

$\mu - \tau$ 対称性はニュートリノ混合 に何を示唆するか？

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This talk is based on work with prof Yasue

はじめに

$$U_{PMNS}^T m_\nu U_{PMNS} = m_\nu^{diag}$$

$$U_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

$$\sin^2\theta_{12} = 0.314(1_{-0.15}^{+0.18})$$

$$\Delta m_{\odot}^2 = 7.92(1 \pm 0.09) \times 10^{-5} eV^2$$

$$\sin^2\theta_{23} = 0.44(1_{-0.22}^{+0.41})$$

$$\Delta m_{atm}^2 = 2.4(1_{-0.26}^{+0.21}) \times 10^{-3} eV^2$$

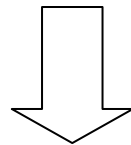
$$\sin^2\theta_{13} = 0.9(1_{-0.9}^{+2.3}) \times 10^{-2}$$

振動実験から

$$\sin^2 \theta_{13} \ll 1 \quad \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$$

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如何にして小さな値を理解するか

動機

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naturalness *G. 't Hooft ('80)*

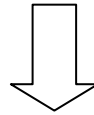
(背後に近似的対称性を示唆する)

動機

如何にして小さな値を理解するか

naturalness *G. 't Hooft ('80)*

(背後に近似的対称性を示唆する)



$\mu - \tau$ 対称性

$\mu - \tau$ 対称性からの帰結

微小な破れによって $\sin^2 \theta_{13} \ll 1$ $\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$ を理解するために

$$\nu_{\mu} \leftrightarrow -\sigma \nu_{\tau} \quad (\sigma = \pm 1)$$

$$m_{\nu} = \begin{pmatrix} a & b & -\sigma b \\ b & d & e \\ -\sigma b & e & -d \end{pmatrix}$$

$$\Downarrow U_{PMNS}^T m_{\nu} U_{PMNS} = m_{\nu}^{diag}$$

$$\sin \theta_{13} = 0 \quad \text{又は} \quad \sin \theta_{12} = 0$$

θ_{23} が最大

$\mu - \tau$ 対称性の微小な破れ

$$m_\nu = \underbrace{\begin{pmatrix} a & b & -\sigma b \\ b & d & e \\ -\sigma b & e & -d \end{pmatrix}}_{\mu - \tau \text{ 対称}} + \epsilon \underbrace{\begin{pmatrix} 0 & b' & \sigma b' \\ b' & d' & 0 \\ \sigma b' & 0 & d' \end{pmatrix}}_{\mu - \tau \text{ 対称性の破れ}}$$

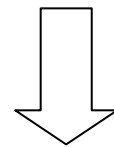
$\mu - \tau$ 対称

$\mu - \tau$ 対称性の破れ

$$(\epsilon \sim 0)$$

$\mu - \tau$ 対称性の微小な破れ

$$m_\nu = \begin{pmatrix} a & b & -\sigma b \\ b & d & e \\ -\sigma b & e & -d \end{pmatrix} + \epsilon \begin{pmatrix} 0 & b' & \sigma b' \\ b' & d' & 0 \\ \sigma b' & 0 & d' \end{pmatrix}$$



$$U_{PMNS}^T m_\nu U_{PMNS} = m_\nu^{diag}$$

$$m_i = f(a, b, \dots)$$

$$\theta_{ij} = h(a, b, \dots)$$

ν 質量と混合角

$$X = \frac{(c_{23} + \sigma s_{23})b + \epsilon(c_{23} - \sigma s_{23})b'}{c_{13}}$$

$$m_1 = \cos^2 \theta_{12} \lambda_1 + \sin^2 \theta_{12} \lambda_2 - \sin 2\theta_{12} X$$

$$m_2 = \sin^2 \theta_{12} \lambda_1 + \cos^2 \theta_{12} \lambda_2 - \sin 2\theta_{12} X$$

$$m_3 = \frac{1}{2} \left(\lambda_3 + a + \frac{\lambda_3 - a}{\cos 2\theta_{13}} \right)$$

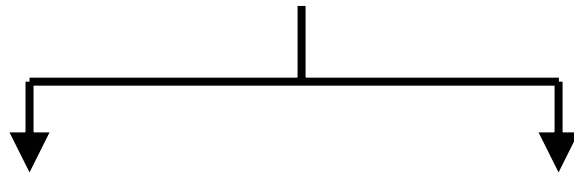
⋮

実験結果を再現するために

$$\Delta m_{\odot}^2 = \frac{2\sqrt{2}(m_1 + m_2)X}{\sin 2\theta_{12}}$$

実験結果を再現するために

$$\Delta m_{\odot}^2 = \frac{2\sqrt{2}(m_1 + m_2)X}{\sin 2\theta_{12}} \ll \Delta m_{atm}^2$$



$$X \sim 0$$

$$(m_1 + m_2) \sim 0$$

ν 質量と混合角 1

C1) $\sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$

$$m_1 \approx \frac{a + d - \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$\cos 2\theta_{23} \approx 2\Delta$$

$$m_2 \approx \frac{a + d - \sigma e + O(\epsilon^2)}{2} + \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$X \approx \sqrt{2}(b + O(\epsilon^2))$$

$$m_3 \approx d + \sigma e + O(\epsilon^2)$$

$$Y \approx \sqrt{2}\sigma(\epsilon b' - b\Delta)$$

$$\tan 2\theta_{12} \approx \frac{2X}{d - \sigma e - a + O(\epsilon^2)}$$

$$\Delta \approx \frac{\sqrt{2}s_{13}b + \sigma\epsilon d'}{2e}$$

$$\tan 2\theta_{13} \approx \frac{2Y}{d + \sigma e - a + O(\epsilon^2)}$$

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1 \quad \text{を得るために} \quad (X \sim 0 \text{ の場合})$$

$$\mathbf{C1)} \sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$$

$$m_1 \approx \frac{a + d - \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

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$$\underline{X \approx \sqrt{2}(b + O(\epsilon^2))}$$

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$$\tan 2\theta_{13} \approx \frac{2Y}{d + \sigma e - a + O(\epsilon^2)}$$

$$\cdot b \sim 0$$

$$(d - \sigma e - a \sim 0)$$

$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$ を得るために ($\cdot m_1 + m_2 \sim 0$ の場合)

C1) $\sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$

$$m_1 \approx \frac{a + d - \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

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$$\tan 2\theta_{13} \approx \frac{2Y}{d + \sigma e - a + O(\epsilon^2)}$$

$$\cdot a + d - \sigma e \sim 0$$

ν 質量と混合角2

C2) $\sin \theta_{12} \rightarrow 0 (\epsilon \rightarrow 0)$

$$m_1 \approx \frac{a + d + \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$\cos 2\theta_{23} \approx 2\Delta$$

$$m_2 \approx \frac{a + d + \sigma e + O(\epsilon^2)}{2} + \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$\underline{X \approx \sqrt{2}\sigma(\epsilon b' + b\Delta)}$$

$$m_3 \approx d - \sigma e + O(\epsilon^2)$$

$$Y \approx -\sqrt{2}(b + O(\epsilon^2))$$

$$\tan 2\theta_{12} \approx \frac{2X}{d + \sigma e - a + O(\epsilon^2)}$$

$$\Delta \approx \frac{\sigma\epsilon d' - \sqrt{2}s_{13}b}{2e + \sqrt{2}s_{13}b}$$

$$\tan 2\theta_{13} \approx \frac{2Y}{d + \sigma e - a + O(\epsilon^2)}$$

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim X \ll 1$$

• $\sin^2 \theta_{13} \ll 1$ を得るために

C2) $\sin \theta_{12} \sim O(\epsilon)$

$$m_1 \approx \frac{a + d + \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$\cos 2\theta_{23} \approx 2\Delta$$

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$$\tan 2\theta_{12} \approx \frac{2X}{d + \sigma e - a + O(\epsilon^2)}$$

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$$\tan 2\theta_{13} \approx \frac{2Y}{d + \sigma e - a + O(\epsilon^2)}$$

$$\bullet b \sim 0$$

$$(\ d + \sigma e - a \sim O(\epsilon))$$

まとめ

$\mu - \tau$ 対称性を基調に $\sin^2 \theta_{13} \ll 1$ $\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$ の導出を考えた

C1). $\sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

● $a + d + \sigma e \sim 0$ $(\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1)$ 又は $\left\{ \begin{array}{l} \bullet b \sim 0 \quad (\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1) \\ \bullet d - \sigma e - a \sim 0 \quad (\tan 2\theta_{12} \sim O(1)) \end{array} \right.$

C2). $\sin \theta_{12} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

$$\left\{ \begin{array}{l} \bullet b \sim 0 \quad (\sin^2 \theta_{13} \ll 1) \\ \bullet d + \sigma e - a \sim 0 \quad (\tan 2\theta_{12} \sim O(1)) \end{array} \right.$$

まとめ

C1). $\sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

● $a + d + \sigma e \sim 0$ ($\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$) 又は $\begin{cases} \bullet b \sim 0 & (\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1) \\ \bullet d - \sigma e - a \sim 0 & (\tan 2\theta_{12} \sim O(1)) \end{cases}$

C2). $\sin \theta_{12} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

$$\begin{cases} \bullet b \sim 0 & (\sin^2 \theta_{13} \ll 1) \\ \bullet d + \sigma e - a \sim 0 & (\tan 2\theta_{12} \sim O(1)) \end{cases}$$

考察

C1). $\sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

$$\mu - \tau \text{ 対称性} \Rightarrow \sin^2 \theta_{13} \ll 1 \quad L_e \Rightarrow \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$$

C2). $\sin \theta_{12} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

$$\mu - \tau \text{ 対称性} \Rightarrow \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1 \quad L_e \Rightarrow \sin^2 \theta_{13} \ll 1$$

ν 質量と混合角 1

C1) $\sin \theta_{13} \sim O(\epsilon)$

$$m_1 \approx \frac{a + d - \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$\underline{\cos 2\theta_{23} \approx 2\Delta}$$

$$m_2 \approx \frac{a + d - \sigma e + O(\epsilon^2)}{2} + \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

$$X \approx \sqrt{2}(b + O(\epsilon^2))$$

$$m_3 \approx d + \sigma e + O(\epsilon^2)$$

$$\underline{Y \approx \sqrt{2}\sigma(\epsilon b' - b\Delta)}$$

$$\tan 2\theta_{12} \approx \frac{2X}{d - \sigma e - a + O(\epsilon^2)}$$

$$\Delta \approx \frac{\sqrt{2}s_{13}b + \sigma\epsilon d'}{2e}$$

$$\tan 2\theta_{13} \approx \frac{2Y}{d + \sigma e - a + O(\epsilon^2)}$$

$$\sin \theta_{13} \sim O(\epsilon) \sim \cos 2\theta_{23}$$

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1 \quad \text{を得るために} \quad (X \sim 0 \text{ の場合})$$

$$\mathbf{C1)} \sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$$

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$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim X \sim O(\epsilon^2) \sim \sin^2 \theta_{13}$$

$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \ll 1$ を得るために ($m_1 + m_2 \sim 0$ の場合)

C1) $\sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$

$$m_1 \approx \frac{a + d - \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

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$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim m_1 + m_2 \sim O(\epsilon^2) \sim \sin^2 \theta_{13}$$

ν 質量と混合角2

C2) $\sin \theta_{12} \sim O(\epsilon)$

$$m_1 \approx \frac{a + d + \sigma e + O(\epsilon^2)}{2} - \frac{\sqrt{2}X}{\sin 2\theta_{12}}$$

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$$\tan 2\theta_{13} \approx \frac{2Y}{d + \sigma e - a + O(\epsilon^2)}$$

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim X \sim O(\epsilon) \sim \cos 2\theta_{23}$$

近似的 $\mu - \tau$ 対称性から導かれる 関係式

C1). $\sin \theta_{13} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

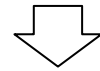
$$\sin \theta_{13} \sim \cos 2\theta_{23} \quad \left(\text{かつ } \frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim \sin^2 \theta_{13} \right)$$

C2). $\sin \theta_{12} \rightarrow 0 (\epsilon \rightarrow 0)$ の場合

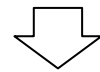
$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim \cos 2\theta_{23}$$

実験結果を再現するために

$$b \sim 0$$



近似的に L_e が保存 (normal hierarchy)



$$X \approx \sqrt{2}(b + O(\epsilon^2)) \sim 0$$

$$\frac{\Delta m_{\odot}^2}{\Delta m_{atm}^2} \sim X \ll 1$$