matrices and quark mass ratios will be described by quite few parameters, e.g. five observable quantities (two up-quark mass ratios and three neutrino mixing parameters) will be excellently fitted by two parameters. Also, the Cabibbo-Kobayayashi-Maskawa (CKM) mixing parameters and down-quark mass ratios will be given under the other two parameters.

First, let us give a short review of the yukawaon model. In the standard model of quarks and leptons, the Yukawa coupling constants Y_f are fundamental constants in the theory. Even if we assume flavor symmetries in order to reduce the number of the fundamental constants $(Y_f)_{ij}$, some of $(Y_f)_{ij}$ will still remain as fundamental constants in the theory. If we make a multi-Higgs extension in which Higgs scalars have flavor quantum numbers, we will encounter some of troubles, e.g. a flavor changing neutral current problem, unwelcome behavior of the SU(2)$_L$ β-function, and so on. In contrast to such a standard model, in the yukawaon model, effective Yukawa coupling constants

Y_f^{eff} are given by vacuum expectation values (VEVs) of gauge singlet scalars Y_f:

$$(Y_f^{eff})_{ij} = \frac{y_f}{\Lambda} \langle (Y_f)_{ij} \rangle, \tag{1}$$

where Λ is a scale of an effective theory which is valid at $\mu \leq \Lambda$, and we assume $\langle Y_f \rangle \sim \Lambda$. Thus, in the yukawaon model, an origin of the mass spectra and mixings is attributed to the VEV structures $\langle Y_f \rangle$, while an origin of the quark and lepton masses is still attributed to the standard Higgs scalars H_u and H_d with $\langle H_u \rangle \sim 10^2$ GeV (here, we have assumed a supersymmetric scenario). We refer to the fields Y_f as "yukawaons" [2] hereafter. Differently from a Froggatt-Nielsen type model [3], we introduce a separate yukawaon Y_f for each fermion sector f ($f = u, d, \ldots$). A unified description of quark and lepton mass matrices is realized by introducing a further fundamental scalar field Φ_e and by considering that each VEV structure $\langle Y_f \rangle$ is given by a bilinear form of the VEV matrix $\langle \Phi_e \rangle$ as we show later. We refer to the field Φ_e as "ur-yukawaon" hereafter.

Those VEVs can, in principle, be calculated dynamically, although dynamics for the yukawaons is, at present, not yet established. Meanwhile, we have a hint for this dynamics: In the charged lepton sector, we know that an empirical relation [4]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3} \tag{2}$$

is satisfied with an order of 10^{-5} for the pole masses, i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ [5], while it is only valid with an order of 10^{-3} for the running masses, i.e. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$ [6]. In conventional mass matrix models, "masses" mean not "pole masses", but "running masses". Why is the mass formula (2) so remarkably satisfied for the pole masses? This had been a mysterious problem as to the relation (2) for long years. Recently, Sumino [7] has proposed a very interesting model for the charged lepton mass relation by assuming U(3)×O(3) flavor gauge symmetries. The deviation of $K(\mu)$ from K^{pole} is caused by a term $m_{ei} \log(\mu/m_{ei})$ in the running mass terms. In his model, the logarithmic term $m_{ei} \log(\mu/m_{ei})$ in the electromagnetic correction is exactly canceled by that due to the family gauge interactions, so that the charged lepton mass relation $K(\mu)$ for the running masses are given by the same form as K^{pole} for the pole masses, i.e. $K(\mu) = K^{pole}$. He speculated $\Lambda \sim 10^3$ TeV.

In the present paper, however, for the time being, without adhering to Sumino's scenario, we will adopt a conventional scenario given in a series of yukawaon models: (i) Masses which we deal with are not "pole masses", but "running masses". (ii) We assume a global O(3) flavor symmetry, which is completely broken at $\mu \sim \Lambda$. (ii) We consider $\Lambda \sim 10^{14-16}$ GeV, and Y_f^{eff} evolve as those in the standard model below the scale Λ. (iv) In order to obtain VEV relations, we will use supersymmetric vacuum conditions, so that the supersymmetry (SUSY) are still unbroken for $\mu \sim \Lambda$.

In the present model, we assume an O(3) flavor symmetry. Would-be Yukawa interactions are given by

$$H_Y = \sum_{i,j} \frac{y_u}{\Lambda} u_i^c (Y_u)_{ij} q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} d_i^c (Y_d)_{ij} q_j H_d$$

$$+\sum_{i,j}\frac{y_\nu}{\Lambda}\ell_i(Y_\nu)_{ij}\nu^c_jH_u+\sum_{i,j}\frac{y_e}{\Lambda}\ell_i(Y_e)_{ij}e^c_jH_d+h.c.+\sum_{i,j}y_R\nu^c_i(Y_R)_{ij}\nu^c_j, \tag{3}$$

where q and ℓ are SU(2)$_L$ doublet fields, and f^c $(f=u,d,e,\nu)$ are SU(2)$_L$ singlet fields. All of the yukawaons Y_f belong to $(\mathbf{3}\times\mathbf{3})_S=\mathbf{5}+\mathbf{1}$ of O(3). In order to distinguish each Y_f from others, we assume a U(1)$_X$ symmetry (i.e. "sector charge") in addition to the O(3) symmetry, and we have assigned U(1)$_X$ charges as $Q_X(Y_f)=x_f$, $Q_X(f^c)=-x_f$ and $Q_X(\nu^c)=2x_\nu$. (The SU(2)$_L$ doublet fields q, ℓ, H_u and H_d are assigned to sector charges $Q_X=0$.) For the neutrino sector, we assume $Q_X(\nu^c)=Q_X(e^c)$, so that the yukawaon Y_e can also couple to the neutrino sector as $(\ell Y_e\nu^c)H_u$ instead of $(\ell Y_\nu\nu^c)H_u$ in Eq.(3). We do not need a yukawaon Y_ν in the present model.

Then, we obtain VEV relations as follows: (i) We give an O(3) and U(1)$_X$ invariant superpotential for yukawaons Y_f. (ii) We solve SUSY vacuum conditions $\partial W/\partial Y_f=0$. (iii) Then, we obtain VEV relations among Y_f. For example, we have assume the following superpotential

$$W_e=\lambda_e\mathrm{Tr}[\Phi_e\Phi_e\Theta_e]+\mu_e\mathrm{Tr}[Y_e\Theta_e]+W_\Phi, \tag{4}$$

where we have assumed $Q_X(\Phi_e)=\frac{1}{2}Q_X(Y_e)=-\frac{1}{2}Q_X(\Theta_e)$ and the term W_Φ has been introduced in order to determine a VEV spectrum $\langle\Phi_e\rangle$ completely. Then, from a SUSY vacuum condition

$$\frac{\partial W}{\partial\Theta_e}=\lambda_e\Phi_e\Phi_e+\mu_eY_e=0, \tag{5}$$

we obtain a VEV relation

$$\langle Y_e\rangle=-\frac{\lambda_e}{\mu_e}\langle\Phi_e\rangle\langle\Phi_e\rangle. \tag{6}$$

In other words, in the present model, we assume

$$\langle\Phi_e\rangle_e=\mathrm{diag}(v_1,v_2,v_3)\propto\mathrm{diag}(\sqrt{m_e},\sqrt{m_\mu},\sqrt{m_\tau}). \tag{7}$$

Here, the notation $\langle A\rangle_f$ denotes a form of a VEV matrix $\langle A\rangle$ in the diagonal basis of $\langle Y_f\rangle$ (we refer to it as f basis). The scalar Θ_e does not have a VEV, i.e. $\langle\Theta_e\rangle=0$. Therefore, terms which include more than two of Θ_e do not play any physical role, so that we do not consider such terms in the present effective theory. [Hereafter, we will denote fields whose VEV values are zero as notations Θ_A $(A=u,d,\cdots)$.]

In the yukawaon model, from the interaction (3), the neutrino mass matrix M_ν is given by a seesaw-type mass matrix, $M_\nu\propto\langle Y_\nu\rangle\langle Y_R\rangle^{-1}\langle Y_\nu\rangle^T$. In a previous work [8], the author has obtained the following VEV relations

$$\langle Y_R\rangle\propto\langle Y_e\rangle\langle\Phi_u\rangle+\langle\Phi_u\rangle\langle Y_e\rangle \tag{8}$$

together with $\langle Y_u\rangle\propto\langle\Phi_u\rangle\langle\Phi_u\rangle$. Here, the relation (8) has been derived from a superpotential

$$W_R=\mu_R\mathrm{Tr}[Y_R\Theta_R]+\lambda_R\mathrm{Tr}[(Y_e\Phi_u+\Phi_uY_e)\Theta_R]. \tag{9}$$

As a result, the neutrino mass matrix is given by a form

$$\langle M_\nu\rangle_e\propto\langle Y_e\rangle_e\left\{\langle Y_e\rangle_e\langle\Phi_u\rangle_e+\langle\Phi_u\rangle_e\langle Y_e\rangle_e\right\}^{-1}\langle Y_e\rangle_e, \tag{10}$$

where

$$\langle\Phi_u\rangle_u \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t}). \tag{11}$$

We can obtain a form $\langle\Phi_u\rangle_d = V(\delta)^T\langle\Phi_u\rangle_u V(\delta)$ from the definition of the CKM matrix $V(\delta)$ (δ is a *CP* violating phase parameter), but we do not know an explicit form of $\langle\Phi_u\rangle_e$. Since Φ_u must be real in the O(3) model, in a previous work [8], we put an ansatz

$$\langle\Phi_u\rangle_e = V(\pi)^T\langle\Phi_u\rangle_u V(\pi) \tag{12}$$

by supposing $\langle\Phi_u\rangle_e \simeq \langle\Phi_u\rangle_d$, and we obtained excellent predictions of the neutrino oscillation parameters $\sin^2 2\theta_{atm} = 0.995$, $|U_{13}| = 0.001$ and $\tan^2\theta_{solar} = 0.553$, without assuming any discrete symmetry.

However, there is no theoretical ground for the ansatz (12) for the form $\langle\Phi_u\rangle_e$. The purpose of the present work is to investigate a quark mass matrix model in order to predict neutrino mixing parameters on the basis of a yukawaon model (3), without such the ad hoc ansatz, because if we give a quark mass matrix model where mass matrices (M_u, M_d) are given on the e basis, then, we can obtain the form $\langle\Phi_u\rangle_e$ by using a transformation

$$\langle\Phi_u\rangle_e = U_u\langle\Phi_u\rangle_u U_u^T, \tag{13}$$

where U_u is defined by $U_u^T M_u U_u = D_u \equiv \mathrm{diag}(m_u, m_c, m_t)$.

YUKAWAONS IN THE QUARK SECTOR

We pay attention to the fact that M_d should be corresponding to $M_u^{1/2}$ (not to M_u) as far as the mass spectra (mass ratios) are concerned, e.g. $\sqrt{m_c/m_t} \sim m_s/m_b$. We also consider that the quark mass matrices should be in terms of the charged lepton mass spectrum Φ_e. Therefore, in this paper, by way of trial, we assume the following superpotential [9] in the quark sector:

$$\begin{aligned} W_q = {} & \mu_u \mathrm{Tr}[Y_u\Theta_u] + \lambda_u \mathrm{Tr}[\Phi_u\Phi_u\Theta_u] + \mu'_u \mathrm{Tr}[\Phi_u\Theta'_u] + \mu_d \mathrm{Tr}[Y_d\Theta_d] \\ & + \frac{\xi_u}{\Lambda}\mathrm{Tr}[\Phi_e(X + a_u E)\Phi_e\Theta'_u] + \frac{\xi_d}{\Lambda}\mathrm{Tr}[\Phi_e(X + a_d E)\Phi_e\Theta_d]. \end{aligned} \tag{14}$$

Here, we have assigned U(1)$_X$ charges as follows: $Q_X(Y_e) = -Q_X(\Theta_e) = x_e$, $Q_X(\Phi_e) = \frac{1}{2}x_e$, $Q_X(Y_u) = -Q_X(\Theta_u) = x_u$, $Q_X(\Phi_u) = \frac{1}{2}x_u$ and $Q_X(Y_d) = -Q_X(\Theta_d) = x_d$. When we denote $Q_X(X) = Q_X(E) = x_X$, U(1)$_X$ charges of Θ'_u and Θ_d have to be $Q_X(\Theta'_u) = Q_X(\Theta_d) = -(x_X + x_e)$. Then, we also have to assume that the coefficients μ'_u and μ_d have U(1)$_X$ charges $Q_X(\mu'_u) = x_d$ and $Q_X(\mu'_u) = \frac{1}{2}x_u$, respectively, where $x_X = x_d + \frac{1}{2}x_u - x_e$. Although Θ'_u and Θ_d have the same U(1)$_X$ charges, those fields are separate fields each other. Then, without losing generality, we can define Θ'_u and Θ_d as fields which couple to Φ_u and Y_d, respectively.

From SUSY vacuum conditions $\partial W/\partial\Theta_u = 0$, $\partial W/\partial\Theta'_u = 0$ and $\partial W/\partial\Theta_d = 0$, we obtain $\langle Y_u\rangle \propto \langle\Phi_u\rangle\langle\Phi_u\rangle$,

$$M_u^{1/2} \propto \langle\Phi_u\rangle_e \propto \langle\Phi_e\rangle_e\left(\langle X\rangle_e + a_u\langle E\rangle_e\right)\langle\Phi_e\rangle_e, \tag{15}$$

$$M_d \propto \langle Y_d \rangle_e \propto \langle \Phi_e \rangle_e \left(\langle X \rangle_e + a_d e^{i\alpha_d} \langle E \rangle_e \right) \langle \Phi_e \rangle_e, \tag{16}$$

respectively. Here, $\langle X \rangle_e$ and $\langle E \rangle_e$ are given by

$$\langle X \rangle_e \propto X \equiv \frac{1}{3} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad \langle E \rangle_e \propto \mathbf{1} \equiv \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{17}$$

(Here, the VEV form $\langle X \rangle_e$ breaks the O(3) flavor symmetry into S_3.) Therefore, we obtain quark mass matrices

$$M_u^{1/2} \propto M_e^{1/2} \left(X + a_u \mathbf{1} \right) M_e^{1/2}, \quad M_d \propto M_e^{1/2} \left(X + a_d e^{i\alpha_d} \mathbf{1} \right) M_e^{1/2}, \tag{18}$$

on the e basis. (This quark mass matrix form has first been proposed in Ref.[10] as a “democratic universal seesaw mass matrix model".) Note that we have assumed that the O(3) relations are valid only on the e and u bases, so that $\langle Y_e \rangle$ and $\langle Y_u \rangle$ must be real. [The VEV matrix $\langle \Phi_u \rangle$ must satisfy the relation (15) on the e basis, while $\langle \Phi_u \rangle$ must also satisfy the relation $\langle Y_u \rangle \propto \langle \Phi_u \rangle_u \langle \Phi_u \rangle_u$ on the u basis. However, for the down-quark sector, such a condition is not required, because $\langle Y_d \rangle$ is given by Eq.(16) only on the e basis.]

A case $a_u \simeq -0.56$ can give a reasonable up-quark mass ratios

$$\sqrt{\frac{m_{u1}}{m_{u2}}} = 0.043, \quad \sqrt{\frac{m_{u2}}{m_{u3}}} = 0.057, \tag{19}$$

which are in favor of the observed values [11] $\sqrt{m_u/m_c} = 0.045^{+0.013}_{-0.010}$, and $\sqrt{m_c/m_t} = 0.060 \pm 0.005$ at $\mu = m_Z$.

Meanwhile, in this paper, we will carry out parameter-fitting at $\mu = m_Z$, because we interest in the mixing values $\mu = m_Z$. Exactly speaking, fitting for the mass ratios must be done at $\mu = \Lambda \sim 10^{14-16}$ GeV. However, at present, our model does not intend to give so precise predictions of the quark mass ratios. For example, we know [11] $\sqrt{m_u/m_c} = 0.046^{+0.013}_{-0.012}$ and $\sqrt{m_c/m_t} = 0.051^{+0.002}_{-0.006}$ even at $\mu = 2 \times 10^{16}$ GeV ($\tan\beta = 10$). Even in $\sqrt{m_c/m_t}$, the discrepancy is smaller than 20%. Besides, the mass values are dependent on the value of $\tan\beta$ in the SUSY model. Therefore, for simplicity, in this paper, we will carry out the parameter-fitting at $\mu = m_Z$.

YUKAWAONS IN THE NEUTRINO SECTOR

However, the up-quark mass matrix (18) failed to give reasonable neutrino oscillation parameter values although it can give reasonable up-quark mass ratios. Therefore, we will slightly modify the model (8) in the neutrino sector.

Note that signs of the eigenvalues of $M_u^{1/2}$ given by Eq.(18) are $(+,-,+)$ for the case $a_u \simeq -0.56$. If we assume that the eigenvalues of $\langle \Phi_u \rangle_u$ must be positive, so that $\langle \Phi_u \rangle_u$ in Eq.(8) is replaced as $\langle \Phi_u \rangle_u \to \langle \Phi_u \rangle_u \cdot \mathrm{diag}(+1,-1,+1)$, then we can obtain successful results except for $\tan^2 \theta_{solar}$, i.e. predictions $\sin^2 2\theta_{atm} = 0.984$ and $|U_{13}| = 0.0128$ and

TABLE 1. Predicted values for the neutrino oscillation parameters

ξ	$\sin^2 2\theta_{atm}$	$\tan^2\theta_{solar}$	$\lvert U_{13}\rvert$
0	0.9848	0.7033	0.0128
$+0.004$	0.9825	0.4891	0.0123
$+0.005$	0.9819	0.4486	0.0122
$+0.006$	0.9812	0.4123	0.0120
-0.0011	0.9897	0.4854	0.0142
-0.0012	0.9900	0.4408	0.0143
-0.0013	0.9904	0.4008	0.0144

TABLE 2. Predicted values for the CKM mixing parameters

a_u	a_d	α_d	$\lvert m_{d1}/m_{d2}\rvert$	$\lvert m_{d2}/m_{d3}\rvert$	$\lvert V_{us}\rvert$	$\lvert V_{cb}\rvert$	$\lvert V_{ub}\rvert$	$\lvert V_{td}\rvert$
-0.56	-0.620	4°	0.1078	0.0273	0.2035	0.0666	0.0101	0.0178
-0.56	-0.625	6°	0.0783	0.0313	0.2187	0.0818	0.0123	0.0190
-0.56	-0.630	8°	0.0542	0.0362	0.2222	0.0977	0.0146	0.0194
-0.58	-0.630	2°	0.1959	0.0195	0.2272	0.0448	0.0088	0.0163

an unfavorable prediction $\tan^2\theta_{solar} = 0.7033$ (see predicted values in a case of $\xi = 0$ in Table 1).

When we introduce a new field P_u with a VEV

$$\langle P_u\rangle_u \propto \mathrm{diag}(+1,-1,+1), \tag{20}$$

we must consider an existence of $P_u Y_e \Phi_u + \Phi_u Y_e P_u$ in addition to $Y_e P_u \Phi_u + \Phi_u P_u Y_e$ [9], because they have the same $\mathrm{U}(1)_X$ charges. Therefore, we modify Eq.(9) into

$$\begin{aligned} W_R = \mu_R \mathrm{Tr}[Y_R \Theta_R] + \frac{\lambda_R}{\Lambda}\{&\mathrm{Tr}[(Y_e P_u \Phi_u + \Phi_u P_u Y_e)\Theta_R] \\ &+\xi\,\mathrm{Tr}[(P_u Y_e \Phi_u + \Phi_u Y_e P_u)\Theta_R]\}, \end{aligned} \tag{21}$$

which leads to VEV relation

$$Y_R \propto Y_e P_u \Phi_u + \Phi_u P_u Y_e + \xi\,(P_u Y_e \Phi_u + \Phi_u Y_e P_u). \tag{22}$$

We list numerical results from the model (22) in Table 1. The results at $a_u \simeq -0.56$ are excellently in favor of the observed neutrino oscillation parameters $\sin^2\theta_{atm} = 1.00_{-0.13}$ [12] and $\tan^2 2\theta_{solar} = 0.469^{+0.047}_{-0.041}$ [13] by taking a small value of $|\xi|$.

Also, we can calculate the down-quark sector. The observed values of m_d/m_s and m_s/m_b at $\mu = m_Z$ [11] are $m_d/m_s = 0.0527^{+0.0508}_{-0.0285}$ and $m_s/m_b = 0.0190^{+0.0063}_{-0.0056}$, respectively. On the other hand, we have two parameters (a_d, α_d) in the down-quark sector given in Eq.(16). As seen in Table 2, the results are roughly reasonable, although $|V_{ub}|$ and $|V_{td}|$ are somewhat larger than the observed values. Those discrepancies will be improved in future version of the model.

TABLE 3. Summary table

Sector	Parameters	Predictions		
		$\sin^2\theta_{atm}$	$\tan^2\theta_{solar}$	$\vert U_{13}\vert$
M_ν	$\xi = +0.0005$	0.982	0.449	0.012
	$\xi = -0.0012$	0.990	0.441	0.014
$M_u^{1/2}$	$a_u = -0.56$	$\sqrt{\frac{m_u}{m_c}} = 0.0425$	$\sqrt{\frac{m_c}{m_t}} = 0.0570$	
	two parameters	5 observables: fitted excellently		
M_d	$a_d e^{i\alpha_d}$	$\sqrt{\frac{m_d}{m_s}},\ \sqrt{\frac{m_s}{m_b}},\ \vert V_{us}\vert,\ \vert V_{cb}\vert,\ \vert V_{ub}\vert,\ \vert V_{td}\vert$		
	two parameters	6 observables: not always excellent		

CONCLUDING REMARKS

The numerical results are summarized in Table 3. The model predicts masses and mixings for quarks and leptons with quite few parameters: Five observable quantities (two up-quark mass ratios and three neutrino mixing parameters) are excellently fitted by the two parameters a_u and ξ. Also, the CKM mixing parameters and down-quark mass ratios are given under the other two parameters a_d and α_d.

In the present paper, we have not mentioned how to obtain the VEV spectrum $\langle\Phi_e\rangle$. The VEV values $\langle\Phi_e\rangle$ play an essential role in this model: The values determine the charged lepton mass spectrum $\langle Y_e\rangle$ through the relation (6), the quark mass spectra (M_u, M_d) through the relations (18) and the neutrino mass matrix M_ν through the relation (22). In the present paper, we have used the observed values $(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$ at $\mu = m_Z$ as the values of $\langle\Phi_e\rangle_e = \mathrm{diag}(v_1, v_2, v_3)$. For a possible mechanism for $\langle\Phi_e\rangle$, for example, see Refs.[7, 14].

Also, we have not discussed neutrino mass spectrum. In the present model, we can add a term

$$\frac{y'_R}{\Lambda}\nu^c Y_e Y_e \nu^c, \tag{23}$$

to the would-be Yukawa interactions (3) under the O(3) and U$(1)_X$ symmetries. Since the term (23) gives only contribution which is proportional to a unit matrix, the term does not affect the neutrino mixing matrix U_ν. Therefore, we can always fit the observed neutrino mass ratio $R \equiv \Delta m^2_{21}/\Delta m^2_{32}$ by adjusting the parameter y'_R/y_R suitably. In other words, there is no predictability as far as the ratio R is concerned.

Here, we would like to give some comments on the present works.

(i) We have obtained a nearly tribimaximal mixing without assuming any discrete symmetry for the neutrino mass matrix, but note that we have assumed S_3 symmetry in the quark sector.

(ii) Note that it is essential that the quark mass matrices are given in the e basis in which the charged lepton mass matrix takes a diagonal form. We think that the e basis has a specific and fundamental status in the flavor physics just like absolute rest frame (inertial frame of reference) in the classical theory of motion. For example, only in the e basis, the charged lepton mass matrix is diagonal, and the quark matrices take simple forms. Of course, a form of the superpotential W is independent of the flavor bases, i.e., W is

invariant under the O(3) flavor symmetry. Only the VEV matrix relations take special forms on the *e* basis.
(iii) How can we detect a signature of the yukawaon model? In the present paper, we have assumed that the energy scale Λ is of the order of 10^{14} GeV, so that most yukawaon effects will be invisible[15]. On the other hand, Sumino has speculated $\Lambda \sim 10^3$ TeV in his model [7]. A yukawaon model with a lower energy scale Λ is a future task.
(iv) In the present scenario, an O(3) global symmetry has been assumed as a flavor symmetry. However, Sumino [7] has recently assumed a U(3) gauge symmetry in his model. His scenario is very attractive. Is family symmetry global or gauged? The symmetry is O(3) or U(3)? At present, those are open questions.

The present approach will shed new light on unified understanding of the masses and mixings. At least, the present model will provide a suggestive hint on a unification model for quarks and leptons.

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