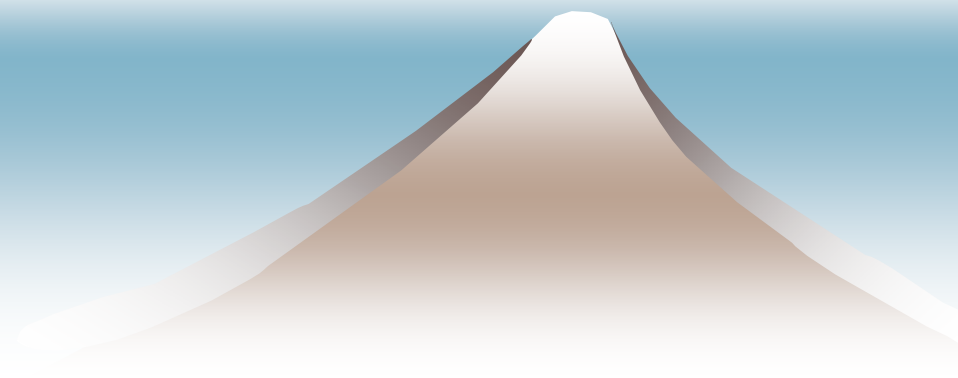


複素ニュートリノ質量行列と PMNSユニタリー行列

安江正樹(東海大)



導入

未知のニュートリノ物理

ニュートリノフレーバー質量行列: M_{flavor}

$$M_{\text{flavor}} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix} \text{ for } \begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} \leftarrow M_{\text{charged}} = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}$$

PMNSユニタリ行列: U_{PMNS}

実験との比較

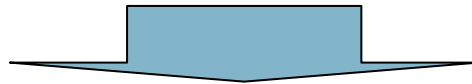
$$\Delta m_{\text{atm}}^2 = 2.4 \begin{pmatrix} 1^{+0.21} \\ -0.26 \end{pmatrix} \times 10^{-5} \text{ eV}^2 \gg \Delta m_e^2 = 7.92(1 \pm 0.09) \times 10^{-5} \text{ eV}^2$$
$$\sin^2 \theta_{12} = 0.314 \begin{pmatrix} 1^{+0.18} \\ -0.15 \end{pmatrix} \cdot \sin^2 \theta_{23} = 0.44 \begin{pmatrix} 1^{+0.41} \\ -0.22 \end{pmatrix} \gg \sin^2 \theta_{13} = 0.9_{-0.9}^{+2.3} \times 10^{-2}$$

比較

未知のニュートリノ物理



現在、知っている形を導くように調節する



Normal mass hierarchy

$$m_0 \begin{pmatrix} : 0 & : \varepsilon & : -\sigma\varepsilon \\ : \varepsilon & : 1 & : \sigma \\ : -\sigma\varepsilon & : \sigma & : 1 \end{pmatrix}$$

Inverted mass hierarchy

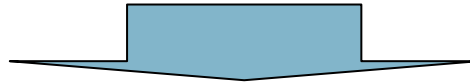
$$m_0 \begin{pmatrix} : 2 & : \varepsilon & : -\sigma\varepsilon \\ : \varepsilon & : 1 & : -\sigma \\ : -\sigma\varepsilon & : -\sigma & : 1 \end{pmatrix}$$

($\sigma = \pm 1$)

もっと一般的な制限をみつけたあ～♪いい～♪

方法

ニュートリノフレーバー質量行列: M_{flavor}



PMNSユニタリ行列: U_{PMNS}

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

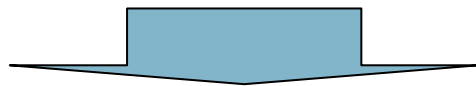
M_{flavor} の固有ベクトル $|\lambda_{1,2,3}\rangle$



$$U_{\text{PMNS}} = (|\lambda_1\rangle, |\lambda_2\rangle, |\lambda_3\rangle)$$

複素行列の対角化

$$M_{\text{flavor}} = \begin{pmatrix} a & b & c \\ b & d & e \\ c & e & f \end{pmatrix}$$



$$M = M_{\text{flavor}}^\dagger M_{\text{flavor}} = \begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix}$$

$$A = |a|^2 + |b|^2 + |c|^2, \quad B = a^*b + b^*d + c^*e, \quad C = a^*c + b^*e + c^*f$$

$$D = |b|^2 + |d|^2 + |e|^2, \quad E = b^*c + d^*e + e^*f, \quad F = |c|^2 + |e|^2 + |f|^2$$

μ - τ 対称性とその破れ

$$M_{\text{flavor}} = \underbrace{\begin{pmatrix} a & b_+ & -\sigma b_+ \\ b_+ & d_+ & e \\ -\sigma b_+ & e & d_+ \end{pmatrix}}_{\mu\text{-}\tau\text{対称}} + \underbrace{\begin{pmatrix} 0 & b_- & \sigma b_- \\ b_- & d_- & 0 \\ \sigma b_- & 0 & d_- \end{pmatrix}}_{\mu\text{-}\tau\text{非対称}}$$

$$M = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & \text{Re}(E) \\ -\sigma B_+^* & \text{Re}(E) & D_+ \end{pmatrix} + \begin{pmatrix} 0 & B_- & \sigma B_- \\ B_-^* & D_- & i \text{Im}(E) \\ \sigma B_-^* & -i \text{Im}(E) & -D_- \end{pmatrix}$$

$$A = |a|^2 + 2(|b_+|^2 + |b_-|^2)$$


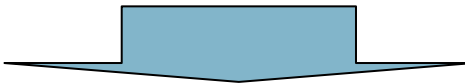
$$B_+ = a^* b_+ + b_+^* (d_+ - \sigma e) + b_-^* d_-, \quad B_- = a^* b_- + b_-^* (d_+ + \sigma e) + b_+^* d_-$$

$$D_+ = |b_+|^2 + |b_-|^2 + |d_+|^2 + |d_-|^2 + |e|^2, \quad D_- = 2 \text{Re}(b_-^* b_+ + d_-^* d_+)$$

$$\text{Re}(E) = \sigma(|b_-|^2 - |b_+|^2) + 2 \text{Re}(d_+^* e), \quad \text{Im}(E) = 2 \text{Im}(d_-^* e - \sigma b_-^* b_+)$$

μ - τ 対称部の対角化

$$\mathbf{M}_{\text{sym}} = \begin{pmatrix} A & B_+ & -\sigma B_+ \\ B_+^* & D_+ & \text{Re}(E) \\ -\sigma B_+^* & \text{Re}(E) & D_+ \end{pmatrix}$$


$$B_+ = |B_+| e^{i\rho}$$


$$\begin{pmatrix} A & |B_+| e^{i\rho} & -\sigma |B_+| e^{i\rho} \\ |B_+| e^{-i\rho} & D_+ & \text{Re}(E) \\ -\sigma |B_+| e^{-i\rho} & \text{Re}(E) & D_+ \end{pmatrix} \Rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \text{ for } \lambda = D_+ + \sigma \text{Re}(E)$$

残りの2つの固有ベクトル...

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \xrightarrow{\text{直交性}} \frac{1}{\sqrt{2(|w|^2 + |z|^2)}} \begin{pmatrix} \sqrt{2}w \\ -z \\ \sigma z \end{pmatrix} \text{ with } w, z = \text{complex}$$

$$\begin{pmatrix} A & |B_+|e^{i\rho} & -\sigma|B_+|e^{i\rho} \\ |B_+|e^{-i\rho} & D_+ & \text{Re}(E) \\ -\sigma|B_+|e^{-i\rho} & \text{Re}(E) & D_+ \end{pmatrix} \begin{pmatrix} \sqrt{2}w \\ -z \\ \sigma z \end{pmatrix} = \lambda \begin{pmatrix} \sqrt{2}w \\ -z \\ \sigma z \end{pmatrix}$$

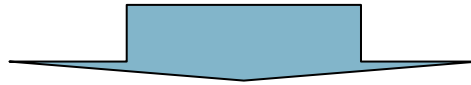
$$(*) \cos \theta = \frac{|w|}{\sqrt{|w|^2 + |z|^2}}$$

$$w = |w|, \quad ze^{i\rho} = |z|$$

$$\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \cos \theta \\ -e^{-i\rho} \sin \theta \\ \sigma e^{-i\rho} \sin \theta \end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2}e^{i\rho} \sin \theta \\ \cos \theta \\ -\sigma \cos \theta \end{pmatrix} \text{ for } (A - \lambda)(D_+ - \sigma E_+ - \lambda) = 2|B_+|^2$$

μ - τ 対称部の U_{PMNS}

$$\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \cos \theta \\ -e^{-i\rho} \sin \theta \\ \sigma e^{-i\rho} \sin \theta \end{pmatrix}, \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} e^{i\rho} \sin \theta \\ \cos \theta \\ -\sigma \cos \theta \end{pmatrix}$$



$$U_{PMNS} = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \cos \theta \\ -e^{-i\rho} \sin \theta \\ \sigma e^{-i\rho} \sin \theta \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} e^{i\rho} \sin \theta \\ \cos \theta \\ -\sigma \cos \theta \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \right)$$

$$U_{PMNS} = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \cos \theta \\ -e^{-i\rho} \sin \theta \\ \sigma e^{-i\rho} \sin \theta \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \quad \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} e^{i\rho} \sin \theta \\ \cos \theta \\ -\sigma \cos \theta \end{pmatrix} \right)$$

μ - τ 対称部の混合角 I

$$U_{PMNS} = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \cos \theta \\ -e^{-i\rho} \sin \theta \\ \sigma e^{-i\rho} \sin \theta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} e^{i\rho} \sin \theta \\ \cos \theta \\ -\sigma \cos \theta \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \right)$$

$$U_{PMNS} = \begin{pmatrix} c_{12} & s_{12} e^{i\rho} & 0 \\ -c_{23} s_{12} e^{-i\rho} & c_{23} c_{12} & s_{23} \\ s_{23} s_{12} e^{-i\rho} & -s_{23} c_{12} & c_{23} \end{pmatrix}$$

$$\theta_{12} = \theta, \quad \cos \theta_{23} = \sigma \sin \theta_{23} = \frac{1}{\sqrt{2}}, \quad \sin \theta_{13} = 0, \quad B_0 = |B_0| e^{i\rho}$$

μ - τ 対称部の混合角 II

$$U_{PMNS} = \left(\frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} \cos \theta \\ -e^{-i\rho} \sin \theta \\ \sigma e^{-i\rho} \sin \theta \end{pmatrix} \frac{\sigma}{\sqrt{2}} \begin{pmatrix} 0 \\ \sigma \\ 1 \end{pmatrix} \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} e^{i\rho} \sin \theta \\ \cos \theta \\ -\sigma \cos \theta \end{pmatrix} \right)$$


$$U_{PMNS} = \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ s_{23} s_{13} e^{i\delta} & c_{23} & s_{23} c_{13} \\ c_{23} s_{13} e^{i\delta} & -s_{23} & c_{23} c_{13} \end{pmatrix}$$

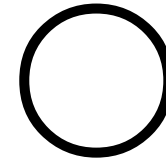
$$\sin \theta_{12} = 0, \quad \cos \theta_{23} = -\sigma \sin \theta_{23} = \frac{1}{\sqrt{2}}, \quad \theta_{13} = \theta, \quad \rho = -\delta$$

μ - τ 非対称部の影響


摂動による解析: ε

$$\theta_{12} = \theta, \cos \theta_{23} = \sigma \sin \theta_{23} = \frac{1}{\sqrt{2}}, \sin \theta_{13} = 0, B_0 = |B_0| e^{i\rho}$$


$$\cos 2\theta_{23} = O(\varepsilon), \sin \theta_{13} = O(\varepsilon)$$



$$\sin \theta_{12} = 0, \cos \theta_{23} = -\sigma \sin \theta_{23} = \frac{1}{\sqrt{2}}, \theta_{13} = \theta, \rho = -\delta$$


$$\cos 2\theta_{23} = O(\varepsilon), \sin \theta_{12} = O(\varepsilon)$$



μ - τ 非対称部の影響:摂動の1次

$$|1^{(1)}\rangle = -\frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}c_{12}B_- e^{-i\rho} - s_{12}(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_1^{(0)2}} \right)^* e^{-i\rho} \begin{pmatrix} 0 \\ 1 \\ \sigma \end{pmatrix}$$

$m_{1,2,3}^{(0)2}$: treeでの質量

$m_3^{(0)2} = D_+ + \sigma \text{Re}(E)$ 等

$$|2^{(1)}\rangle = -\frac{1}{\sqrt{2}} \left(\frac{\sqrt{2}s_{12}B_- e^{-i\rho} + c_{12}(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_2^{(0)2}} \right)^* \begin{pmatrix} 0 \\ 1 \\ \sigma \end{pmatrix}$$

$$|3^{(1)}\rangle = \frac{1}{\sqrt{2}} \left[\begin{aligned} & \sqrt{2}\sigma \left(\frac{\sqrt{2}c_{12}^2 B_- e^{-i\rho} - c_{12}s_{12}(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_1^{(0)2}} + \frac{\sqrt{2}s_{12}^2 B_- e^{-i\rho} + c_{12}s_{12}(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_2^{(0)2}} \right) e^{i\rho} \\ & - \sigma \left(\frac{\sqrt{2}c_{12}s_{12}B_- e^{-i\rho} - s_{12}^2(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_1^{(0)2}} - \frac{\sqrt{2}c_{12}s_{12}B_- e^{-i\rho} + c_{12}^2(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_2^{(0)2}} \right) \\ & \frac{\sqrt{2}c_{12}s_{12}B_- e^{-i\rho} - s_{12}^2(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_1^{(0)2}} - \frac{\sqrt{2}c_{12}s_{12}B_- e^{-i\rho} + c_{12}^2(D_- + \sigma i \text{Im}(E))}{m_3^{(0)2} - m_2^{(0)2}} \end{aligned} \right]$$

摂動の1次を含めた U_{PMNS} v.s. $|3\rangle$

$$|3\rangle = \frac{1}{\sqrt{2}} \sigma \left(\begin{array}{l} 4\sigma \frac{(2\Delta m_{atm}^2 + \Delta m_e^2 \cos 2\theta_{12}) \boxed{B_-} e^{-i\rho} + \Delta m_e^2 \sin 2\theta_{12} \boxed{D_- + \sigma i \text{Im}(E)}}{(2\Delta m_{atm}^2 - \Delta m_e^2)(2\Delta m_{atm}^2 + \Delta m_e^2)} e^{i\rho} \\ 1 + 2 \frac{(2\Delta m_{atm}^2 - \Delta m_e^2 \cos 2\theta_{12}) \boxed{D_- + \sigma i \text{Im}(E)} + \sqrt{2}\Delta m_e^2 \sin 2\theta_{12} \boxed{B_-} e^{-i\rho}}{(2\Delta m_{atm}^2 - \Delta m_e^2)(2\Delta m_{atm}^2 + \Delta m_e^2)} \\ 1 - 2 \frac{(2\Delta m_{atm}^2 - \Delta m_e^2 \cos 2\theta_{12}) \boxed{D_- + \sigma i \text{Im}(E)} + \sqrt{2}\Delta m_e^2 \sin 2\theta_{12} \boxed{B_-} e^{-i\rho}}{(2\Delta m_{atm}^2 - \Delta m_e^2)(2\Delta m_{atm}^2 + \Delta m_e^2)} \end{array} \right)$$

$$(*) \Delta m_e^2 = m_2^{(0)2} - m_1^{(0)2} > 0, \quad \Delta m_{atm}^2 = m_3^{(0)2} - \frac{m_1^{(0)2} + m_2^{(0)2}}{2}$$

$$c_{12}^2 m_1^{(0)2} + s_{12}^2 m_2^{(0)2} - m_3^{(0)2} = \frac{1}{2} (\Delta m_e^2 \cos 2\theta_{12} - 2\Delta m_{atm}^2)$$

$$s_{12}^2 m_1^{(0)2} + c_{12}^2 m_2^{(0)2} - m_3^{(0)2} = -\frac{1}{2} (\Delta m_e^2 \cos 2\theta_{12} + 2\Delta m_{atm}^2)$$

$$m_3^{(0)2} - m_2^{(0)2} = \frac{1}{2} (2\Delta m_{atm}^2 - \Delta m_e^2), \quad m_3^{(0)2} - m_1^{(0)2} = \frac{1}{2} (2\Delta m_{atm}^2 + \Delta m_e^2)$$

この $|3\rangle$ を要素として持つ U_{PMNS} の形

$$|3\rangle = \begin{pmatrix} s_{13}e^{-i\delta} \\ s_{23}e^{i\gamma} \\ c_{23}e^{-i\gamma} \end{pmatrix} \approx \begin{pmatrix} s_{13}e^{-i\delta} \\ \sigma \frac{1-\Delta+i\gamma}{\sqrt{2}} \\ \frac{1+\Delta-i\gamma}{\sqrt{2}} \end{pmatrix} \quad \text{with } |\Delta| = O(\varepsilon) \ \& \ |\gamma| = O(\varepsilon)$$

$|1\rangle, |2\rangle$ も同様に...

$$s_{13}e^{-i(\rho+\delta)} \approx \frac{\sqrt{2}\sigma e^{-i\rho} B_-}{\Delta m_{atm}^2} \Rightarrow B_- \approx \pm |B_-| e^{-i\delta}$$

$$s_{13}e^{-i(\rho+\delta)} \approx \sigma \frac{2\sqrt{2}e^{-i\rho} B_- + R \sin 2\theta_{12} (D_- + \sigma i \text{Im}(E))}{2\Delta m_{atm}^2} \quad R = \frac{\Delta m_e^2}{\Delta m_{atm}^2} \ll 1$$

$$\Delta \approx -\frac{2D_- + \sigma s_{13} \cos(\rho+\delta) \sin 2\theta_{12} \Delta m_e^2}{2\Delta m_{atm}^2} \quad J = -c_{23}c_{12}s_{13}s_{23}c_{13}s_{12}c_{13}s_{\rho+\delta} \Rightarrow \rho+\delta$$

$$\gamma \approx \frac{2\sigma \text{Im}(E) - \sigma s_{13} \sin(\rho+\delta) \sin 2\theta_{12} \Delta m_e^2}{2\Delta m_{atm}^2}$$

U_{PMNS} への示唆

摂動範囲を超えて・・・ U_{PMNS} の形

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} e^{i\rho} \\ -\sin \theta_{12} e^{-i\rho} & \cos \theta_{12} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

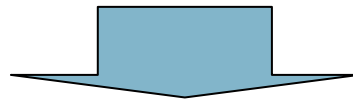
$$\begin{pmatrix} \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} e^{i\gamma} & 0 \\ 0 & e^{-i\gamma} \end{pmatrix} \begin{pmatrix} \cos \theta_{23} & \sin \theta_{23} \\ -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} \nu_2 \\ \nu_3 \end{pmatrix}$$

$$\begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} \cos \theta_{13} & \sin \theta_{13} e^{-i\delta} \\ -\sin \theta_{13} e^{i\delta} & \cos \theta_{13} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_3 \end{pmatrix}$$

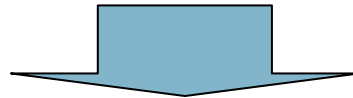
(*) Majorana phases for $\nu_{1,2,3}$

U_{PMNS} からの示唆

$$\begin{pmatrix} A & B & C \\ B^* & D & E \\ C^* & E^* & F \end{pmatrix} \text{ with } U_{PMNS}(\rho, \gamma, \delta)$$



$$\begin{pmatrix} A & e^{i(\gamma-\rho)}B & e^{-i(\gamma+\rho)}C \\ e^{-i(\gamma-\rho)}B^* & D & e^{-2i\gamma}E \\ e^{i(\gamma+\rho)}C^* & e^{2i\gamma}E^* & F \end{pmatrix} \text{ with } U_{PMNS}^{PDG}(\rho + \delta)$$



$$\begin{pmatrix} e^{2i\rho}a & e^{i(\gamma+\rho)}b & e^{-i(\gamma-\rho)}c \\ e^{i(\gamma+\rho)}b & e^{2i\gamma}d & e \\ e^{-i(\gamma-\rho)}c & e & e^{-2i\gamma}f \end{pmatrix} \text{ with } U_{PMNS}^{PDG}(\rho + \delta)$$

応用

Mohapatra & Rodejohann, PRD 72 ('05)

$$\begin{pmatrix} 0 & \beta\varepsilon & -\sigma\beta\varepsilon e^{i\alpha} \\ \beta\varepsilon & 1+\varepsilon & \sigma \\ -\sigma\beta\varepsilon e^{i\alpha} & \sigma & 1+\varepsilon \end{pmatrix}$$

$$a = 0, \quad b_+ = \frac{1+e^{i\alpha}}{2} \beta\varepsilon$$

$$d_+ = 1+\varepsilon$$

$$\sigma e = 1 \quad d_- = 0$$

$$\begin{pmatrix} A & e^{i(\gamma-\rho)} B & e^{-i(\gamma+\rho)} C \\ e^{-i(\gamma-\rho)} B^* & D & e^{-2i\gamma} E \\ e^{i(\gamma+\rho)} C^* & e^{2i\gamma} E^* & F \end{pmatrix} \text{ with } U_{PMNS}^{PDG}(\rho + \delta)$$

$$\boxed{B_+ = |B_+| e^{i\rho}} = 0b_+ + b_+^*(d_+ - \sigma e) + b_-^* 0 = \frac{1+e^{-i\alpha}}{2} \beta\varepsilon^2 = e^{-i\alpha/2} \beta\varepsilon^2 \cos\left(\frac{\alpha}{2}\right)$$

$$\boxed{B_- = \pm |B_+| e^{-i\delta}} = a^* b_- + b_-^*(d_+ + \sigma e) + b_+^* d_- = \frac{1-e^{-i\alpha}}{2} 2\beta\varepsilon = 2ie^{-i\alpha/2} \beta\varepsilon \sin\left(\frac{\alpha}{2}\right)$$

$$\rho = -\frac{\alpha}{2} \quad -\delta = \frac{\pi}{2} - \frac{\alpha}{2}$$

$$\rho \approx -\frac{\alpha}{2}, \gamma \approx 0, \delta \approx \frac{\alpha - \pi}{2} \Rightarrow \rho + \delta \approx -\frac{\pi}{2}$$

$$\begin{pmatrix} e^{2i\rho} a & e^{i(\gamma+\rho)} b & e^{-i(\gamma-\rho)} c \\ e^{i(\gamma+\rho)} b & e^{2i\gamma} d & e \\ e^{-i(\gamma-\rho)} c & e & e^{-2i\gamma} f \end{pmatrix} \rightarrow \begin{pmatrix} 0 & e^{-i\alpha/2} \beta \varepsilon & -e^{-i\alpha/2} \sigma \beta \varepsilon e^{i\alpha} \\ e^{-i\alpha/2} \beta \varepsilon & 1 + \varepsilon & \sigma \\ -e^{-i\alpha/2} \sigma \beta \varepsilon e^{i\alpha} & \sigma & 1 + \varepsilon \end{pmatrix}$$

with $U_{PMNS}^{PDG}(\rho + \delta)$

$$z = e^{-i\alpha/2} \beta \varepsilon$$

$$\begin{pmatrix} 0 & z & -\sigma z^* \\ z & 1 + \varepsilon & \sigma \\ -\sigma z^* & \sigma & 1 + \varepsilon \end{pmatrix}$$

α に依らずに(ほぼ)最大のCPの破れ

with $U_{PMNS}^{PDG}(\rho + \delta)$

まとめ

$$\begin{pmatrix} e^{2i\rho} a & e^{i(\gamma+\rho)} b & e^{-i(\gamma-\rho)} c \\ e^{i(\gamma+\rho)} b & e^{2i\gamma} d & e \\ e^{-i(\gamma-\rho)} c & e & e^{-2i\gamma} f \end{pmatrix} \Leftrightarrow \begin{pmatrix} A & e^{i(\gamma-\rho)} B & e^{-i(\gamma+\rho)} C \\ e^{-i(\gamma-\rho)} B^* & D & e^{-2i\gamma} E \\ e^{i(\gamma+\rho)} C^* & e^{2i\gamma} E^* & F \end{pmatrix}$$

with $U_{PMNS}^{PDG}(\rho + \delta)$

$$\begin{aligned} B_+ &\approx |B_+| e^{i\rho} & B_- &\approx \pm |B_-| e^{-i\delta} \text{ for } R \approx 0 \\ s_{13} e^{-i(\rho+\delta)} &\approx \sigma \frac{2\sqrt{2} e^{-i\rho} B_- + R \sin 2\theta_{12} (D_- + \sigma i \text{Im}(E))}{2\Delta m_{atm}^2} \\ \Delta &\approx -\frac{2D_- + \sigma s_{13} \cos(\rho + \delta) \sin 2\theta_{12} \Delta m_e^2}{2\Delta m_{atm}^2} \\ \gamma &\approx \frac{2\sigma \text{Im}(E) - \sigma s_{13} \sin(\rho + \delta) \sin 2\theta_{12} \Delta m_e^2}{2\Delta m_{atm}^2} \end{aligned}$$

$$R = \frac{\Delta m_e^2}{\Delta m_{atm}^2} \ll 1$$