Maximal CP Violation Hypothesis and a Lepton Mixing Matrix

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Abstract

Maximal CP violation hypothesis is applied to a simple lepton mixing matrix form $U = V_{CKM}^\dagger U_{TB}$, which has recently been speculated under an ansatz that U becomes an exact tribimaximal mixing U_{TB} in a limit of the quark mixing matrix $V_{CKM} \to \mathbf{1}$. The prediction $\tan^2 \theta_{12} = 1/2$ in the case of the exact tribimaximal mixing $U = U_{TB}$ is considerably spoiled in the speculated mixing $U = V_{CKM}^\dagger U_{TB}$. However, the application of the hypothesis to the lepton sector can again recover the spoiled value to $\tan^2 \theta_{12} \simeq 1/2$ if the original Kobayashi-Maskawa phase convention for V_{CKM} is adopted.

1 Introduction

Recently, an interesting form of the lepton mixing matrix U has been proposed [1]:

$$U = V^\dagger U_{TB}, \tag{1.1}$$

which was speculated under an ansatz that U becomes an exact tribimaximal mixing [2] U_{TB} in a limit $V \to \mathbf{1}$ (V is the Cabibbo-Kobayashi-Maskawa (CKM) quark mixing matrix). Here, U_{TB} is given by

$$U_{TB} = P^\dagger(\gamma)U_{TB}^0 P(\sigma), \tag{1.2}$$

where

$$P(\gamma) = \text{diag}(e^{i\gamma_1}, e^{i\gamma_2}, e^{i\gamma_3}), \quad P(\sigma) = \text{diag}(e^{i\sigma_1}, e^{i\sigma_2}, e^{i\sigma_3}), \tag{1.3}$$

$$U_{TB}^0 = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}. \tag{1.4}$$

A brief description of derivation of the relation (1.1) is as follows: the up-quark and neutrino mass matrices in the limit of $U_u \to \mathbf{1}$ are given by $M_u^0 = D_u$ and $M_\nu^0 = U_{TB}D_\nu U_{TB}^T$ ($D_f = \text{diag}(m_{f1}, m_{f2}, m_{f3})$), and those in the observed world with a realistic small deviation $V \neq \mathbf{1}$ from $V = \mathbf{1}$ become modified as $M_u^0 \to M_u = U_u D_u U_u^\dagger$ and $M_\nu^0 \to U_u M_\nu^0 U_u^\dagger$ (we use a mass matrix convention $U_f^\dagger M_f U_f = D_f$). Therefore, we obtain $U_\nu = U_u U_{TB}$ and $U = U_e^\dagger U_\nu = U_e^\dagger U_d V^\dagger U_{TB}$, which leads to the relation (1.1) by using an additional ansatz "$U_d \to \mathbf{1}$ and $U_e \to \mathbf{1}$ in the limit of $U_u \to \mathbf{1}$" demanding approximately $U_e = U_d$. (For an explicit neutrino

1

mass matrix model which gives the relation (1.1), see, for example, Refs.[3, 4].) Note that we have assumed a hypothetical limit $V \to \mathbf{1}$ which is realized by switching off terms giving $V \neq \mathbf{1}$, irrespectively of an energy scale.

The pure tribimaximal mixing $U = U_{TB}$ predicts $\tan^2 \theta_{12} = 1/2$, $\sin^2 2\theta_{23} = 1$ and $\sin^2 \theta_{13} = 0$ even if we consider a degree of freedom due to the phase convention given by (1.2). In contrast to the case $U = U_{TB}$, the predictions in the case $U = V^\dagger U_{TB}$ are spoiled by the presence of $P(\gamma)$. Especially, the strict prediction $\tan^2 \theta_{12} = 1/2$ is considerably spoiled by the presence of a phase parameter $\beta \equiv \gamma_2 - \gamma_1$: The predicted deviations of $\sin^2 2\theta_{23}$ and $\sin^2 \theta_{13}$ from those in the exact tribimaximal mixing $U = U_{TB}$ are small, i.e. $0.024 \leq \sin^2 \theta_{13} \leq 0.028$ and $0.94 \leq \sin^2 2\theta_{23} \leq 0.95$ depending on a phase parameter $\alpha \equiv \gamma_3 - \gamma_2$, while the prediction $\tan^2 \theta_{12} = 1/2$ becomes vague, i.e. $0.24 \leq \tan^2 \theta_{12} \leq 1.00$ depending on the phase parameter β (see Fig.3 in Ref.[1]). Here, the parameters α and β are not observable parameters in the mixing matrix U, but they are "model-parameters". However, since we fix the matrix V in the ansatz (1.1) by the observed CKM matrix parameters, the rotation angles and CP violation phase parameter δ_ℓ in the lepton mixing matrix are completely determined by the parameters α and β under the ansatz (1.1). (Note that the phase parameters σ_i, which are the so-called Majorana phases, do not affect neutrino oscillation phenomena.) If we take $\beta \simeq \pi/2$, we can again predict $\tan^2 \theta_{12} \simeq 1/2$. This was pointed out by Plentinger and Rodejohann [3], and also by the authors [1]. However, it is not clear whether the choice $\beta = \pi/2$ means really a case of the maximal CP violation or not, because there are three CP violating phases in the present scenario, i.e. α, β and δ_q (δ_q is a CP violating phase parameter in the CKM matrix $V(\delta_q)$).

Since we apply the maximal CP violation hypothesis to the phenomenological ansatz (1.1), here, let us present a short review of the hypothesis. Usually, the maximal CP violation hypothesis is defined as follows: the nature takes values of CP violating phases so that a magnitude of the rephasing invariant quantity J [5] takes its maximal value. Generally, the CKM matrix $V(\delta_q)$ is described by 4 phase-convention-dependent parameters (there are, in general, 9 phase conventions of the CKM matrix [6]), i.e. three rotation parameters $(\theta_1, \theta_2, \theta_3)$ and one CP violating phase parameter δ_q. We may choose the observable values $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and $|V_{td}|$ straightforwardly, instead of three rotation parameters and one phase parameter. In fact, as we demonstrate in the next section, we can fix the three rotation parameters and one phase parameter from the observed 4 values of $|V_{ij}|$ when we adopt some phase convention (but signs of the rotation parameters remain as unsettled ones). The rephasing invariant quantity J is expressed by $J \propto \sin \delta_q$ in any phase convention [6] of the CKM matrix, so that the maximal CP violation means $\delta_q = \pi/2$. The requirement of this maximal CP violation, in general, put an over-constraint on the CKM parameters, because we already know the four independent values of the CKM matrix $|V_{us}|$, $|V_{cb}|$, $|V_{ub}|$ and $|V_{td}|$. As we demonstrate in the next section, we find that only the original Kobayashi-Maskawa (KM) phase convention [7] can satisfies the maximal CP violation hypothesis [8]. We know that the physics in the CKM mixing are invariant under the rephasing. On the other hand, we know that the phase conventions of the CKM matrix are deeply related to explicit mass matrix forms in the models. This suggests that the hypothesis is not for parameters in the CKM matrix, but for those in a mass matrix model. It should be noted that the maximal CP violation hypothesis is not one based on a theoretical ground but

a phenomenological one.

In the present paper, we extend the maximal CP violation hypothesis to the following hypothesis: When the rephasing invariant quantity J is a function of CP violating phases δ_1, δ_2, $\cdots$, i.e. $J = J(\delta_1, \delta_2, \cdots)$, the maximal CP violation hypothesis requires

$$\frac{\partial J}{\partial \delta_1} = \frac{\partial J}{\partial \delta_2} = \cdots = 0, \tag{1.5}$$

under the condition that rotation parameters are fixed. Here, δ_1, δ_2, $\cdots$ are CP violating phase parameters in a mass matrix model. Note that the mixing matrix V (U) can always be expressed by three rotation parameters and one phase parameter δ_q (δ_ℓ), and they can become observable parameters when we adopt some phase convention. In contrast to these four parameters in the mixing matrix, the phases δ_i are not observable even when we adopt a phase convention. The CP violating parameter δ_q (δ_ℓ) is given by a function of δ_i and other mass matrix parameters. By abbreviating δ_q (δ_ℓ) to δ we have

$$\frac{\partial J}{\partial \delta} = \frac{\partial J}{\partial \delta_1}\frac{\partial \delta_1}{\partial \delta} + \frac{\partial J}{\partial \delta_2}\frac{\partial \delta_2}{\partial \delta} + \cdots . \tag{1.6}$$

Therefore, it turns out that the requirement (1.5) is considerably stronger than the constraint $\partial J/\partial \delta = 0$.

First, let us demonstrate that even when J involves only one CP violating phase δ, results based on the above definition of the maximal CP violation hypothesis depend on phase conventions of the flavor mixing matrix [8]. For example, in the standard expression [9] $V_{SD}(\delta_{SD})$ and original Kobayashi-Maskawa (KM) expression [7] $V_{KM}(\delta_{KM})$ of V, the rephasing invariant quantity J is given by

$$J_{SD} = c_{13}^2 s_{13} c_{12} s_{12} c_{23} s_{23} \sin \delta_{SD}, \tag{1.7}$$

and

$$J_{KM} = c_1 s_1^2 c_2 s_2 c_3 s_3 \sin \delta_{KM}, \tag{1.8}$$

respectively. Here, V_{SD} and V_{KM} are explicitly given by

$$V_{SD} = R_1(\theta_{23})P_3(\delta_{SD})R_2(\theta_{13})P_3^\dagger(\delta_{SD})R_3(\theta_{12})$$

$$= \begin{pmatrix} c_{13}c_{12} & c_{13}s_{12} & s_{13}e^{-i\delta_{SD}} \\ -c_{23}s_{12} - s_{23}c_{12}s_{13}e^{i\delta_{SD}} & c_{23}c_{12} - s_{23}s_{12}s_{13}e^{i\delta_{SD}} & s_{23}c_{13} \\ s_{23}s_{12} - c_{23}c_{12}s_{13}e^{i\delta_{SD}} & -s_{23}c_{12} - c_{23}s_{12}s_{13}e^{i\delta_{SD}} & c_{23}c_{13} \end{pmatrix}, \tag{1.9}$$

$$V_{KM} = R_1^T(\theta_2)P_3(\delta_{KM}+\pi)R_3(\theta_1)R_1(\theta_3)$$

$$= \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta_{KM}} & c_1c_2s_3 + s_2c_3e^{i\delta_{KM}} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta_{KM}} & c_1s_2s_3 - c_2c_3e^{i\delta_{KM}} \end{pmatrix}, \tag{1.10}$$

respectively, where

$$R_1(\theta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix}, \quad R_2(\theta) = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ -s & 0 & c \end{pmatrix}, \quad R_3(\theta) = \begin{pmatrix} c & s & 0 \\ -s & c & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (1.11)$$

$$P_3(\delta) = \mathrm{diag}(1, \ 1, \ e^{i\delta}), \tag{1.12}$$

$s = \sin\theta$ and $c = \cos\theta$. It is well known [8] that the standard expression $V_{SD}(\delta_{SD})$ with $\delta_{SD} = \pm\pi/2$ cannot describe the observed CKM matrix parameters, while $V_{KM}(\delta_{KM})$ with $\delta_{KM} = \pm\pi/2$ can well describe the observed those. (In the standard phase convention $V_{SD}(\delta_{SD})$, a case with $\delta_{SD} \simeq 70°$ is in favor of the observed data.) Thus the requirement of the maximal CP violation in the quark sector can give a reasonable value for the CKM phase parameter only when the original KM matrix phase convention is adopted. From such a phenomenological point of view, we adopt this convention not only for the quark sector but also for the lepton sector [i.e. V in the lepton mixing matrix U given by Eq.(1.1)] in order to ensure consistency.

In this paper, we assume the maximal CP violation hypothesis for both the quark and lepton sectors. In this scenario, since the matrix $V(\delta_q)$ in Eq.(1.1) is already fixed by the observed data in the quark sector, the rephasing invariant quantity J is only a function of α and β. In Sec.2, we re-investigate the CKM mixing parameters from the data in the quark sector, and fix mixing parameters in the phase convention $V = V_{KM}$ at $\mu \simeq m_Z$ under the maximal CP violation hypothesis. Here we use an energy scale $\mu = m_Z$ at which the maximal CP violation hypothesis in the quark sector seems to work out. In Sec.3, we will apply the maximal CP violation hypothesis to the lepton mixing $U = V^\dagger U_{TB}$ with $V = V_{KM}$. We find that the maximal value of $|J(\alpha, \beta)|$ takes place at $\beta \simeq \pm\pi/2$ and $\alpha \simeq 0$ (or $\alpha \simeq \pi$), so that we can again obtain $\tan^2\theta_{12} \simeq 1/2$. (Note that the definition of the parameter α and β in the present paper are different from those in the previous paper [1], because the CKM matrix V in $U = V^\dagger U_{TB}$ was V_{SD} in the previous paper, while the present one is V_{KM}.) Finally, Sec.4 is devoted to the summary and concluding remarks.

2 Maximal CP violation hypothesis in the quark sector

First, we estimate the CKM matrix parameters in the original KM matrix $V_{KM}(\delta_{KM})$ without assuming the maximal CP violation. Using input values [10] $|V_{us}| = 0.2255 \pm 0.0019$, $|V_{ub}| = 0.00393 \pm 0.00036$ and $|V_{td}| = 0.0081 \pm 0.0006$, we obtain the rotation parameters

$$|s_1| = 0.2255 \pm 0.0019, \quad |s_2| = 0.0359^{+0.0030}_{-0.0029}, \quad |s_3| = 0.0174^{+0.0018}_{-0.0017}. \tag{2.1}$$

By fitting the value of δ_{KM} to the observed value $|V_{cb}| = 0.0412 \pm 0.0011$, we obtain $\delta_{KM} = (84^{+16}_{-22})°$. The present observed values do not give an exact value $\delta_{KM} = \pi/2$, but it is not ruled out.

Inversely, if we assume the maximal CP violation, i.e. $\delta_{KM} = \pm\pi/2$, we can fix the parameters s_1, s_2 and s_3 from the observed values of $|V_{us}|$, $|V_{cb}|$ and $|V_{ub}|$, and can predict the value of $|V_{td}|$. (Although the value of s_2 is readily fixed from the relation $V_{td} = s_1 s_2$ in the

4

original KM matrix, we use the value $|V_{cb}|$ as an input value, because the accuracy of $|V_{td}|$ is not so precise compared with that of $|V_{cb}|$.) For convenience, we define $V_{us} > 0$, so that we take $s_1 = -\sqrt{|V_{us}|^2 + |V_{ub}|^2} < 0$ and $s_3 = V_{ub}/\sqrt{|V_{us}|^2 + |V_{ub}|^2}$. We also define that all c_i ($i = 1, 2, 3$) are positive, i.e. $c_i = \sqrt{1 - s_i^2} > 0$. For input values [10], $V_{us} = 0.2255 \pm 0.0019$, $V_{ub} = -s_1 s_3 = \pm(0.00393 \pm 0.00036)$, $|V_{cb}| = 0.0412 \pm 0.0011$, we obtain reasonable CKM parameter values only for cases with $s_3 s_\delta / s_2 > 0$ ($s_\delta = \sin \delta_{KM} = \pm 1$):

$$s_1 = -(0.2255 \pm 0.0019), \quad |s_2| = 0.0376^{+0.0019}_{-0.0021}, \quad |s_3| = 0.0174^{+0.0018}_{-0.0017}, \tag{2.2}$$

$$|V_{td}| = 0.0085 \pm 0.0005, \tag{2.3}$$

$$\phi_1 = (24.4^{-3.5}_{+3.2})^\circ, \quad \phi_2 = (89.963 \mp 0.004)^\circ, \quad \phi_3 = (65.7^{+3.1}_{-3.5})^\circ, \tag{2.4}$$

where the angles ϕ_i of the unitary triangle have been defined by

$$\phi_1 = \arg\left(-\frac{V_{21}V_{23}^*}{V_{31}V_{33}^*}\right), \quad \phi_2 = \arg\left(-\frac{V_{31}V_{33}^*}{V_{11}V_{13}^*}\right), \quad \phi_3 = \arg\left(-\frac{V_{11}V_{13}^*}{V_{21}V_{23}^*}\right). \tag{2.5}$$

Those predicted values are in agreement with the observed CKM matrix data [10].

Next we consider the case in which we adopt $V = V_{SD}(\delta_{SD})$ instead of using V_{KM}. In this standard phase convention, by using the global fit values, $|V_{us}| = 0.2257 \pm 0.0010$, $|V_{cb}| = 0.0415^{+0.0010}_{-0.0011}$, $|V_{ub}| = 0.00359 \pm 0.00016$ and $|V_{td}| = 0.00874^{+0.00026}_{-0.00037}$, reported by Particle Data Group [10], we obtain

$$\delta_{SD} = (68.9^{+\ 9.1}_{-10.7})^\circ. \tag{2.6}$$

Thus, for the standard phase convention V_{SD}, we cannot demand the maximal CP violation hypothesis consistently, because the value $\delta_{SD} = (68.9^{+\ 9.1}_{-10.7})^\circ$ is far from the value $\delta_{SD} = \pi/2$ in the maximal CP violation hypothesis.

3 Maximal CP violation hypothesis in the lepton sector

We assume that the lepton mixing matrix U is given by Eq.(1.1). Although the observable parameters in the matrix U are three rotation parameters and one phase parameter, we practically have two parameters $\alpha \equiv \gamma_3 - \gamma_2$ and $\beta \equiv \gamma_2 - \gamma_1$ as adjustable parameters, because we fix the values of the CKM matrix V by the observed one $V = V_{KM}$. We apply the ansatz (1.5) to the lepton mixing matrix U with the free parameters α and β. The parameters α and β correspond to δ_1 and δ_2 in Eq.(1.5). Of course, the observable parameter in CP violation is only δ_ℓ in the present model (1.1), although it is not explicitly given in the present paper.

Now, we calculate the rephasing invariant quantity J in the lepton sector using the relation

$$J = \text{Im}(U_{23}U_{12}U_{22}^*U_{13}^*), \tag{3.1}$$

where

$$\begin{aligned}
U_{12} &= \tfrac{1}{\sqrt{3}}\left(V_{11}^*e^{-i\gamma_1} + V_{22}^*e^{-i\gamma_2} + V_{32}^*e^{-i\gamma_3}\right)e^{i\sigma_2}, \\
U_{22}^* &= \tfrac{1}{\sqrt{3}}\left(V_{12}e^{i\gamma_1} + V_{21}e^{i\gamma_2} + V_{31}e^{i\gamma_3}\right)e^{-i\sigma_2}, \\
U_{23} &= \tfrac{1}{\sqrt{2}}\left(-V_{22}^*e^{-i\gamma_2} + V_{32}^*e^{-i\gamma_3}\right)e^{i\sigma_3}, \\
U_{13}^* &= \tfrac{1}{\sqrt{2}}\left(-V_{21}e^{i\gamma_2} + V_{31}e^{i\gamma_3}\right)e^{-i\sigma_3}.
\end{aligned} \tag{3.2}$$

Here, the lepton mixing matrix U is given by the form (1.1), i.e. $U = V^\dagger U_{TB}$. Note that the CKM mixing matrix V should be estimated at energy scale $\mu \simeq m_Z$ by using a specific phase convention. Since we assume the maximal CP violation hypothesis for the quark sectors, too, we adopt the CKM matrix parameters θ_1, θ_2, θ_3 and $\delta_{KM} = \pm\pi/2$ in the original KM phase convention as we discussed in the previous section. Since the numerical results for the mixing U are dependent on the phase convention of V, the predicted values of the neutrino mixing parameters in the present paper are different from those in the previous paper [1], where the phase convention $V = V_{SD}$ was adopted. For reference, we illustrate the numerical results of the neutrino mixing parameters $\sin^2\theta_{13}$, $\sin^2 2\theta_{23}$ and $\tan^2\theta_{12}$ in Figs.1-3, correspondingly to Figs.1-3 in the previous paper. Although the numerical results are almost similar to the previous ones, a value (α, β) which gives a maximal $|J|$ is changed from the previous one.
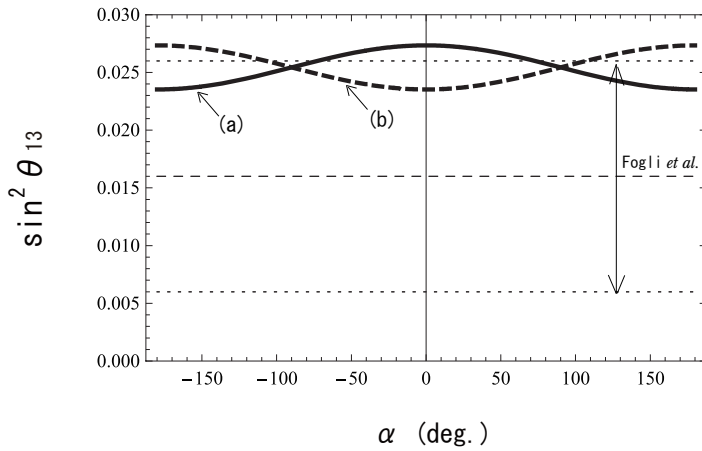
Fig. 1 Behavior of $\sin^2\theta_{13}$ versus α. Curves are drawn for inputs (a) $(s_\delta, s_1, s_2, s_3) = (+1, -0.2255, -0.0376, -0.0174)$ and (b) $(s_\delta, s_1, s_2, s_3) = (+1, -0.2255, +0.0376, +0.0174)$, where $s_\delta = \sin\delta_{KM}$.
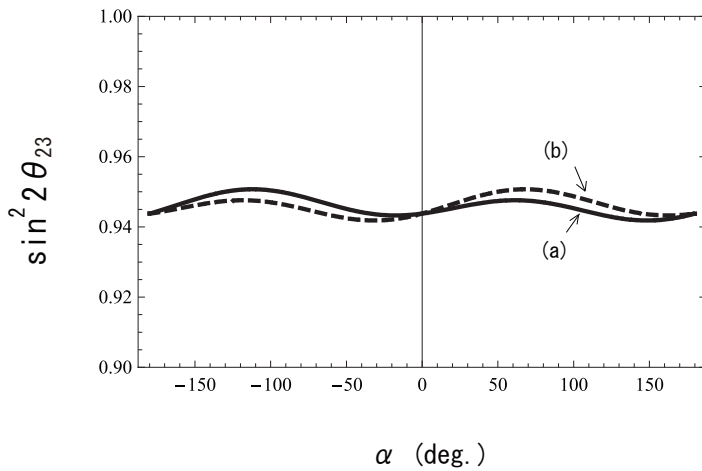
Fig. 2 Behavior of $\sin^2 2\theta_{23}$ versus α. Curves are drawn for inputs (a) $(s_\delta, s_1, s_2, s_3) = (+1, -0.2255, -0.0376, -0.0174)$ and (b) $(s_\delta, s_1, s_2, s_3) = (+1, -0.2255, +0.0376, +0.0174)$, where $s_\delta = \sin\delta_{KM}$.
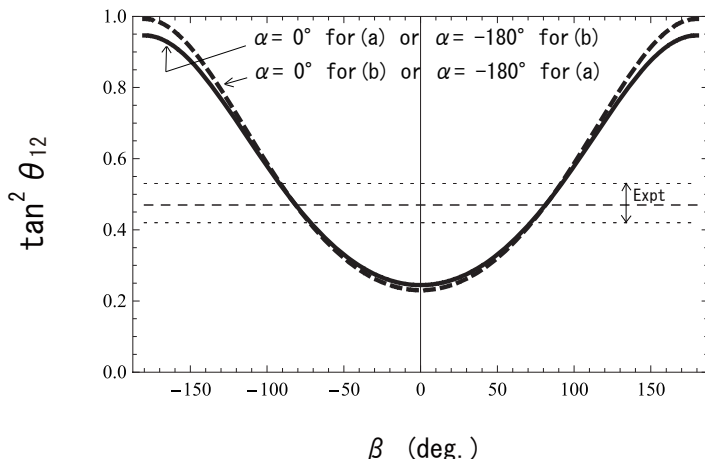
Fig. 3 Behavior of $\tan^2\theta_{12}$ versus β for typical values of α. Curves are drawn by taking $\alpha = 0°$ and $-180°$ for inputs (a) $(s_\delta, s_1, s_2, s_3) = (+1, -0.2255, -0.0376, -0.0174)$ and (b) $(s_\delta, s_1, s_2, s_3) = (+1, -0.2255, +0.0376, +0.0174)$, where $s_\delta = \sin\delta_{KM}$.

From Eq.(2.1), we obtain

$$J \simeq \frac{1}{6}s_1(s_\beta + s_2 s_\alpha c_\beta - s_2 c_\alpha s_\beta + 2s_3 c_\alpha c_\beta s_\delta), \tag{3.3}$$

where $\alpha = \gamma_3 - \gamma_2$, $\beta = \gamma_2 - \gamma_1$, $s_\delta = \sin\delta_{KM} = \pm 1$, and $c_\alpha = \cos\alpha$ and so on, and we have used the observed fact $1 \gg |s_1| \simeq |V_{us}| \gg |s_2| \simeq |V_{td}|/|V_{us}| \sim |s_3| \simeq |V_{ub}|/|V_{us}|$. The value J is approximately given by $J \simeq (1/6)\sin\theta_1 \sin\beta$, so that the maximal CP violation hypothesis demands $\beta \simeq \pm\pi/2$ (however, the small deviation from $\beta = \pm\pi/2$ is crucial). More precisely speaking, from $\partial J/\partial\beta = 0$, we obtain

$$\cot\beta \simeq 2s_3 c_\alpha s_\delta + s_2 s_\alpha. \tag{3.4}$$

Similarly, we obtain

$$\tan\alpha \simeq \frac{s_2 c_\beta}{2s_3 c_\beta s_\delta - s_2 s_\beta}, \tag{3.5}$$

from $\partial J/\partial\alpha = 0$ (but with a rough approximation). Since $\beta \simeq \pm\pi/2$ from Eq.(3.4), we obtain $\alpha \simeq 0$ or π from Eq.(3.5). We emphasize that the maximal CP violation hypothesis can determine values of the phase parameters α and β simultaneously. The numerical results obtained with use of no approximation are given in Table 1. As an example of the behavior of $|J(\alpha,\beta)|$, J versus α in a typical case $(s_\delta, s_2, s_3) = (+, -, -)$ in Table 1 is illustrated in Fig. 4. As seen in Table 1, the value of α takes 0 or π according as $s_2 < 0$ or $s_2 > 0$, i.e. $V_{td} < 0$ or $V_{td} > 0$. For comparison, we show the results for the case of $V = V_{SD}$ in Table 2. In this case, we obtain $\alpha \simeq 25°$, although we can still obtain $\beta \simeq \pi/2$.

s_δ	s_2	s_3	$(\pm J)_{max}$	α	β	Case
$+$	$+$	$+$	0.03772 ± 0.00037	$-(175.6^{-1.1}_{+0.4})^\circ$	$-(87.93 \mp 0.21)^\circ$	A_1
			$-(0.03800^{-0.00037}_{+0.00034})$	$+(179.3^{+0.6}_{-1.3})^\circ$	$+(91.88^{+0.20}_{-0.22})^\circ$	A_2
$+$	$-$	$-$	0.03772 ± 0.00037	$+(3.64^{+0.46}_{-0.44})^\circ$	$-(87.96 \mp 0.21)^\circ$	B_1
			$-(0.03801^{-0.00035}_{+0.00034})$	$+(2.86^{+0.29}_{-0.30})^\circ$	$+(92.01^{+0.21}_{-0.20})^\circ$	B_2
$-$	$+$	$-$	0.03772 ± 0.00037	$-(175.6^{-1.1}_{+0.4})^\circ$	$-(87.93 \mp 0.21)^\circ$	A_1
			$-(0.03800^{-0.00037}_{+0.00034})$	$+(179.3^{+0.6}_{-1.3})^\circ$	$+(91.88^{+0.20}_{-0.22})^\circ$	A_2
$-$	$-$	$+$	0.03772 ± 0.00037	$+(3.64^{+0.46}_{-0.44})^\circ$	$-(87.96 \mp 0.21)^\circ$	B_1
			$-(0.03801^{-0.00035}_{+0.00034})$	$+(2.86^{+0.29}_{-0.30})^\circ$	$+(92.01^{+0.21}_{-0.20})^\circ$	B_2

Table 1: Possible solutions of CP violating phase factors $\alpha = \gamma_3 - \gamma_2$ and $\beta = \gamma_2 - \gamma_1$ under the maximal CP violation hypothesis. We obtain four sets of (α, β), which are denoted by A_1, A_2, B_1, and B_2.

$(\pm J)_{max}$	α	β
$+(0.0378 \pm 0.0002)$	$+(25.4^{+2.4}_{-3.6})^\circ$	$-(88.2^{+0.1}_{-0.2})^\circ$
$-(0.0381 \pm 0.0002)$	$+(24.5^{+1.8}_{-1.7})^\circ$	$+(91.8 \pm 0.1)^\circ$

Table 2: Possible values of CP violating phase factors $\alpha = \gamma_3 - \gamma_2$ and $\beta = \gamma_2 - \gamma_1$ for the case $V = V_{SD}(\delta_{SD})$ with $\delta_{SD} = (68.9^{+9.1}_{-10.7})^\circ$.
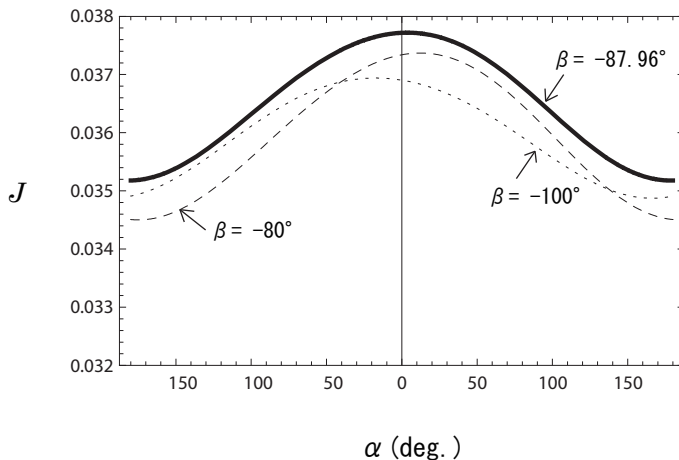

Fig. 4 An example of the behavior of $J(\alpha, \beta)$ versus α for typical values of β. The case corresponds to the case with $(s_\delta, s_2, s_3) = (+, -, -)$ in Table 1.

4 Summary

In conclusion, we have applied an extended "maximal CP violation hypothesis" (1.5) to a simple lepton mixing matrix form $U = V^\dagger U_{TB}$, which has recently been speculated under an ansatz that U becomes an exact tribimaximal mixing U_{TB} in a limit of the quark mixing matrix $V \to \mathbf{1}$. The mixing matrix U_{TB} includes two phase parameters $\alpha = \gamma_3 - \gamma_2$ and $\beta = \gamma_2 - \gamma_1$ due to the phase convention of the tribimaximal mixing. Therefore, the rephasing invariant quantity J in the lepton sector is a function of phase parameters α, β and δ_q (δ_q is a CP violating phase parameter in the quark mixing matrix $V(\delta_q)$). We have demanded the maximal CP violation

Case	$\sin^2 2\theta_{23}$	$\sin^2 \theta_{13}$	$\tan^2 \theta_{12}$
A_1	0.944 ± 0.001	0.0273 ± 0.0005	0.507 ± 0.001
A_2	0.944 ± 0.001	0.0273 ± 0.0005	0.530 ± 0.001
B_1	0.944 ± 0.001	0.0273 ± 0.0006	0.5083 ± 0.0003
B_2	0.944 ± 0.001	0.0273 ± 0.0006	0.529 ± 0.001

Table 3: Predicted values of neutrino oscillation parameters in the cases defined in Table 1.

hypothesis for the quark sector too. Thus, we have taken the original KM phase convention $V_{KM}(\delta_{KM})$ with $\delta_{KM} = \pm\pi/2$ as the CKM matrix V in Eq.(1.1), because the standard phase convention $V = V_{SD}(\delta_{SD})$ with $\delta_{SD} = \pm\pi/2$ cannot reproduce the observed CKM parameters consistently under the hypothesis (1.5). Then, the quantity J in the lepton sector is a function of only α and β. We have regarded the parameters α and β as the independent CP violation parameters in applying the maximal CP violation hypothesis (1.5) to the lepton mixing matrix (1.1), although the observable CP violation parameter is still a parameter δ_ℓ which is given by a function of α and β. (For example, we can choose $\sin^2 2\theta_{23}$, $\tan^2 \theta_{12}$, $|U_{13}|$ and δ_ℓ as the four observable quantities in the lepton mixing matrix U except for Majorana phase parameters.)

We have found that only for the case $V = V_{KM}$, the maximal CP violation hypothesis leads to interesting results, $\delta_{KM} = \pm\pi/2$ in the quark sector, and $\beta \simeq \pm\pi/2$ and $\alpha \simeq 0$ (Cases A_1 and A_2) [or $\alpha \simeq \pi$ (Cases B_1 and B_2)] in the lepton sector. The result $\beta \simeq \pm\pi/2$ predicts [1] $\tan^2 \theta_{12} \simeq 1/2$ which is in good agreement with the observed value $\tan^2 \theta_{12} = 0.47^{+0.05}_{-0.04}$ [11]. The result $\alpha \simeq 0$ (or $\alpha \simeq \pi$) means that the neutrino mass matrix $M_\nu^0 = U_{TB} D_\nu U_{TB}^T$ in the limit of $V \to \mathbf{1}$ is nearly $2 \leftrightarrow 3$ symmetric (or antisymmetric). The predicted neutrino oscillation parameters are listed in Table 3 for the possible cases defined in Table 1. The predicted values are consistent with the observed values $\sin^2 2\theta_{23} = 1.00_{-0.13}$ [12], $\tan^2 \theta_{12} = 0.47^{+0.05}_{-0.04}$ [11] and $\sin^2 \theta_{13} = 0.016 \pm 0.010$ (1σ) [13], although the predicted value $\sin^2 \theta_{13} = 0.0273$ is somewhat critical compared with the value [13] $\sin^2 \theta_{13} = 0.016 + 0.010$ reported by Fogli *et al.*

It is worthwhile noticing that the neutrino mixing matrix $U = V^\dagger U_{TB}$ with the realistic $V \neq \mathbf{1}$ spoils the prediction $\tan^2 \theta_{12} = 1/2$ in the pure tribimaximal mixing $U = U_{TB}$ as $0.24 \leq \tan^2 \theta_{12} \leq 1.00$, while the maximal CP violation hypothesis fixes the phase parameter β as $\beta \simeq \pm\pi/2$, so that the hypothesis recovers the spoiled value of $\tan^2 \theta_{12}$ to $\tan^2 \theta_{12} \simeq 1/2$. The parameter β is fixed almost independently of the phase convention of the quark mixing matrix V, while the parameter α is fixed dependently on the phase convention of V: If we take $V = V_{KM}(\delta_{KM})$ with $\delta_{KM} = \pm\pi/2$ under the maximal CP violation hypothesis, we obtain the result $\alpha \simeq 0$ or π. On the other hand, if we take $V = V_{SD}(\delta_{SD})$ with $\delta_{SD} = 68.9°$ (without the maximal CP violation hypothesis in the quark sector), we obtain $\alpha \simeq 25°$, which does not seem to be a suggestive value. Thus, the maximal CP violation hypothesis can lead to phenomenologically interesting results not only in the quark sector, but also in the lepton sector. However, the reason why the hypothesis is so effective only when we take $V = V_{KM}$ has still not been understood. Also, theoretical ground for the maximal CP violation hypothesis has still been unclear. We hope that, by investigating these problems, one will find a promising clue to a unified mass matrix model.

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