S_3 Higgs Potential and Texture zeros in Supersymmetric standard model

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1, Introduction

Texture-zeros in quark–lepton mass matrix are successful to predict masses and mixings.

$$M = \begin{pmatrix} \times & 0 & 0 \\ 0 & \times & \times \\ 0 & \times & \times \end{pmatrix}$$

However, the origin of zero is not clear!

Discrete flavor symmetry approach

The favorable mass matrix structure can be derived when certain Yukawa couplings are forbidden by discrete symmetry.

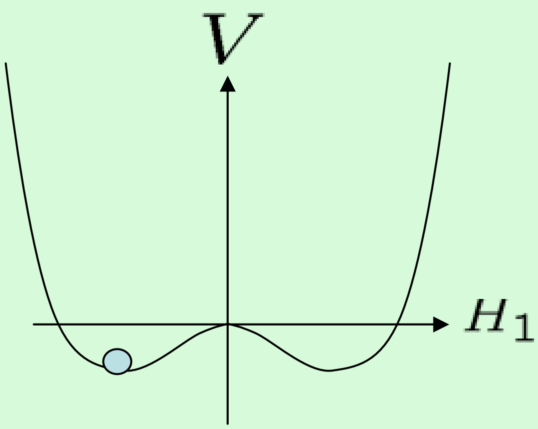
When we consider small discrete flavor Symmetry S_3 ,Texture-zeros can be derived??

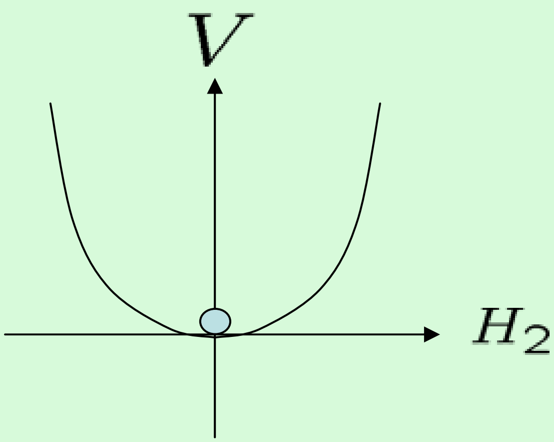
S_3 : permutations of three objects

Mass matrix

$$M_{ij} = y_{ij} \langle H \rangle$$

To derive Texture-zeros, we consider **VEV=0**.





Higgs : All representations are taken into account.

S_3 irreducible representation $\underline{2}$ $\underline{1}_S$ $\underline{1}_A$

To analyze whether **VEV=0** is possible or not,

Higgs potential analysis on SUSY

contents

2, S_3 invariant mass matrix on SUSY

N. Haba and K. Yoshioka, hep-ph/051118

Tensor product of S_3 doublet

$\phi,\ \psi$: S_3 doublet (complex representation)

$$\phi\times\psi \;=\; \underset{\underline{2}}{(\phi_1^*\psi_2,\ \phi_2^*\psi_1)^T} \;+\; \underset{\underline{1}_A}{(\phi_1^*\psi_1-\phi_2^*\psi_2)} \;+\; \underset{\underline{1}_S}{(\phi_1^*\psi_1+\phi_2^*\psi_2)}$$

$$\phi^c\times\psi \;=\; \underset{\underline{2}}{(\phi_2\psi_2,\ \phi_1\psi_1)^T} \;+\; \underset{\underline{1}_A}{(\phi_1\psi_2-\phi_2\psi_1)} \;+\; \underset{\underline{1}_S}{(\phi_1\psi_2+\phi_2\psi_1)}$$

$\phi^c \equiv \sigma_1\phi^*$: S_3 doublet

$$\phi=\begin{pmatrix}\phi_1\\ \phi_2\end{pmatrix},\psi=\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$$

Supersymmetric S_3 mass matrix

ϕ_R , ψ_L : fermion

$\underline{3}$ (= $\underline{2} + \underline{1}_S$)

↑ Third generation

$(\phi_R)^c$

Φ , Ψ : chiral superfield

Higgs
$\underline{2}$: (H_1, H_2)
$\underline{1}_A$: H_A
$\underline{1}_S$: H_S

Supersymmetric Dirac mass matrix

$$W = \Phi M_D \Psi$$

$$M_D = \left(\begin{array}{cc|c} aH_1 & bH_S + cH_A & dH_2 \\ bH_S - cH_A & aH_2 & dH_1 \\ \hline eH_2 & eH_1 & fH_S \end{array}\right)$$

Supersymmetric Majorana mass matrix

$$W = \Phi M_R \Phi$$

$$M_R = \left(\begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array}\right)$$

3, S_3 invariant Higgs scalar potential

superfields

$$\hat{H}_{uS},\ \hat{H}_{dS} \quad : \quad \mathrm{S_3\ symmetric\ singlet}$$

$$\hat{H}_{uA},\ \hat{H}_{dA} \quad : \quad \mathrm{S_3\ antisymmetric\ singlet}$$

$$\hat{H}_{u1},\ \hat{H}_{d1},\ \hat{H}_{u2},\ \hat{H}_{d2} \quad : \quad \mathrm{S_3\ doublet}$$

hat : superfield

gauge groups

$$U(1)_Y \quad ; \quad \text{up type Higgs} : \frac{1}{2},\ \text{down type Higgs} : -\frac{1}{2}$$

$$SU(2)_L \quad ; \quad \text{all Higgses are } \mathrm{SU(2)_L} \text{ doublet}$$

Higgs potential

$$
\begin{aligned}
V &= V_{\mathrm{SUSY}} + V_{\mathrm{soft}} \\
&= \left(|\mu_S|^2 + m_{uS}^2\right) v_{uS}^2 + \left(|\mu_S|^2 + m_{dS}^2\right) v_{dS}^2 - 2b_S v_{uS} v_{dS} \\
&\quad + \left(|\mu_A|^2 + m_{uA}^2\right) v_{uA}^2 + \left(|\mu_A|^2 + m_{dA}^2\right) v_{dA}^2 - 2b_A v_{uA} v_{dA} \\
&\quad + \left(|\mu_D|^2 + m_{uD}^2\right) v_{u1}^2 + \left(|\mu_D|^2 + m_{dD}^2\right) v_{d2}^2 - 2b_D v_{u1} v_{d2} \\
&\quad + \left(|\mu_D|^2 + m_{uD}^2\right) v_{u2}^2 + \left(|\mu_D|^2 + m_{dD}^2\right) v_{d1}^2 - 2b_D v_{u2} v_{d1} \\
&\quad + \frac{2}{g_Y^2 + g_2^2} A^2
\end{aligned}
$$

$$
A \equiv \frac{g_Y^2 + g_2^2}{4} \left\{ (v_{uS}^2 - v_{dS}^2) + (v_{uA}^2 - v_{dA}^2) + (v_{u1}^2 - v_{d2}^2) + (v_{u2}^2 - v_{d1}^2) \right\}
$$

$$
v_{(u,d)S} \equiv \left|\left\langle H^0_{(u,d)S} \right\rangle\right|, \; v_{(u,d)A} \equiv \left|\left\langle H^0_{(u,d)A} \right\rangle\right|, \; v_{(u,d)1} \equiv \left|\left\langle H^0_{(u,d)1} \right\rangle\right|, \; v_{(u,d)2} \equiv \left|\left\langle H^0_{(u,d)2} \right\rangle\right|
$$

$\ast$ assumption   Charged Higgs VEVs are zero.

4, Potential analysis

Equations at vacua

$$S \left(\left(|\mu_S|^2 + m_{uS}^2\right) v_{uS} \; = \; b_S v_{dS} - A v_{uS} \qquad \left(|\mu_S|^2 + m_{dS}^2\right) v_{dS} \; = \; b_S v_{uS} + A v_{dS} \right.$$

$$A \left(\left(|\mu_A|^2 + m_{uA}^2\right) v_{uA} \; = \; b_A v_{dA} - A v_{uA} \qquad \left(|\mu_A|^2 + m_{dA}^2\right) v_{dA} \; = \; b_A v_{uS} + A v_{dA} \right.$$

$$D \left(\begin{array}{l} \left(|\mu_D|^2 + m_{uD}^2\right) v_{u1} \; = \; b_D v_{d2} - A v_{u1} \qquad \left(|\mu_D|^2 + m_{dD}^2\right) v_{d2} \; = \; b_D v_{u1} + A v_{d2} \\ \left(|\mu_D|^2 + m_{uD}^2\right) v_{u2} \; = \; b_D v_{d1} - A v_{u2} \qquad \left(|\mu_D|^2 + m_{dD}^2\right) v_{d1} \; = \; b_D v_{u2} + A v_{d1} \end{array} \right.$$

$$A \equiv \tfrac{1}{4}(g_Y^2 + g_2^2)\left\{(v_{uS}^2 - v_{dS}^2) + (v_{uA}^2 - v_{dA}^2) + (v_{u1}^2 - v_{d2}^2) + (v_{u2}^2 - v_{d1}^2)\right\}$$

4.1, Classification of vacua about S_3 symmetric singlet, S_3 antisymmetric singlet

* Both up-type and down-type Higgs VEVs are zero

This solution always exists. $v_{uS} = v_{dS} = 0$

$v_{uA} = v_{dA} = 0$

$$\left(|\mu_S|^2 + m_{uS}^2\right) v_{uS} = b_S v_{dS} - A v_{uS} \quad \left(|\mu_S|^2 + m_{dS}^2\right) v_{dS} = b_S v_{uS} + A v_{dS}$$

* One of up-type and down-type Higgs VEVs is zero

$v_{uS} = 0 \;\; v_{dS} \neq 0$ or $v_{uS} \neq 0 \;\; v_{dS} = 0$

$b_S = 0$ is necessary for this solution to exist.

For antisymmetric singlet $S \rightarrow A$

Proof

When $b_S = 0$, the following solutions exist.

		condition
$v_{uS} = 0$	$v_{dS} = 0$	
$\left(\lvert\mu_S\rvert^2 + m_{uS}^2\right) + A\rvert_{v_{dS}=0} = 0$	$v_{dS} = 0$	
$v_{uS} = 0$	$\left(\lvert\mu_S\rvert^2 + m_{dS}^2\right) - A\rvert_{v_{uS}=0} = 0$	
$\left(\lvert\mu_S\rvert^2 + m_{uS}^2\right) + A = 0$		$2\lvert\mu_S\rvert^2 + m_{uS}^2 + m_{dS}^2 = 0$

$$A \equiv \frac{g_Y^2 + g_2^2}{4}\left\{(v_{uS}^2 - v_{dS}^2) + (v_{uA}^2 - v_{dA}^2) + (v_{u1}^2 - v_{d2}^2) + (v_{u2}^2 - v_{d1}^2)\right\}$$

The solution that ***one of two VEVs is zero*** is contained.

Oppositely, when $v_{uS} = 0$, $v_{dS} \neq 0$,

$$b_S = 0 \quad \left(\lvert\mu_S\rvert^2 + m_{dS}^2\right) - A\rvert_{v_{uS}=0} = 0$$

Therefore $b_S = 0$ is ***the necessary condition*** of $v_{uS} = 0$ $v_{dS} \neq 0$ or $v_{uS} \neq 0$ $v_{dS} = 0$.

4,2 Classification of vacua about S_3 *doublet*

Because b_D is common to v_{u1}, v_{d2} and v_{u2}, v_{d1}, it is necessary to discuss four VEVs simultaneously.

Independent candidates of VEV=0 are below.

	b_D dependence
# all of VEVs are zero	
$v_{u1}=0,\ v_{d2}=0,\ v_{u2}=0,\ v_{d1}=0$	
# three of VEVs are zero	
$v_{u1}=0,\ v_{d2}=0,\ v_{u2}=0,\ v_{d1}\neq 0$	$b_D=0$
# two of VEVs are zero	
$v_{u1}=0,\ v_{d2}=0,\ v_{u2}\neq 0,\ v_{d1}\neq 0$	
$v_{u1}=0,\ v_{d2}\neq 0,\ v_{u2}=0,\ v_{d1}\neq 0$	$b_D=0$
△ $v_{u1}=0,\ v_{d2}\neq 0,\ v_{u2}\neq 0,\ v_{d1}=0$	$b_D=0$
# one of VEVs is zero	
△ $v_{u1}\neq 0,\ v_{d2}\neq 0,\ v_{u2}\neq 0,\ v_{d1}=0$	$b_D=0$
# all of VEVs are non zero	
$v_{u1}\neq 0,\ v_{d2}\neq 0,\ v_{u2}\neq 0,\ v_{d1}\neq 0$	

∗ Why are the followings not taken?

$$v_{u1} = 0,\ v_{d2} \neq 0,\ v_{u2} \neq 0,\ v_{d1} = 0$$
$$v_{u1} \neq 0,\ v_{d2} \neq 0,\ v_{u2} \neq 0,\ v_{d1} = 0$$

These solutions satisfy $2|\mu_D|^2 + m_{uD}^2 + m_{dD}^2 = 0$.
$$b_D = 0$$

↓

The condition could be satisfied at certain energy scales
but RGE (Renormalization Group Equation) dependent.

↓

It is difficult to construct models
that satisfy that condition.

Summary of potential analysis

symmetric singlet

$* \; v_{uS} = v_{dS} = 0$

$* \; v_{uS} \neq 0, \;\; v_{dS} \neq 0$

antisymmetric singlet

$* \; v_{uA} = v_{dA} = 0$

$* \; v_{uA} \neq 0, \;\; v_{dA} \neq 0$

doublet

v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0	0	0
0	0		
		0	0

Consequently, there are 14 patterns in terms of Texture-zeros.

: (2 × 2 × 4) - 2

assumption

$b_S \neq 0, \;\; b_A \neq 0, \;\; b_D \neq 0$

5, Phenomenology

Quark/Lepton mass matrix structure

* left handed neutrino mass matrix analysis

Supersymmetric Dirac mass matrix

$$M_{(u,d)D} = \left(\begin{array}{cc|c} av_{(u,d)1} & bv_{(u,d)S} + cv_{(u,d)A} & dv_{(u,d)2} \\ bv_{(u,d)S} - cv_{(u,d)A} & av_{(u,d)2} & dv_{(u,d)1} \\ \hline ev_{(u,d)2} & ev_{(u,d)1} & fv_{(u,d)S} \end{array}\right)$$

Supersymmetric Majorana mass matrix

$$M_R = \left(\begin{array}{cc|c} 0 & M & 0 \\ M & 0 & 0 \\ \hline 0 & 0 & M' \end{array}\right)$$

see-saw mechanism

$$-M_\nu = M_D^T M_R^{-1} M_D$$

⇓

As results of potential analysis, there are two interesting solutions in terms of Texture-zeros.

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0			0	0		

$$M_u = \begin{pmatrix} 0 & c v_{uA} & d v_{u2} \\ -c v_{uA} & a v_{u2} & 0 \\ e v_{u2} & 0 & 0 \end{pmatrix} M_{d,e} = \begin{pmatrix} a' v_{d1} & c' v_{dA} & 0 \\ -c' v_{dA} & 0 & d' v_{d1} \\ 0 & e' v_{d1} & 0 \end{pmatrix} M_\nu = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

v_{uS}	v_{dS}	v_{uA}	v_{dA}	v_{u1}	v_{d2}	v_{u2}	v_{d1}
0	0					0	0

$$M_u = \begin{pmatrix} a v_{u1} & c v_{uA} & 0 \\ -c v_{uA} & 0 & d v_{u1} \\ 0 & e v_{u1} & 0 \end{pmatrix} M_{d,e} = \begin{pmatrix} 0 & c' v_{dA} & d' v_{d2} \\ -c' v_{dA} & a' v_{d2} & 0 \\ e' v_{d2} & 0 & 0 \end{pmatrix} M_\nu = \begin{pmatrix} \times & \times & \times \\ \times & \times & \times \\ \times & \times & 0 \end{pmatrix}$$

6, Discussion

＊Problem

Global symmetry → massless Higgs??

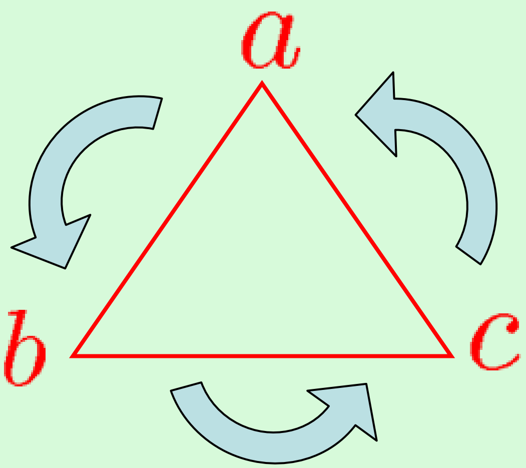
- Higgs mass spectrum
- S_3 breaking in Lagrangian

＊Future work

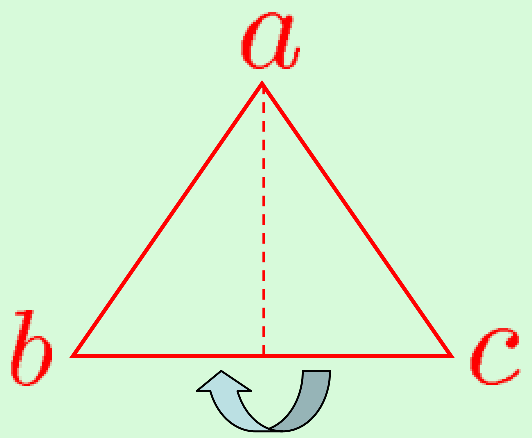
- $S_{3L} \times S_{3R}$ symmetry

S_3 : permutations of three objects

(a, b, c)



3 elements

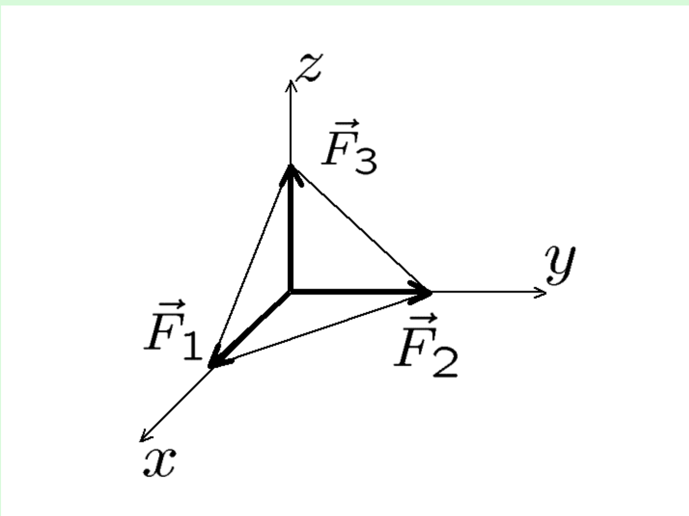


3 elements

S_3 is the smallest group of non abelian finite groups.

S_3 irreducible representation

$$\underline{2} \quad \underline{1}_S \quad \underline{1}_A$$



$$\underline{3} = \underline{2} + \underline{1}_S$$