Neutrino Mass Matrix Related to Up-Quark Masses and Nearly Tribimaximal Mixing – Based on a Yukawaon model –[1]

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Abstract

Based on a new approach (the so-called Yukawaon model) to the mass spectra and mixings, a neutrino mass matrix which is described in terms of the up-quark masses and CKM matrix parameters is proposed. The mass matrix successfully leads to a nearly tribimaximal mixing without assuming any discrete symmetry.

1 What is a Yukawaon model?

In the standard model of the quarks and leptons, we have many parameters in the theory. Especially, since the Yukawa coupling constants Y_f $(f = u, d, e, \nu)$ are entirely free, we have no predictions for the mass spectra and mixings. Therefore, usually, we assume a flavor symmetry, and thereby, we discussed relations among the mass spectra and mixings. However, if we want to bring a flavor symmetry into the standard model, we encounter a no-go theorem [1, 2] on the flavor symmetry, so that we cannot impose any flavor symmetry on the standard model. Of course, we can evade[2] the no-go theorem if we consider a multi-Higgs model in which Higgs scalars have flavor quantum numbers. However, such multi-Higgs models will newly bring some troubles, e.g. flavor-changing neutral current problem, rapid evolutions of coupling constants, and so on.

There is another idea for the origin of the mass spectra and mixings: In the Yukawa interactions

$$H_Y = \sum_{i,j} \overline{q}_L^i (Y_u)_i^j u_{Rj} H_u + \cdots , \tag{1.1}$$

we regard the Yukawa coupling constants Y_f as "effective" coupling constants Y_f^{eff} in an effective theory, and we consider that Y_f^{eff} originate in vacuum expectation values (VEVs) of new scalars Y_f, i.e.

$$Y_f^{eff} = \frac{y_f}{\Lambda} \langle Y_f \rangle, \tag{1.2}$$

where Λ is a scale of the effective theory, i.e.

$$H_Y = \sum_{i,j} \frac{y_u}{\Lambda} \overline{q}_L^i (Y_u)_i^j u_{Rj} H_u + \cdots . \tag{1.3}$$

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We refer the fields Y_f as "Yukawaons" [3] hereafter. Note that in the Yukawaon model, the Higgs scalars are the same as ones in the conventional model, i.e. we consider only two Higgs scalars H_u and H_d as the origin of the masses (not as the origin of the mass spectra). It should be noted that the Yukawaons Y_f are gauge singlets.

The VEVs of Yukawaons are obtained from supersymmetric (SUSY) vacuum conditions for a superpotential W. In the conventional approach, the masses and mixings are calculated by diagonalizing the mass matrices M_f which are constrained by flavor symmetries, while, in the Yukawaon approach, those are obtained by writing a superpotential under flavor symmetries and by solving simultaneous equations from the SUSY vacuum conditions. For example, we assume an O(3) flavor symmetry [4] and we consider that the Yukawaons Y_f are $(\mathbf{3}\times\mathbf{3})_S = \mathbf{1}+\mathbf{5}$ of $\mathrm{O}(3)_F$:

$$W_Y = \sum_{i,j} \frac{y_u}{\Lambda} u_i^c (Y_u)_{ij} q_j H_u + \sum_{i,j} \frac{y_d}{\Lambda} d_i^c (Y_d)_{ij} q_j H_d$$

$$+ \sum_{i,j} \frac{y_\nu}{\Lambda} \ell_i (Y_\nu)_{ij} \nu_j^c H_u + \sum_{i,j} \frac{y_e}{\Lambda} \ell_i (Y_e)_{ij} e_j^c H_d + h.c. + \sum_{i,j} y_R \nu_i^c (Y_R)_{ij} \nu_j^c, \qquad (1.4)$$

where q and ℓ are $\mathrm{SU}(2)_L$ doublet fields, and f^c $(f = u, d, e, \nu)$ are $\mathrm{SU}(2)_L$ singlet fields. Here, in order to distinguish each Y_f from others, we have assigned $\mathrm{U}(1)_X$ charges as $Q_X(f^c) = -x_f$, $Q_X(Y_f) = +x_f$ and $Q_X(Y_R) = 2x_\nu$. We also write superpotential terms W_f which are introduced in order to fix the VEVs of Y_f under the O(3) flavor symmetry and $\mathrm{U}(1)_X$ symmetry, and we obtain simultaneous equations for the VEVs $\langle Y_f \rangle$ by calculating SUSY vacuum conditions for $W = W_Y + W_u + W_d + \cdots$.

In the next section, we give a short review of a mass spectrum of the charged leptons as an example of the supersymmetric Yukawaon approach. We will discuss ratios

$$R_e \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2}, \qquad (1.5)$$

and

$$r_e \equiv \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3}. \qquad (1.6)$$

In Sec.3, we propose a curious neutrino mass matrix [2]

$$M_\nu = k_\nu M_e (M_e M_u^{1/2} + M_u^{1/2} M_e)^{-1} M_e, \qquad (1.7)$$

which is related to up-quark mass matrix M_u, and which can leads to a nearly tribimaximal mixing without assuming any discrete symmetry. (The tribimaximal mixing has usually been explained by assuming a discrete symmetry for the lepton mass matrices.[5]) Finally, Sec.4 is devoted to summary and concluding remarks.

[2]The form (1.7) has already been proposed in Ref. 3. Although five Yukawaons Y_u, Y_d, Y_e, Y_ν and Y_R have been assumed in Ref. 3, in the present scenario, we will assume only four Yukawaons Y_u, Y_d, Y_e and Y_R.

2 Mass spectrum

In order to see how to evaluate the mass spectra in the Yukawaon model, let us show an example in the charged lepton sector.

Under the O(3) flavor symmetry and U$(1)_X$ symmetry, we can write the following superpotential terms[6]

$$W_A = \lambda_A[\Phi_e\Phi_e A_e] + \mu_A[Y_e A_e] + \lambda'_A[\Phi_e\Phi_e A'_e] + \mu'_A[Y_e A'_e]$$
$$+\lambda''_A[\hat{\Phi}_e\hat{\Phi}_e A'_e] + \mu''_A[Y'_e A'_e], \tag{2.1}$$

where, for simplicity, we have denoted Tr$[X]$ as $[X]$ concisely. Here, the field Φ_e has been introduced in order to fix the VEV values of Y_e. We will refer Φ_e as an "ur-Yukawaon". The field $\hat{\Phi}_e$ denotes a traceless part of the ur-Yukawaon Φ_e, i.e. $\hat{\Phi}_e = \Phi_e - \frac{1}{3}[\Phi_e]$, and we have assigned U$(1)_X$ charges as $Q_X(Y_e) = x_e$, $Q_X(\Phi_e) = \frac{1}{2}x_e$ and $Q_X(A_e) = Q_X(A'_e) = -x_e$. (In order to prevent $(Y'_e)_{ij}$ from coupling with $\ell_i e^c_j$, we have to assign a different U$(1)_X$ charge to the field Y'_e from the Yukawaon Y_e, so that we replace [6] the coefficient μ''_A with $\lambda'''_A\phi_x$, where $Q_X(\phi_x) = x_\phi$ and $Q_X(Y'_e) = x_e - x_\phi$. However, for simplicity, hereafter, we use the expression μ''_A with U$(1)_X$ charge x_ϕ instead of $\lambda'''_A\phi_x$.) In Eq.(2.1), we have assumed non-existence of a term $[Y'_e A_e]$. This is an ad hoc assumption, but it is a crucial assumption to obtain a charged lepton mass relation (2.10) later.

From the SUSY vacuum condition $\partial W/\partial A_e = 0$, and $\partial W/\partial A'_e = 0$, we obtain the VEV relations

$$Y_e = k\Phi_e\Phi_e, \tag{2.2}$$

and

$$Y'_e = k'(\Phi_e\Phi_e + \xi\hat{\Phi}_e\hat{\Phi}_e), \tag{2.3}$$

respectively, where $k = -\lambda_A/\mu_A$ and

$$k' = -\frac{1}{\mu''_A}\left(\lambda'_A - \frac{\mu'_A}{\mu_A}\lambda_A\right), \quad \xi = \frac{\lambda''_A}{\lambda'_A - \frac{\mu'_A}{\mu_A}\lambda_A}. \tag{2.4}$$

Here, in Eq.(2.3), we have used the relation (2.2).

Next, we introduce a field B_e with $Q_X = -\frac{3}{2}x_e + x_\phi$, and we write a superpotential term

$$W_B = \lambda_B[\Phi_e Y'_e B_e] + \varepsilon_1\lambda_B[\Phi_e][Y'_e][B_e]. \tag{2.5}$$

(For a more general form, see Ref.[6].) The SUSY vacuum condition $\partial W/\partial B_e = 0$ ($W = W_e = W_A + W_B$) gives

$$\Phi_e(\Phi_e\Phi_e + \xi\hat{\Phi}_e\hat{\Phi}_e) + \varepsilon_1[\Phi_e][\Phi_e\Phi_e + \xi\hat{\Phi}_e\hat{\Phi}_e]\mathbf{1}$$
$$= (1+\xi)\Phi_e^3 - \frac{2}{3}\xi[\Phi_e]\,\Phi_e^2 + \frac{1}{9}\xi[\Phi_e]^2\,\Phi_e$$
$$+\varepsilon_1[\Phi_e]\left((1+\xi)[\Phi_e\Phi_e] - \frac{\xi}{3}[\Phi_e]^2\right)\mathbf{1} = 0, \tag{2.6}$$

from Eq.(2.3). (Other SUSY vacuum conditions $\partial W/\partial Y_e = 0$ and $\partial W/\partial \Phi_e = 0$ lead to $A_e = B_e = 0$.) On the other hand, in general, in a cubic equation

$$\Phi^3 + c_2\Phi^2 + c_1\Phi + c_0\mathbf{1} = 0, \tag{2.7}$$

the coefficients c_i have the following relations:

$$c_2 = -[\Phi], \quad c_1 = \frac{1}{2}\left([\Phi]^2 - [\Phi\Phi]\right), \quad c_0 = -\mathrm{det}\Phi. \tag{2.8}$$

In order to get non-zero a solution $[\Phi_e] \neq 0$, we must take

$$\xi = -3, \tag{2.9}$$

from the coefficient c_2. Then, we can obtain the ratios

$$R_e \equiv \frac{m_e + m_\mu + m_\tau}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} = \frac{[\Phi_e\Phi_e]}{[\Phi_e]^2} = 1 - \frac{2\xi}{9(1+\xi)} = \frac{2}{3}, \tag{2.10}$$

and

$$r_e = \frac{\sqrt{m_e m_\mu m_\tau}}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^3} = \frac{\mathrm{det}\Phi_e}{[\Phi_e]^3} = -\frac{1}{6}\varepsilon_1, \tag{2.11}$$

from the coefficients c_1 and c_0, respectively. Thus, we can obtain the successful relation [7] (2.10) for the charged lepton masses. At present, the parameter ε_1 is free, so that we cannot predict the value of r_e.

By the way, since we have successfully obtained the relation (2.10) without any adjustable parameter, another problem has risen in the present scenario: We know that $R = 2/3$ is valid only for the charged lepton masses, and the observed masses for another sectors do not satisfy $R = 2/3$. For example, the ratio R_u for the up-quark masses is $R_u \simeq 8/9$ [8]. Can we modify the present scenario as it leads to $R_u \simeq 8/9$? At present it seems to be impossible, because there is no adjustable parameter in the present scenario. We expect that all Yukawaons are related to each other and those VEVs are described in terms of the VEV of the ur-Yukawaon Φ_e. (A possibility that $\langle Y_u \rangle$ is described in terms of $\langle Y_e \rangle$ is discussed in Ref. [6]). However, the purpose of the present talk is not to give the mass spectra, we do not discuss more details of this topic here. Our goal is to give a unified description of all Yukawaons. As the first step, we discuss a phenomenological relation between the Yukawaons Y_e and Y_u in the next section.

3 Neutrino mass matrix

Based on a supersymmetric Yukawaon model, a curious neutrino mass matrix has recently been proposed.[4]. The mass matrix M_ν is related to up-quark masses as follows:

$$M_\nu = M_D M_R^{-1} M_D^T, \tag{3.1}$$

where the neutrino Dirac mass matrix M_D is given by

$$M_D \propto \langle Y_\nu \rangle \propto \langle Y_e \rangle \propto M_e, \tag{3.2}$$

and the right-handed neutrino Majorana mass matrix M_R is given by

$$M_R \propto \langle Y_R \rangle \propto \langle Y_e \rangle \langle \Phi_u \rangle + \langle \Phi_u \rangle \langle Y_e \rangle \propto M_e M_u^{1/2} + M_u^{1/2} M_e, \tag{3.3}$$

where the ur-Yukawaon Φ_u has a relation $Y_u = k_u \Phi_u \Phi_u$ similar to Eq.(2.2).

In the present model, differently from the model given in Ref. [4], we assume that ν^c and e^c have the same $\mathrm{U}(1)_X$ charge x_e, so that the Yukawaon Y_e couples not only to the charged lepton sector, but also to the neutrino sector:

$$W_Y = \frac{y_\nu}{\Lambda}(\ell Y_e \nu^c) H_u + \frac{y_e}{\Lambda}(\ell Y_e e^c) H_d + y_R(\nu^c Y_R \nu^c) + \frac{y'_R}{\Lambda}(\nu^c Y_e Y_e \nu^c). \tag{3.4}$$

Next, we assume additional fields A_R with $Q_X = -(\frac{1}{2}x_u + x_e)$, so that we obtain the superpotential terms for A_R as follows:[3]

$$W_R = \lambda_R[(Y_e \Phi_u + \Phi_u Y_e) A_R] + \mu_R [Y_R A_R]. \tag{3.5}$$

Then, from the SUSY vacuum condition $\partial W / \partial A_R = 0$, we can obtain the relation (3.3).

In order to calculate lepton mixing matrix U, we have to know a matrix form M_ν at the diagonal basis of $\langle Y_e \rangle$. Hereafter, we will denote a VEV matrix of a field A at the diagonal basis of $\langle Y_f \rangle$ as $\langle A \rangle_f$. From the definition of the diagonal basis, we can express

$$\langle Y_e \rangle_e \propto D_e = \mathrm{diag}(m_e, m_\mu, m_\tau), \tag{3.6}$$

$$\langle \Phi_u \rangle_e \neq \langle \Phi_u \rangle_u \propto D_u^{1/2} = \mathrm{diag}\left(\sqrt{m_u}, \sqrt{m_c}, \sqrt{m_t}\right). \tag{3.7}$$

Our concern is in the form of $\langle \Phi_u \rangle_e$. The Cabibbo-Kobayashi-Maskawa (CKM) matrix V satisfies the following relation[4]

$$\langle Y_u \rangle_d = V^T(\delta_q) \langle Y_u \rangle_u V(\delta_q). \tag{3.8}$$

On the analogy of the form of $\langle Y_u \rangle_d$, (3.8), we assume a form of $\langle \Phi_u \rangle_e$ as

$$\langle \Phi_u \rangle_e = V^T(\delta_\ell) \langle \Phi_u \rangle_u V(\delta_\ell), \tag{3.9}$$

where we have assumed that δ_ℓ is a free parameter. Then, from observed values of the up-quark masses[9] and CKM parameters[10], we obtain numerical results of the neutrino mixing parameters $\sin^2 2\theta_{23}$, $\tan^2 \theta_{12}$ and $|U_{13}|$ as shown in Table 1.

The observed value[10] of δ_q in the quark sector is $\delta_q = 68.9^\circ$, but the value $\delta_\ell = \delta_q = 68.9^\circ$ can not give a reasonable value of $\sin^2 2\theta_{23}$. On the other hand, the value $\delta_\ell = 180^\circ$ can

[3]Exactly speaking, we have to read μ_R as $\lambda'_R \phi_R$. Otherwise, the field Y_R has the $\mathrm{U}(1)_X$ charge $Q_X(Y_R) = 2x_e = \frac{1}{2}x_u + x_e$, so that we are obliged to accept the relation $x_e = \frac{1}{2}x_u$. This means that Y_e and Φ_u have the same charge, a mixing between Y_e and Φ_u is caused.

[4]In the present O(3) flavor model, the mass matrix M_f is diagonalized as $U_f^T M_f U_f = D_f$.

Table 1. Numerical results for neutrino mixing parameters versus δ_ℓ.

δ_ℓ	$\sin^2 2\theta_{23}$	$\tan^2\theta_{12}$	$\|U_{13}\|$	$\Delta m^2_{21}/\Delta m^2_{32}$
0	0.4803	0.4745	0.01042	0.00196
60°	0.7631	0.4801	0.00844	0.00139
68.9°	0.8127	0.4851	0.00781	0.00127
90°	0.9028	0.5017	0.00615	0.00102
120°	0.9688	0.5277	0.00386	0.00081
180°	0.9952	0.5525	0.00094	0.00068

successfully give a nearly tribimaximal mixing. Since, in the present O(3) model, the parameter δ_ℓ must be 0 or π because we have assumed that $\langle\Phi_e\rangle$ and $\langle\Phi_u\rangle$ are real. This is in favor of the results in Table 1. Although the values of $\Delta m^2_{21}/\Delta m^2_{32}$ in Table 1 are too small compared with the observed value[12, 11] $\Delta m^2_{21}/\Delta m^2_{32} = 0.028 \pm 0.004$, the values can suitably be adjusted by considering the y'_R-term in Eq.(3.4) without changing the predictions of the neutrino mixing parameters.

4 Concluding remarks

In conclusion, we have obtained a curious neutrino mass matrix (1.7) which is related to up-quark mass matrix, and which can leads to a nearly tribimaximal mixing without assuming any discrete symmetry. (Although the form (1.7) has already been given in Ref. [4], the superpotential form in the present Yukawaon model is somewhat different from that in Ref. [4].) However, at present, there is no theoretical ground for the relation (3.9). The relation (3.9) is merely a phenomenological assumption. Nevertheless, we have successfully obtained a nearly tribimaximal mixing under this assumption without assuming any discrete symmetry. It will offer an important clue to a unified understanding of quarks and leptons to investigate why the phenomenological assumption (3.9) is so effectual.

The approach based on a Yukawaon model to masses and mixings will provide a new view different from conventional mass matrix models. Especially, the approach seems to be powerful to the predictions of mass relations R_f and r_f similar to R_e and r_e defined in Eqs.(1.5) and (1.6), respectively.

The neutrino mass matrix (1.7) can successfully give a tribimaximal mixing without assuming any discrete symmetry. Here, the neutrino mass matrix has been described by VEVs of the ur-Yukawaons Φ_e and Φ_u. This suggests a possible relation between Yukawaons in the lepton and quark sectors. Our goal is to give a unified description of all Yukawaons. There is a possibility that VEVs of all Yukawaons Y_f are described only one VEV matrix of the ur-Yukawaon Y_e. So far, we did not discuss the down-quark Yukawaon Y_d. Whether a unified Yukawaon model is possible or not is dependent on whether a Y_d can also reasonably be described in terms of Φ_e (and also Φ_u). This will be a touchstone of the Yukawaon approach.

At present, the calculations have been done in the supersymmetric limit. We have considered $\langle\Phi_f\rangle \sim \Lambda \sim 10^{15}$ GeV, so that almost Yukawaon components are massive and invisible in the low energy phenomena. However, some of Yukawaons are massless in the supersymmetric limit. Since we consider an explicitly broken term of the flavor symmetry which effectively appears when SUSY is broken, those massless particles will acquire masses of TeV scale, so that the effects will be able to be observed in the TeV region physics.

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