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Neutrino Mass Matrix Speculated from Sumino's U(3)×O(3) Family Symmetries

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Abstract

A model with U(3)×O(3) family symmetries, which has been recently proposed by Sumino in order to solve a problem as to a charged lepton mass formula, is not anomaly free. It is pointed out that the model can be anomaly free by considering a SUSY version of the Sumino model. Then, the model suggests an interesting neutrino mass matrix form, which predicts a nearly tribimaximal mixing without assuming any discrete symmetry for the neutrino sector.

1. Introduction

One of the most challenging problems in contemporary particle physics is to clarify the origin of flavors. For such a purpose, it is interesting to investigate whether the observed flavor physics phenomena can be understood or not from a concept of a family gauge symmetry. The present data [?] seem to suggest that numbers of lepton- and quark-families are both three. Then, from a point of view of a unification model, and also from a point of view of economy of the number of fundamental particles, it will be natural to consider that the lepton- and quark-families are identical. However, at present, this is experimentally not yet confirmed. In this paper, we investigate a possibility that both families are different from each other. A hint for this idea is in a Sumino mechanism which has recently been proposed by Sumino [?, ?] on the basis of U(3)×O(3) family symmetries. First, let us give a short review of the Sumino mechanism.

Recently, Sumino [?, ?] has proposed a U(3) family gauge symmetry with a special purpose to solve a mystery in a charged lepton mass relation. His interest was in the charged lepton mass relation [?]

$$K \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{2}{3}. \tag{1}$$

The relation (1) is satisfied with the order of 10^{-5} with the pole masses, i.e. $K^{pole} = (2/3) \times (0.999989 \pm 0.000014)$ [?], while it is only valid with the order of 10^{-3} with the running masses, i.e. $K(\mu) = (2/3) \times (1.00189 \pm 0.00002)$ at $\mu = m_Z$. In conventional mass matrix models, "mass" means not "pole mass" but "running mass." Sumino has seriously taken why the mass formula is so remarkably satisfied with the pole masses. The deviation of $K(\mu)$ from K^{pole} is caused by a logarithmic term $m_{ei} \log(\mu/m_{ei})$ in the radiative correction term due to photon. (Note that the value K is invariant under a scale transformation $m_{ei} \to m_{ei}(1 + \varepsilon_0)$, where ε_0 is a constant independent of the family number i.) Therefore, he assumed that a family symmetry is gauged, and that the logarithmic term in the radiative correction due to photon is canceled by that due to family gauge bosons. As a result, we can obtain $K(\mu) = K^{pole}$.

In the Sumino model, the charged lepton mass term is generated by a would-be Yukawa interaction term

$$H_e = \frac{y_e}{\Lambda^2} \sum_{i,j} \sum_{\alpha} \bar{\ell}_L^i (\Phi_e)_{i\alpha} (\Phi_e^T)_{\alpha j} e_R^j H, \tag{2}$$

where $\ell = (\nu_L, e_L)$ and the indeces i, j and α are those of U(3) and O(3), respectively. The charged lepton masses are acquired from the vacuum expectation value (VEV) of the scalar Φ_e, i.e. the VEV is given by $\langle \Phi_e \rangle_e \propto \mathrm{diag}(\sqrt{m_e}, \sqrt{m_\mu}, \sqrt{m_\tau})$, where the sufix "$e$" denotes that a VEV matrix $\langle A \rangle$ takes a form $\langle A \rangle_e$ in a flavor basis in which the charged lepton mass matrix M_e is diagonal. Here, the fields ℓ_L and e_R belong to $(3, 1)$ and $(3^*, 1)$ of U(3)×O(3) family symmetries, respectively, and Φ_e belongs to $(3, 3)$. [Sumino mechanism with U(3)×O(3) has been derived from a U(9) family gauge symmetry. The formula (1) is derived at a scale Λ, at which the symmetry U(9) is broken into U(3)×O(3). However, in this paper, we will confine ourself to the model with the U(3)×O(3) symmetries.]

In his model, in order that the cancellation works correctly, it is essential that the left-handed lepton field e_L and its right-handed partner e_R are assigned to $\mathbf{3}$ and $\mathbf{3}^*$ of U(3), respectively. However, such an assignment destroys anomaly free of the model. Of course, the model is effective theory, so that it does not need to require that the theory is anomaly free below a scale Λ of the effective theory. Nevertheless, it is interesting to require that the model should be anomaly free even below the scale Λ. (For example, we usually regard the standard model for quarks and leptons as an effective theory model, and, at the same time, we know that the model is anomay free.) We note that if we suppose a supersymmetric (SUSY) version of the Sumino mechanism, the fermion set (ℓ, e^c, Φ_e) becomes anomaly free. (Here, we have taken Φ_e as $(3^*, 3)$ of U(3)×O(3).) Then, we cannot regard the SU(2)$_L$ singlet neutrino ν^c as ν_i^c, i.e. $(3, 1)$ of U(3)×O(3), because the model with (ℓ_i, e_i^c, ν_i^c) cannot becomes anomaly free. We must regard ν^c as ν_α^c, i.e. $(1, 3)$ of U(3)×O(3) if we require that the model should be anomaly free in the lepton sector itself. (Sumino has not mentioned any assignments in U(3)×O(3) not only for quarks but also for neutrinos, because his interest centers only on the charged lepton sector.) Therefore, we can consider the following model with anomaly free for lepton sector:

$$W = \frac{y_e}{\Lambda^2} \ell_i (\Phi_e)^{i\alpha} (\Phi_e^T)^{\alpha j} e_j^c H_d + \frac{y_\nu}{\Lambda} \ell_i (\Phi_e)^{i\alpha} \nu_\alpha^c H_u + y_R \nu_\alpha^c (Y_R)^{\alpha\beta} \nu_\beta^c, \tag{3}$$

where Y_R plays a role in generating a Majorana mass matrix M_R of the right-handed neutrinos via its VEV $\langle Y_R \rangle$. The characteristic of this modified model (3) is that the Dirac neutrino mass matrix m_D is given by $\langle \Phi_e \rangle$, i.e. by $M_e^{1/2}$. That is, the seesaw-type neutrino mass matrix M_ν is given by

$$M_\nu \propto M_e^{1/2} \langle Y_R \rangle^{-1} M_e^{1/2}. \tag{4}$$

The purpose of the present paper is to investigate a possible structure of $\langle Y_R \rangle$ which can give reasonable neutrino mixing parameters, based on the basic idea described in Eq.(3) with keeping anomaly free. Hereafter, for convenience, we will rewrite the first term in the superpotential (3) as

$$\frac{y_e}{\Lambda}\ell_i(Y_e)^{ij}e^c_j H_d + \mu_e (Y_e)^{ij}(\Theta_e)_{ji} + \lambda_e (\Phi_e)^{i\alpha}(\Phi_e^T)^{\alpha j}(\Theta_e)_{ji}. \tag{5}$$

Such a model is called "yukawaon" model [?], in which all yukawa coupling constants are regarded as effective coupling constants Y_f^{eff}, and the constants are given by VEVs of scalars Y_f (we refer to them as "yukawaon") as $Y_f^{eff} = y_f\langle Y_f\rangle/\Lambda$. (Thus, by separating the origin of the flavor mixing from the origin of the quark and lepton masses, we can be freed from the flavor changing neutral current (FCNC) problem in a multi-Higgs model.) Relations among such VEV matrices are obtained by solving supersymmetric vacuum conditions. For example, a SUSY vacuum condition $\partial W/\partial \Theta_e = 0$ leads to the VEV relation

$$\langle (Y_e)^{ij}\rangle = -\frac{\lambda_e}{\mu_e}\langle (\Phi_e)^{i\alpha}\rangle\langle (\Phi_e^T)^{\alpha j}\rangle, \tag{6}$$

from Eq.(5). Since the new fields Y_e and Θ_e are $\mathbf{6}^*$ and $\mathbf{6}$ of U(3) gauge family, they do not affect anomaly free condition. Another SUSY vacuum conditions do not yield any additional constraints on the VEV relations because we alway take a vacuum with $\langle\Theta_e\rangle = 0$. [Hereafter, we will denote such a field with vanishing VEV as a notation Θ_A $(A = u, d, \cdots)$.]

2. A hint for Y_R from a yukawaon model

For a structure of $\langle Y_R\rangle$, there is a hint: Previously, the author [?] has proposed a neutrino mass matrix model based on a yukawaon model with an O(3) family symmetry, in which the neutrino mass matrix is given by a seesaw-type mass matrix $M_\nu = m_D M_R^{-1} m_D^T$, where the Dirac and Majonara mass matrices are given by

$$m_D = M_e, \qquad M_R \propto M_u^{1/2} M_e + M_e M_u^{1/2}, \tag{7}$$

respectively, and M_u is given by $M_u^{1/2} = U_u (M_u^{1/2})^{diag} U_u^T$ and $(M_u^{1/2})^{diag} \propto \mathrm{diag}(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t})$. If we take a form $U_u^T = V_{CKM}(\delta = \pi)$ (δ is a CP violating phase in the standard expression of the Cabiboo-Kobayashi-Maskawa (CKM) mixing matrix $V_{CKM}(\delta)$), we can successfully obtain [?] the observed "nearly tribimaximal neutrino mixing" without assuming any discrete symmetry [?] for neutrino sector. Furthermore, by assuming that the quark mass matrix $M_u^{1/2}$ and M_d are given by

$$M_u^{1/2} = M_e^{1/2} S_u M_e^{1/2}, \tag{8}$$

$$M_d = M_e^{1/2} S_d M_e^{1/2}, \tag{9}$$

where S_q $(q = u, d)$ have forms

$$S_u = \mathbf{1} + a_u X, \quad S_d = \mathbf{1} + a_d X, \tag{10}$$

$$\mathbf{1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad X = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}. \tag{11}$$

we have also obtained not only the nearly tribimaximal neutrino mixing, but also reasonable CKM quark mixing [**?**] (however, the fitting of CKM mixing is not so excellent compared with that of neutrino mixing). Since we take interest only in the relative ratios among the matrix elements, here and hereafter, for convenience, we drop numerical coefficients even if they have mass dimensions. (Therefore, for simplicity, the notation "$\propto$" will be given by "$=$".) If we take a value $a_u \simeq -1.8$ in the up-quark mass matrix model (8), we can give reasonable up-quark mass ratios, but signs of the eigenvalues of $M_u^{1/2}$ show $(+,-,+)$, i.e. $(M_u^{1/2})^{diag} \propto \mathrm{diag}(\sqrt{m_u}, -\sqrt{m_c}, \sqrt{m_t})$ for $a_u \simeq -1.8$. Therefore, in Eq.(8), we replace $M_u^{1/2}$ with $M_u^{1/2} P_u$, where P_u takes a form

$$\langle P_u \rangle_u = v_P \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \tag{12}$$

where the index "u" denotes that a VEV matrix $\langle A \rangle$ takes form $\langle A \rangle_u$ at the diagonal basis of the up-quark mass matrix M_u. Then, we can obtain [**?**] the observed nearly tribimaximal mixing by taking $a_u \simeq -1.8$.

In the present model, since $m_D = M_e^{1/2}$, and considering

$$M_\nu = M_e (M_u^{1/2} M_e + M_e M_u^{1/2})^{-1} M_e = M_e^{1/2} \left(M_e^{-1/2} M_u^{1/2} M_e^{1/2} + M_e^{1/2} M_u^{1/2} M_e^{-1/2} \right)^{-1} M_e^{1/2}, \tag{13}$$

we may consider $M_R = M_e^{-1/2} M_u^{1/2} M_e^{1/2} + M_e^{1/2} M_u^{1/2} M_e^{-1/2}$, i.e.

$$\Phi_e Y_R \Phi_e = \Phi_u Y_e + Y_e \Phi_u. \tag{14}$$

Therefore, we assume the following superpotential terms for Y_R sector;

$$W_R = \frac{1}{\Lambda^3} \left\{ \lambda_R (\Phi_e)^{i\alpha} (Y_R)^{\alpha\beta} (\Phi_e^T)^{\beta j} \right.$$

$$\left. + \lambda'_R \left[(\Phi_u)^{i\alpha} (E^T)_{\alpha k} (Y_e)^{kj} + (Y_e)^{ik} E_{k\alpha} (\Phi_u^T)^{\alpha j} \right] \right\} E_{j\gamma} (\Theta_R)_{\gamma\delta} (E^T)_{\delta i}, \tag{15}$$

where E is a field which has a VEV $\langle E \rangle = v_E \mathbf{1}$ and $\langle \Phi_u \rangle$ corresponds to $M_u^{1/2}$. Here, we have assumed somewhat factitious form $E_{j\gamma} (\Theta_R)_{\gamma\delta} (E^T)_{\delta i}$ instead of $(\Theta_R)_{ji}$ in order to keep the correspondence of Θ_R to Y_R. We will find that the field E plays an essential role in building an anomaly-free model as we state later. Since the fields Φ_u and E are not peculiar fields in the lepton sectors, we do not count as fields in the right-handed neutrino sector. Then, anomalies are free in the right-handed neutrino sector, too.

3. Model for quark sector

Next, we investigate possible superpotential forms for Φ_u and Y_d which lead to the relations (8) and (9). Although, correspondingly to the lepton sector with $(\ell_i, e_i^c, \nu_\alpha)$, we can naively consider a model with (q_i, u_i^c, d_α^c), the number of $\mathbf{3}$ of U(3) is three times compared with the lepton sector because these fields have the degree of color. Since we wish that the number of **3** (and also $\mathbf{3}^*$) is as few as possible, in this paper, we propose another model for the quark sector, i.e. with $(q_\alpha, u_i^c, d_\alpha^c)$:

$$W_q = \frac{y_u}{\Lambda} u_i^c (Y_u)^{i\beta} q_\beta H_u + \frac{y_d}{\Lambda} d_\alpha^c (Y_d^T)^{\alpha\beta} q_\beta H_d, \tag{16}$$

where $q = (u_L, d_L)$. Although we consider $(\Phi_u)^{i\alpha}$ similar to Eq.(6), we do not identify Φ_u as Y_u differently from the case in the lepton sector given in Eq.(3). Note that in this model, the U(3) gauge bosons A_i^j which couple to charged lepton sector cannot couple to the down-quark sector. Then, for yukawaons, we assume the following superpotential terms:

$$W_u = \mu_u (Y_u)^{i\alpha} (\Theta_u)_{\alpha i} + \frac{\lambda_u}{\Lambda} (\Phi_u)^{i\beta} (E^T)_{\beta j} (\Phi_u)^{j\alpha} (\Theta_u)_{\alpha i}$$

$$+ \left\{ \frac{\lambda'_u}{\Lambda^2} (\Phi_u)^{i\alpha} (P_u)^{\alpha j} + \frac{\lambda''_u}{\Lambda^3} (\Phi_e^T)^{i\alpha} (S_u)_{\alpha\beta} (\Phi_e)^{\beta j} \right\} E_{j\gamma} (\Theta'_u)_{\gamma\delta} E_{\delta i}, \tag{17}$$

$$W_d = \mu_d (Y_d)^{\alpha\beta} (\Theta_d)_{\beta\alpha} + \frac{\lambda_d}{\Lambda^3} (\Phi_e^T)^{\alpha i} E_{i\gamma} (S_d)_{\gamma\delta} (\Phi_e^T)^{\delta j} E_{j\beta} (\Theta_d)_{\beta\alpha}. \tag{18}$$

Here, since we have take anomaly free conditions into consideration, we have again be obligated to adopt somewhat factitious forms, e.g. $(E^T)_{j\gamma}(\Theta'_u)_{\gamma\delta} E_{\delta i}$ instead of $(\Theta'_u)_{ji}$, and so on. Anyhow, since we have three **3** (u^c, E and Θ_u) and three $\mathbf{3}^*$ (Y_u, Φ_u and P_u), so that quark sector is also anomaly free.

4. Neutrino mass matrix

In this paper, we do not discuss phenomenological results for the CKM mixing, because the model for quark sector is effectively the same as that given in Ref.[?] (we refer to it as the model A). As stated in Ref.[?], the model A for down-quark sector may be improved. On the other hand, the present model for neutrino sector is somewhat different from the model A. In the model A with O(3) family symmetry, the Majorana mass matrix $\langle Y_R \rangle$ was given by

$$\langle (Y_R)_i^j \rangle = \langle (\Phi_u)_i^k \rangle \langle (P_u)_k^l \rangle \langle (Y_e)_l^j \rangle + \langle (Y_e)_i^k \rangle \langle (P_u)_k^l \rangle \langle (\Phi_u)_l^j \rangle$$

$$+ \xi_\nu \left(\langle (\Phi_u)_i^k \rangle \langle (Y_e)_k^l \rangle \langle (P_u)_l^j \rangle + \langle (P_u)_i^k \rangle \langle (Y_e)_k^l \rangle \langle (\Phi_u)_l^j \rangle \right), \tag{19}$$

where $\langle \Phi_u \rangle_u = \mathrm{diag}(\sqrt{m_u}, -\sqrt{m_c}, \sqrt{m_t})$, so that $\langle \Phi_u \rangle_u \langle P_u \rangle_u$ in the model A corresponds to $\langle \Phi_u \rangle_u = \mathrm{diag}(\sqrt{m_u}, +\sqrt{m_c}, \sqrt{m_t})$ in the present model. In the model A which was a O(3) model, when a term ABC (A, B and C are fields with $\mathbf{5}+\mathbf{1}$ of O(3)) is allowed in a superpotential, then a term ACB is also allowed. Therefore, the ξ_ν term appeared in Eq.(19). However, in the present model, we cannot consider such a term which corresponds to the ξ_ν term in the model A. In the model A, in order to give the observed value [?] $\tan^2\theta_{solar} \simeq 1/2$, it was indispensable that we take a non-vanishing value of ξ_ν, although we could give the observed values [?] $\sin^2 2\theta_{atm} \simeq 1$ and $|U_{13}|^2 \simeq 0$ even if $\xi_\nu = 0$. Therefore, in the present model, we assume an alternative term (ξ_ν-term) as follows:

$$\lambda_R (\Phi_e Y_R \Phi_e^T)^{ij} + \lambda'_R \left\{ \left[(\Phi_u E^T)_k^i (Y_e)^{kj} + (Y_e)^{ik} (E \Phi_u^T)_k^j \right] + \xi_\nu (\Phi_u E^T)_k^k (Y_e)^{ij} \right\} = 0, \tag{20}$$

ξ_ν	$\tan^2\theta_{solar}$	$\sin^2 2\theta_{atm}$	$|U_{13}|^2$
0	0.6995	0.9872	1.72×10^{-4}
0.009	0.4610	0.9902	2.28×10^{-4}
0.010	0.4408	0.9905	2.35×10^{-4}

Table 1: ξ_ν dependence of the neutrino parameters. The value of a_u is taken as $a_u = -1.78$ which can give reasonable up-quark mass ratios.

Here, we have denoted a VEV relation after the SUSY vacuum condition $\partial W/\partial\Theta_R = 0$ was carried out.

According to the modified relation (20) for the form $\langle Y_R\rangle$, we obtain the following neutrino mass matrix M_ν:

$$M_\nu \propto M_e^{1/2}\left\{M_e^{-1/2}\left[M_u^{1/2}M_e + M_e M_u^{1/2} + \xi_\nu \mathrm{Tr}[M_u^{1/2}]M_e\right]M_e^{-1/2}\right\}^{-1} M_e^{1/2}, \tag{21}$$

where $M_u^{1/2}$ is given by Eq.(8) (but with $M_u \to M_u^{1/2}\langle P_u\rangle$. In Table 1, we demonstrate ξ_ν-dependence of the neutrino mixing parameters in a case with $a_u = -1.78$, which gives reasonable up-quark mass ratios

$$\sqrt{\frac{m_u}{m_c}} = 0.04389, \qquad \sqrt{\frac{m_c}{m_t}} = 0.05564. \tag{22}$$

The predicted values (22) are in good agreement with the observed values at $\mu = m_Z$ [?]

$$\sqrt{\frac{m_u}{m_c}} = 0.045^{+0.013}_{-0.010}, \qquad \sqrt{\frac{m_c}{m_t}} = 0.060 \pm 0.005. \tag{23}$$

As seen in Table 1, the neutrino mixing parameters $\sin^2 2\theta_{atm}$ and $|U_{13}|^2$ are almost independent of the parameter ξ_ν, while the value of $\tan^2\theta_{solar}$ is somewhat dependent on the value of ξ_ν.

5. Concluding remarks

In conclusion, stimulated by the Sumino mechanism [?, ?] for the charged lepton mass relation, we have considered a SUSY version of his model with U(3)×O(3) family symmetries. The original Sumino model is a model for the charged leptons, so that he has mentioned nothing as to quark and neutrino sectors. However, if we require that the model should be anomaly free in individual sectors of leptons and quarks, the model predicts an interesting form (4) of the neutrino mass matrix from the superpotential (3) for the lepton sector with anomaly free. We have investigated a possible form of the Majorana mass matrix M_R ($\propto \langle Y_R\rangle$) of the right-handed neutrinos by referring to a supersymmetric yukawaon model (model A) [?] for the neutrino sector. The present form (21) of M_R is similar to the form (19) in the model A, but the ξ_ν term is different from the model A. Nevertheless, in this model, too, we can successfully obtain the observed (nearly) tribimaximal neutrino mixing by adjusting the parameter ξ_ν as seen in Table 1.

Although the present model for the quark sector is also anomaly free, the model is somewhat factitious, e.g. introducing of a field $E_{i\alpha}$ with $\langle E\rangle = v_E \mathbf{1}$, and so on. When we build a more realistic model for the down-quark sector in future, we will need more fields. Then, the anomaly-free problem for quark sector will be solved naturally.

In the evolution equation for the U(3) family gauge coupling constatnt $\alpha_f \equiv g_f^2/4\pi$, $d\alpha_f^{-1}/dt = b_f/2\pi$, the coefficient b_f (beta function) is given by

$$b_f = \frac{3}{2}\ell_{adj} - \frac{1}{2}\sum \ell_3 - \frac{1}{2}\sum \ell_6, \tag{24}$$

under one-loop approximation, where ℓ_{adj}, ℓ_3 and ℓ_6 are indices for **8**, **3** and **6** of U(3), respectively, and they are given by $\ell_{adj} = 6$, $\ell_3 = 1$ and $\ell_6 = 5$. In this model, since we have twelve **3** $(\ell, e^c, u^c, E, \Theta_u)$, twelve $\mathbf{3}^*$ $(\Phi_e, Y_u, \Phi_u, P_u)$, one **6** (Θ_R) and one $\mathbf{6}^*$ (Y_R), we obtain $b_f = 9 - 12 - 5 = -8 < 0$. This is somewhat troublesome. If we take it seriously, we must consider a direct coupling of Φ_u and Φ_e to the up-quarks and charged leptons without mediating Y_u and Y_e, respectively, as Eq.(3). Then, we can build a model without (Y_e, Θ_e) and (Y_u, Θ_u), so that we can obtain $b_f = 9 - 9 = 0$.

Finally, we would like to emphasize that the U(3) gauge bosons cannot couple to the down-quark sector and do only to u^c. Therefore, the gauge boson masses are free from constraints [**?**] from the observed kaon physics. Also, physics in quark sector does not spoil the original Sumono's idea for the charged lepton sector. Phenomenological meanings of the present model in TeV region physics will be discussed elsewhere.

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